β-supergravity: a ten-dimensional theory with non-geometric fluxes

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arXiv:1306.4381 by D. A. and André Betz

(and see today's talks...)

StringPheno 2013,
17/07/2013, DESY, Hamburg, Germany

Introduction

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⇔ some "structure constants" in gauging algebra

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Here: new 10D $\tilde{\mathcal{L}}_\beta(Q_a{}^{bc},\ R^{abc},\ f^a{}_{bc})$ on standard geometry

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NSNS of " β-supergravity ", a reformulation of standard sugra
Aim: $\tilde{\mathcal{L}}_\beta$ provides a clear 10D uplift of some 4D gauged sugra

Obtaining the Lag. $\tilde{\mathcal{L}}_\beta$ in curved indices

Reformulation of $\mathcal{L}_{\rm NSNS}$ via a field redefinition

Build on earlier results: arXiv:1303.0251 by D. A.

arXiv:1106.4015 by D. A., M. Larfors, D. Lüst, P. Patalong

arXiv:1202.3060, arXiv:1204.1979 by D. A., O. Hohm, M. Larfors, D. Lüst, P. Patalong

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$$\begin{aligned} \tilde{g}^{-1} &= (g+b)^{-1} g (g-b)^{-1} \\ \beta &= -(g+b)^{-1} b (g-b)^{-1} \end{aligned} \Leftrightarrow (g+b)^{-1} = (\tilde{g}^{-1} + \beta) \ , \ \frac{e^{-2\tilde{\phi}}}{e^{-2\phi}} = \frac{\sqrt{|g|}}{\sqrt{|\tilde{g}|}}$$

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$\Leftrightarrow$ reparametrization of gen. metric $\mathcal{H}$

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$$\mathcal{E} = \begin{pmatrix} e & 0 \\ e^{-T}b & e^{-T} \end{pmatrix},\ \tilde{\mathcal{E}} = \begin{pmatrix} \tilde{e} & \tilde{e}\beta \\ 0 & \tilde{e}^{-T} \end{pmatrix},\ \mathbb{I} = \begin{pmatrix} \eta_d & 0 \\ 0 & \eta_d^{-1} \end{pmatrix},\quad \begin{aligned} g &= e^T \eta_d e \\ \tilde{g} &= \tilde{e}^T \eta_d \tilde{e} \end{aligned}$$

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β w.r.t. Q, R: motivations from Gen. Complex Geom./sugra

hep-th/0609084, arXiv:0708.2392 by P. Grange, S. Schäfer-Nameki

arXiv:0807.4527 by M. Graña, R. Minasian, M. Petrini, D. Waldram

Apply the field redefinition to $\mathcal{L}_{\rm NSNS}$

$$\mathcal{L}_{\rm NSNS} = e^{-2\phi}\sqrt{|g|}\left(\mathcal{R}(g) + 4(\partial\phi)^2 - \frac{1}{2}H^2\right),\ H_{mnp} \equiv 3\partial_{[m}b_{np]}$$

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$$\mathcal{L}_{\rm NSNS} + \partial(\dots) = \tilde{\mathcal{L}}_0 + \partial(\dots) = = = = = =$$

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$$\begin{aligned}\tilde{\mathcal{L}}_0 = e^{-2\tilde{\phi}}\sqrt{|\tilde{g}|}\ \Big(&\mathcal{R}(\tilde{g}) + 4(\partial\tilde{\phi})^2 - \frac{1}{2}R^2 \\ &+ 4\tilde{g}_{mn}\beta^{mp}\beta^{nq}\partial_p d\ \partial_q d - 2\partial_p d\ \partial_q\left(\tilde{g}_{mn}\beta^{mp}\beta^{nq}\right) \\ &- \frac{1}{4}\tilde{g}_{mp}\tilde{g}_{nq}\tilde{g}^{rs}\ \partial_r\beta^{pq}\ \partial_s\beta^{mn} + \frac{1}{2}\tilde{g}_{mn}\partial_p\beta^{qm}\ \partial_q\beta^{pn} \\ &+ \tilde{g}_{nq}\tilde{g}_{rs}\beta^{nm}\left(\partial_p\beta^{qr}\ \partial_m\tilde{g}^{ps} + \partial_p\tilde{g}^{qr}\ \partial_m\beta^{ps}\right) \\ &- \frac{1}{4}\tilde{g}_{mp}\tilde{g}_{nq}\tilde{g}_{rs}\left(\beta^{ru}\beta^{sv}\partial_u\tilde{g}^{pq}\ \partial_v\tilde{g}^{mn} - 2\beta^{mu}\beta^{nv}\partial_u\tilde{g}^{qr}\ \partial_v\tilde{g}^{ps}\right)\Big)\end{aligned}$$

