New N=2, d=3 M-theory solutions

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Introduction and motivation

- important for F-theory
- 3d M-theory vacua studied by Becker & Becker in 1996
- 8d manifolds with SU(4) structure [Martelli & Sparks, Cvetic et al]
- does not seem to cover all the type II posibilities of compactifications on $SU(3) \times SU(3)$ structure [McOrist, Morrison, Sethi]
- D3 brane potential – minimum at points of SU(3) structure [Martucci]
- M2 branes in B & B compactifications are N=2 supersymmetric
- $\rightarrow$ find similar setup to type IIB

Varying structure group in 8d

All M-theory compactifications on 8-manifolds consider Majorana–Weyl spinors (for simplicity)

Tsimpis 2007 $\rightarrow$ only need Majorana spinors ξ

Supersymmetry equation $\mathcal{D}\eta = 0$, η – 11d Majorana spinor

Compactification Ansatz $M_{1,10} = M_{1,2} \times K_8$

$$\eta = \epsilon \otimes \xi$$

- ϵ – 3d Majorana spinor
- ξ – 8d Majorana spinor

$\rightarrow$ no need of MW spinors $\rightarrow$ fixed chirality in 8d - unnecessary condition $\rightarrow$ special case!

Varying structure group in 8d

Each Majorana spinor has 2 M-W component spinors

$$\xi = \xi_+ + \xi_- \ , \qquad \xi_\pm = \frac{1}{2}(1 \pm \gamma_9)\xi$$

$$||\xi||^2 = ||\xi_+||^2 + ||\xi_-||^2 \neq 0$$

$\xi_\pm$ may vanish at certain points.

Non-vanishing MW spinors $\to$ reduction of structure group

- One non-vanishing MW spinor $\to$ Spin(7) structure
- 2 non-vanishing MW spinors of same chirality $\to$ SU(4) structure
- 2 non-vanishing MW spinors of different chiralities $\to$ G_2 structure

Non-vanishing Majorana spinor ξ $\to$ no reduction of structure group

- in general $||\xi_+|| \neq 0$ and $||\xi_-|| \neq 0 \;\sim G_2$ structure
- $||\xi_\pm|| = 0 \quad \sim Spin(7)_\mp$ structure

Same behavior as in manifolds with SU(3) $\times$ SU(3) structure.

N=2 susy in 3d $\rightarrow$ need 2 Majorana spinors in 8d $\sim$ 4 MW spinors

- in general SU(3) structure
- 2 MW components of the same chirality vanish $\sim$ SU(4) structure
- 2 MW components of different chiralities vanish $\sim G_2$ structure

there exist a unitary description in 9d (no chirality) $\rightarrow$ only Majorana spinors
$\rightarrow$ manifolds with G_2 structure

9d manifolds with G_2 structure

Characterize the structure with the help of spinor bilinears

$$(V_1)_m = \xi_1^T \gamma_m \xi_1 \ , \quad (V_2)_m = \xi_2^T \gamma_m \xi_2 \ , \quad (V_3)_m = \xi_1^T \gamma_m \xi_2 \ ,$$

$$K_{mn} = \xi_1^T \gamma_{mn} \xi_2 \ , \quad \Psi_{mnp} = \xi_1^T \gamma_{mnp} \xi_2$$

$$(\Phi_1)_{mnpq} = \xi_1^T \gamma_{mnpq} \xi_1 \ , \quad (\Phi_2)_{mnpq} = \xi_2^T \gamma_{mnpq} \xi_2 \ ,$$

$$(\Phi_3)_{mnpq} = \xi_1^T \gamma_{mnpq} \xi_2 \ ,$$

Obey Fierz identities

SU(4) structure in 8d

$$V_1 = V_2 \neq 0 \; ; \quad V_3 = 0$$

$$\Psi \sim K \wedge V_1 \; , \quad K - \text{almost complex structure}$$

$$\Phi_+ = K \wedge K \; , \quad \Phi_- + i\Phi_3 \sim \text{holomorphic 4} - \text{form}$$

