Discrete Gauge Symmetries, Tachyon Condensation, and Topology of Extra Dimensions

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Discrete gauge symmetries from (closed string) tachyon condensation, by M. Berasaluce-González, M.Montero, A.R. & A. Uranga

String Pheno '13, DESY, 16th July 2013

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CSIC Consejo Superior de Investigaciones Científicas

Motivation

- *Discrete* symmetries necessary for model building
  - Phenomenologically motivated, but theoretically poorly understood
- Consistent theories of QG require symmetries to be *gauged*
- Usually discrete gauge symmetries arise as subgroups of broken continuous ones see e.g. [Berasaluce-Gonzalez, Ibanez, Soler, Uranga, '11]
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  - Example: breaking of U(1)s in intersecting D-brane models by Stückelberg couplings
- Anyway, many discrete symmetries not obviously embedded into continuous ones (e.g. large isometries of compact space)

Proposal: Extend theory to *supercritical* to make symmetries continuous, then truncate using *closed string tachyons*

How to build a Critical String Theory:

- Decide worldsheet SUSY level $(\mathcal{N}_L, \mathcal{N}_R)$ and put an arbitrary number of worldsheet (super)fields
- Worldsheet matter content fixed by consistency conditions. These show up differently depending on quantization approach.
- In BRST quantization the condition is (for example in Type II):

$$0 = \tilde{c}_{total} = c_{total} = c_{matter} + c_{ghosts} \qquad = \frac{3}{2}D - 15 \;\Rightarrow\; D = 10$$

- Build modular invariant partition function to obtain: GSO-like conditions, spectrum, symmetry groups...

How to build a SuperCritical String Theory: [Chamseddine, '92]

- Decide worldsheet SUSY level $(\mathcal{N}_L, \mathcal{N}_R)$ and put an arbitrary number of worldsheet (super)fields
- Couple worldsheet to a linear dilaton background: $\Phi = \Phi_0 + V_\mu X^\mu$
- Worldsheet matter content fixed by consistency conditions. These show up differently depending on quantization approach.
- In BRST quantization the condition is (for example in Type II):

$$0 = \tilde{c}_{total} = c_{total} = c_{matter} + c_{ghosts} + c_\Phi = \frac{3}{2}D - 15 + 6\alpha' V_\mu V^\mu$$

- Build modular invariant partition function to obtain: GSO-like conditions, spectrum, symmetry groups...

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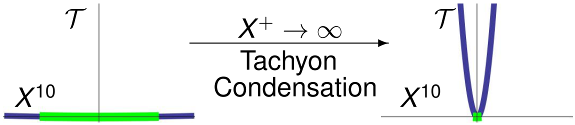
- All supercritical string theories have *closed string tachyons*.
- The tachyon coupling to the worldsheet results in a worldsheet potential
  For time dep. tachyons, theory is solvable [Hellerman et al]

$$\int d\sigma^2 \mathcal{T}(X) \;\Rightarrow\; \int d\sigma^2 \frac{\mu^2}{2\alpha'} \exp(\beta X^+)(X^{10})^2 \quad \text{(example in 10+1}D\text{)}$$



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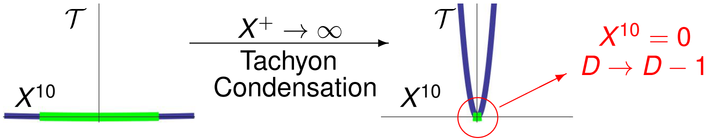
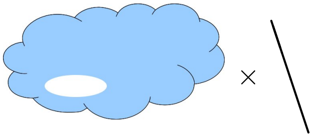


We want to exploit supercritical dimensions to make discrete symmetries continuous. We will use a construction based on...

The *Mapping Torus*:



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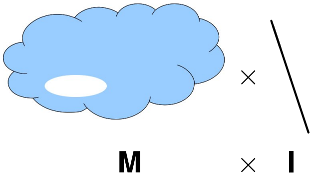

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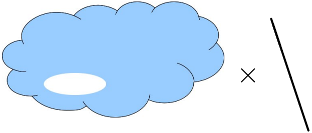

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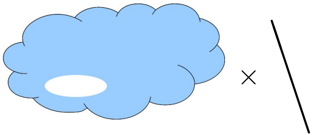

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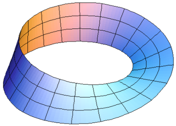
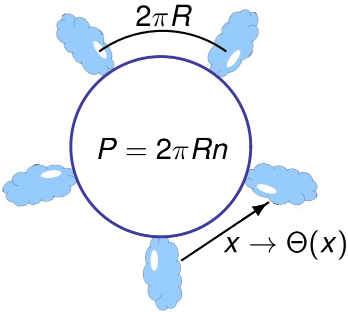
For $\mathbf{M} = [-1, 1]$ and $\Theta(x) = -x \longrightarrow$



Another way to see this ($\Theta^n = 1$):



