Towards Blow-up Amplitudes

Toward one-loop corrections for the LVS

Mark D. Goodsell

Ecole Polytechnique, Paris

Work with L. Witkowski ...

Overview

- Some loose ends regarding loops and instantons
- Sequestering and loop corrections in the LVS
- Toward a desequestering calculation for blow-up moduli

E-instantons at one loop

Distant gaugino condensates/Euclidean D-branes (E-instantons) are known to generate a potential for D3/D7 branes.

- [Bauman et al '06, hep-th/0607050] showed this was present and could be understood via a flux potential in 10D.
- The potential can also be understood as the position-dependence of the gauge kinetic function, i.e.

$$W \supset Ae^{-aT} = A(U)e^{-a(f^0+f^1(y))}$$

See also talk by [Martucci]

E-instantons at one loop II

Similarly, they can lead to corrections to perturbatively allowed couplings:

- [Abel, MDG '06] showed this could be the case in the case of E-instantons in IIA:

$$W \supset Y^{ijk}\phi_i\phi_j\phi_k + Y_E^{ijk}\phi_i\phi_j\phi_k e^{-\mathcal{V}_E}$$

- [Marchesano, Martucci '09] provided evidence for this effect in F-theory for gaugino condensates/E-instantons via flux potentials:

$$W \supset A(U)e^{-a(f^0+f^1(y))} \supset \frac{1}{6}A(U)\phi_i\phi_j\phi_k\partial^3_{ijk}e^{-a(f^0+f^1(\phi))}$$

- [Berg, Conlon, Marsh, Witkowski '12] considered gaugino condensate corrections to Yukawa couplings and A-terms in IIB for branes at singularities and explicitly calculated the result.
- It seems obvious that a similar result should be true for E-instantons.
- In principle, we could also have additional couplings that are not from the gauge kinetic function if we have $A(U,\phi)$.
- Restrictions arise from discrete symmetries in IIA $\rightarrow$ (see talk by [Marchesano] and many others) prompts re-exmination.

Instanton calculus

Instanton amplitudes in type II strings have several components (derived by several authors here):

1. Classical action given by volume of E-instanton, $e^{-\mathcal{V}_E}$ from infinite sum of disconnected disk diagrams with boundary on E-brane and no vertex operators.
2. Product/sum over disconnected disk diagrams containing matter fields and two charged zero modes (i.e. stretching from branes to the E-instanton)
3. Infinited product/sum over disconnected annulus/möbius strip diagrams with any number of matter fields/uncharged fermionic zero modes (stretching from E-brane to itself) inserted, excluding zero modes in the loop → one-loop Pfaffian from sum over empty diagrams $e^{-Z'}$.
4. Integrate the whole thing over all fermionic zero modes

Adapted from [Blumenhagen et al, hep-th/0609191]:

$$\langle \prod \Phi_i \rangle_E = \int d^4x\, d^2\theta \prod \sum_a \left(\prod d\lambda_{aE}\right)\left(\prod d\overline{\lambda}_{Ea}\right) e^{-\mathcal{V}_E} e^{-Z'} \\ \times \prod \langle \lambda_{aE} \overline{\lambda}_{bE} \prod \Phi \rangle_{tree} \times \prod \langle \prod \Phi \rangle_{aE,\ loop}$$

Loop integrals

- The role of the disk diagrams and charged zero modes is fairly evident: if the instanton intersects the visible sector then charged couplings arise.
- For sequestering, we are clearly interested in separated instantons $\rightarrow$ no charged zero modes.
- The loop diagrams with no matter field insertions, i.e. the Pfaffian, were shown to be related to gauge threshold corrections, using the identity

$$\left(\frac{\vartheta_\nu(z)\vartheta_1'(0)}{\vartheta_1(z)\vartheta_\nu(0)}\right)^2 = \frac{\vartheta_\nu''(0)}{\vartheta_\nu(0)} - \partial^2 \log \vartheta_1(z)$$

