Patching-Up Non-Geometric Backgrounds

Felix Rennecke
Max-Planck-Institut für Physik

In collaboration with Ralph Blumenhagen, Andreas Deser , Erik Plauschinn and Christian Schmid

Max-Planck-Institut für Physik
(Werner-Heisenberg-Institut)

Outline

- Motivation
- O(d,d)'s & field redefinitions
  see talks by Blumenhagen & Plauschinn
- Symmetries
- Patching of non-geometric backgrounds
- Simplify backgrounds
- Conclusion & outlook

Motivation

- The String sigma-model is

$$S(G,B) = \frac{1}{2\pi\alpha'} \int_\Sigma d^2z\,(G_{ab} + B_{ab})\,\partial X^a \bar{\partial} X^b$$

Motivation

- The String sigma-model is

$$S(G,B) = \frac{1}{2\pi\alpha'} \int_\Sigma d^2z \, (G_{ab} + B_{ab}) \, \partial X^a \bar{\partial} X^b$$

determine background

Motivation

- The String sigma-model is

$$S(G,B)=\frac{1}{2\pi\alpha'}\int_\Sigma d^2z\,(G_{ab}+B_{ab})\,\partial X^a\bar{\partial}X^b$$

- T-duality: spacetime $\mathbb{T}^d$

Motivation

- The String sigma-model is

$$S(G,B) = \frac{1}{2\pi\alpha'}\int_\Sigma d^2z\,(G_{ab}+B_{ab})\,\partial X^a\bar{\partial}X^b$$

- T-duality: spacetime $\mathbb{T}^d$

$$\begin{tikzcd} S(G,B) \arrow[rr, leftrightarrow, "\text{Buscher rules}"] & & S(\widetilde{G},\widetilde{B}) \end{tikzcd}$$

(Buscher, Roček, Verlinde...)

Motivation

- The String sigma-model is

$$S(G,B) = \frac{1}{2\pi\alpha'} \int_\Sigma d^2z \, (G_{ab} + B_{ab}) \, \partial X^a \bar{\partial} X^b$$

- T-duality: spacetime $\mathbb{T}^d$

$$S(G,B) \xleftrightarrow{\text{Buscher rules}} S(\widetilde{G},\widetilde{B})$$

Buscher rules phrased nicely in generalized geometry:

$$\mathcal{H}(G,B) = \begin{pmatrix} G - BG^{-1}G & BG^{-1} \\ -G^{-1}B & G^{-1} \end{pmatrix} \qquad , \quad \mathcal{T} \in O(d,d;\mathbb{Z})$$

Motivation

- The String sigma-model is

$$S(G,B) = \frac{1}{2\pi\alpha'} \int_\Sigma d^2 z \, (G_{ab} + B_{ab}) \, \partial X^a \bar{\partial} X^b$$

- T-duality: spacetime $\mathbb{T}^d$

$$\begin{tikzcd} S(G,B) \arrow[rrr, leftrightarrow, "\text{Buscher rules}"] & & & S(\widetilde{G},\widetilde{B}) \end{tikzcd}$$

Buscher rules phrased nicely in generalized geometry:

$$\Longrightarrow \quad \mathcal{H}(\widetilde{G},\widetilde{B}) = \widetilde{\mathcal{H}}(G,B) = \mathcal{T}^t \mathcal{H}(G,B) \mathcal{T} \,, \quad \mathcal{T} \in O(d,d;\mathbb{Z})$$

Motivation

- The String sigma-model is

$$S(G,B) = \frac{1}{2\pi\alpha'} \int_\Sigma d^2z \, (G_{ab} + B_{ab}) \, \partial X^a \bar\partial X^b$$

- T-duality: spacetime $\mathbb{T}^d$

$$S(G,B) \xleftrightarrow{\text{Buscher rules}} S(\widetilde{G},\widetilde{B})$$

Buscher rules phrased nicely in generalized geometry:

$$\Longrightarrow \quad \mathcal{H}(\widetilde{G},\widetilde{B}) = \widetilde{\mathcal{H}}(G,B) = \mathcal{T}^t \mathcal{H}(G,B) \mathcal{T}, \quad \mathcal{T} \in O(d,d;\mathbb{Z})$$

field redefinition

Motivation

- The String sigma-model is

$$S(G,B) = \frac{1}{2\pi\alpha'} \int_{\Sigma} d^2z \, (G_{ab} + B_{ab}) \, \partial X^a \bar{\partial} X^b$$

- T-duality: spacetime $\mathbb{T}^d$

$$\begin{tikzcd}
S(G,B) \arrow[rr, leftrightarrow, "\text{Buscher rules}"] \arrow[d, "\text{eff. action}"'] & & S(\widetilde{G},\widetilde{B}) \arrow[d, "\text{eff. action}"] \\
S_0^d(G,B) = \frac{1}{2\kappa^2} \int_{\mathbb{T}^d} d^d x \sqrt{|G|} \left(R - \frac{1}{12} H^2\right) & & S_0^d(\widetilde{G},\widetilde{B})
\end{tikzcd}$$

Motivation

- The String sigma-model is

$$S(G,B) = \frac{1}{2\pi\alpha'}\int_{\Sigma} d^2z\,(G_{ab}+B_{ab})\,\partial X^a\bar{\partial}X^b$$

