

# Heterotic Calabi-Yau Compactifications with Flux

Eirik Eik Svanes  
(University of Oxford)

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## Motivation

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Standard Setup: Calabi-Yau

Relaxing Maximal  
Symmetric  $\mathcal{M}_4$

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# Motivation: Why Calabi-Yau with Flux?

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Usually, heterotic compactifications are of the form

$$\mathcal{M}_{10} = \underbrace{\mathcal{M}_4}_{\text{maximally symmetric}} \times \underbrace{\mathcal{M}_6}_{\text{Calabi-Yau}}$$

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Here  $(\mathcal{M}_6, \Omega, J)$  is a Calabi-Yau with globally defined holomorphic three-form  $\Omega$  and Kähler-form  $J$ ,  $\bar{\partial}\Omega = \bar{\partial}J = 0$ .

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- $\delta\Omega, \bar{\partial}\delta\Omega = 0 \Rightarrow h^{2,1}$  complex structure moduli.
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- Such compactifications are very attractive, as the gauge group  $E_8 \times E_8$  easily fits the Standard Model.
- Recently, hundreds of Standard Models have been discovered, [Anderson, Gray, Lukas, Palti; 1106.4804].
- Powerful tools of Algebraic Geometry and Kähler Geometry available.

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Main problem of these compactifications is moduli stabilisation.

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- If we want to use flux to stabilise moduli, we next relax the condition of a maximally symmetric space-time.

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Simplest generalisation is a domain wall,

$$\mathcal{M}_{10} = \underbrace{\mathcal{M}_3}_{\text{maximally symmetric}} \times \underbrace{\mathcal{M}_7}_{\text{Non-compact}}$$

where

$$\mathcal{M}_7 = \mathbb{R}_y \times \underbrace{\mathcal{M}_6}_{\text{Compact}}$$

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We will take  $\mathcal{M}_6$  to be a Calabi-Yau. Supersymmetry requires  $\mathcal{M}_6$  half-flat.

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# Supersymmetric Calabi-Yau Domain Walls



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Relax maximally symmetric space-time to allow for flux. Next: Supersymmetry.

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Relax maximally symmetric space-time to allow for flux. Next: Supersymmetry. Require  $\mathcal{M}_6$  Calabi-Yau. Supersymmetry then requires

$$\partial_y \Omega_+ = 2\partial_y \phi \Omega_+ - H$$

$$J \wedge \partial_y J = \partial_y \phi J \wedge J$$

$$\Omega_- \wedge *H = 2\partial_y \phi *1,$$

where  $y$  is the direction parametrising  $\mathbb{R}$ .

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Dilaton is always a runaway direction. Complex structure moduli approach constants. Consistent with complex structure dependent 4d superpotential.

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# Four-Dimensional Phenomenology

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We now move to consider the low-energy four-dimensional effective theory.

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We now move to consider the low-energy four-dimensional effective theory.

It consists of:



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We now move to consider the low-energy four-dimensional effective theory.

It consists of:

- $N = 1$  SUGRA fields  $(S, T^i, X^A)$ ,

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We now move to consider the low-energy four-dimensional effective theory.

It consists of:

- $N = 1$  SUGRA fields  $(S, T^i, X^A)$ ,
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- the general case of half-flat: Large complex structure limit [Lukas et al., 1005.5302].
- Calabi-Yau: Everywhere in moduli space [Klaput et al., 1305.0594].

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So for any  $10d$  Calabi-Yau domain-wall with harmonic flux, may be matched to  $4d$  domain wall solution.

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- Domain wall: Lift to maximally symmetric (stable?) vacuum.  
Shown for the case of half-flat domain walls in [Klaput, Lukas, Matti, Svanes 1210.5933].

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If possible to lift: May use flux in heterotic to stabilize moduli.

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Problem: Hard to stabilise moduli.

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## Conclusions:

- Heterotic Calabi-Yau compactifications provide a fertile ground for phenomenology. Problem: Hard to stabilise moduli.
- Use flux to stabilise moduli by relaxing assumption of maximally symmetric  $4d$  space-time.

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- Lift to maximally symmetric space-time with non-perturbative effects.  
Same effects used to stabilise runaway moduli in conventional compactifications.

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## Future directions:

- Study moduli stabilisation and lifting of an explicit model.

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- What about chiral matter and the bundle moduli?

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## Future directions:

- Study moduli stabilisation and lifting of an explicit model.
- What about chiral matter and the bundle moduli?
- Try other compactifications: cosmic string, black hole, ...



Thank you!

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Thank you very much!