

Quantum Corrected Effective Action of String and F-theory

Gary Shiu

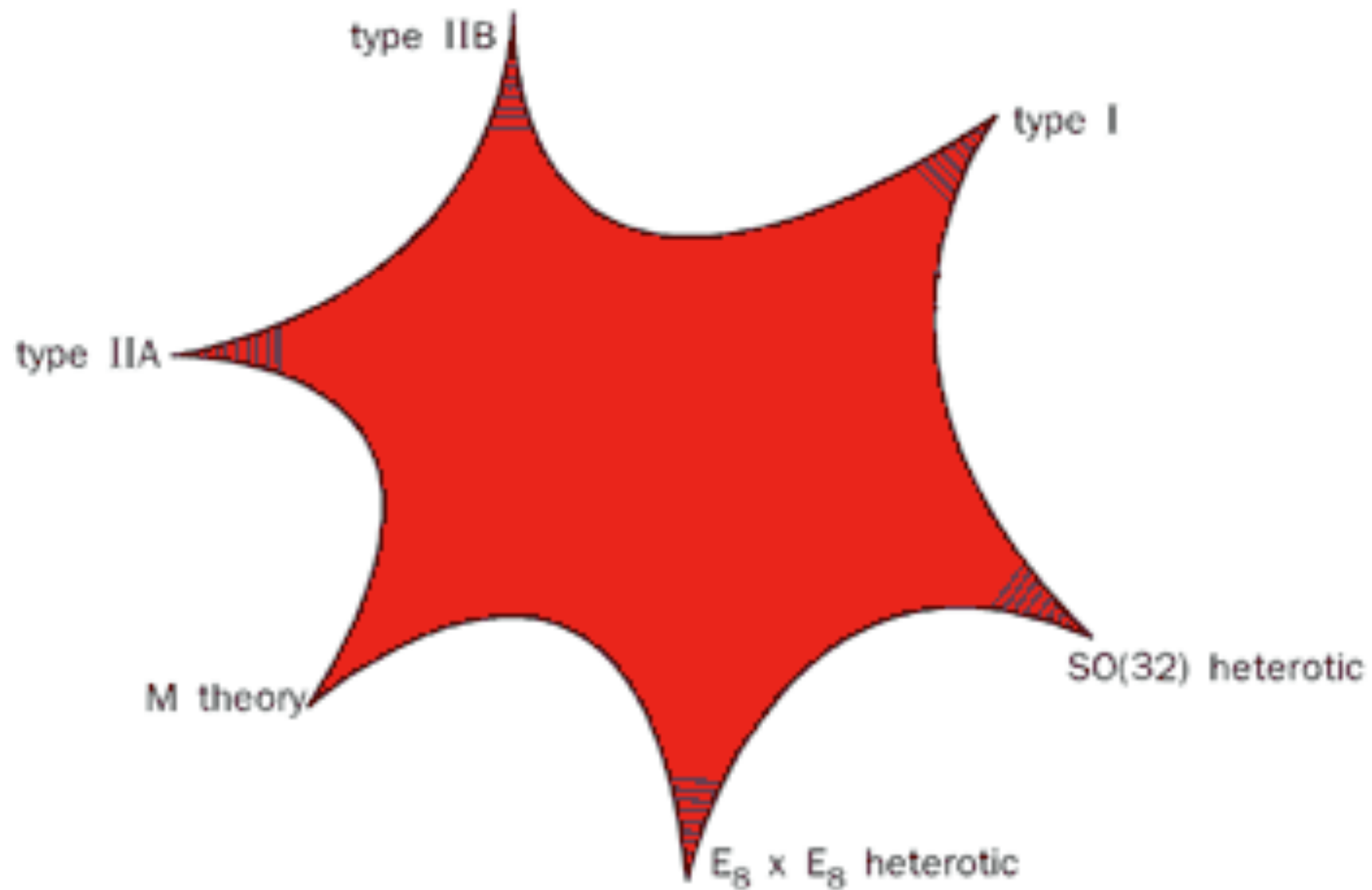


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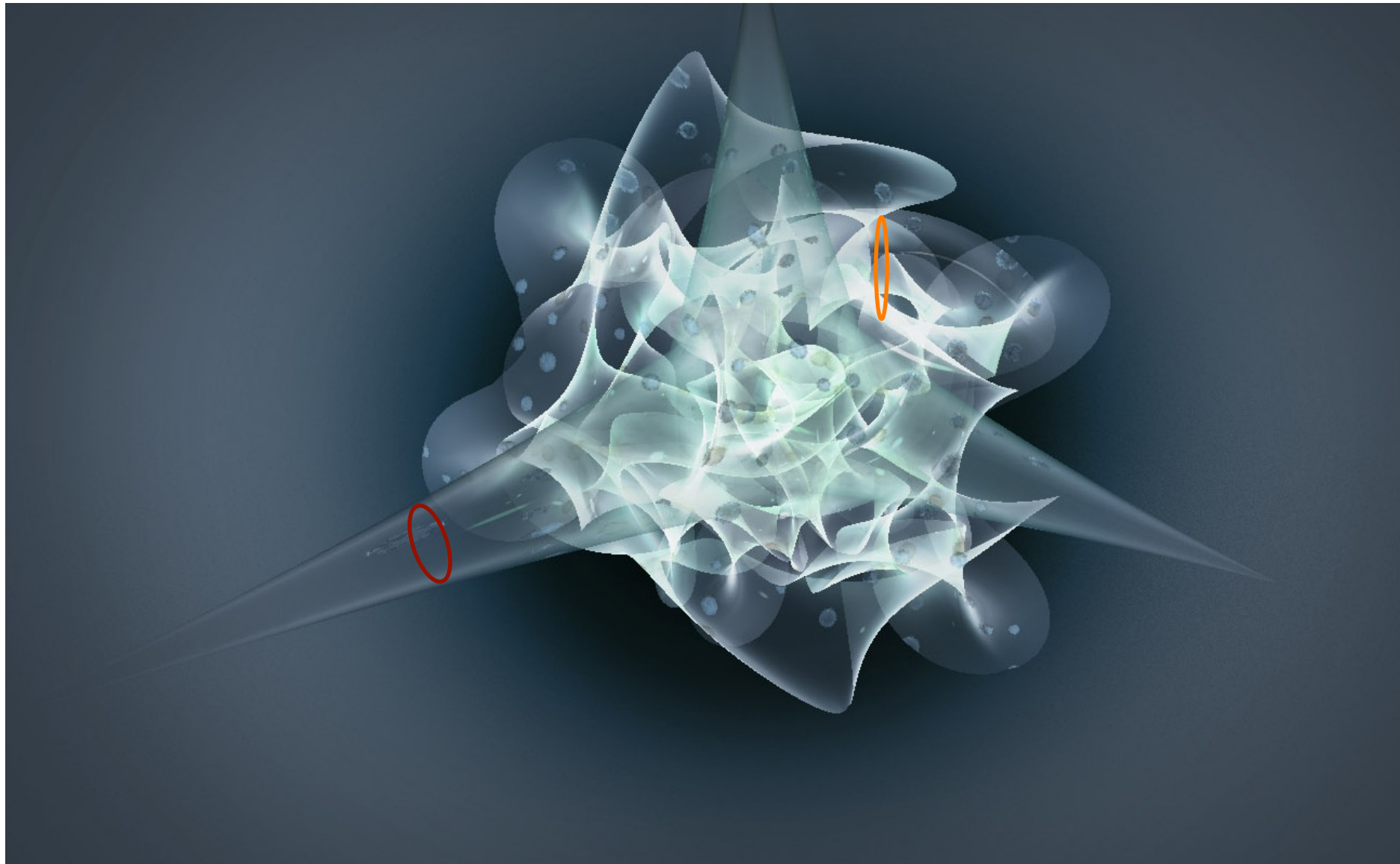
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Prologue



Mid to late 90s: a sense of unity and optimism

Prologue



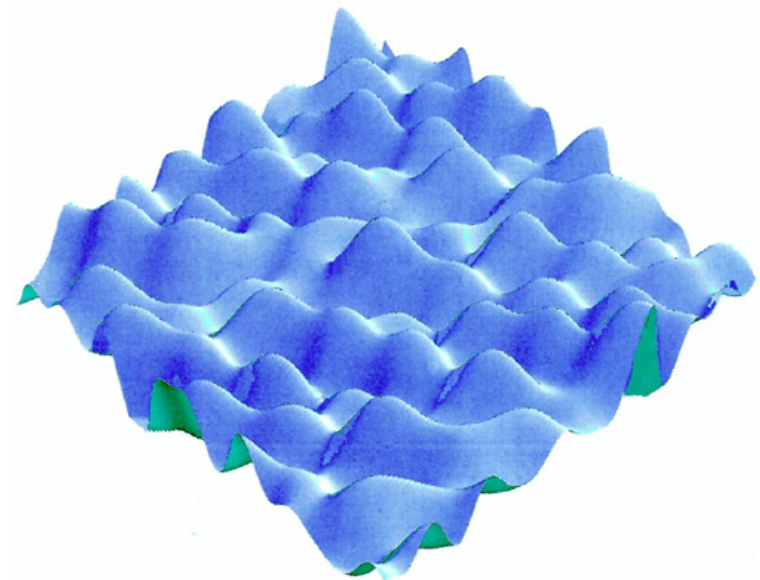
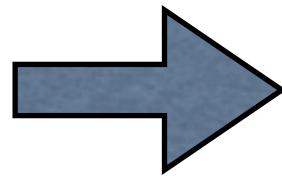
Prologue



Post mid 2000s: full of uncertainty!

Motivation

- 10^{500} vacua or not, need to have control of quantum corrections (in g_s and α') to understand the *vacuum structure* of string theory

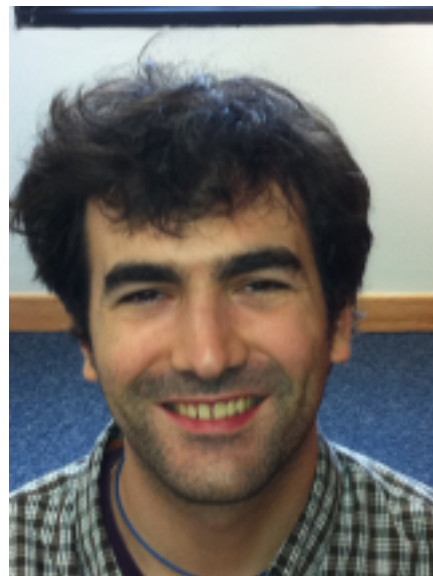


- Some key issues in particle physics and cosmology are sensitive to Planck suppressed operators (e.g., gravity mediated ~~SUSY~~, inflation,..).
- Non-perturbatively generated and higher dim. operators may explain small #s in Nature (e.g., Yukawa/ flavor hierarchies, ν -masses, μ -term).