SU(4) structure in 8d times an additional direction V_1

G_2 structure in 7d

$$V_1 = -V_2 \neq 0\,, \qquad V_3 \neq 0$$

$$K = V_1 \wedge V_3\,, \qquad \Psi \sim \text{associative } 3-\text{form}$$

$$\Phi_{1,2,3} \text{ combination of } V_1 \wedge \Psi,\ V_3 \wedge \Psi \text{ and } *(V_1 \wedge V_3 \wedge \Psi)$$

G_2 structure in 7d times two additional directions V_1 and V_3

General case

Parametrization in terms of $SU(3)$ structure in 6d
Free parameter $\alpha =< V_1, V_2 >$ angle between V_1 and V_2
α varies on the 9d manifold $\rightarrow$ no fixed structure group in 8d

- $\alpha = 1 \ \rightarrow$ SU(4) point
- $\alpha = -1 \ \rightarrow G_2$ point
- $\alpha \neq \pm 1 \ \rightarrow$ SU(3) point

$$V_\pm = V_1 \pm V_2 \ , \qquad < V_+, V_- >=< V_\pm, V_3 >= 0$$

SU(3) structure orthogonal to $V_\pm$ and V_3.
Define SU(3) structure forms J and φ as the parts of K and Ψ orthogonal to $V_\pm$ and V_3.

$$J^2 \sim -\mathbf{1} \ , \qquad J \wedge J \wedge J \sim \varphi \wedge \rho \ , \quad \rho_{mnp} = J_{qm} \varphi^q{}_{np}$$

$$\Omega = \phi + i\rho \ , \quad \bar{\Omega} = \phi - i\rho = J \cdot \Omega$$

Supersymmetry equations

Fluxes $\hat{F}_4 = F_4 + f \wedge \text{Vol}_3$
Spinorial susy equations

$$\mathcal{D}(F, f)\xi_i = 0$$

can be translated into tensorial equations for the spinor bilinears

$$\nabla_m V_{i\,n} = 2\lambda\theta_n V_{i\,m} - 2\lambda(\theta\cdot V_i)\delta_{mn} - \frac{1}{12}F_{mpqr}\Phi_{i\,n}{}^{pqr} + \frac{1}{2}\Phi_{i\,mnpq}f^p\theta^q$$

$$\nabla_m K_{np} = -4\lambda K_{m[n}\theta_{p]} + 4\lambda\delta_{m[n}K_{p]q}\theta^q + \frac{1}{2}F_{m[n}{}^{qr}\Psi_{p]qr} + \Psi_{m[n}{}^q\theta_{p]}f_q$$

Supersymmetry equations

$$\nabla_m \Psi_{npr} = 6\lambda \Psi_{m[np}\theta_{r]} - 6\lambda \delta_{m[n}\Psi_{pr]q}\theta^q + \frac{1}{72} F_m{}^{stu} \epsilon_{nprstuvwz} \Psi^{vwz}$$
$$+ \frac{3}{2} F_{m[np}{}^q K_{r]q} - \frac{1}{12} \epsilon_{mnprklqst} \Psi^{klq} f^s \theta^t - 3K_{m[n} f_p \theta_{r]}$$
$$+ 3\delta_{m[n} f_p K_{r]q} \theta^q - 3\delta_{m[n}\theta_p K_{r]q} f^q$$

$$\nabla_m (\Phi_i)_{npqr} = -8\lambda \Phi_{i\ m[npq}\theta_{r]} + 8\lambda \delta_{m[n}\Phi_{i\ pqr]s}\theta^s + F_{m[n}{}^{st} (*\Phi_i)_{pqr]st}$$
$$- 2F_{m[npq} V_{i\ r]} + 2(*\Phi_i)_{m[npq}{}^s \theta_{r]} f_s - 2(*\Phi_i)_{m[npq}{}^s f_{r]} \theta_s$$
$$+ 2\delta_{m[n} (*\Phi_i)_{pqr]st} f^s \theta^t - 12\delta_{m[n} V_{i\ p} f_q \theta_{r]} \ ,$$