- T-duality: spacetime $\mathbb{T}^d$

$$\begin{tikzcd}
S(G,B) \arrow[rr, leftrightarrow, "\text{Buscher rules}"] \arrow[d, "\text{eff. action}"'] & & S(\widetilde{G},\widetilde{B}) \arrow[d, "\text{eff. action}"] \\
S_0^d(G,B) = \frac{1}{2\kappa^2}\int_{\mathbb{T}^d} d^dx\sqrt{|G|}\left(R-\frac{1}{12}H^2\right) & = & S_0^d(\widetilde{G},\widetilde{B})
\end{tikzcd}$$

(Bergshoeff, Entrop)

Motivation

- The String sigma-model is

$$S(G,B) = \frac{1}{2\pi\alpha'} \int_\Sigma d^2z \, (G_{ab} + B_{ab}) \, \partial X^a \bar{\partial} X^b$$

- T-duality: → maps solutions to solutions;
  "background-generator"
  → well-arranged within generalized geometry

Motivation

- The String sigma-model is

$$S(G,B)=\frac{1}{2\pi\alpha'}\int_{\Sigma}d^2z\,(G_{ab}+B_{ab})\,\partial X^a\bar{\partial}X^b$$

- T-duality: ⟶ maps solutions to solutions;
  "background-generator"
  ⟶ well-arranged within generalized geometry

- non-geometric backgrounds:
  ⟶ from T-duality of geom. ones
  (Kachru, Schulz, Tripathy, Trivedi; Dabholkar, Hull; Hellermann, McGreevy, Williams;...)

Motivation

- The String sigma-model is

$$S(G,B) = \frac{1}{2\pi\alpha'}\int_\Sigma d^2z\,(G_{ab}+B_{ab})\,\partial X^a\bar{\partial}X^b$$

- T-duality: → maps solutions to solutions; "background-generator"
  → well-arranged within generalized geometry

- non-geometric backgrounds:
  → from T-duality of geom. ones
  (Kachru, Schulz, Tripathy, Trivedi; Dabholkar, Hull; Hellermann, McGreevy, Williams;...)

  → asymmetric CFT's, orbifolds
  (Blumenhagen, Deser, Lüst, Plauschinn, FR; Condeescu, Florakis, Kounnas, Lüst)

Motivation

- The String sigma-model is

$$S(G,B) = \frac{1}{2\pi\alpha'} \int_\Sigma d^2z\, (G_{ab} + B_{ab})\, \partial X^a \bar{\partial} X^b$$

- non-geometric backgrounds: ?

Motivation

- The String sigma-model is

$$S(G,B)=\frac{1}{2\pi\alpha'}\int_\Sigma d^2z\,(G_{ab}+B_{ab})\,\partial X^a\bar{\partial}X^b$$

→ diffeomorphism-invariance
→ gauge invariance $B\rightarrow B+d\xi$

- non-geometric backgrounds: ?

Motivation

- The String sigma-model is

$$S(G,B) = \frac{1}{2\pi\alpha'}\int_\Sigma d^2z\,(G_{ab}+B_{ab})\,\partial X^a\bar{\partial}X^b$$

$$\left.\begin{array}{l}\longrightarrow \text{diffeomorphism-invariance}\\ \longrightarrow \text{gauge invariance } B \to B + d\xi\end{array}\right\}\ G_{\text{geom}}$$

- non-geometric backgrounds: ?

Motivation

- The String sigma-model is

$$S(G,B) = \frac{1}{2\pi\alpha'} \int_\Sigma d^2 z \, (G_{ab} + B_{ab}) \, \partial X^a \bar{\partial} X^b$$

$$\left.\begin{array}{l} \longrightarrow \text{diffeomorphism-invariance} \\ \longrightarrow \text{gauge invariance } B \to B + d\xi \end{array}\right\} G_{\text{geom}}$$

- non-geometric backgrounds: ?

geometries which require transformations beyond G_{geom} to glue together local patches

Motivation

Aim: clarify the structure of non-geometric backgrounds in effective string theory

Motivation

Aim: clarify the structure of non-geometric backgrounds in effective string theory by employing

- → field redefinitions
  (Halmagyi; Andriot, Betz, Hohm, Lafors, Lüst, Patalong)
- → the relations between *O(d,d)*'s and Lie algebroids
  (Blumenhagen, Deser, Plauschinn, FR, Schmid)

see talks by Lüst, Blumenhagen, Andriot, Betz, Plauschinn

O(d,d)’s & field redefinitions

- Field-redefinitions from O(d,d)’s $\mathcal{T} = \begin{pmatrix} t_{11} & t_{12} \\ t_{21} & t_{22} \end{pmatrix} \in O(d,d)$

O(d,d)'s & field redefinitions

- Field-redefinitions from O(d,d)'s $\mathcal{T} = \begin{pmatrix} t_{11} & t_{12} \\ t_{21} & t_{22} \end{pmatrix} \in O(d,d)$

$$\mathcal{H}(g,b) = \mathcal{H}'(G,B) = \mathcal{T}^t \mathcal{H}(G,B)\mathcal{T}$$