Summary of Our Work

- A modest attempt to compute *exact results* on quantum corrected EFT of string compactifications (GHSS):

I. Garcia-Etxebarria, H. Hayashi, R. Savelli, GS, JHEP **1303**, 005 (2013).



I. Garcia-Etxebarria



H. Hayashi

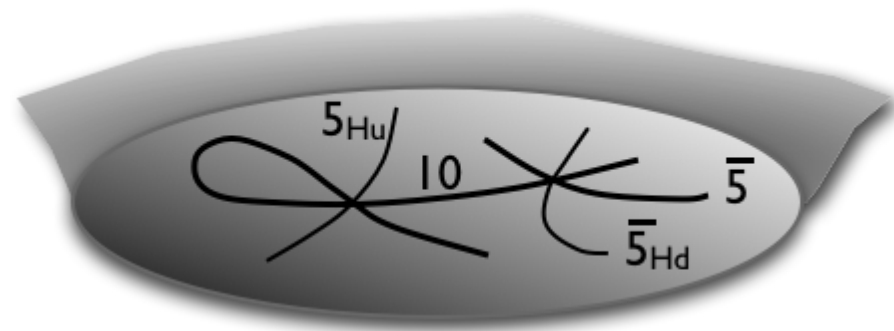
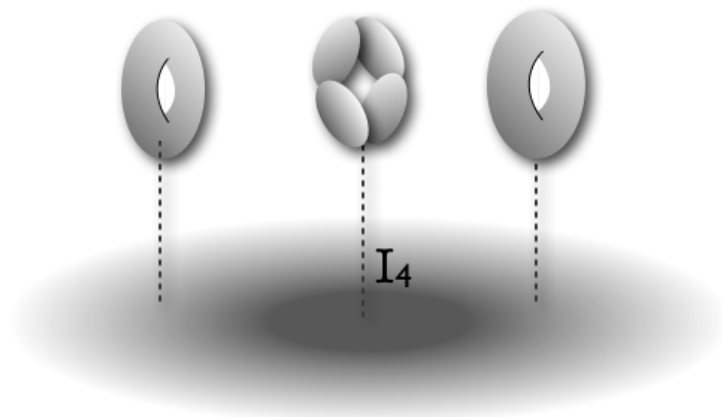


R. Savelli

- Make use of string dualities to compute exact in g_s (albeit perturbatively exact in α') Kahler potential in String/F-theory.
- See also Savelli's parallel talk on Tuesday.

F-theory

- Non-perturbative formulation of IIB string theory [Weigand's talk]
- $SL(2, \mathbb{Z})$ symmetry on axiodilaton $\tau = C_0 + i e^{-\Phi} \Leftrightarrow$ geometrize τ as complex structure of a two-torus [Vafa]; [Morrison, Vafa]; ...



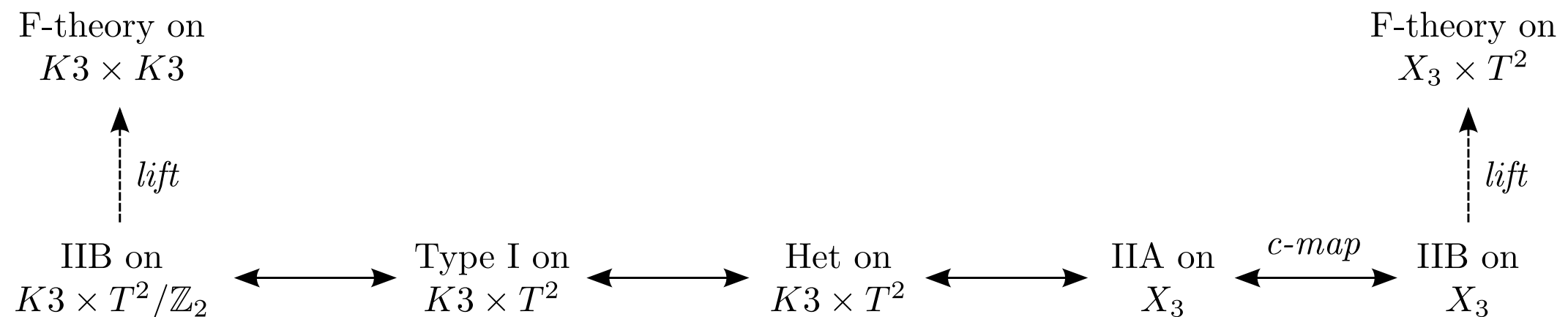
- Aspects of F-theory EFT obtained so far have not fully exploited the geometrical nature of F-theory to describe non-perturbative physics; *non-perturbative symmetries not manifest*.
- **GHSS**: Geometric interpretation of quantum corrections in terms of Gopakumar-Vafa invariants; $SL(2, \mathbb{Z})$ symmetry manifest.

Moduli Stabilization

- Perturbative α' corrections to K have played a key role in moduli stabilization, e.g, [Balasubramanian, Berglund, Conlon, Quevedo]
- $O(\alpha'^3)$ correction to K [Becker², Haack, Louis] was heavily used.
- $O(g_s^2 \alpha'^2)$ corrections to K were found for some $N=1,2$ toroidal orientifolds, cancellations in V (“extended no-scale”) [Berg, Haack, Kors]; [Hebecker, von Gersdorff]; [Cicoli, Conlon, Quevedo]
- We found an *exact in g_s and all orders in α'* generalization of these results. [Garcia-Etxebarria, Hayashi, Savelli, GS]
- Generalization is particularly important for moduli stabilization in F-theory GUTs as strong g_s coupling effects are invoked. [Cicoli, Mayrhofer, Valandro]; [Cicoli, Krippendorff, Mayrhofer, Quevedo, Valandro]

Summary of Strategy

- Consider a particular N=2 model: F-theory on $K3 \times K3$
 - ➔ obtain exact in g_s , perturbatively exact in α' Kahler potential
 - ➔ more constraining than N=1 but yet exhibit similar features
 - ➔ pattern of corrections $N=4 \rightarrow N=2$ as guide for N=1 case.
- Extensive use of **string dualities** and the **c-map**:



F-theory Effective Action

Effective Action

- F-theory is an **exact completion (in g_s)** of IIB but is perturbatively defined in α' . At tree-level in α' : $\mathcal{K} = \mathcal{K}_K + \mathcal{K}_c$,

where $\mathcal{K}_K = -3 \log \mathcal{V}_{\text{CY}_4}$, $\mathcal{K}_c = -\log \int_{\text{CY}_4} \Omega_4 \wedge \bar{\Omega}_4$,

- In the weakly coupled IIB orientifold limit (Sen's limit):

$$\mathcal{K}_c = -\log(\text{Im}S_0) - \log i \int_{\text{CY}_3} \Omega_3 \wedge \bar{\Omega}_3 + \frac{g_s}{2i \int_{\text{CY}_3} \Omega_3 \wedge \bar{\Omega}_3} \mathcal{K}_{\text{D7}} + \mathcal{O}(g_s^2)$$

- \mathcal{K}_c contains all g_s corrections sum in an **SL(2,Z) invariant** way.
- Physics depends on **intrinsic** fibration structure! \Leftrightarrow Labels (monodromies) of 7-branes change, not their mutual relations.
- Target space duality in M-theory when CY4 is trivially fibered but this symmetry holds generally for elliptically fibered CY4.