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$$+ 4\tilde{g}_{mn}\beta^{mp}\beta^{nq}\partial_p d\ \partial_q d - 2\partial_p d\ \partial_q\left(\tilde{g}_{mn}\beta^{mp}\beta^{nq}\right)$$

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Comparison of $\tilde{\mathcal{L}}_0$ to 4D potential: Q-flux term, ${Q_m}^{np} \sim \partial_m\beta^{np}$?

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Double Field Theory (DFT): better organisation, diffeom. cov.

arXiv:0904.4664, arXiv:0908.1792 by C. Hull, B. Zwiebach

arXiv:1003.5027, arXiv:1006.4823 by O. Hohm, C. Hull, B. Zwiebach

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$$\begin{array}{ccccc} \mathcal{L}_{\rm DFT}(g,b,\phi) & = & \mathcal{L}_{\rm DFT}(\tilde{g},\beta,\tilde{\phi}) & = & \\ \tilde{\partial}=0 \Big\| & & \tilde{\partial}=0 \Big\| & & \| \\ \mathcal{L}_{\rm NSNS} + \partial(\dots) & = & \tilde{\mathcal{L}}_0 + \partial(\dots) & = = = = = = & \end{array}$$

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arXiv:1003.5027, arXiv:1006.4823 by O. Hohm, C. Hull, B. Zwiebach

Apply the field redefinition to $\mathcal{L}_{\rm NSNS}$

$$\mathcal{L}_{\rm NSNS} = e^{-2\phi}\sqrt{|g|}\left(\mathcal{R}(g) + 4(\partial\phi)^2 - \frac{1}{2}H^2\right), \ H_{mnp} \equiv 3\partial_{[m}b_{np]}$$

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\end{array}$$

$$\tilde{\mathcal{L}}_0 = e^{-2\tilde{\phi}}\sqrt{|\tilde{g}|}\left(\mathcal{R}(\tilde{g}) + 4(\partial\tilde{\phi})^2 - \frac{1}{2}R^2 + 4 \text{ lines}\right), \ R^{mnp} = 3\beta^{q[m}\nabla_q\beta^{np]}$$

Comparison of $\tilde{\mathcal{L}}_0$ to 4D potential: Q-flux term, ${Q_m}^{np} \sim \partial_m\beta^{np}$?

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$$2\check{\Gamma}^{mn}_p = \tilde{g}_{pq}\left(\beta^{rm}\partial_r\tilde{g}^{nq} + \beta^{rn}\partial_r\tilde{g}^{mq} - \beta^{rq}\partial_r\tilde{g}^{mn}\right) + 2\tilde{g}_{pq}\tilde{g}^{r(m}\partial_r\beta^{n)q} - \partial_p\beta^{mn}$$

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Where/what is the 10D Q-flux?

The Lag. $\tilde{\mathcal{L}}_\beta$ in flat indices and the Q-flux

Higher dimensional realisation of 4D fluxes: with flat indices
Arguments in favor: "structure constants" of gauging algebra...

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Finally get a 10D theory (β-supergravity)
with non-geometric fluxes precisely identified

David ANDRIOT

More structure in $\tilde{\mathcal{L}}_\beta$...

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- For standard ∇_m and Levi-Civita connection Γ^m_{np}

$$\tilde{e}^a{}_m \tilde{e}^n{}_b \nabla_n V^m = \nabla_b V^a \equiv \partial_b V^a + \omega^a_{bc} V^c$$

$$\Leftrightarrow \omega^a_{bc} \equiv \tilde{e}^n{}_b \tilde{e}^a{}_m \left(\partial_n \tilde{e}^m{}_c + \tilde{e}^p{}_c \Gamma^m_{np}\right)$$

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Pushing the analogy further:

$$\mathcal{R}(\tilde{g}) = 2\eta^{bc} \partial_a \omega^a_{bc} + \eta^{bc} \omega^a_{ad} \omega^d_{bc} - \eta^{bc} \omega^a_{db} \omega^d_{ac}$$

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More structure in $\tilde{\mathcal{L}}_\beta$... $\Rightarrow$ Generalized Geometry formalism!

