Electrically gauged $N = 4$ supergravities with $N = 2$ vacua

Christoph Horst

II. Institute of Theoretical Physics,
University of Hamburg

in collaboration with Jan Louis and Paul Smyth
based on JHEP 1303 (2013) 144

String Pheno, Hamburg
July 17, 2013

Introduction

Spontaneously breaking supersymmetry:

- construct $N = 4$ supergravities in $D = 4$ with $N = 2$ vacuum
  $\rightsquigarrow$ super-Higgs mechanisms with partial supersymmetry breaking:

$$\boxed{N = 4 \xrightarrow{\text{spontaneously}} N = 2}$$

- early examples known [de Roo & Wagemans '86, Wagemans '88],
  work towards classification of such theories
- check consistency of super-Higgs mechanism and compute the effective $N = 2$ action below the scale of partial SUSY breaking
  ($\rightsquigarrow$ geometry of the scalar manifold)

Why $N = 4$ supergravity?

- non-trivial matter sector (in contrast to $N = 8$):

  $$\boxed{\text{1 gravity multiplet, } n \in \mathbb{N} \text{ vector multiplets}}$$

- very restrictive couplings

  Bergshoeff, Koh, Sezgin '85; de Roo '85; Schön, Weidner '06

- $N = 4$ gauged supergravities can arise from flux compactifications of string theory

  Schön '06; Kashani-Poor, Minasian, Triendl '13

- also motivated by analysis of $N = 2 \rightarrow N = 1$

  Cortés, Louis, Smyth, Triendl '10 & '13

see also talks by Dall'Agata and Kashani-Poor

Representation theory of $N = 4$ in Minkowski space

$$\{Q^A, \bar{Q}_B\} \sim \delta^A{}_B \Gamma^\mu P_\mu \,, \text{etc.} \qquad A, B = 1, \ldots, N = 4$$

- massless $N = 4$ gravity multiplet:

1 graviton	$g_{\mu\nu}$	
4 gravitini	ψ^i_μ	$(i = 1, \ldots, 4) \rightsquigarrow SU(4)$
6 vectors	$A^{\mu m}$	$(m = 1, \ldots, 6) \rightsquigarrow SO(6)$
4 hel-1/2 fermions	χ^i	
2 real scalars		

- n copies of massless $N = 4$ vector multiplets:

n vectors	$A^{\mu a}$	$(a = 1, \ldots, n) \rightsquigarrow SO(n)$
4n hel-1/2 fermions	λ^{ai}	
6n real scalars		

The scalars described by a non-linear σ-model on the homogeneous space:

$$\underbrace{SL(2)/SO(2)}_{\text{dim } 2} \times \underbrace{SO(6,n)/SO(6) \times SO(n)}_{\text{dim } 6n}$$

Combine all vectors to $A^{\mu M} = (A^{\mu m}, A^{\mu a})$ where M is an $SO(6, n)$ index.

Electrically gauged $N = 4$

Lagrangian:

$$\mathcal{L} = \mathcal{L}(\text{fields}, f_{MNP}) = \dots$$

- deformation/gauging parameters $\quad f_{MNP} = f_{[MNP]} \in \mathbb{R}$
- quadratic consistency constraints $\quad \boxed{f_{R[MN}\, f_{PQ]}{}^{R} = 0}$

All theories classified by $SO(6, n)$-tensor equations

- however, for large $n \gg 1$

$$\# \text{ quadratic equations} \sim n^4$$

$$\# \text{ variables} \sim n^3$$

$$\# \text{ summands in equation} \sim n$$

$\rightsquigarrow$ numerical solutions only for small n

Why $N = 2$ vacua?

- important to understand vacuum structure. Ultimately, in a quantum theory many diverse vacua may influence the physics (via instanton effects).

  $\rightsquigarrow$ in addition to $N = 0$ or 1 consider also "exotic" vacua with $N > 1$

$N = 0$	not automatically stable
$N = 1$	another possible choice, but harder
$N = 2$	our choice
$N = 3$	harder to construct
$N = 4$	no super-Higgs mechanism

- consider only maximally-symmetric vacua!

Local supersymmetry & Killing spinor equations:

- parametrized by local spinors $\epsilon^i(x)$ $(i = 1, \ldots, 4)$
- here: focus on supersymmetry transformations of maximally symmetric background with vanishing fermions $F = 0$ (B = boson)

$$\delta_\epsilon B \sim \epsilon F = 0$$

$$\delta_\epsilon F \sim \begin{cases} D\epsilon + \epsilon B & \text{for } F = \text{gravitino} \\ \epsilon B & \text{for } F \neq \text{gravitino} \end{cases}$$

$\rightsquigarrow$ sufficient to consider variations of fermions:

$$\delta_\epsilon \psi^i_\mu = D_\mu \epsilon^i + A_1^{ij} \bar{\sigma}_\mu \epsilon \, (\epsilon^j)^*$$

$$\delta_\epsilon \chi^i = A_2^{ji} \epsilon \, (\epsilon^j)^*$$

$$\delta_\epsilon {\lambda_a}^i = A^i_{2aj} \, \epsilon^j$$

$\rightsquigarrow$ Killing spinor equations

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$$0 \overset{?}{=} \delta_\epsilon {\lambda_a}^i = A^i_{2aj} \, \epsilon^j$$

