Statistical Preference for a Vanishingly Small Cosmological Constant in Stringy Landscape

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July 16, 2013
StringPheno 2013, DESY Hamburg, Germany

Hong Kong University of Science and Technology

This talk is based on work done with **Yoske Sumitomo** :

arXiv:1204.5177, arXiv:1209.5086, arXiv:1211.6858
arXiv:1305.0753 (also with Sam Wong)

Some of the works relevant to us :
Bousso and Polchinski, hep-th/0004134
Kachru, Kallosh, Linde and Trivedi, hep-th/0301240
Balasubramanian, Berglund, Conlon and Quevedo, hep-th/0502058
Westphal, hep-th/0611332
Denef and Douglas, hep-th/0404116
Douglas and Kachru, hep-th/0610102
Becker, Becker, Haack and Louis, hep-th/0204254
Rummel and Westphal, arXiv:1107.2115 [hep-th]
de Alwis and Givens, arXiv:1106.0759 [hep-th]

Aazami and Easther, hep-th/051205
Chen, Shiu, Sumitomo and Tye, arXiv:1112.3338 [hep-th]
Bachlechner, Marsh, McAllister and Wrase, arXiv:1207.2763 [hep-th]
Blanco-Pillado, Gomez-Reino and Metallinos, arXiv:1209.0796 [hep-th]
Martinez-Pedrera, Mehta, Rummel and Westphal, arXiv:1212.4530 [hep-th]
Danielsson and Dibitetto, arXiv:1212.4984 [hep-th]

Challenge

- There is very strong evidence that we are living in a de-Sitter vacuum with a positive cosmological constant Λ,

$$\Lambda \sim +10^{-122} M_P^4$$

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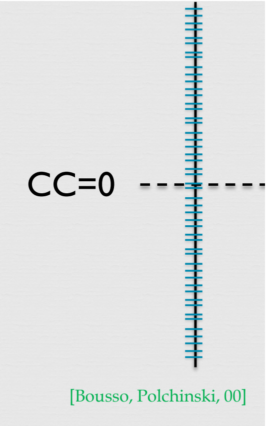
- Since we can always introduce an arbitrary Λ into Einstein's relativity theory, this very small value can either be obtained by fine-tuning there, or explained by "the anthropic principle".
- Since Λ is calculable in string theory, string theory is the place to search for an explanation beyond "the anthropic principle".
- Our universe has probably gone through an inflationary period, when the vacuum energy is much higher than today's value.

Bousso and Polchinski observed that fluxes in string theory are quantized. E.g., J types of quantized 4-form fluxes $F^i_{\mu\nu\rho\sigma}$ contribute to the Λ.



$$\lambda = \lambda_{\rm bare} + \frac{1}{2}\sum_{i=1}^{J} n_i^2 q_i^2 \ .$$

Pressing Question and our Proposal

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Why nature picks such a very small positive Λ ?

We argue that there may be a **statistical preference** for a very small (either positive or negative) Λ. We'll illustrate with some examples in Type IIB string theory.

Approach in IIB

- Consider a string model with a set of moduli $\{u_i\}$ and 2-form fields C_2 and B_2. The 3-form fluxes $F_3 = dC_2$ and $H_3 = dB_2$ wrap cycles in a Calabi-Yau like manifold. The quantized fluxes lead to a set of discrete values labelled as $\{n_j\}$, yielding $V(n_j, u_i)$, where each n_j takes a discretum of values.

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- Solve $V(n_j, u_i)$ for all the meta-stable vacua. For every meta-stable vacuum with a given set $\{n_j\}$, each u_i is determined in terms of $\{n_j\}$: $u_{i,min}(n_j)$.
  So $\Lambda(n_j) = V_{min}(n_j, u_{i,min})$.

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- Treat each $\{n_j\}$ as a random variable with some uniform probability distribution $P_j(n_j)$. Find the probability distribution $P(\Lambda)$ for $\Lambda(n_j)$ as we sweep through allowed $\{n_j\}$.
- As we shall see, $P(\Lambda)$ tends to peak at $\Lambda = 0$.

This peaking behavior of $P(\Lambda)$ at $\Lambda = 0$

The Basic Idea is very simple :

It is based on the properties of the probability distribution of functions of random variables.