to give

$$\begin{aligned} Z(E2, D_a) =& N_a \int \frac{dt}{t} \sum_\nu \delta_\nu \frac{\vartheta_\nu^2(\frac{it}{4})}{\vartheta_1^2(\frac{it}{4})} \frac{\eta^3(it/2)}{\vartheta_\nu(0)} Z_\nu^{int} \\ =& - N_a \int \frac{dt}{4\pi^2 t} \sum_\nu \delta_\nu \frac{\vartheta_\nu''(0)}{\eta^3(it/2)} Z_\nu^{int} \\ \langle V_A^0(k) V_A^0(-k)\rangle =& - \epsilon_\mu \epsilon_\nu (k^2 \eta^{\mu\nu} - k^\mu k^\nu) N_a \int \frac{dt}{16\pi^4 t} \sum_\nu \delta_\nu \frac{\vartheta_\nu''(0)}{\eta^3(it/2)} Z_\nu^{int} \\ \rightarrow Z(E2, D_a) =& \Delta\left[\frac{8\pi^2}{g^2}\right] \end{aligned}$$

Corrections to bosonic couplings

The above calculation can be almost trivially extended by adding matter fields: for 1PI diagrams should not have factorisation of matter operators, so write

$$\langle \prod \phi \rangle_{loop} \Big|_{k=0} = \langle \prod V^0_\phi (k=0) \rangle_{loop}$$

i.e. we picture-change vertex operators on internal directions. Then since $V^0_\phi(k=0)$ is just the identity on the non-compact directions, we have

$$\langle \prod V^0_\phi (k=0) \rangle_{aE,\ loop} = N_a \int \frac{dt}{t} \sum_\nu \delta_\nu \frac{\vartheta^2_\nu(\frac{it}{4})}{\vartheta^2_1(\frac{it}{4})} \frac{\eta^3(it/2)}{\vartheta_\nu(0)} \langle \prod V^{int}_\phi \rangle_\nu$$

which is just the same as $4\pi^2$ times the amplitude from $\langle A_\mu(k) A_\nu(-k) \prod V^0_\phi(k=0) \rangle_{ab}$ modulo the Lorentz factor.

Corrections to Yukawa couplings

- We have shown that instanton amplitudes are all prefaced by $e^{-\frac{8\pi^2}{g^2}}$ where the gauge couplings may depend on bosonic fields.
- We expect that the same result should hold for actual Yukawa couplings by supersymmetry.
- For this we should compute $\langle \phi\psi\psi\rangle_{aE,\ \mathrm{loop}}$; this is just the calculation of [Abel, MDG '06], but we want to check for IIB.
- It requires inserting bosonic zero modes in the amplitude; in IIA the result was

$$
\begin{aligned}
&\langle V^0_{\phi\, ab} V^{1/2}_{\psi\, bc} V^{1/2}_{\psi\, ca} V_\theta^{-1/2} V_\theta^{-1/2}\rangle_{a,E2} = \phi_{(ab)}\psi_{(bc)\alpha} C^{\alpha\beta}\psi_{(ca)\beta}\ \theta_1\theta_2 \\
&\times \int dt\ it \prod_{i=2}^{3}\int_0^{it} dz_i \prod_{j=1}^{2}\int_{1/2}^{1/2+it} dw_j \langle e^{\frac{\phi}{2}(z_2)} e^{\frac{\phi}{2}(z_3)} e^{-\frac{\phi}{2}(w_1)} e^{-\frac{\phi}{2}(w_2)}\rangle \lim_{x_1\to z_1}\lim_{x_2\to z_2}\lim_{x_3\to z_3} \\
&\times (x_1-z_1)(x_2-z_2)^{1/2}(x_3-z_3)^{1/2}\langle \tilde{S}^1(z_2)\tilde{S}^1(z_3)\tilde{S}^2(w_1)\tilde{S}^2(w_2)\rangle\langle \prod_{i=1}^{3} e^{ik_i\cdot X(z_i)}\rangle \\
&\sum_{\{y_1,y_2,y_3\}=P(x_1,x_2,x_3)} \prod_{\kappa=1}^{3}\sqrt{\frac{2}{\alpha'}}\langle \partial\bar{X}^\kappa(y_\kappa)\sigma_{\phi^\kappa_{ab}}(z_1)\sigma_{\phi^\kappa_{bc}}(z_2)\sigma_{\phi^\kappa_{ca}}(z_3)\rangle \\
&\times\langle e^{iH^\kappa(y_\kappa)} e^{i(\phi^\kappa_{(ab)}-1)H^\kappa(z_1)} e^{i(\phi^\kappa_{(bc)}-1/2)H^\kappa(z_2)} e^{i(\phi^\kappa_{(ca)}-1/2)H^\kappa(z_3)} e^{-\frac{i}{2}H^\kappa(w_1)} e^{-\frac{i}{2}H^\kappa(w_2
\end{aligned}
$$