SL(2,Z)

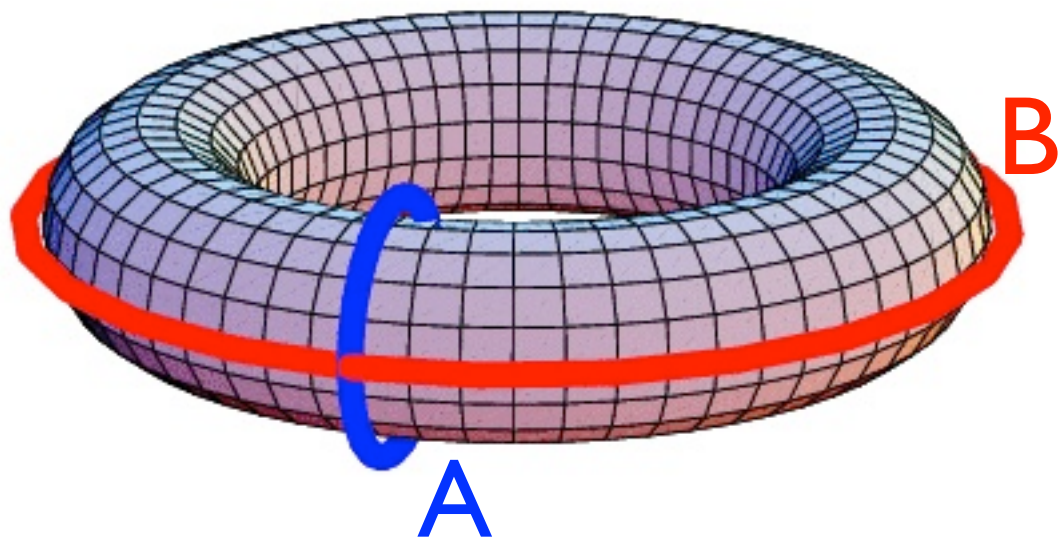
- Strong-weak coupling transformation: Local Lagrangian?
- Gauge couplings of gauge fields from KK reduction of bulk SUGRA or 7-branes do not change under SL(2,Z), e.g.,

$$\frac{1}{g_{\text{YM}}^2} \sim \frac{\text{Vol}(4\text{-cycle})}{g_s} = \widehat{\text{Vol}}(4\text{-cycle}),$$

only Einstein frame volumes are involved, SL(2,Z) inv.

- We shall verify that each α' -tower of corrections displays an SL(2,Z) symmetry for F-theory on K3xK3.
- More general than the case in [Collinucci, Soler, Uranga] as fibration is non-trivial; yet enjoy the non-renormalization theorems for N=2, and admits multiple dual descriptions.

F/M-theory Picture



- Reduce M to IIA along **A**.
- T-dualize IIA to IIB along **B**.
- Take the $v^0 = \text{Vol}(T^2) \rightarrow 0$ limit.

➔ Type IIB string theory with varying axio-dilaton.

- IIB metric in **Einstein frame** and **10d Poincaré** $r = (v^0)^{-3/4}$.

- The finite quantity $v_s^\alpha = \sqrt{v^0} v_M^\alpha$ is the 2-cycle size in string units.

- M-theory scale is small compared to string length $l_M/l_s = (v^0)^{1/4}$.

➔ Look for **higher derivative corrections to 11d sugra** which survive **the F-theory limit** $v^0 = \text{Vol}(T^2) \rightarrow 0$.

Our Model & Its Heterotic Dual

Our Model

- Consider Type IIB orientifold on $K3 \times T^2/Z_2$
- 4 Fixed points: positions of 4 O7-planes wrapping $R^{1,3} \times K3$
- $N=2, D=4$ Compactification:
 - ❖ **Vector multiplets:**
 - ▶ **3 bulk:** axio-dilaton (**S**), K3 volume (**T**), T^2 cplx str. (**U**).
 - ▶ **16 brane:** transverse positions of D7-branes (**Cⁱ**).
 - ▶ **No D3-moduli** (**16 rigid**, space-time-filling, “half” **D3s**).
 - ❖ **Hypermultiplets:** complex structure of K3 and Kahler modulus of T^2 .
- Focus on quantum corrections to the metric of the vector multiplet moduli space (a **Special Kahler manifold**).

Duality Dictionary

- We computed such corrections by following the duality



- Heterotic & Type I are a strong-weak coupling dual-pair in 10d:

$$\begin{aligned}
 \phi_{10}^H &= -\phi_{10}^I, \\
 G^H &= e^{-\phi_{10}^I} G^I,
 \end{aligned}
 \quad \text{[Witten]}$$

- Upon further T-duality, the vector moduli space gets mapped to:

$$\begin{array}{l}
 S_H = B^d + ie^{-2\phi_{10}^H} \text{Vol}(T^2 \times K3)^H \\
 T_H = \int_{T^2} B_{45} + i \text{Vol}(T^2)^H \\
 U_H = \frac{G_{45}^H + i\sqrt{G_{T^2}^H}}{G_{44}^H} \\
 A_H^i = U_H A_4^i - A_5^i \quad (16 \text{ Wilson lines})
 \end{array}
 \longleftrightarrow
 \begin{array}{l}
 T_{I'} = \int_{K3} C_4 + ie^{-\phi_{10}^{I'}} \text{Vol}(K3)^{I'} \\
 S_{I'} = C_0 + ie^{-\phi_{10}^{I'}} \\
 U_{I'} = U_I \\
 C_I^i = U_{I'} p_4^i - p_5^i \quad (16 \text{ D7 positions})
 \end{array}$$

- 5-brane instantons & worldsheet instantons in heterotic become **D3 instantons** and **D(-1) instantons** in Type I'.