Does $\Lambda(n_j)$ has the right functional form ? Do the parameters n_j have the right distribution ?

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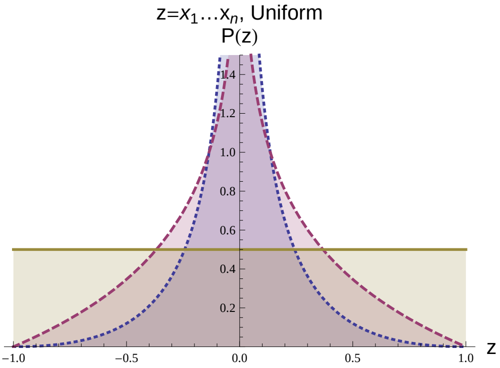
An example :

Consider a set of random variables x_i $(i = 1, 2, ..., n)$. Let the probability distribution of each x_i be uniform in the range $[-L, +L]$. What is the probability distribution of their product z ?

Probability distribution of $z = x_1 x_2$ and $z = x_1 x_2 x_3$



$$P(z) = \frac{1}{2(n-1)!}\left(\ln\frac{1}{|z|}\right)^{n-1}$$

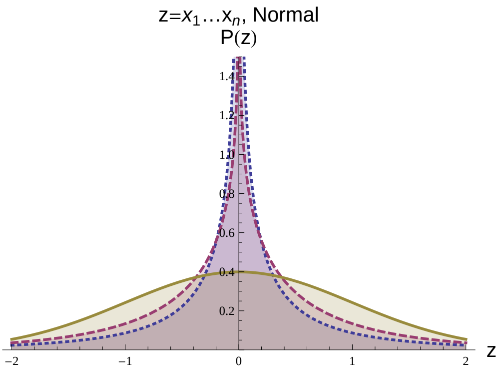
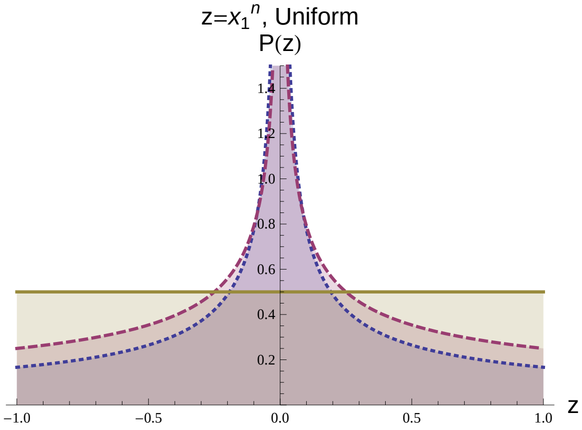


Figure: The product distribution $P(z)$ is for $z = x_1$ (solid brown curve for normal distribution), $z = x_1 x_2$ (red dashed curve), and $z = x_1 x_2 x_3$ (blue dotted curve), respectively. In general, the curves are given by the Meijer-G function.

Probability distribution $P(z)$ for $z = x_1^n$



$$P(z) \sim z^{-1+1/n}$$

Probability distribution $P(z)$

z	Asymptote of $P(z)$ at $z=0$
$x_1 \cdots x_n$	$(\ln(1/\|z\|))^{n-1}$
x_1^n	$z^{-1+1/n}$
$x_1^n \cdots x_m^n$	$z^{-1+1/n}(\ln(1/\|z\|))^{m-1}$
$x_1^m x_2^n$	$(z^{-1+1/m} - z^{-1+1/n})/(m-n)$
$x_1 \cdots x_m \,/\, y_1 \cdots y_n$	$(\ln(1/\|z\|))^{m-1}$
x_1^m / y_1^n	$z^{-1+1/m}$
$x_1^{n_1} + \cdots + x_m^{n_m}$	$z^{-1+1/n_1+\cdots 1/n_m}$
$x_1 x_2$, $0 < c = x_1/x_2 < \infty$	smooth
$x_1 x_2$, $0 \leq c = x_1/x_2$ or $c \leq \infty$	$\ln(1/\|z\|)$

Example

$P(z)$ of $z = f(x_j)$ can always be properly normalized, even when $P(z)$ diverges at $z = 0$.