O(d,d)'s & field redefinitions

- Field-redefinitions from O(d,d)'s $\mathcal{T} = \begin{pmatrix} t_{11} & t_{12} \\ t_{21} & t_{22} \end{pmatrix} \in O(d,d)$

$$\mathcal{H}(g,b) = \mathcal{H}'(G,B) = \mathcal{T}^t \mathcal{H}(G,B)\mathcal{T}$$

recall: T-duality can be described likewise

O(d,d)'s & field redefinitions

- Field-redefinitions from O(d,d)'s $\mathcal{T} = \begin{pmatrix} t_{11} & t_{12} \\ t_{21} & t_{22} \end{pmatrix} \in O(d,d)$

$$\mathcal{H}(g,b) = \mathcal{H}'(G,B) = \mathcal{T}^t \mathcal{H}(G,B) \mathcal{T}$$

$$\Longrightarrow \quad G = \gamma\, g\, \gamma^t\,, \quad \gamma^{-1} = t_{11}^t + (g-b) t_{12}^t$$

$$B = \gamma\, \mathfrak{b}\, \gamma^t\,, \quad \mathfrak{b} = \gamma^{-1}\big(t_{21} + t_{22}(g+b)\big) - g$$

O(d,d)'s & field redefinitions

- Field-redefinitions from O(d,d)'s $\mathcal{T} = \begin{pmatrix} t_{11} & t_{12} \\ t_{21} & t_{22} \end{pmatrix} \in O(d,d)$

$$\mathcal{H}(g,b) = \mathcal{H}'(G,B) = \mathcal{T}^t \mathcal{H}(G,B) \mathcal{T}$$

$$\Longrightarrow \quad \begin{aligned} G &= \gamma\, g\, \gamma^t\,, \quad & \boxed{\gamma^{-1} = t_{11}^t + (g-b)t_{12}^t} \\ B &= \gamma\, \mathfrak{b}\, \gamma^t\,, \quad & \mathfrak{b} = \gamma^{-1}\big(t_{21} + t_{22}(g+b)\big) - g \end{aligned}$$

O(d,d)'s & field redefinitions

- Field-redefinitions from O(d,d)'s $\mathcal{T} = \begin{pmatrix} t_{11} & t_{12} \\ t_{21} & t_{22} \end{pmatrix} \in O(d,d)$

$$\mathcal{H}(g,b) = \mathcal{H}'(G,B) = \mathcal{T}^t \mathcal{H}(G,B)\mathcal{T}$$

$$\Longrightarrow \quad G = \gamma\, g\, \gamma^t\,, \quad \boxed{\gamma^{-1} = t_{11}^t + (g-b)t_{12}^t}$$

$$B = \gamma\, \mathfrak{b}\, \gamma^t\,, \quad \mathfrak{b} = \gamma^{-1}\big(t_{21} + t_{22}(g+b)\big) - g$$

provides transpose of anchor of the Lie algebroid

$$\big(TM, [\![\cdot,\cdot]\!], \rho = (\gamma^{-1})^t\big)$$

O(d,d)'s & field redefinitions

- Field-redefinitions from O(d,d)'s $\mathcal{T} = \begin{pmatrix} t_{11} & t_{12} \\ t_{21} & t_{22} \end{pmatrix} \in O(d,d)$

$$\mathcal{H}(g,b) = \mathcal{H}'(G,B) = \mathcal{T}^t \mathcal{H}(G,B) \mathcal{T}$$

$$\Longrightarrow \quad G = \gamma\, g\, \gamma^t\,, \quad \gamma^{-1} = t_{11}^t + (g-b) t_{12}^t$$

$$B = \gamma\, \mathfrak{b}\, \gamma^t\,, \quad \boxed{\mathfrak{b} = \gamma^{-1}\big(t_{21} + t_{22}(g+b)\big) - g}$$

O(d,d)’s & field redefinitions

- Field-redefinitions from O(d,d)’s $\mathcal{T} = \begin{pmatrix} t_{11} & t_{12} \\ t_{21} & t_{22} \end{pmatrix} \in O(d,d)$

$$\mathcal{H}(g,b) = \mathcal{H}'(G,B) = \mathcal{T}^t \mathcal{H}(G,B)\mathcal{T}$$

$$\Longrightarrow \quad G = \gamma\, g\, \gamma^t\,, \quad \gamma^{-1} = t_{11}^t + (g-b)t_{12}^t$$

$$B = \gamma\, \mathfrak{b}\, \gamma^t\,, \quad \boxed{\mathfrak{b} = \gamma^{-1}\big(t_{21} + t_{22}(g+b)\big) - g}$$

“gauge field” in the Lie algebroid: analogue of B

O(d,d)'s & field redefinitions

- Field-redefinitions from O(d,d)'s $\mathcal{T} = \begin{pmatrix} t_{11} & t_{12} \\ t_{21} & t_{22} \end{pmatrix} \in O(d,d)$

$$\mathcal{H}(g,b) = \mathcal{H}'(G,B) = \mathcal{T}^t \mathcal{H}(G,B)\mathcal{T}$$

$$\Longrightarrow \quad \begin{aligned} G &= \gamma\, g\, \gamma^t\,, \quad & \gamma^{-1} &= t_{11}^t + (g-b)t_{12}^t \\ B &= \gamma\, \mathfrak{b}\, \gamma^t\,, \quad & \mathfrak{b} &= \gamma^{-1}\big(t_{21} + t_{22}(g+b)\big) - g \end{aligned}$$

$$\boxed{\mathcal{T} \in O(d,d) \quad \Longrightarrow \quad \begin{aligned} &E = \big(TM, [\![\cdot,\cdot]\!], \rho\big) \\ &G = \otimes^2 \rho^* g\,, \quad B = \otimes^2 \rho^* \mathfrak{b} \end{aligned}}$$