Yukawas cont'd

- Above is somewhat puzzling, since we now expect a clean result.
- Furthermore, we have now five vertex operators; when we integrate over their positions we naively expect that the integrand should go like $\frac{dt}{t}t^5$ rather than $\frac{dt}{t}t^3$ as we require.
- Let's look at the case of IIB with D3 branes at singularities.
- Then do the integral in both IIA and IIB toroidal models generally.

Corrections to Yukawas due to instantons in IIB

Want to calculate $\langle V^0_{\phi_i} V^{1/2}_{\psi_j} V^{1/2}_{\psi_k} V^{-1/2}_{\theta} V^{-1/2}_{\theta}\rangle_{Ae}$; vertex operators are

$$
\begin{aligned}
V^0_{\phi} &= \partial Z^3 \\
V^{1/2}_{\psi} &= \lambda_\alpha e^{\phi/2} S^\alpha e^{-\frac{i}{2}H_1} e^{-\frac{i}{2}H_2} e^{-\frac{i}{2}H_3} \partial Z^3 \\
V^{-1/2}_{\theta} &= \theta_\alpha e^{-\phi/2} S^\alpha e^{\frac{i}{2}H_1} e^{\frac{i}{2}H_2} e^{\frac{i}{2}H_3}.
\end{aligned}
$$

Two different Lorentz structures:

$$
\begin{aligned}
\langle S^\alpha S^\beta S^\gamma S^\delta \rangle &= A_1 \epsilon^{\alpha\beta}\epsilon^{\gamma\delta} + A_2 \epsilon^{\alpha\gamma}\epsilon^{\beta\delta} \\
A_1 &= \langle S^+ S^- S^+ S^- \rangle \\
A_2 &= \langle S^+ S^+ S^- S^- \rangle
\end{aligned}
$$

- Get $\phi_i A_1 (\psi_j \psi_k)(\theta\theta) + \phi_i A_2 (\psi_j \theta)(\psi_k \theta)$.
- We must add the spin-structure part of the amplitude, as well as the bosonic part $\langle (\partial Z^3)^3 \rangle$ - which has <u>different flavour structure to the Tree-level Yukawas</u>.

Spin structure summation and result

Find

$$A_1 \propto -2\int \frac{dt}{t}\int_0^{it/2} dx_1 dx_2 dy_1 dy_2 \exp[2\pi i(\frac{x_1+x_2-y_1-y_2}{2})]$$
$$\times \frac{\vartheta_1(it/4)\vartheta_1(it/4+x_1+x_2-y_1-y_2)\vartheta_1(x_1-x_2)\vartheta_1(y_1-y_2)}{\vartheta_1(x_1-y_1)\vartheta_1(x_1-y_2)\vartheta_1(x_2-y_1)\vartheta_1(x_2-y_2)}$$
$$A_2 \propto -2\int \frac{dt}{t}\int_0^{it/2} dx_1 dx_2 dy_1 dy_2 \exp[2\pi i(\frac{x_1-x_2+y_1-y_2}{2})]$$
$$\times \frac{\vartheta_1(it/4+y_1-x_2)\vartheta_1(it/4+x_1-y_2)\vartheta_1'(0)^2}{\vartheta_1(y_1-x_2)\vartheta_1(x_1-y_2)\vartheta_1(it/4)^2}$$

Key observation is that both integrands are holomorphic in y_1, y_2, periodic in $y_i \to y_i + it/2$ and antiperiodic in $y \to y+1$. Then

$$\left[\int_{1/2}^{1/2+it/2} dy_1 + \int_{1/2+it/2}^{-1/2+it/2} dy_1 + \int_{-1/2+it/2}^{-1/2} dy_1 + \int_{-1/2}^{1/2} dy_1\right] f = 2\int_{1/2}^{1/2+it/2} dy_1 f$$

and we can evaluate both by just finding the poles! We obtain

$$A_1 = 0$$
$$A_2 = -\pi^2 \int \frac{dt}{t} t^3 \int_0^1 \langle (\partial Z)^3 \rangle$$

i.e. we find the same result as for the bosonic case exactly and the Yukawa couplings become Kähler-modulus dependent.

