Generalised Geometry of Supergravity

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Introduction

- Reformulation of supergravity using:

  $O(d,d) \times \mathbb{R}^+$ generalised geometry $\leftrightarrow$ Type II
  $E_{d(d)} \times \mathbb{R}^+$ generalised geometry $\leftrightarrow$ 11D

- Bosonic sector purely geometrical (exactly like GR)

  [Bosonic fields $\leftrightarrow$ "Generalised metric"]

- SUSY and fermion equations naturally included

- Bosonic symmetries unified and hidden symmetries appear

Ordinary Geometry and GR

- Manifold M of dimension d, with tangent bundle TM
- Diffeomorphisms $\quad [\frac{\partial x'}{\partial x} \in GL(d,\mathbb{R}) \Rightarrow GL(d,\mathbb{R})$ bundle]

  $\rightarrow$ generated by vector fields $v \in TM$

  $\rightarrow$ acts by Lie derivative $\mathcal{L}_v = \partial_v - (\partial \otimes v)\cdot$
- Connection

$$\nabla : Q \rightarrow T^*M \otimes Q, \qquad Q \sim GL(d,\mathbb{R}) \text{ tensor bundle}$$

- Torsion

$$T(v) = \mathcal{L}_v^{\nabla} - \mathcal{L}_v$$

Ordinary Geometry and GR

Metric structure

- Gravity field $\leftrightarrow$ metric on TM, $\quad g_{\mu\nu} \in \frac{GL(d,\mathbb{R})}{O(d-1,1)}$
- Metric equivalent to $O(d-1,1)$ structure

  $\leftrightarrow$ Othonormal frames $g(\hat{e}_a, \hat{e}_b) = \eta_{ab}$

Levi-Civita Connection

- $\exists\, \nabla$ torsion-free $O(d-1,1)$ compatible connection
- Note: Levi-Civita connection exists uniquely
- Curvatures $[\nabla, \nabla]$ give action & eqns of motion

Generalised Geometry: The Plan

- Define generalised tangent bundle as generators of bosonic symmetries of SUGRA [Structure group: $O(d,d)$ or $E_{d(d)}$]
- Define analogues of Lie derivative, connections and torsion
- Define reduced structure group using bosonic fields of SUGRA [Structure group: $O(d) \times O(d)$ or H_d]
- Find analogue of Levi-Civita and resulting curvatures

Type II NS-NS Sector

- NS-NS sector bosons $(g_{(\mu\nu)}, B_{[\mu\nu]}, \phi)$
- Symmetries

Symmetry	Generator	Action
Diffeo	$v \in TM$	$\delta_v = \mathcal{L}_v$
Gauge	$\xi \in T^*M$	$\delta B = -\mathrm{d}\xi$

- B only defined locally on patches $U_{(i)} \subset M$

$$B_{(i)} = B_{(j)} - \mathrm{d}\Lambda_{(ij)}$$

- $\Rightarrow \xi$ also locally defined

$$\xi_{(i)} = \xi_{(j)} - i_v \Lambda_{(ij)}$$

Generalised Tangent Space [Hitchin '02 & Gualtieri '04]

- Define E by exact sequence:

$$0 \to T^*M \to E \to TM \to 0$$

- I.e. $V \in E$ locally represented as $v + \xi_{(i)} \in TU_{(i)} \oplus T^*U_{(i)}$

$$\xi_{(i)} = \xi_{(j)} - i_v \Lambda_{(ij)}$$

- Sections of E generate the NS-NS bosonic symmetries

- Split signature norm $\langle V, V \rangle = \xi(v)$

  $\to$ view E as $O(10, 10)$ bundle

Dorfman Derivative

- Lie derivative $\to$ Dorfman derivative for $V \in E$

$$L_V = \partial_V - (\partial \times_{\text{ad}} V) \cdot$$

- Generates diffeo + gauge transformation by $V = v + \xi \in E$

$$L_V \sim \left[\mathcal{L}_v - (\mathrm{d}\xi) \cdot \right]$$

Generalised Connections and Torsion [Alekseev & Xu '01; Gualtieri '07]

- Generalised Connection is linear differential operator

$$D : \mathcal{Q} \to E^* \otimes \mathcal{Q}$$

  for $\mathcal{Q}$ a tensor of $O(10, 10)$

- Torsion is defined (for $U \in E$) by

$$T_U = (L_U^D - L_U)$$

- Find that $T \in \Lambda^3 E \subset E^* \otimes \mathrm{ad}(E)$

$O(10,10) \times \mathbb{R}^+$

- Can also define another bundle $\tilde{E}$ in same way from

$$|\det T^*M| \otimes \left(TM \oplus T^*M\right)$$

- This has structure group $O(10,10) \times \mathbb{R}^+$
- Torsion space enlarged to $E \oplus \Lambda^3 E$

NS-NS fields and $O(9,1) \times O(9,1)$ structure

Generalised Metric Structure

- NS-NS fields $(g_{\mu\nu}, B_{\mu\nu}, \phi)$ parametrise $\frac{O(10,10)\times\mathbb{R}^+}{O(9,1)\times O(9,1)}$
- $E = C_+ \oplus C_-$ becomes an $O(9,1) \times O(9,1)$ structure
- Spin bundles $S(C_\pm)$, (one for each $SO(9,1)$ group)

Generalised Levi-Civita

- Examine $O(9,1) \times O(9,1)$ compatible connections D
- Find that torsion-free D always exists, but not unique