To every *O(d,d)* there is an associated Lie algebroid;
the corresponding fields on the bundles are associated by the anchor

O(d,d)'s & field redefinitions

- Field-redefinitions from O(d,d)'s $\mathcal{T} = \begin{pmatrix} t_{11} & t_{12} \\ t_{21} & t_{22} \end{pmatrix} \in O(d,d)$

$$\mathcal{H}(g,b) = \mathcal{H}'(G,B) = \mathcal{T}^t \mathcal{H}(G,B)\mathcal{T}$$

$$\Longrightarrow \; G = \gamma\, g\, \gamma^t\,, \quad \gamma^{-1} = t_{11}^t + (g-b)t_{12}^t$$

$$B = \gamma\, \mathfrak{b}\, \gamma^t\,, \quad \mathfrak{b} = \gamma^{-1}(t_{21} + t_{22}(g+b)) - g$$

$$G_{ab} = \rho^{\alpha}{}_{a}\rho^{\beta}{}_{b}\, g_{\alpha\beta}$$

$$\Big\Updownarrow$$

$$\mathcal{T} \in O(d,d) \quad \Longrightarrow \quad E = (TM, [\![\cdot,\cdot]\!], \rho)$$

$$G = \otimes^2 \rho^* g\,, \quad B = \otimes^2 \rho^* \mathfrak{b}$$

To every *O(d,d)* there is an associated Lie algebroid;
the corresponding fields on the bundles are associated by the anchor

O(d,d)'s & field redefinitions

- Field-redefinitions from O(d,d)'s $\mathcal{T} = \begin{pmatrix} t_{11} & t_{12} \\ t_{21} & t_{22} \end{pmatrix} \in O(d,d)$

$$\mathcal{H}(g,b) = \mathcal{H}'(G,B) = \mathcal{T}^t \mathcal{H}(G,B)\mathcal{T}$$

$$\begin{aligned} \Longrightarrow \quad G &= \gamma\, g\, \gamma^t\,, \quad \gamma^{-1} = t_{11}^t + (g-b)t_{12}^t \\ B &= \gamma\, \mathfrak{b}\, \gamma^t\,, \quad \mathfrak{b} = \gamma^{-1}\big(t_{21} + t_{22}(g+b)\big) - g \end{aligned}$$

$$\boxed{\mathcal{T} \in O(d,d) \quad \Longrightarrow \quad \begin{aligned} &E = \big(TM, [\![\cdot,\cdot]\!], \rho\big) \\ &G = \otimes^2 \rho^* g\,, \quad B = \otimes^2 \rho^* \mathfrak{b} \end{aligned}}$$

To every *O(d,d)* there is an associated Lie algebroid;
the corresponding fields on the bundles are associated by the anchor

Symmetries: gauge transformations

- The geometric construction above is very general: anchor of Lie algebroid $\rho : E \to TM$

Symmetries: gauge transformations

- The geometric construction above is very general: anchor of Lie algebroid $\rho : E \to TM$

$$\left.\begin{array}{c} \big(TM, [\cdot,\cdot], id\big) \\ G \in \Gamma(S^2T^*M) \\ B \in \Gamma(\Lambda^2T^*M) \end{array}\right\} \quad \begin{array}{c} \rho(\llbracket s,t \rrbracket) = [\rho(s), \rho(t)] \\ G = \otimes^2\rho^* g \\ B = \otimes^2\rho^* \mathfrak{b} \end{array} \quad \left\{\begin{array}{c} \big(E, \llbracket\cdot,\cdot\rrbracket, \rho\big) \\ g \in \Gamma(S^2E^*) \\ \mathfrak{b} \in \Gamma(\Lambda^2E^*) \end{array}\right.$$

Symmetries: gauge transformations

- The geometric construction above is very general: anchor of Lie algebroid $\rho : E \to TM$

$$\left.\boxed{\begin{array}{c} \big(TM,[\cdot,\cdot],id\big) \\ G\in\Gamma(S^2T^*M) \\ B\in\Gamma(\Lambda^2T^*M)\end{array}}\right\} \quad \begin{array}{c} \rho([\![s,t]\!]) = [\rho(s),\rho(t)] \\ G = \otimes^2\rho^* g \\ B = \otimes^2\rho^*\mathfrak{b}\end{array} \quad \left\{ \begin{array}{c}\big(E,[\![\cdot,\cdot]\!],\rho\big)\\ g\in\Gamma(S^2E^*)\\ \mathfrak{b}\in\Gamma(\Lambda^2E^*)\end{array}\right.$$