Threshold Corrections

- **Special Kahler geometry** (N=2 prepotential):

$$K = -\log i \left[2\mathcal{F} - 2\bar{\mathcal{F}} - \sum_{\alpha} (\phi^{\alpha} - \bar{\phi}^{\alpha}) (\partial_{\phi^{\alpha}} \mathcal{F} + \partial_{\bar{\phi}^{\alpha}} \bar{\mathcal{F}}) \right]$$

- Use CFT techniques to compute all α' corrections in heterotic dual by going to orbifold limit (2-cycle volumes in hypers.)

➔ Kahler potential for Type IIB orientifold, **exact in g_s**

- First, ignore the Wilson Lines: [Harvey, Moore]; [Henningson, Moore]

$$\mathcal{F}_H(S_H, T_H, U_H) = \hat{S}_H T_H U_H + h(T_H, U_H),$$

$$\hat{S}_H = S_H + \frac{1}{2} \partial_{T_H} \partial_{U_H} h(T_H, U_H),$$

exact in α' , perturbatively exact in g_s .

- $S_H \rightarrow S_H + \lambda$ is an exact symmetry in perturbation theory. \mathcal{F}_H (and S_H) has no perturbative corrections beyond one loop in $g_s = 1/\text{Im } S_H$.

Threshold Corrections

- Prepotential for Type IIB Orientifold:

$$\mathcal{F}(S, \hat{T}, U) = S\hat{T}U + h(S, U),$$

$$\hat{T} = T + \frac{1}{2}\partial_S\partial_U h(S, U).$$

exact in $g_s = 1/\text{Im } S$, perturbatively exact in $\alpha'^2 = 1/\text{Im } T$.

- In the region $\text{Im } S > \text{Im } U$

$$h(S, U) = -\frac{i}{(2\pi)^4} \left[\text{Li}_3 \left(e^{2\pi i(S-U)} \right) + \sum_{\substack{k, l \geq 0 \\ (k, l) \neq (0, 0)}} c(kl) \text{Li}_3 \left(e^{2\pi i(kS+lU)} \right) \right] + \frac{15i}{2\pi^4} \zeta(3) + \frac{U^3}{12\pi},$$

where

$$\text{Li}_m(z) = \sum_{n=1}^{\infty} \frac{z^n}{n^m}, \quad \sum_{n=-1}^{\infty} c(n)z^n = \frac{E_6 E_4}{\eta^{24}}(z)$$

S, U interchanged when $\text{Im } U > \text{Im } S$.

Threshold Corrections

- Kahler potential is invariant under shift of F (also h) by a polynomial at most quadratic in Φ^a with real coefficients.
- Ambiguity in h is related to non-trivial monodromies at special regions of moduli space.
- h develops co-dim. 1,2 singularities due to *enhanced gauge symmetry*.
- Classical duality group is modified, but K should be unaffected.

Perturbative α' Corrections

- Expand the log to get perturbative α' corrections to K:

$$K(S, T, U) = K^{(0)}(S, T, U) + \sum_{n=1}^{\infty} \frac{1}{n} K^{(n)}(S, T, U),$$

$$K^{(0)}(S, T, U) = -\log [-i(S - \bar{S})(T - \bar{T})(U - \bar{U})],$$

$$K^{(n)}(S, T, U) = -\frac{(-1)^n}{(T - \bar{T})^n} \left[\frac{2h - 2\bar{h}}{(S - \bar{S})(U - \bar{U})} - \frac{\partial_S h + \partial_{\bar{S}} \bar{h}}{U - \bar{U}} - \frac{\partial_U h + \partial_{\bar{U}} \bar{h}}{S - \bar{S}} - \frac{1}{2}(\partial_S \partial_U h - \partial_{\bar{S}} \partial_{\bar{U}} \bar{h}) \right]^n.$$

- Only $(g_s \alpha')^{2n}$ terms survive since $\partial_S h \rightarrow 0$ exponentially for $S \rightarrow i\infty$
- α'^3 term of BBHL is absent since $\chi(\text{K3} \times \text{T}^2) = 0$.
- Odd powers of g_s are absent as open string moduli are frozen.

Perturbative α' Corrections

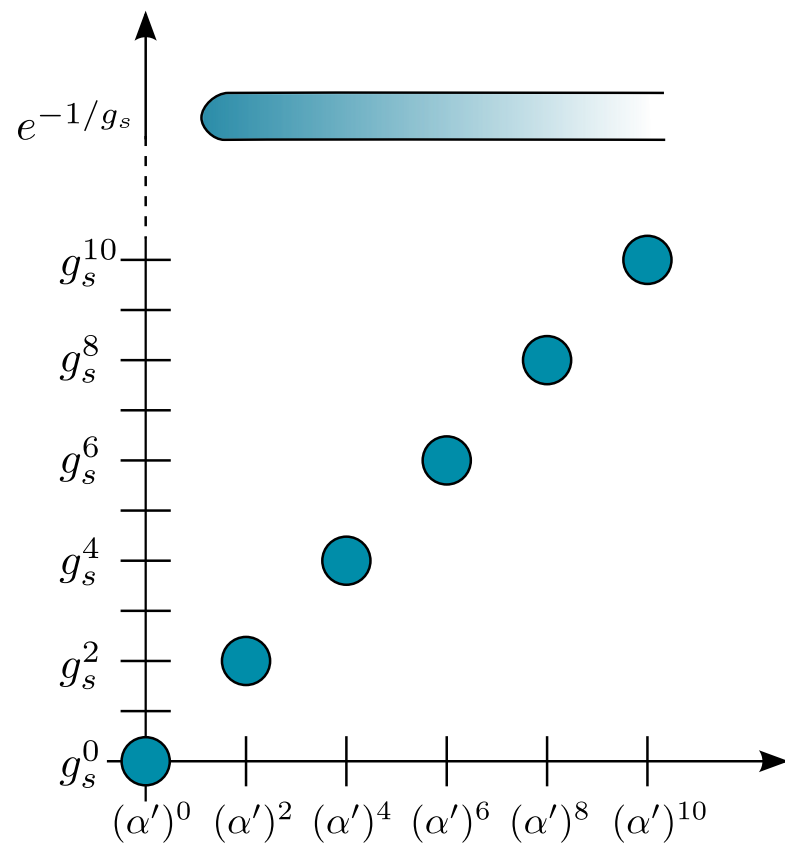
- Kahler modulus and axio-dilaton **mix** at α'^2 order (n=1):