Uncharged zero modes

Can we do this more generally?

- Recall that uncharged fermionic vertex operators are just the supercharges for the broken supersymmetries:

$$V_\theta^{-1/2}(y) = e^{-\phi/2(y)}\theta_\alpha(y)S^\alpha(y)\Sigma^{int}(y)$$
$$V_\theta^{-1/2}(y)V_\psi^{-1/2}(z) \sim (y-z)^{-1}V_\phi^{-1}$$

- These fields are located on opposite boundary to matter fields; so the application of the OPE was not immediately obvious
- Amplitude for one complex dimension is

$$\langle e^{i a_\kappa^\theta H_\kappa(y)} \prod_{i\neq\theta} e^{i a_\kappa^i H_\kappa(z_i)}\rangle_\nu = e^{2\pi i h_\kappa[a_\kappa^\theta y + \sum_{j\neq\theta} a_\kappa^j z_j]}\vartheta_\nu(h_\kappa it/2 + g_\kappa + a_\kappa^\theta y + \sum_{i\neq\theta} a_\kappa^i z_i)$$
$$\times \prod_{i\neq\theta}\vartheta_1(y-z_i)^{a_\kappa^\theta a_\kappa^i}\prod_{i<j\neq\theta}\vartheta_1(z_i-z_j)^{a_\kappa^i a_\kappa^i}.$$

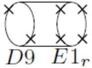
- When we construct an amplitude containing these, we must sum over the spin structures ν.
- When we translate $y \to y+1, y \to y + it/2$ we rotate the spin-structures into each other: for a brane-brane amplitude the spin-structure-dependent part picks up a minus-sign.
- For the brane-E-brane amplitude it is instead invariant (due to the two complex ND directions).
- The spin-structure independent part from $\prod_{i\neq\theta} \vartheta_1(y-z_i)^{a_\kappa^\theta a_\kappa^i}$ picks up a phase -1 due to locality ...
- So the amplitude is always antiperiodic and we always have the result that

$$\langle V_\theta \prod V_\psi \prod V_\phi \rangle = \langle \sum V\Big(\{Q, \psi_i\}\Big) \prod_{j\neq i} V_{\psi_j} \prod V_\phi \rangle$$

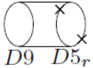
Consequences

- Result relevant for [Berasaluce-Gonzalez et al, 1206.2383], [Marchesano et al, 1306.1284].
- A similar result is now true for additional fermionic zero modes. In the toroidal orbifold CFT these just come from extra broken supersymmetries.
- Hence if we have an annulus diagram containing extra fermionic zero modes and the boundary conditions of the annulus preserve the additional supersymmetries (i.e. for D3-D7 systems this only applies to the extra SUSYs in the DD direction) then we can treat them as SUSY generators.
- Furthermore, we have proved via the CFT calculus the several assertions in [Blumenhagen, Schmidt-Sommerfeld '08] of the form

$$\Re\left[\int d^2\theta_r d^2\mu_r\right.$$



$$\left.\vphantom{\int}\right] = \frac{\partial^2}{\partial m^2}$$



$$\Big|_{m=m_0}$$

Sequestering and the LVS

- The string scale depends on the compactification volume as $m_s \sim \frac{M_P}{\sqrt{\mathcal{V}}}$
- In the LARGE volume scenario, gravitino mass is given by $m_{3/2} \sim \frac{|W_0|}{\mathcal{V}} M_P$
- If we want a GUT theory, we need a large string scale $\sim 10^{14\div16}$ GeV and thus $\mathcal{V} \sim 10^{4\div8}$, meaning $m_{3/2} \sim 10^{10\div14}$ GeV.
- Original LVS with the SM on a geometric cycle supporting an instanton or gaugino condensate lead to $M_{\text{SOFT}} \sim \frac{m_{3/2}}{\log \mathcal{V}} \rightarrow$ soft masses only as low as 10^9 GeV.