Riemannian geometry
+ de Rahm cohomology

Symmetries: gauge transformations

- The geometric construction above is very general:
  anchor of Lie algebroid $\rho : E \to TM$

$$\left.\boxed{\begin{array}{c}(TM, [\cdot,\cdot], id)\\ G \in \Gamma(S^2T^*M)\\ B \in \Gamma(\Lambda^2T^*M)\end{array}}\right\} \quad \begin{array}{c}\rho([\![s,t]\!]) = [\rho(s),\rho(t)]\\ G = \otimes^2\rho^* g\\ B = \otimes^2\rho^*\mathfrak{b}\end{array} \quad \left\{\boxed{\begin{array}{c}(E, [\![\cdot,\cdot]\!], \rho)\\ g \in \Gamma(S^2E^*)\\ \mathfrak{b} \in \Gamma(\Lambda^2E^*)\end{array}}\right.$$

Riemannian geometry
+ de Rahm cohomology

Lie algebroid geometry
+ Lie algebroid cohomology

Symmetries: gauge transformations

- The geometric construction above is very general:
  anchor of Lie algebroid $\rho : E \to TM$

$$\begin{gathered} (TM, [\cdot,\cdot], id) \\ G \in \Gamma(S^2 T^* M) \\ B \in \Gamma(\Lambda^2 T^* M) \end{gathered}$$

$$\begin{gathered} \rho(\llbracket s, t \rrbracket) = [\rho(s), \rho(t)] \\ G = \otimes^2 \rho^* g \\ B = \otimes^2 \rho^* \mathfrak{b} \end{gathered}$$

$$\begin{gathered} (E, \llbracket \cdot,\cdot \rrbracket, \rho) \\ g \in \Gamma(S^2 E^*) \\ \mathfrak{b} \in \Gamma(\Lambda^2 E^*) \end{gathered}$$

Riemannian geometry
+ de Rahm cohomology

$$d\xi(X,Y) = X^a \partial_a \xi(Y) - Y^a \partial_a \xi(X) \\ - [X,Y]$$

Lie algebroid geometry
+ Lie algebroid cohomology

$$d_E \mathfrak{a}(s,t) = s^\alpha D_\alpha \mathfrak{a}(t) - t^\alpha D_\alpha \mathfrak{a}(s) \\ - \llbracket s, t \rrbracket$$

Symmetries: gauge transformations

- The geometric construction above is very general:
  anchor of Lie algebroid $\rho : E \to TM$

$$\boxed{\begin{array}{c} (TM, [\cdot,\cdot], id) \\ G \in \Gamma(S^2 T^* M) \\ B \in \Gamma(\Lambda^2 T^* M) \end{array}} \left. \vphantom{\begin{array}{c} a \\ a \\ a \end{array}} \right\} \quad \begin{array}{c} \rho([\![s,t]\!]) = [\rho(s), \rho(t)] \\ G = \otimes^2 \rho^* g \\ B = \otimes^2 \rho^* \mathfrak{b} \end{array} \quad \left\{ \vphantom{\begin{array}{c} a \\ a \\ a \end{array}} \right. \boxed{\begin{array}{c} (E, [\![\cdot,\cdot]\!], \rho) \\ g \in \Gamma(S^2 E^*) \\ \mathfrak{b} \in \Gamma(\Lambda^2 E^*) \end{array}}$$

Riemannian geometry
+ de Rahm cohomology

$$d\xi(X,Y) = X^a \partial_a \xi(Y) - Y^a \partial_a \xi(X) - [X,Y]$$

Lie algebroid geometry
+ Lie algebroid cohomology

$$d_E \mathfrak{a}(s,t) = s^\alpha D_\alpha \mathfrak{a}(t) - t^\alpha D_\alpha \mathfrak{a}(s) - [\![s,t]\!]$$

differentials are related via:

$$(\otimes^2 \rho^* \, d_E \mathfrak{a})(X,Y) = d(\rho^* \mathfrak{a})(X,Y)$$

Symmetries: gauge transformations

- The geometric construction above is very general: anchor of Lie algebroid $\rho : E \to TM$
  → geometry on $(TM, [\cdot,\cdot], id)$ and on $(E, [\![\cdot,\cdot]\!], \rho)$
  translate via "anchoring"

Symmetries: gauge transformations

- The geometric construction above is very general: anchor of Lie algebroid $\rho : E \to TM$
  → geometry on $(TM, [\cdot,\cdot], id)$ and on $(E, [\![\cdot,\cdot]\!], \rho)$
  translate via "anchoring"

- In particular $\otimes^2 \rho^* (\mathfrak{b} + d_E \mathfrak{a}) = B + d\xi$

Symmetries: gauge transformations

- The geometric construction above is very general: anchor of Lie algebroid $\rho : E \to TM$
  → geometry on $(TM, [\cdot,\cdot], id)$ and on $(E, [\![\cdot,\cdot]\!], \rho)$
  translate via "anchoring"

- In particular $\otimes^2 \rho^* (\mathfrak{b} + d_E \mathfrak{a}) = B + d\xi$

→ the "gauge invariant" analogue of $H = dB$ is

$$\Theta = d_E \mathfrak{b}$$

→ they are related as all other geometric objects:

$$\Theta_{\alpha\beta\gamma} = \rho^a{}_\alpha \rho^b{}_\beta \rho^c{}_\gamma \, H_{abc}$$