$\rightsquigarrow$ Killing spinor equations

Fermion shift-matrices

- here,

$$A_1^{ij} = (\mathcal{V}_-)^* \, \mathcal{V}^M{}_{[kl]} \, \mathcal{V}_N{}^{[ik]} \, \mathcal{V}_P{}^{[jl]} \, f_M{}^{NP}$$
$$A_2^{ij} = \mathcal{V}_- \, \mathcal{V}^M{}_{[kl]} \, \mathcal{V}_N{}^{[ik]} \, \mathcal{V}_P{}^{[jl]} \, f_M{}^{NP}$$
$$A_{2ai}{}^j = \mathcal{V}_- \, \mathcal{V}^M{}_a \, \mathcal{V}^N{}_{[ik]} \, \mathcal{V}_P{}^{[jk]} \, f_{MN}{}^P$$

  expressed in terms of scalar "vielbeins" for $SL(2)$ and $SO(6, n)$

- $\delta_\epsilon \psi^i_\mu = 0$ yields integrability condition

  $\rightsquigarrow$ $N \geq 1$ vacuum $\Rightarrow$ no de Sitter-vacuum
  $\rightsquigarrow$ (A_1^{ij}) has eigenvalue zero for each unbroken supersymmetry direction i

Quadratic constraints for $N = 2$ vacua

- need $n \geq 1$
- hard to solve in full generality for $n > 7$
- for $n > 7$ set $f_{a26} \stackrel{!}{=} 0$ by hand
  $\rightsquigarrow$ many consistent solutions can be constructed for any $n \in \mathbb{N}$
- a large subset of equations for any n fully solved thanks to Lie's theorem in representation theory of solvable Lie algebras

Physical implications:

$\rightsquigarrow$ 2 massless gravitini,
2 massive gravitini of degenerate mass $m_{3/2} > 0$

$\rightsquigarrow$ no $N = 3$ vacua and only Minkowski vacua (electric gaugings)

$\rightsquigarrow$ stability

$\rightsquigarrow$ super-Higgs mechanism
(massive gravitini "eat" Goldstini, $N = 2$ mass degeneracies of superpartners, etc.)

$\rightsquigarrow$ ...

Aspects of $N = 2$ low-energy effective theory

Mass terms:

- gravity/Goldstini sector:

$N = 2$ multiplets	mass squared
gravity	0
BPS gravitino	$m_{3/2}^2$
$2\times$ vector	0

- matter sector:

block	$N = 2$ multiplets	mass squared
$G_1^{(ij)} = G_4^{(ij)} = 0 \cdot \mathbb{1}_l$	$(l)\times$ massless vector $(l)\times$ BPS hyper	0 $m_{3/2}^2$
$(G_1^{(ij)})^2 = -x^2\, \mathbb{1}_{2l'}$, $(G_4^{(ij)})^2 = -y^2\, \mathbb{1}_{2l'}$ with $x \neq 0$ or $y \neq 0$	$(2l')\times$ BPS vector $(l')\times$ BPS hyper $(l')\times$ (BPS) hyper	$(x^2 + y^2)$ $x^2 + (m_{3/2} + \|y\|)^2$ $x^2 + (m_{3/2} - \|y\|)^2$

all fields fit into complete $N = 2$ multiplets!
massive $N = 2$ multiplets are BPS!

Aspects of $N = 2$ low-energy effective theory

Unbroken gauge group:

- leaves background invariant
- possible to construct compact, reductive Lie group

$$U(1)^3 \times G_{\text{vac}}$$

$N = 2$ **effective theory:**

- below scale $m_{3/2}$ integrate out massive gravitini and their superpartners, etc.
- integrating out massive vectors affect scalar geometry since would-be Goldstone bosons are eliminated
- $N = 2$ scalar manifold expected to be

$$M_{\text{special Kähler}} \times M_{\text{quaternionic Kähler}}$$

- only possibility of special Kähler manifold with factor $SL(2)/SO(2)$ is
  Ferrara, van Proeyen '89

$$SL(2)/SO(2) \times SO(2,k)/SO(2)\times SO(k)$$

Conclusion & outlook

Conclusion:

- studied gauged $N = 4$ supergravities with $N = 2$ vacua
- classification of vacua amounts to solving a system of algebraic, quadratic equations
- while unable to fully solve constraints, constructed many electrically gauged $N = 4$ with $N = 2$ vacuum
- checked consistency of super-Higgs mechanism

Outlook:

- interesting to also find solutions in magnetically gauged $N = 4$, but more difficult constraints!
- construct vacua with $N = 4, 3, 1$ or $N = 0$ (e.g. with de Sitter-background)
- in principle, methods transferable to $N = 8$ supergravity with scalar manifold

  $$E_{7,7}/(SU(8)/\mathbb{Z}_2)$$

  of dimension 70
  $\rightsquigarrow$ no free parameter $n \in \mathbb{N}$ but exceptional Lie group symmetry!