$\rightarrow$ Some cpts fixed to be $H = \mathrm{d}B$ or $\mathrm{d}\phi$

SUSY and fermions

- For $\epsilon^{\pm}, \rho^{\pm} \in S(C_{\pm})$ and $\psi^{\pm} \in C_{\mp} \otimes S(C_{\pm})$:
- Unique operators $\rightarrow$ SUSY:

$$\delta\psi^{+}_{\bar{a}} = D_{\bar{a}}\epsilon^{+} \qquad \delta\rho^{+} = \gamma^{a} D_{a}\epsilon^{+}$$
$$\delta\psi^{-}_{a} = D_{a}\epsilon^{-} \qquad \delta\rho^{-} = \gamma^{\bar{a}} D_{\bar{a}}\epsilon^{-}$$

- Unique operators $\rightarrow$ Fermion equations:

$$\gamma^{b} D_{b}\psi^{+}_{\bar{a}} - D_{\bar{a}}\rho^{+} = 0 \qquad \gamma^{a} D_{a}\rho^{+} - D^{\bar{a}}\psi^{+}_{\bar{a}} = 0$$
$$\gamma^{\bar{b}} D_{\bar{b}}\psi^{-}_{a} - D_{a}\rho^{-} = 0 \qquad \gamma^{\bar{a}} D_{\bar{a}}\rho^{-} - D^{a}\psi^{-}_{a} = 0$$

- Closure of SUSY algebra $\rightarrow$ Curvature (Bosonic e.o.m.)

$$\gamma^{a}(D_{a}D_{\bar{b}} - D_{\bar{b}}D_{a})\epsilon^{+} = \tfrac{1}{2}\mathrm{Ric}_{a\bar{b}}\gamma^{a}\epsilon^{+}$$

Summary

- The Einstein equation is

$$\mathrm{Ric}_{a\bar{b}} = 0$$

- SUSY and fermion equations are unique generalised operators

$$\delta\psi^{+}_{\bar{a}} = D_{\bar{a}}\epsilon^{+}, \qquad \gamma^{b} D_{b}\psi^{+}_{\bar{a}} - D_{\bar{a}}\rho^{+} = 0, \text{ etc.}$$

- Can also define a scalar curvature R s.t.

$$S = \frac{1}{2\kappa^2}\int \Phi R$$

Restrictions of 11D SUGRA

- Warped metric ansatz $(m, n = 1, \ldots, d \leq 7)$

$$\mathrm{d}s_{11}^2 = \mathrm{e}^{2\Delta(x)}\eta_{\mu\nu}\mathrm{d}y^{\mu}\mathrm{d}y^{\nu} + g_{mn}(x)\mathrm{d}x^m\mathrm{d}x^n$$

- Internal fields are $\{g_{mn}, A_{mnp}, \tilde{A}_{m_1\ldots m_6}, \Delta; \psi_m, \rho\}$
- Gauge transformation: $(\Lambda \in \Lambda^2 T^* M, \tilde{\Lambda} \in \Lambda^5 T^* M)$

$$A' = A + \mathrm{d}\Lambda \qquad \tilde{A}' = \tilde{A} + \mathrm{d}\tilde{\Lambda} - \tfrac{1}{2}\mathrm{d}\Lambda \wedge A$$

- Generalised tangent bundle (for $d \leq 7$)

$$E \simeq TM \oplus \Lambda^2 T^* M \oplus \Lambda^5 T^* M \oplus (T^* M \otimes \Lambda^7 T^* M)$$

$$\rightarrow E_{d(d)} \times \mathbb{R}^+ \text{ structure}$$

SUGRA from $\tilde{H}_d$ structure

- Bosonic fields define $\tilde{H}_d$ structure on E (e.g. $\tilde{H}_7 = SU(8)$)
- Fermions $\rightarrow$ sections of $\tilde{H}_d$ bundles $\psi \in J$, $\rho \in S$
- Fermion SUSY variations and equations of motion are

$$\delta\psi = (D \times_J \varepsilon) \qquad (D \times_J \psi) + (D \times_J \rho) = 0$$
$$\delta\rho = (D \times_S \varepsilon) \qquad (D \times_S \psi) + (D \times_S \rho) = 0$$

- Closure of SUSY algebra $\rightarrow$ Curvatures

$$D \times_J (D \times_J \varepsilon) + D \times_J (D \times_S \varepsilon) = R^0 \cdot \varepsilon,$$
$$D \times_S (D \times_J \varepsilon) + D \times_S (D \times_S \varepsilon) = R\, \varepsilon,$$

Compactifications

- Globally defined spinors $\{\varepsilon^i\}$ on M_{int}
  $\rightarrow$ SUSY in effective theory
- E.g. N=1 in 4d $\Leftrightarrow$ $SU(7) \subset SU(8)$ structure on M_7
  N=2 in 4d $\Leftrightarrow$ $SU(6) \subset SU(8)$ structure on M_7
- For identity structure case (maximal SUSY)

  Embedding tensor $\leftrightarrow$ Generalised torsion

  [Aldazabal, Grana, Marqués & Rosabal '13]

Supersymmetric Backgrounds (with fluxes)

Result:

Killing spinor equations $\Leftrightarrow \exists$ torsion-free D s.t. $D_M \varepsilon^i = 0$

- Analogue of special holonomy for D
- SUSY background $=$ Torsion-free generalised G-structure

[Usually see $\nabla\varepsilon \sim F\varepsilon$ so integrability broken by flux]

Related topics and further questions

- Non-geometric backgrounds? [Andriot & Betz '13; Lüst et al.; Blumenhagen et al.]

  (Genuine $O(d,d)$ / $E_{d(d)}$ structures?)

- Higher order corrections? [Hohm, Siegel & Zwiebach '13; Godazger^2 '13]

- Finding new flux backgrounds?

- Superalgebras / SUSY more manifest?

  (Seems like torsion-free $\leftrightarrow$ SUSY)

- (Non-linear) Dual gravity? (Needed for $d \geq 8$)