Symmetries

- effective actions: talks of Blumenhagen & Plauschinn

$$S^d_{\text{eff}}(G,B) = \frac{1}{2\kappa^2}\int d^dx\sqrt{|G|}e^{-2\phi}\left(R - \tfrac{1}{12}H_{abc}\,H^{abc} + 4\,\partial_a\phi\,\partial^a\phi\right)$$

$$\begin{aligned} G &= \otimes^2\rho^* g \\ B &= \otimes^2\rho^* \mathfrak{b} \end{aligned} \;\Bigg\updownarrow\; \mathcal{T}$$

$$S^d_{\text{eff}}(g,b) = \frac{1}{2\kappa^2}\int d^dx\sqrt{|g|}|\rho^*|e^{-2\phi}\left(\widehat{R} - \tfrac{1}{12}\Theta_{\alpha\beta\gamma}\,\Theta^{\alpha\beta\gamma} + 4\,D_\alpha\phi\,D^\alpha\phi\right)$$

Symmetries

- effective actions:

$$\begin{tikzcd}
S^d_{\text{eff}}(G,B) = \frac{1}{2\kappa^2}\int d^d x\sqrt{|G|}\,e^{-2\phi}\left(R - \tfrac{1}{12}H_{abc}\,H^{abc} + 4\,\partial_a\phi\,\partial^a\phi\right) \arrow[d, leftrightarrow, "\mathcal{T}", "{\begin{array}{l} G = \otimes^2\rho^* g \\ B = \otimes^2\rho^*\mathfrak{b} \end{array}}"'] \\
S^d_{\text{eff}}(g,b) = \frac{1}{2\kappa^2}\int d^d x\sqrt{|g|}\,|\rho^*|\,e^{-2\phi}\left(\widehat{R} - \tfrac{1}{12}\Theta_{\alpha\beta\gamma}\,\Theta^{\alpha\beta\gamma} + 4\,D_\alpha\phi\,D^\alpha\phi\right)
\end{tikzcd}$$

G_{geom} : diffeomorphisms
gauge transformations $B \to B + d\xi$

G'_{geom} : diffeomorphisms
gauge transformations $\mathfrak{b} \to \mathfrak{b} + d_E\mathfrak{a}$

Symmetries

- effective actions:

$H = dB$

$$S^d_{\text{eff}}(G,B) = \frac{1}{2\kappa^2}\int d^dx\sqrt{|G|}\,e^{-2\phi}\left(R - \tfrac{1}{12}\boxed{H}_{abc}\,H^{abc} + 4\,\partial_a\phi\,\partial^a\phi\right)$$

G_{geom} : diffeomorphisms
gauge transformations $B \to B + d\xi$

$$G = \otimes^2\rho^* g \qquad B = \otimes^2\rho^*\mathfrak{b} \qquad \Big\updownarrow\, \mathcal{T}$$

$$S^d_{\text{eff}}(g,b) = \frac{1}{2\kappa^2}\int d^dx\sqrt{|g|}\,|\rho^*|\,e^{-2\phi}\left(\widehat{R} - \tfrac{1}{12}\Theta_{\alpha\beta\gamma}\,\Theta^{\alpha\beta\gamma} + 4\,D_\alpha\phi\,D^\alpha\phi\right)$$

G'_{geom} : diffeomorphisms
gauge transformations $\mathfrak{b} \to \mathfrak{b} + d_E\mathfrak{a}$

Symmetries

- effective actions:

$H = dB$

$$S^d_{\text{eff}}(G,B) = \frac{1}{2\kappa^2}\int d^dx\sqrt{|G|}e^{-2\phi}\left(R - \tfrac{1}{12}\boxed{H}_{abc}\,H^{abc} + 4\,\partial_a\phi\,\partial^a\phi\right)$$

G_{geom} : diffeomorphisms

gauge transformations $B \to B + d\xi$

$$G = \otimes^2\rho^* g \qquad B = \otimes^2\rho^*\mathfrak{b} \qquad \Big\updownarrow\,\mathcal{T}$$

$\Theta = d_E\mathfrak{b}$

$$S^d_{\text{eff}}(g,b) = \frac{1}{2\kappa^2}\int d^dx\sqrt{|g|}|\rho^*|e^{-2\phi}\left(\widehat{R} - \tfrac{1}{12}\boxed{\Theta}_{\alpha\beta\gamma}\,\Theta^{\alpha\beta\gamma} + 4\,D_\alpha\phi\,D^\alpha\phi\right)$$

G'_{geom} : diffeomorphisms

gauge transformations $\mathfrak{b} \to \mathfrak{b} + d_E\mathfrak{a}$

Symmetries

- translation of symmetries:
  - ⟶ diffeomorphisms by construction
  - ⟶ gauge transformations via algebroid-differential

$$\otimes^2\rho^*(\mathfrak{b} + d_E\mathfrak{a}) = B + d\xi$$

Symmetries

- translation of symmetries:
  - ⟶ diffeomorphisms by construction
  - ⟶ gauge transformations via algebroid-differential

$$\otimes^2 \rho^* (\mathfrak{b} + d_E \mathfrak{a}) = B + d\xi$$

- in terms of geometric group

$$G'_{\text{geom}} = \mathcal{T}^{-1}\, G_{\text{geom}}\, \mathcal{T}\,, \quad \mathcal{T} \in O(d,d)$$

Symmetries

- translation of symmetries:
  → diffeomorphisms by construction
  → gauge transformations via algebroid-differential

$$\otimes^2 \rho^* (\mathfrak{b} + d_E \mathfrak{a}) = B + d\xi$$

- in terms of geometric group

$$G'_{\text{geom}} = \mathcal{T}^{-1} \, G_{\text{geom}} \, \mathcal{T} \, , \quad \mathcal{T} \in O(d,d)$$

Field redefinitions mix gauge fields and tensors!
→ geometric group changes
→ Lie algebroids provide geometric interpretation

So what?