Hence we either

1. Consider intermediate string scales and LHC-accessible SUSY.
2. Have a high string and intermediate SUSY-breaking scale (nb the gauginos would not be light $\rightarrow$ not unified).
3. A GUT and a SUSY solution to the hierarchy problem, if we can suppress soft masses to be $M_P/\mathcal{V}^{3/2}$ or $M_P/\mathcal{V}^2 \rightarrow$ sequestering.

Kähler corrections and Yukawa couplings

Following [Cicoli's] talk:

- Since physical Yukawa couplings on magnetised branes, at least to leading order, should not depend on the overall volume, and $Y^{phys}_{ijk} = \frac{e^{K/2} Y^{hol}_{ijk}}{\sqrt{K_{ii} K_{jj} K_{kk}}}$, $K = -2 \log \mathcal{V}$ at leading order, so matter Kähler metrics at leading order are $\sim \mathcal{V}^{-2/3}$.
- Tree-level Kähler potential has known α' corrections

$$-2 \log(\mathcal{V} + \xi/2) = -2 \log \mathcal{V} - \frac{\xi}{\mathcal{V}} + \ldots$$

- Hence tree-level Kähler metric should have the form [Blumenhagen et al, 0906.3297]

$$K_{ii} = \frac{k}{\mathcal{V}^{2/3}} (1 - \delta \frac{Re(S)^{3/2}}{\mathcal{V}} + \ldots)$$

- Equivalently, if Kähler metric is $e^{K/3}$ the Yukawa couplings would be invariant.
- Furthermore, since soft masses are $m^2_{\tilde{q}_i} = m^2_{3/2} + V_0 - F^I \overline{F}^J \partial_I \overline{\partial}_J \log K_{ii}$ the above form leads to vanishing soft masses if $V_0 = 0$ (F-term uplifting).
- This cancels the $(\alpha')^3$ correction to soft to soft terms from $e^{K/2}$ for appropriate δ.
- $\rightarrow$ Kähler modulus dependence of physical Yukawa couplings implies corrections scalar masses.
- We currently do not know at what order this occurs $\rightarrow$ we do not know what the soft masses are in the LVS!

Known knowns and known unknowns

- (We think we) Know the Kähler potential to one-loop:

$$K_0 = -2\log(\mathcal{V} + \xi/2 + c\tau_i \cap \tau_j) + k_a^{0,2}\frac{\tau_a^2}{\mathcal{V}} + \sum \frac{\mathcal{E}(U,\overline{U})}{\mathrm{Re}(S)\tau_i} + \sum \frac{\tilde{\mathcal{E}}(U,\overline{U})}{\tau_i\tau_j} + \ldots$$
$$+ \sum_{n=1,m=1}^{\infty} g_s^m \frac{k_a^{m,n}}{\mathcal{V}}\tau_a^n$$

- We know the Kähler metric at tree-level

$$K_{ii} = \frac{k}{\mathcal{V}^{2/3}}\left[1 - \delta\frac{\mathrm{Re}(S)^{3/2}}{\mathcal{V}} + \delta^{(1)}\frac{1}{\mathrm{Re}(S)}\left(\frac{\mathrm{Re}(S)}{\mathcal{V}^{2/3}}\right)^{n/2} + \sum_{n,m} g_s^m \epsilon^{m,n}\tau_s^n + \ldots\right]$$

- May also be non-perturbative corrections to superpotential and A-terms which can lead to desequestering [Berg,Conlon,Marsh,Witkowski '12].

Unknowns

See talks by [Pedro, Aparicio, Cicoli,Krippendorf,Quevedo]

- Soft scalar masses schematically, $V_0 \sim \frac{m_{3/2}^2}{\tau_s g_s^{3/2} \mathcal{V}}$ without uplifting:

$$m_{\tilde{q}}^2 = \frac{2}{3} V_0 + c \frac{m_{3/2}^2}{g_s^{3/2} \mathcal{V}} (\delta - \xi/3)$$