$$K^{(1)} = -\frac{\mathcal{E}(U, \bar{U})}{(T - \bar{T})(S - \bar{S})} \quad \mathcal{E} := \lim_{S \rightarrow i\infty} \frac{2h - 2\bar{h}}{U - \bar{U}} - \partial_U h - \partial_{\bar{U}} \bar{h}.$$

agrees with 1-loop open string computation **[Berg, Haack, Kors]**

- Such corrections come from (i) KK exchange between D7s and non-mobile D3s, or (ii) Mobius amplitude between parallel D7s.
- $(1/\text{Im } T)^2 (1/\text{Im } S)^0$ correction which comes from exchange of strings wound around the intersection of D7s is absent as two D7s either do not intersect or coincide, and K3 has no 1-cycle.
- At each perturbative α' order, the Kahler potential is inv. under
$$O(2, 2, \mathbb{Z}) = SL(2, \mathbb{Z})_S \times SL(2, \mathbb{Z})_U \rtimes \mathbb{Z}_2$$
- Generalized to include Wilson lines & checked $SL(2, \mathbb{Z})$ invariance.

Summary of Corrections



- ✓ Terms non-perturbative in g_s from D(-1) instantons.
- ✓ Only $(g_s \alpha')^{2n}$ terms survive at perturbative level
- ✓ α'^3 term of BBHL is absent since $\chi(K3 \times T^2) = 0$.
- ✓ No odd powers of g_s , as open string moduli are frozen.

Non-perturbative in α' corrections are not included in this analysis:

- WS & D1 instantons are absent due to orientifold projection.
- ED3 wrapping $T^2 \times$ (2-cycle of K3) correct hyper. moduli sp. metric.
- $SL(2, \mathbb{Z})$ invariant ED3 branes wrapping K3 (more later) not included here.

Non-perturbative α' Corrections

- ED3 wrapping K3 are non-perturbative in both α' and g_s .
- Their contribution is trivially $SL(2, \mathbb{Z})$ invariant.
- The exact prepotential:

$$\mathcal{F} = \hat{S}\hat{T}U - \frac{\hat{T}}{2} \sum_{i=1}^{16} (C^i)^2 + \tilde{h}(\hat{S}, U, C^i) + \sum_m \mathcal{A}_m(\hat{S}, U, C^i) e^{2\pi i m T}$$

- The A-factor can be computed from the Type IIA dual on a CY_3 which admits a K3 fibration over S^2 .
- D3 instantons \rightarrow worldsheet instantons wrapping S^2 on the IIA side. Partial results were obtained in [\[Berglund, Mayr\]](#).

Lift to F-theory

Heterotic Picture

- Type IIB orientifold on $K3 \times T^2/\mathbb{Z}_2$ is related by two T-dualities to the BSGP model [Bianchi,Sagnotti];[Gimon,Polchinski]
- The BSGP model is S-dual to heterotic $SO(32)$ theory w/o vector structure [Bianchi];[Witten], though its maximal gauge group is $U(16)$.
- Heterotic $SO(32)$ without vector structure is dual to heterotic $E_8 \times E_8$ with instanton embedding $(12,12)$.
 - ❖ At generic points of the hypermultiplet moduli space, only $U(1)^4$ is left, corresponding to S_H, T_H, U_H , and the graviphoton.
- Heterotic $SO(32)$ with vector structure is dual to heterotic $E_8 \times E_8$ with different instanton embedding.
 - ❖ Not enough instantons for complete Higgsing to $U(1)$ s. Need to turn on WLs, e.g., $(24,0)$ instanton embedding requires 8 WLs.

Heterotic Picture

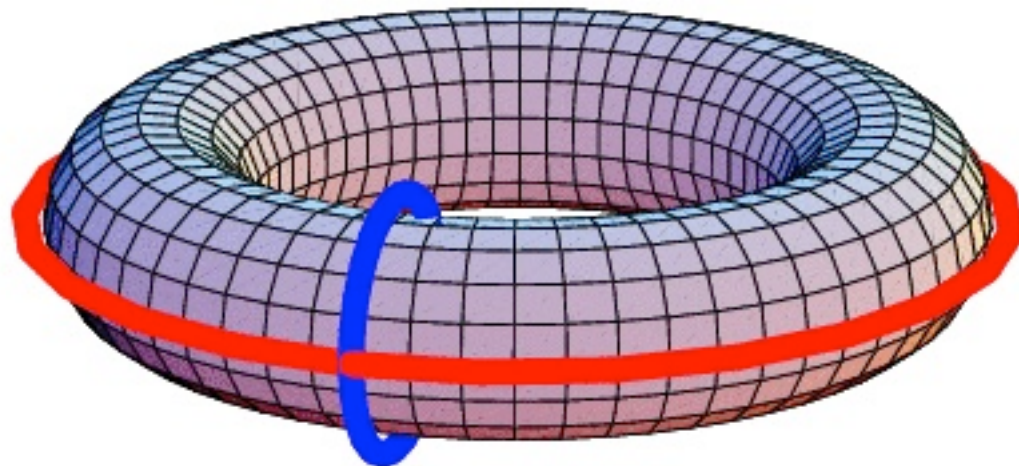
- We focus on the **vector multiplet moduli space**.
- When WFs are turned off, prepotential of the theory w/o vector structure matches that w/ vector structure.
- This insensitivity to instanton embedding (and hence 4D gauge groups) can be explained using the relation to SUSY index [**Lopes Cardoso, Curio, Lust**]. We also showed this by explicit computation.
- Expressions for heterotic $SO(32)$ and $E_8 \times E_8$ with general instanton embeddings & WFs can be found in [**Henningson, Moore**].