Consider again $z = x_1 x_2 ... x_n$
where each x_i has a uniform distribution in the range $[-L, +L]$.
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For $z = (x_1 x_2 ... x_n)^2$, we have

$$\frac{z_{50\%}}{\langle z \rangle} = 10^{0.28-0.39n} \qquad \frac{z_{10\%}}{\langle z \rangle} = 10^{-2.3-0.52n}$$

Introduce $z_{Y\%}$: $Y\%$ of the solutions have a value below $z_{Y\%}$.

Median as a useful measure for the expected values

For our purpose, $\frac{|\Lambda|_{50\%}}{\langle|\Lambda|\rangle}$ is a good measure of the preference for a small Λ.

Example : 10^6 solutions at $\Lambda = 10^{-9}$ and one solution at $\Lambda = 1$.

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Then $\Lambda_{50\%} = 10^{-20}$ while $\langle\Lambda\rangle$ does not change.

In this special case, $\Lambda_{10\%} = \Lambda_{90\%} = 10^{-20}$ also.

In general, if

$$V = V_1(n_j, u_i) + V_2(m_k, v_l)$$

where the 2 terms in V do not couple, then

$$\Lambda = \Lambda_1(n_j) + \Lambda_2(m_k)$$

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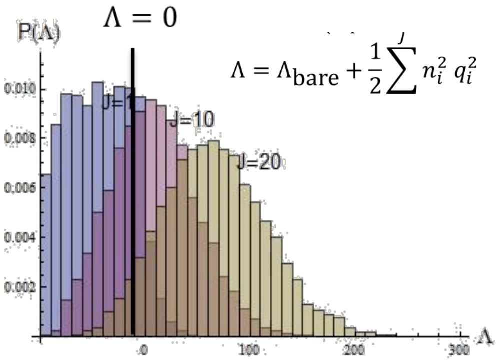
Fortunately, gravity couples to all sectors, so this decoupling should not happen.

But it may happen in over-simplified models.

Example : Bousso-Polchinski Model

No peaking behavior for $P(\Lambda)$



Type IIB String Theory ($M_P = 1$)

Consider the superpotential W_0 (Gukov-Vafa-Witten)

$$W_0(U_i, S) = \sum_{cycles} \int G_3 \wedge \Omega = (F_3 - iSH_3) \cdot \Pi(U_i)$$

$$= (f_{3j} - iSh_{3j})\mathcal{F}_j(U_i)$$

$$\simeq c_1 + \sum_j b_j U_j - S(c_2 + \sum_j d_j U_j)$$

where f_{3j} and h_{3j} take discrete flux values.

E.g., Only linear terms in U_j in orientifolded toroidal orbifolds
(Font, ..., Lust, Reffert, Schulgin, Stieberger, ...).

$$
\begin{aligned}
V =& e^K \left(K^{J\bar{I}} D_J W D_{\bar{I}} \bar{W} - 3|W|^2 \right), \\
K =& -2\ln(\mathcal{V} + \xi/2) - \ln(S + \bar{S}) - \sum_j \ln(U_j + \bar{U}_j) \\
\mathcal{V} =& Vol/\alpha'^3 = \gamma_1 (T_1 + \bar{T}_1)^{3/2} - \sum_{i=2} \gamma_i (T_i + \bar{T}_i)^{3/2}, \\
W =& W_0(U_j, S) + \sum_{i=1}^{N_K} A_i e^{-a_i T_i}, \\
W_0(U_j, S) =& c_1 + \sum_j b_j U_j - S(c_2 + \sum_j d_j U_j)
\end{aligned}
$$

where ξ is the α' correction (Becker, Becker, Haack and Louis: Pedro, Rummel and Westphal) that can provide the Kähler uplift to de Sitter solutions (Rummel and Westphal, deAlwis and Givens.)

We shall illustrate the statistical preference for a small Λ with 2 types of examples :

(1) A single Kähler modulus T in a racetrack model with Kähler uplift : $W = W_0 + Ae^{-aT} + Be^{-bT}$ (Yoske Sumitomo's talk) :

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(2) The many complex structure moduli $\{U_j\}$ case $\rightarrow W_0(U_j, S)$.