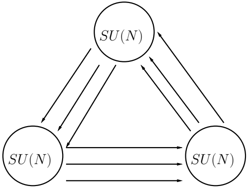
- Gaugino masses depending on anomaly-mediation contribution, uplifting and thus matter Kähler metrics:

$$M_\lambda = \frac{M_P}{\mathcal{V}^{3/2}} \div \frac{M_P}{\mathcal{V}^2}$$

- Tree-level Kähler metrics: [Conlon, Witkowski '11] showed no dependence of the physical Yukawas on blow-up moduli, $\delta = \xi/3$, $\epsilon^{(1)} = 3k_a^{0,2}/\mathcal{V}$ etc (or more generally $e^{\sum k_a^{0,m} \tau_a^m / \mathcal{V}} (1 + \sum_{n,m} g_s^m \epsilon^{m,n} \tau_s^n k / \mathcal{V}^{2/3})^{-3/2} = 1$).
- [Lawrence, Sever '07] attempted to calculate the disk correction $k_a^{1,2} \rightarrow$ non-zero and finite as $\mathcal{V} \rightarrow \infty$, so can absorb into $k_a^{0,2}$.
- For ultra-local models (e.g. locally $\mathbb{C}/\mathbb{Z}_3$ orbifold) $\delta^{(1)} = 0$, no BHK corrections.
- The actual values of the soft masses in the sequestered scenario are therefore not known. It is necessary to calculate then the additional pieces. Hence we need to work "toward blow-up amplitudes" at one-loop.

Toroidal orbifolds as prototypes

- Orbifolds $\mathbb{T}^2 \times \mathbb{T}^2 \times \mathbb{T}^2/\mathbb{Z}_N$ are useful as prototypes for real compactifications: they contain bulk moduli (up to three Kähler moduli) and blow-ups with isolated singularities.
- Matter fields ϕ_i, ψ_i are projected out of $N = 4$ adjoints: have Chan-Paton matrices λ_i, $i = 1, 2, 3$ for each torus and projection $\lambda = e^{2\pi i b_r/N}\gamma_\theta \lambda \gamma_\theta^{-1}$ for $\sum_r b_r = 0$.
- These are particularly simple to calculate with as we have already seen ...



Corrections to Yukawa couplings

- Want to calculate corrections to Kähler potential and metric involving blow-up modes.
- Attempt to go straight for the goal: calculate Yukawa couplings to see whether there are modulus-dependent corrections

Recall that for ultra-local models, e.g. $\mathbb{Z}_3$ orbifolds, have no corrections at one loop for several reasons:

1. In twisted sector, the only amplitude possible is connects the same stack - so cannot feel distant branes
2. Furthermore, in twisted sector, amplitude vanishes due to tadpole cancellation:

$$\langle \phi_i \psi_j \psi_k \rangle \propto \mathrm{tr}_L(\gamma_\theta \lambda_i \lambda_j \lambda_k) \mathrm{tr}_R(\gamma_\theta) = 0$$

3. In untwisted sector, amplitude vanishes due to effective $N = 4$ supersymmetric states running in the loop: amplitude is proportional to a sum over spin structures

$$\sum_\nu \delta_\nu \prod_{i=1}^{4} \vartheta_\nu (\sum_{j=1}^{3} a_i^j z_j) = 0 \text{ if } \sum_i a_i^j \ \forall j$$

Amplitudes with blow-up moduli

- Want to calculate

$$\langle \phi_i \psi_j \psi_k \prod_{m=1}^{L} \tau_{\theta_m} \rangle$$

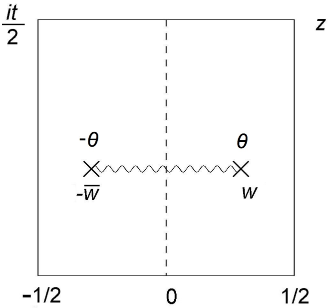
- The vertex operator for blow-up modes τ_{θ_m} contains bosonic twist fields which greatly complicate the analysis:

$$V_{\sigma_\theta}^{-1,-1}(w,\overline{w}) = e^{-\phi} e^{-\tilde{\phi}} e^{ik\cdot X} \prod_{\kappa=1}^{3} e^{i\theta_\kappa H(w)} e^{-i\theta_\kappa \tilde{H}\overline{w}} \sigma_{\theta_\kappa}(w,\overline{w})$$

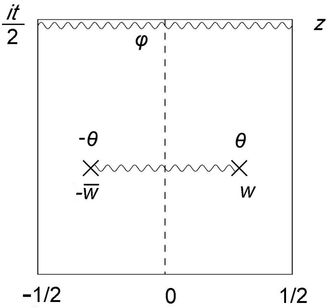
- We are interested in the case where the blow-ups are associated with <u>distant</u> fixed-points under the orbifold group.
- The bosonic twists introduce branch cuts on the worldsheet which are accompanied by chan-paton rotations γ_θ. We must also consider diagrams in twisted sectors which introduce a further twist. Tadpole cancellation then eliminates many diagrams.