- KK decomposition:

$$\Psi(x, y) = \sum_{k \in \mathbb{Z}} \Psi_k(x) e^{i \frac{k}{n} y}$$

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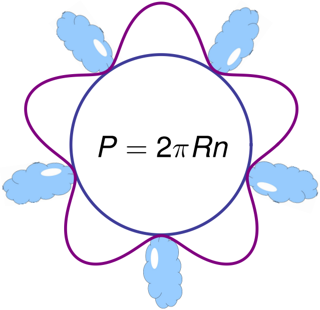

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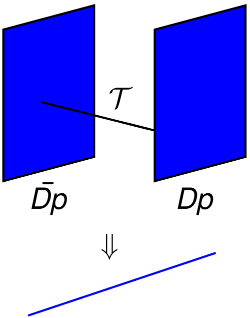
- Sum over disconnected theories
- Quantum states are:

$$|\alpha\rangle = \frac{1+\theta+\ldots+\theta^n}{n}|\alpha\rangle$$

- Break U(1) $\xrightarrow{\mathcal{T}}$ $\mathbb{Z}_n$ (abelian)

We are removing d.o.f... What do we gain???



First consider Type I/II with additional brane-antibrane pairs: [Sen '98]

Solitons of open string tachyon field

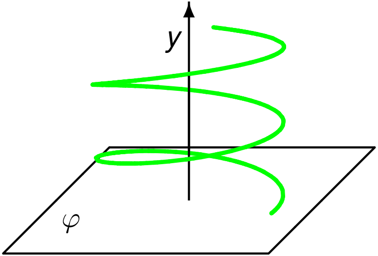
$\Downarrow$

Remnants after brane-antibrane anhilation

Going back to closed string tachyon condensation...

Construction of solitons (*Quenched fluxbranes*):

[Costa, Gutperle '01; Saffin '01; Gutperle, Strominger '01; Uranga '01]



1. $X^{D-1} + iX^{D-2} \equiv re^{i\varphi}$
2. For mapping torus identify
$$y \sim y + 2\pi R \quad ; \quad \varphi \sim \varphi + 2\pi/n$$
3. Consider Killing vector field
$$\mathbf{q} = \partial_y + \frac{1}{Rn}\partial_\varphi$$
4. Couple $\mathcal{T}$ to y' ($\mathbf{q}y' = 0$)

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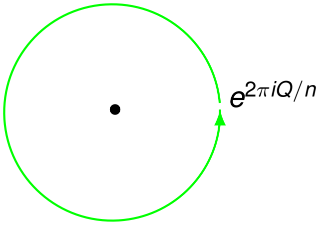
Charged under $\mathbb{Z}_n$!!!



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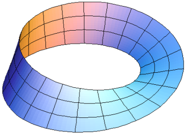
5. 1 orbit along $\mathbf{q} \Rightarrow \; e^{2\pi i Q/n}$

Examples

Spacetime parity

Bosonic supercritical in $26 + 1$D



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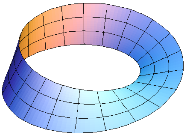

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A discrete $\mathbb{Z}_5$ isometry of the quintic

- $\mathbf{X}_6 = \mathbf{P}_5[5]$ at the Fermat point

$$z_1^5 + z_2^5 + z_3^5 + z_4^5 + z_5^5 = 0$$

- After mapping torus extension

$$wz_1^5 + z_2^5 + z_3^5 + z_4^5 + z_5^5 = 0$$

- $w \to e^{i\delta y} w \;\Rightarrow\; z_1 \to e^{-i\delta y/5} z_1$
- Break U(1) $\xrightarrow{\mathcal{T}} \mathbb{Z}_5$

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We want a theory $\mathbf{X}_N$ with non-abelian discrete gauge group Γ

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  1. Extend $\mathbf{X}_N \to \mathbf{X}_N \times \mathbf{Y}$
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- Instead, consider G to be a group action: a symmetry that became massive. G has an unbroken discrete subgroup Γ.
- The tachyon $\mathcal{T}$ just truncates to the critical slice.

Non-abelian example: magnetized torus

Let's build a theory $\mathbf{X}_N$ with:

- U(1) gauge symmetry
- Discrete gauge symm. with generators A, B s.t.

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1. Extend $\mathbf{X}_N \to \mathbf{X}_N \times \mathbf{T}^2$
   - A, B generate (continuous) translations along U'(1)s of $\mathbf{T}^2$
   - $[A, B] = C \in \mathrm{U}(1) \Rightarrow$ U(1) magnetic field ON: This U(1) breaks KK $[\mathrm{U'(1)} \times \mathrm{U'(1)}] \to \mathrm{H}_n \rtimes \mathrm{U}(1)$
2. $\mathcal{T}$ truncates theory to critical (breaks no symmetry)

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- After tachyon condensation, critical theory contains charged defects: solitons of closed string tachyon condensation

Thank you :)