Typical Manifolds Studied

$$\chi(M) = 2(h^{1,1} - h^{2,1})$$

Manifold	$h^{1,1}$	$h^{2,1}$	χ
$\mathcal{P}^4_{[1,1,1,6,9]}$	2	272	-540
$\mathcal{F}_{11}$	3	111	-216
$\mathcal{F}_{18}$	5	89	-168
$\mathcal{CP}^4_{[1,1,1,1,1]}$	1	$\mathcal{O}(100)$	$\mathcal{O}(-200)$

A manifold has $h^{1,1}$ number of Kähler moduli and $h^{2,1}$ number of complex structure moduli.

Approach for the Multi-Complex Structure Moduli case

- Consider the above model
$W_0(U_i, S) = c_1 + \sum_j b_j U_j - S(c_2 + \sum_j d_j U_j)$
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  with the dilation S and $h^{2,1}$ number of complex structure moduli U_i.
- All flux parameters b_i, c_i and d_i are treated as real random variables with some uniform probability distributions.
- Find the supersymmetric solution $w_0 = W_0|_{\min}$ of W_0 for the complex structure moduli and the dilaton and insert this w_0 into V to stabilize the Kähler modulus.
- The functional form of $\Lambda = V_{\min}$ in terms of the parameters allow us to find $P(\Lambda)$.

$$D_S W_0 = \partial_S W_0 + K_S W_0 = 0, \qquad D_i W_0 = 0$$
$$W_0(u_i, s) = c_1 + \sum_j b_j u_j - s(c_2 + \sum_j d_j u_j)$$

Solution : $u_i = -(c_1 - sc_2)/(h^{2,1} - 2)(b_i - sd_i)$

$$(h^{2,1} - 2)\frac{c_1 + sc_2}{c_1 - sc_2} = \sum_{i=1}^{h^{2,1}} \frac{b_i + sd_i}{b_i - sd_i}$$

$$w_0 = W_0|_{\min} = -\frac{2(c_1 - sc_2)}{h^{2,1} - 2} = \frac{2(c_1 + sc_2)\Pi_i(b_i - sd_i)}{\sum_i (b_i + sd_i)\Pi_{j\neq i}(b_j - sd_j)}$$

Then insert w_0 into the V for the Kähler modulus and find the solution :

$$\Lambda = \frac{e^{-5/2}}{9}\left(\frac{2}{5}\right)^2 \frac{-w_0 a^3 A}{\gamma^2}\left(x_m - \frac{5}{2}\right)$$

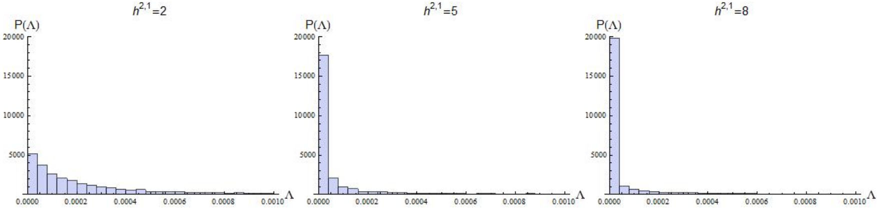


Figure: The probability distribution $P(\Lambda)$ of Λ at meta-stable vacua as a function of $h^{2,1} = 2, 5, 8$ number of complex structure moduli and a single Kähler modulus ($h^{1,1} = 1$). Although the range is $0 \leq \Lambda \lesssim 1$, the probability distributions for only $0 \leq \Lambda \leq 10^{-3}$ are shown.

$P(\Lambda)$ becomes more peaked at $\Lambda = 0$ as $h^{2,1}$ increases.