Twisted worldsheets

Consider e.g. $\mathbb{Z}_3$, where $\mathrm{tr}(\gamma_\theta) = \mathrm{tr}(\gamma_\theta^2) = 0 = \sum_{\mathrm{perms}(\gamma)} \mathrm{tr}(\gamma_\theta \lambda_1 \lambda_2 \lambda_3)$. Insert matter fields at $\mathrm{Re}(z) = -0.5$:



Vanishes through $\mathrm{tr}_{\mathrm{R}}(\gamma_\theta) = 0$



Vanishes when $\mathrm{tr}_{\mathrm{R}}(\gamma_\theta \gamma_\varphi) = 0$ and always through $\sum_{\mathrm{perms}(\gamma)} \mathrm{tr}_{\mathrm{L}}(\gamma_\varphi \lambda_1 \lambda_2 \lambda_3) = 0$

So we have no twisted-sector amplitudes that can contribute to the Yukawa coupling, and there is no contribution to the matter Kähler metric linear in τ_θ.

Untwisted sectors

- Recall that for the Yukawa couplings it was the Riemann summation part of the amplitude that caused the amplitude to vanish.
- Let us calculate an untwisted amplitude with a twist and antitwist: we need

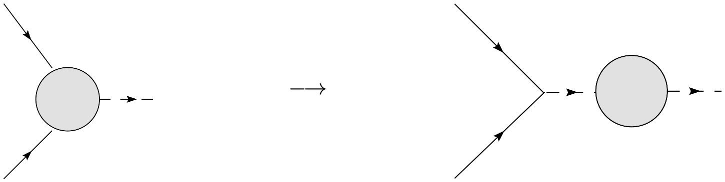
$$\langle V_{\phi}^{0} V_{\psi}^{-1/2} V_{\psi}^{1/2} V_{\tau_{\theta}}^{0,0} V_{\tau_{1-\theta}}^{0,0} \rangle$$

- Note that we insert six Picture-Changing Operators (PCOs). These must be in pairs for each complex dimension.
- We want to extract momentum-independent piece; thus we either need a contribution from amplitude with PCOs inserted only on internal directions, or need momentum poles (PCOs inserted on non-compact dimensions lead to $k \cdot \psi$ operators).
- We can evaluate the spin-structure sum for the all-internal case and, just as for normal Yukawa couplings, find zero.

$5 \to 4$

- This is not a surprise! Actually it implies that the only corrections to the Yukawa couplings come from factorising down to bosonic propagators (expression of non-renormalisation theorem):

$$\langle V_{\phi}^{0} V_{\psi}^{-1/2} V_{\psi}^{1/2} V_{\tau_\theta}^{0,0} V_{\tau_{1-\theta}}^{0,0} \rangle \to \langle V_{\phi}^{0} V_{\overline{\phi}}^{0} V_{\tau_\theta}^{0,0} V_{\tau_{1-\theta}}^{0,0} \rangle$$



- So in the end we are calculating matter Kähler metrics anyway.
- In fact, it is also important to compute the correction to the blow-up Kähler metric:
- For this we require simply

$$\langle V_{\tau_\theta}^{0,0} V_{\tau_{1-\theta}}^{0,0} \rangle$$

  with two internal PCO insertions
- Evaluating the Riemann summation for both of these we <u>do not find that they vanish</u>.