- For standard ∇_m and Levi-Civita connection Γ^m_{np}

$$\tilde{e}^a{}_m \tilde{e}^n{}_b \nabla_n V^m = \nabla_b V^a \equiv \partial_b V^a + \omega^a_{bc} V^c$$
$$\Leftrightarrow \omega^a_{bc} \equiv \tilde{e}^n{}_b \tilde{e}^a{}_m \left(\partial_n \tilde{e}^m{}_c + \tilde{e}^p{}_c \Gamma^m_{np}\right)$$
$$\omega^a_{bc} = \frac{1}{2}\left(f^a{}_{bc} + \eta^{ad}\eta_{ce} f^e{}_{db} + \eta^{ad}\eta_{be} f^e{}_{dc}\right)$$

- Proceeding similarly for the new $\check{\nabla}^m$

$$\tilde{e}^m{}_a \tilde{e}^b{}_n \check{\nabla}^n V_m = \check{\nabla}^b V_a \equiv -\beta^{bd}\partial_d V_a - \omega_Q{}_a^{bc} V_c$$
$$\Leftrightarrow -\omega_Q{}_a^{bc} \equiv \tilde{e}^b{}_n \tilde{e}^m{}_a \left(-\beta^{nq}\partial_q \tilde{e}^c{}_m + \tilde{e}^c{}_p \check{\Gamma}^{np}_m\right)$$
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Pushing the analogy further:

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- 10D theory (NSNS sector of " β-supergravity ")
  $\tilde{\mathcal{L}}_\beta({Q_a}^{bc},\ R^{abc},\ {f^a}_{bc})$, with ${Q_c}^{ab} = \partial_c \beta^{ab} - 2\beta^{d[a} {f^{b]}}_{cd}$

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- Extensions to/beyond β-supergravity:
  - Other sectors (fermions, Ramond-Ramond, heterotic...)?
  - New (exotic) branes?
    arXiv:1004.2521, arXiv 1209.6056 by J. de Boer, M. Shigemori
    arXiv:1109.4484 by E. A. Bergshoeff, T. Ortin, F. Riccioni
    arXiv:1303.1413 by F. Hassler, D. Lüst
  - Add a b-field?

  $\hookrightarrow$ Get new interesting 10D backgrounds...

The 4d scalar potential

Split 10d $\Rightarrow$ 4d max. sym. space-time $\times$ 6d compact $\mathcal{M}$
Compactification ansatz: $\mathrm{d}s^2_{10} = \mathrm{d}s^2_4 + \mathrm{d}s^2_6$ (no warp factor), b, β purely internal

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$$S_E = M_4^2 \int \mathrm{d}^4x\ \sqrt{|g^E|}\left(\mathcal{R}_4^E + \mathrm{kin} - \frac{1}{M_4^2}\, V(\rho,\sigma)\right)$$

arXiv:0712.1196 by E. Silverstein

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arXiv:0711.2512 by M. P. Hertzberg, S. Kachru, W. Taylor, M. Tegmark

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$$V_{\mathcal{R}} = \frac{M_4^2}{4v_0}\int \mathrm{d}x^6 \sqrt{|\tilde{g}^{(0)}_{(6)}|}\ \eta^{be}\left(\eta_{ad}\eta^{cg}\ f^a{}_{bc} f^d{}_{eg} + 2\ f^a{}_{cb} f^c{}_{ae}\right)$$

$$V_Q = \frac{M_4^2}{4v_0}\int \mathrm{d}x^6 \sqrt{|\tilde{g}^{(0)}_{(6)}|}\ \eta_{be}\left(\eta^{ad}\eta_{cg}\ Q_a{}^{bc} Q_d{}^{eg} + 2\ Q_a{}^{cb} Q_c{}^{ae} + 2\ R^{bcd} f^e{}_{cd}\right)$$

$$V_R = \frac{M_4^2}{4v_0}\int \mathrm{d}x^6 \sqrt{|\tilde{g}^{(0)}_{(6)}|}\ \ \frac{1}{3}\eta_{ad}\eta_{be}\eta_{cg}\ R^{abc} R^{deg}$$

(with $\forall b$, $f^a{}_{ab} = 0$, $Q_a{}^{ab} = 0$, $\mathcal{T}^b = 0$)