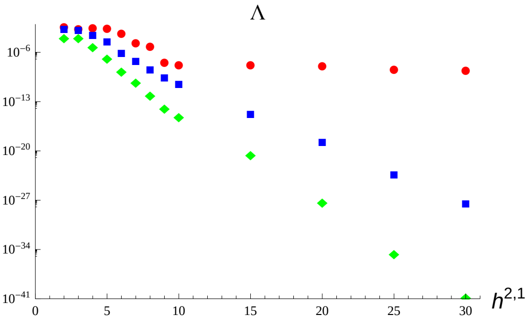
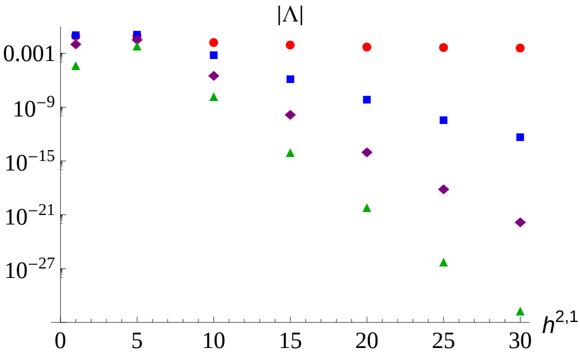


Figure: The figure shows $\langle\Lambda\rangle$ (red circles), $\Lambda^{80\%}$ (blue squares) and $\Lambda^{10\%}$ (green diamonds) as a function of $h^{2,1}$. Here, the b_i parameters are fixed or have limited ranges. At $h^{2,1} = 30$: $\Lambda^{10\%} \simeq 1.5 \times 10^{-41}$ (green diamonds) while $\langle\Lambda\rangle \simeq 10^{-8}$ (red circles).

$$\Lambda_{50\%} \sim 10^{-1.1h^{2,1}} \text{ while } \langle\Lambda\rangle \simeq 10^{-8}$$

The Supersymmetric KKLT Case



$$|\Lambda|_{median} = |\Lambda|_{50\%} \sim 10^{-0.82h^{2,1}+2.7} \text{ while } \langle|\Lambda|\rangle \sim 10^{-3}.$$

$h^{2,1}$	1	5	10	15	20	25
Probability	0.897	0.981	0.984	0.989	0.990	0.994

Table: The probability of having a positive Hessian $(\partial_i \partial_j V)$ at $h^{2,1} = 1, 5, 10, 15, 20, 25$. The probability is approaching unity as $h^{2,1}$ increases.

Summary of the Picture

- Peaking of $P(\Lambda)$ at $\Lambda = 0$ happens for both Λ^+ and Λ^-.
- Introducing "multi-complex structure moduli" into the racetrack potential for a single Kähler modulus can yield a vanishingly small Λ.
- More "Kähler moduli" (in Swiss Cheese type) does not seem to change the picture much.

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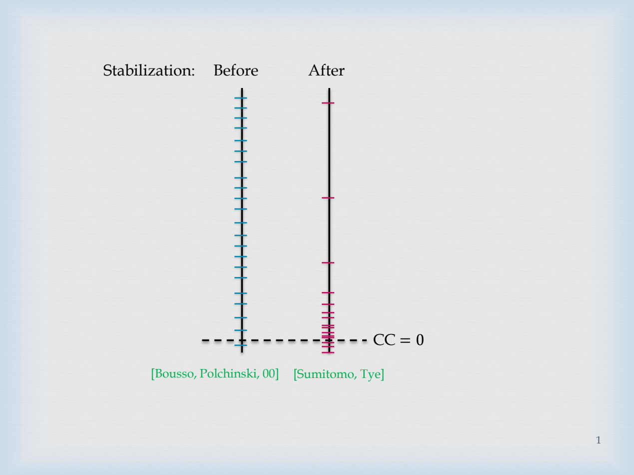
- Introducing "multi-complex structure moduli" into the racetrack potential for a single Kähler modulus can yield a vanishingly small Λ.
- More "Kähler moduli" (in Swiss Cheese type) does not seem to change the picture much.
- At high vacuum energies, hardly any meta-stable vacua exist. Most vacua accumulate around $\Lambda = 0$.

Rolling down after the inflationary epoch, our universe reaches the small positive Λ region before the small negative Λ region.



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Technical challenge :

When we simplify the model too much, the moduli are not coupled to each other so $P(\Lambda)$ does not peak at $\Lambda = 0$. On the other hand, when we include more couplings, the meta-stable vacua can be found only numerically; so it is difficult to find $P(\Lambda)$ for high $h^{2,1}$.