F-theory Picture

- 8D duality: Heterotic on T^2 = F-theory on K3
- Heterotic on $K3 \times T^2$ = F-theory on $K3 \times K3$
- Our model actually admits more F-theory duals.
- If the heterotic K3 admits an elliptical fibration over P^1 , we get an F-theory dual on $X_3 \times T^2$ where X_3 =K3 fibration over P^1 :
 - ➔ Same X_3 on which the dual IIA theory compactified [Louis, Sonnenschein, Theisen, Yankielowicz].
 - ➔ Base of X_3 as an elliptical fibration is an Hirzebruch surface F_n , with n related to the instanton embedding of the dual heterotic $E_8 \times E_8$ theory [Morrison, Vafa].
 - $X_3 = WP_{1,1,2,8,12}(24)$ for $(12,12)$ instanton embedding [Klemm, Lerche, Mayr]; [Hosono, Klemm, Theisen, Yau]

Quantum Corrections

- We checked that the classical vector multiplet moduli space matches the classical moduli space of K3'.
- Quantum corrections → Quantum moduli space of K3'.
 - ▶ Factorization destroyed but SK-geometry preserved.
- Identify BPS objects in M-theory which generate the corrections found. Recall the M-theory definition of F-theory:



T-duality circle S_T

M-theory circle S_M

Quantum Corrections

- ❖ **Non-perturbative g_s corrections:** D(-1) instantons in IIB \rightarrow D0-brane in IIA looping along $S_T \rightarrow$ KK particle in IID with **non-trivial p_M & w_T** .
- ▶ For trivial fibration: higher derivative corrections to IID SUGRA
[Green,Gutperle];[Green,Vanhove];[Green,Gutperle,Vanhove];[Green,Sethi]
 \rightarrow Contribute to R^4 coupling $\rightarrow \alpha'^3$ correction of BBHL
- ▶ Non-trivial fibration: contributions already at order α'^2
- ▶ For these l_M corrections to stay finite in the F-theory limit, they should appear as powers of $l_M^2/\sqrt{v_0}$ because of the relation:

$$l_M/l_s = (v_0)^{1/4}$$

- ▶ A subset of higher derivative corrections to IID SUGRA (F-theory limit)

Quantum Corrections

❖ **Perturbative g_s corrections** when combined with the non-perturbative ones lead to $SL(2, \mathbb{Z})$ invariant sets of g_s corrections for each α' tower. This suggests:

- ▶ I I D supergravitons with **non-trivial w_T but no p_M**
- ▶ I I D supergravitons with **non-trivial w_M , and possibly p_T**

(c.f. [Green, Vanhove])

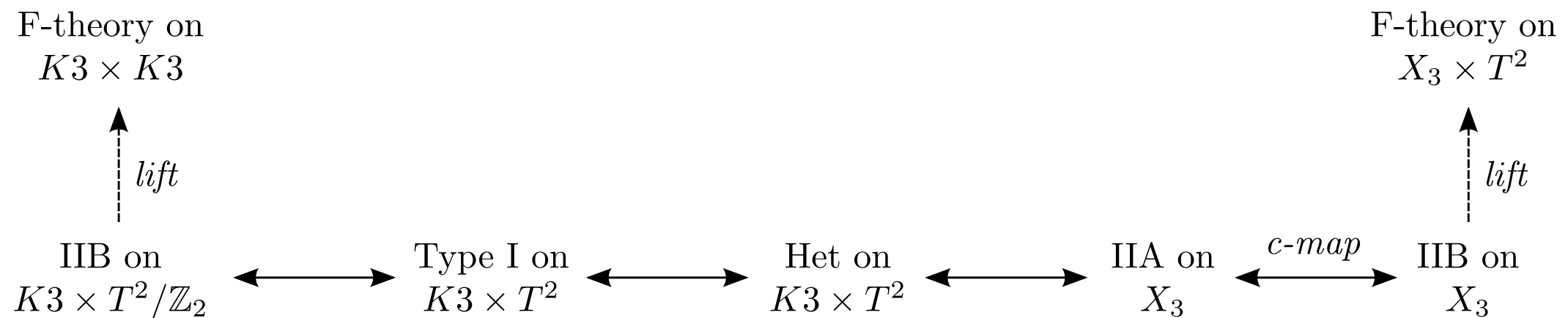
Being loops of 1-cycle of the fiber, these sources should generate corrections proportional to $l_M^2 / \sqrt{v_0}$, thus survive the F-theory limit.

Explicit Computation

Explicit Computation

- We have identified the BPS objects in M-theory responsible for the corrections to the vector multiplet moduli space metric for F-theory on $K3 \times K3'$.
- Direct Schwinger-loop calculation on $K3'$ along the lines of [Collinucci, Soler, Uranga] is hard, due to non-trivial fibration.
- We make use of other F-theory duals to bypass this difficulty.

Chain of Dualities



- Swapping the role of elliptic fiber of X_3 and F-theory fiber
 - ➔ Hypermultiplet moduli space of a *braneless* F-theory
 - ➔ M-theory method [Collinucci, Soler, Uranga] to compute corrections.
 - However, this procedure gives more corrections than needed!
 - ◆ IIA vector multiplet moduli space metric has only α' corrections.
 - ◆ IIB hypermultiplet moduli space receives both α' & g_s corrections.
 - ➔ Extract *tree-level* in g_s part of Schwinger-loop computation
- (Note g_s here is *neither* g_s *nor* α' of the original F-theory on $K3 \times K3$!)