Immediate question: **what is this good for?**

So what?

Immediate question: **what is this good for?**

⟶ control about patching of non-geometric backgrounds

⟶ easy representation for complicated backgrounds

Patching of non-geometric backgrounds

so far:

- to every O(d,d)-induced field redefinition there is an action with geometry described by a Lie algebroid

Patching of non-geometric backgrounds

so far:

- to every O(d,d)-induced field redefinition there is an action with geometry described by a Lie algebroid

- for non-geometric transformations (ß-transformations, T-dualities) the actions are different

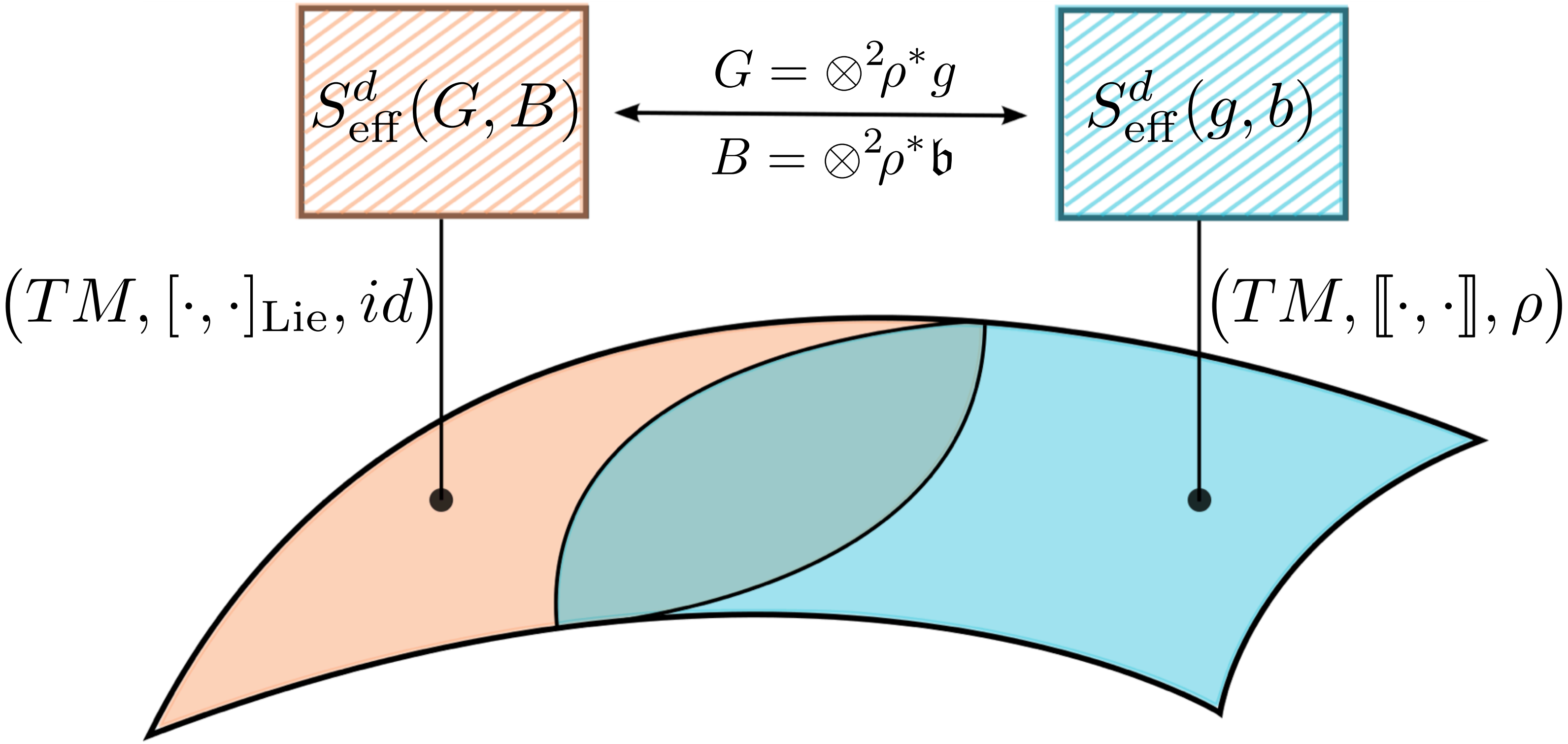
Patching of non-geometric backgrounds

so far:

- to every O(d,d)-induced field redefinition there is an action with geometry described by a Lie algebroid
- for non-geometric transformations (ß-transformations, T-dualities) the actions are different
- non-geometric backgrounds require non-geometric transformations to be patched-up