All spinor bilinears on the 9d manifold can be expressed in terms of $V_\pm$, V_3 and the SU(3) structure forms J and φ. Express susy equations in terms of these

Varying α

Key point: α allowed to vary $\leftrightarrow$ varying structure group.

$$||V_+||^2 = 2(1+\alpha)\ , \qquad \rightarrow \partial_m \alpha = \frac{1}{2}\partial_m ||V_+||^2 = (V_+)^n \nabla_m (V_+)_n$$

$$\boxed{d\alpha = \rho \lrcorner F}$$

Flux responsible for varying α:
$F = h \wedge \rho + g \wedge \varphi$ – $(3,0) + (0,3)$ wrt SU(3) structure

SU(4) limit – $(3,1) + (4,0) +$ c.c. – no $(2,2) \rightarrow$ different from B & B

$$d\alpha = 2(1-\alpha)(\frac{1+\alpha}{2} h + g \lrcorner J) + 2(1+\alpha)\left[(h \cdot V_3)V_3 + \frac{1}{4}(h \cdot V_-)V_-\right] + \frac{1}{2}(1-\alpha)(h \cdot V_+)V_+$$

Susy Equations for particular flux

$$F = h \wedge \rho + g \wedge \varphi \,, \qquad f = 0$$

Solution to external gravitino equation:

$$\begin{aligned} \lambda =& 0 \,, \qquad h \cdot V_+ = g \cdot V_+ = d\Delta \cdot V_+ = 0 \\ g \cdot V_- =& 2h \cdot V_3 \,, \qquad g \cdot V_3 = -\frac{1}{2} h \cdot V_- \,, \\ d\Delta \cdot V_- =& \frac{1-\alpha}{3} h \cdot V_- \,, \qquad d\Delta \cdot V_3 = \frac{1-\alpha}{3} h \cdot V_3 \end{aligned}$$

$$g - h \lrcorner J = \frac{2}{1-\alpha} h \lrcorner (V_- \wedge V_3)$$

$$d\alpha = 4(1+\alpha)(h \cdot V_3) V_3 + (1+\alpha)(h \cdot V_-) V_-$$

$$
\begin{aligned}
dV_+ =& 0 \, , \qquad dK = 0 \\
dV_- =& - \frac{2+\alpha}{1-\alpha}(h \cdot V_3) V_- \wedge V_3 \\
dV_3 =& \frac{2+\alpha}{4(1-\alpha)}(h \cdot V_-) V_- \wedge V_3 \\
d\Psi \sim& \varphi \wedge V_- + \varphi \wedge V_3 + \rho \wedge V_- + \rho \wedge V_3 \\
& + J \wedge V_+ \wedge V_- + J \wedge V_+ \wedge V_3
\end{aligned}
$$

M2 brane potential

- In the varying structure group background M2 branes are not automatically supersymmetric
- M2 brane supersymmetry controlled by the M2 brane chirality operator $\Gamma_{M2} = \gamma_9$

$$\epsilon_1 \otimes \xi_1 + \epsilon_2 \otimes \xi_2 = \epsilon_1 \otimes \gamma_9 \xi_1 + \epsilon_2 \otimes \gamma_9 \xi_2$$

- N=1 $\rightarrow$ one + chirality and one - chirality spinor $\rightarrow \ \alpha = -1 \ \rightarrow \ G_2$ point
- N=2 $\rightarrow$ two + chirality spinors $\rightarrow \ \alpha = +1 \ \rightarrow \ SU(4)$ point

α - cos of the angle between V_1 and $V_2 \rightarrow \alpha = \pm 1$ - extrema
Potential must vanish at susy points

Conclusions

- M-theory on 8-manifold – MW spinors – particular case
- Majorana spinors – varying structure group SU(3), SU(4) and G_2 – analog of SU(3) $\times$ SU(3) structure in type II compactifications
- flux responsible for varying structure – (3,0) + (0,3) in the SU(3) language
- M2 brane potential has extrema at $\alpha = \pm 1$ – $SU(4)$ or G_2 points