M-theory Computation

- Hypermultiplet moduli space of F-theory on $X_3 \times T^2$
- D-instantons \leftrightarrow D-particles via c-map [Seiberg, Shenker]; [Ooguri, Vafa]
- Compute corrections to the hypermultiplet moduli space (equiv. Einstein term ($\sim R$) in 3d effective action)

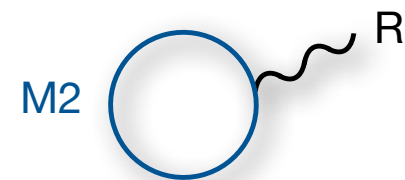
D1/D(-1) instantons in IIB on CY

\Updownarrow c-map

D2/D0 particles in IIA on CY x S

\Uparrow

M2 particles (and gravitons) in M-th on CY x T^2



M-theory Computation

- Compute Schwinger-loops for $X_3 = \text{WP}_{1,1,2,8,12}$ (24)

$$\mathcal{F}_{\text{class}} = \frac{1}{6} \kappa_{\alpha\beta\gamma} t_\alpha t_\beta t_\gamma,$$

$$\mathcal{F}_{\text{pert}} = -\frac{i}{4(2\pi)^3(\tau_2)^{3/2}} \chi(X_3) \sum_{(m,n) \neq (0,0)} \frac{\tau_2^{3/2}}{|m\tau + n|^3},$$

$$\mathcal{F}_{\text{non-pert}} = \frac{i}{2(2\pi)^3(\tau_2)^{3/2}} \sum_{\mathbf{d}} n_{d\alpha} \sum_{(m,n) \neq (0,0)} \frac{\tau_2^{3/2}}{|m\tau + n|^3} e^{2\pi i d_\alpha (m c_\alpha + n b_\alpha + i |m\tau + n| j_\alpha)}$$

- ▶ $\kappa_{\alpha\beta\gamma}$ = classical intersection # of X_3
- ▶ $n_{d\alpha}$ = genus-zero Gopakumar-Vafa invariants of X_3
- ▶ $c_\alpha, b_\alpha, j_\alpha$ = zero modes of the RR 2-form, the B-field, & the Kahler form expanded in a basis of $H^{1,1}(X_3, \mathbb{Z})$
- Displays $SL(2, \mathbb{Z})$ for each perturbative α' tower; also contains non-perturbative α' corrections again in $SL(2, \mathbb{Z})$ inv. manner.

Final Result

- To derive corrections to the vector multiplet moduli space metric of the *original* F-theory on K3xK3, consider the limit:

$$g_s \rightarrow 0, \quad i.e., \quad \tau_2 \rightarrow \infty$$

- Matches with the heterotic computation. Identify:

$$-\frac{1}{2}n_{l+k,-k,l-k} = c(lk)$$

- Counting rational curves via modular forms [Henningson, Moore].
- $c(lk)$ agree with GV invariants in [Hosono, Klemm, Theisen, Yau].
- Given GV invariant for the 0-class is the Euler number:

$$n_{0,\dots,0} = -\frac{\chi(X_3)}{2}$$

- GV invariants \simeq non-perturbative (in g_s) generalization of the BBHL term (albeit here the α' corrections are of order α'^2).

Generalizations

- To generalize this result to include 7-brane moduli C^i , one considers X_3 with the right number of vector multiplet.
- For example, with 8 C^i switched on, use $X_3 = \text{WP}_{1,1,12,28,42}$ (84) as $\mathcal{F}_{\text{vector multiplet}}$. should be insensitive to instanton embedding.
- Repeat the Schwinger-loop computation with a different set of topological invariants.

Summary

- Computed the *exact* in g_s , perturbatively exact in α' , vector multiplet moduli Kahler potential \mathcal{K} of an N=2 F-theory model.
- Provided an M/F-theory interpretation of corrections to the Kahler potential and identified the contributing BPS states.
- Shown explicitly that quantum corrections to the Kahler potential are $SL(2, \mathbb{Z})$ invariant at each α' level.
- Shown that Kahler moduli & complex structure moduli start to mix when perturbative α' corrections are taken into account.
- Provided a genuine (albeit indirect) M-theory computation.

Outlook

- A modest step towards the ambitious goal of computing exact results for realistic compactifications.
- Even within the realm of $N=2$ compactifications, we have focussed on the easiest half of the problem, i.e., *vector multiplet* moduli.
- It'd be interesting to apply the remarkable work on understanding quantum corrections to the *hypermultiplet moduli* space, see e.g.:
[Robles-Ilana,Rocek,Saueressig,Theis,Vandoren];[Alexandrov,Saueressig,Vandoren];
[Pioline,Persson];[Bao,Kleinschmidt,Nilsson,Persson,Pioline];[Alexandrov];
[Alexandrov,Saueressig];[Alexandrov,Manschot,Pioline]; ...
in F-theory as we have done for the vector multiplet moduli.



Outlook

- The Holy Grail is to find non-pert. results for $N \leq 1$ compactifications.
- The pattern of corrections from $N=4 \rightarrow N=2$ that we found may serve as a guide for further breaking to $N=1$. More concretely:
 - ★ Compute directly D-particle loops for non-trivial elliptic fibration
 - ★ Spontaneously breaking $N=2 \rightarrow N=1$ (e.g, by fluxes); sometimes inherit structure of $N=2$ effective action. See, e.g., [Ferrara, Girardello, Porrati]; [Fre, Girardello, Pesando, Trigiante][Louis, Symth, Triendl], ...
- More α' corrections arise in $N=1$ String/F-theory vacua (albeit only tree-level g_s result in [Grimm, Savelli, Weißenbacher]; [Pedro, Rummel, Westphal])
- Any such progress would undoubtedly shed light on the vacuum structure of string theory and its low energy descriptions.



THANKS