Volume dependence

- Now want to find the volume dependence of the amplitude and ensure that it does not decrease exponentially with the volume.
- Amplitude itself is complicated; volume dependence comes from bosonic correlators.
- Two pairs of PCOs are now inserted internally:

$$\mathcal{A} \propto \langle \sigma_{\theta_i} \sigma_{1-\theta_i} \rangle \langle \partial_n X \partial_n \overline{X} \sigma_{\theta_j^1} \sigma_{1-\theta_j} \rangle \langle \partial_n X \partial_n \overline{X} \sigma_{\theta_k} \sigma_{1-\theta_k} \rangle$$

- Volume factors come from classical solutions:

$$\langle \partial_n X \partial_n \overline{X} \sigma_\theta(w_1, \overline{w}_1) \sigma_{1-\theta}(w_2, \overline{w}_2) \rangle =$$

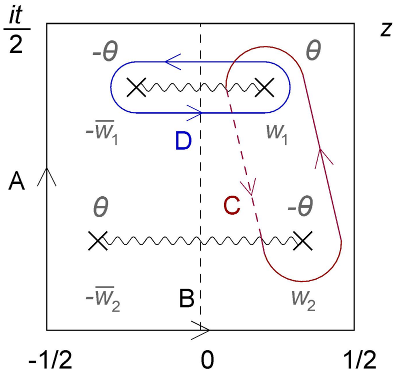
$$\left(\partial_n X_{cl} \partial_n \overline{X}_{cl} f_{qu}(w_i, \overline{w}_i) + \langle \partial_n X \partial_n \overline{X} \sigma_\theta(w_1, \overline{w}_1) \sigma_{1-\theta}(w_2, \overline{w}_2) \rangle_{qu} \right) e^{-S_{cl}}$$

- To find the classical solutions we must construct a basis of cut differentials for the ∂X_{cl} obeying the boundary conditions

$$\oint_\gamma dz \partial X + \oint_\gamma d\overline{z} \overline{\partial} X = \nu_\gamma$$

- ν_γ are given by cosets of the orbifold subject to the condition that the ends of the string attach to separated branes.
- Problem becomes that of finding a set of basis cycles on worldsheet and their associated shifts $\rightarrow$ canonical dissection.

Canonical dissection



$$\partial X(z) \equiv \begin{cases} \partial Z(z) & \mathrm{Re}(z) > 0 \\ \bar\partial Z(-\bar z) & \mathrm{Re}(z) < 0 \end{cases}$$

$$\partial Z(z) = \nu_a (W^{-1})^a_{i'} \omega^{i'}_{1-\theta}(z)$$

$$\bar\partial Z(z) = \nu_a (W^{-1})^a_{i''} \overline{\omega}^{i''}_{\theta}(\bar z)$$

$$\overline{\partial Z}(\bar z) = \Big(\partial Z(z)\Big)^*$$

$$S = \frac{1}{4\pi\alpha'} \int d^2 z \partial X \partial \overline{X} + \bar\partial X \overline{\partial X}$$

$$= \frac{i}{4\pi\alpha'} S_{ab} \nu_a \overline{\nu}_b$$

From this, we construct S_{ab} in terms of integrals of the ω around the above cycles.
Action is substantially simpler than the case for a torus.

Expectations and conjectures

- Expect that, since we are looking at a sequestered situation, that the dominant contribution comes from closed string KK modes
- Go to closed-string channel, where $t \to 0$ and annulus becomes a long thin cylinder.
- Dominant contributions from B-cycle only (also $\nu_A = 0$) to go as $\int_{\gamma_B} \omega \sim 1/t$
- Expect other sums to just give constant and exponentially suppressed contribution
- Expect $\partial X_{cl} \propto \nu_B = 2\pi\sqrt{\frac{T_2}{U_2}}(n + mU + y)$
- For matter Kähler metric, amplitude should be $\sim (\partial X_{cl})^4 e^{-S}$; when we go to closed string channel and Poisson-resum we should then find

$$\mathcal{A} \sim \int \frac{dt}{t^3} t^4 \sum_{n_i, m_i} \frac{1}{R^{10} t^5} \prod_i e^{-\frac{c}{R^2 t}|n_i + m_i U_i|^2 + 2\pi i m d y)}$$
$$\sim \frac{1}{R^4}$$

- Would lead to $\delta K_{ii} \sim K^0_{ii}/\tau_b$.
- This corresponds to the field-theory expectation of exchange of a KK mode in six dimensions.

Conclusions

- Have resolved the role that annulus diagrams play in instanton calculus
- Have developed some substantial technology for calculating amplitudes with twist fields on annulus diagrams
- We have some evidence for the corrections to the Kähler metric corrections involving blow-up modes at one-loop (even if we are some way from claiming a bottle of champagne)
- $\rightarrow$ still going towards blow-up amplitudes ...

Mahlzeit!