Patching of non-geometric backgrounds

Resulting in the following picture:



$$S^d_{\text{eff}}(G,B)$$

$$G = \otimes^2\rho^* g$$

$$B = \otimes^2\rho^* \mathfrak{b}$$

$$S^d_{\text{eff}}(g,b)$$

$$(TM, [\cdot,\cdot]_{\text{Lie}}, id)$$

$$(TM, [\![\cdot,\cdot]\!], \rho)$$

Simplify backgrounds

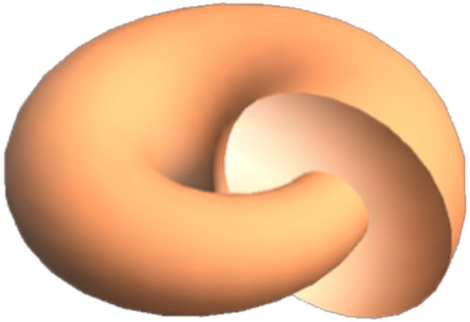
Example: the *Q*-flux – again

$$G = \frac{1}{1+N^2z^2}\left(dx^2+dy^2\right)+dz^2 \qquad B = \frac{Nz}{1+N^2z^2}\,dx\wedge dy$$

Patching of non-geometric backgrounds

Example: the *Q*-flux – again

$$G = \frac{1}{1+N^2z^2}\left(dx^2+dy^2\right)+dz^2 \qquad B = \frac{Nz}{1+N^2z^2}\,dx\wedge dy$$



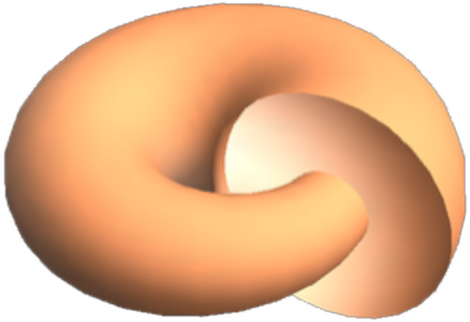
$$z \to z+2\pi \implies \mathcal{H}(G,B) \to T^t\mathcal{H}(G,B)T$$

with $T = \begin{pmatrix} \mathbb{1} & \beta \\ 0 & \mathbb{1} \end{pmatrix}\ ,\ \beta = \begin{pmatrix} 0 & 2\pi N & 0 \\ -2\pi N & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

Patching of non-geometric backgrounds

Example: the *Q*-flux – again

$$G = \frac{1}{1+N^2z^2}\left(dx^2+dy^2\right)+dz^2 \qquad B = \frac{Nz}{1+N^2z^2}\,dx\wedge dy$$



$$z\to z+2\pi \implies \mathcal{H}(G,B)\to T^t\mathcal{H}(G,B)T$$

with T a ß-transfromation $\notin G_{\mathrm{geom}}$

Patching of non-geometric backgrounds

Example: the *Q*-flux – again

$$G = \frac{1}{1+N^2z^2}\left(dx^2+dy^2\right)+dz^2 \qquad B = \frac{Nz}{1+N^2z^2}\,dx\wedge dy$$

G_{geom} : diffeo + gauge trafo ; patching: ß-trafo

Patching of non-geometric backgrounds

Example: the *Q*-flux – again

$$G = \frac{1}{1+N^2z^2}\left(dx^2+dy^2\right)+dz^2 \qquad B = \frac{Nz}{1+N^2z^2}\,dx\wedge dy$$

G_{geom} : diffeo + gauge trafo ; patching: ß-trafo

$$\mathcal{T} = \begin{pmatrix} 0 & (G-BG^{-1}B)^{-1} \\ G-BG^{-1}B & 0 \end{pmatrix}$$

$$g = dx^2+dy^2+dz^2 \qquad \mathfrak{b} = Nz\,dx\wedge dy$$

G_{geom} : diffeo + ß-trafo ; patching: $\mathfrak{b} \to \mathfrak{b} + d\mathfrak{a}$

Patching of non-geometric backgrounds

Example: the *Q*-flux – again

$$G = \frac{1}{1+N^2z^2}\left(dx^2+dy^2\right)+dz^2 \qquad B = \frac{Nz}{1+N^2z^2}\,dx\wedge dy$$

G_{geom} : diffeo + gauge trafo ; patching: ß-trafo

$$\mathcal{T} = \begin{pmatrix} 0 & (G-BG^{-1}B)^{-1} \\ G-BG^{-1}B & 0 \end{pmatrix}$$

$$g = dx^2+dy^2+dz^2 \qquad \mathfrak{b} = Nz\,dx\wedge dy$$

G_{geom} : diffeo + ß-trafo ; patching: $\mathfrak{b} \to \mathfrak{b} + d\mathfrak{a}$

not algebroid gauge trafo!

Conclusion & outlook

- Geometry of Lie algebroids suitable for describing non-geometric backgrounds locally

- Quest for global description:

  → generalized geometry suitable for diffeos + gauge trafos
  or diffeos + ß-trafos

  (Blumenhagen, Deser, Plauschinn, FR: 1205.1522)

  → not both: DFT ?

  (Hull, Hohm, Zwiebach,...)

- Deformation quantization:

  → suitable redefinition introduces (quasi-) Poisson structure

  (Blumenhagen, Deser, Plauschinn, FR: 1211.0030)

Thank you