SO(10) group in F-theory Phenomenology

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2012: a diverse subject covering many directions.

Many ways to engineer Standard Model-like theories in string theory

$SU(3)_c \times SU(2) \times U(1)_Y$ with 3 families can be obtained in brane intersections

What about going to higher GUT energies (10^{16} GeV)?

Impose the condition of Built-in GUT Unification: we can get one of the rank 4,5 or 6 groups: SU(5), SO(10), E_6

Question: SU(5), SO(10), E_6 perturbative Open Strings (D-branes)?

Answer is NO, for various reasons in the case of each group:

1) E_6 (or any other exceptional group) cannot be obtained with D-brane configurations.

2) SO(10): quarks and leptons sit in $\mathbf{16}$ which is a spinor representation

3) $SU(5)$ in D-brane pictures, it has two big problems:

(a) Does not exclude the Proton Decay which is is in disagreement with the life expentancy for the proton

(b) D-branes can't describe the up-type quarks Yukawa couplings

Solution: embed them into at least E_7 group appearing naturally in Heterotic, Fheory, M-theory

We need to describe the breaking of E_8 to $SU(5)_{GUT}$ or $SO(10)_{GUT}$.

Focus on SO(10) GUT group ($SU(5), E_6$ in Sakura talk)

• One advantage: The matter representations can enter a single representations of some larger group of symmetries, containing SU(5) as a subgroup.

Minimal Requirements

• SO(10) GUT group is a SUSY extension of the Standard Model of Particle Physics

• Unify $SU(3) \times SU(2) \times U(1)$ into $SO(10)$

• we have 3 generations of

$16_M = (q, u^c, e^c, d^c, l, \nu_R)$

• contains the Right handed neutrino ν_R useful for Seesaw mechanism

This is an improvement as compared to $SU(5)$ where the right handed neutrino were introduced as complex structure moduli.

Question: How can one see the splitting of moduli when going from $SO(10)$ to $SU(5)$?

• When answering this question one can also address :

The breaking pattern $SO(10) \to SU(5) \times U(1)$ allows for an additional neutral gauge boson Z , which could be almost as light as the SM gauge bosons, e.g., at the TeV scale

• One needs Higgs particles for particle masses. What representations for the Higgs field?

$16 \times 16 = 10 + 120 + 126$ so Higgs can be in $10, 120, 1\bar{2}6$

• The simplest choice is when Higgses H are in 10 representation.

- Yukawa couplings are $f_{ij} 16_i 16_j 10_H$.

- This leads to similar equalities as in $SU(5)$ case between the masses of down quarks and charged leptons which:

$m_b = m_\tau$ but fails for the first 2 families $m_s = m_\mu, m_d = m_e$.

- One way to solve this paradox is to consider the 126 HIggs field: 1616126_H which would introduce a see-saw mass for the RH neutrinos

- Problem in String Theory: how to include the 126 Higgs?

• hep-th/9604112 (March-Russell and Dienes) : free-field heterotic string models can't give a massless 126 representation of SO(10) for level 1.

One needs to go to higher levels to get an aswer in Heterotic strings.

• What about 126 in F-theory?

One can have:

• F-theory models dual to Heterotic String

• F-theory which are not dual to Heterotic string

For the first type, we use the Heterotic/F-theory duality to map vector bundles from heterotic strings into F-theory

In Heterotic string, the representations of SU(5) are obtained from the representation of the vector bundle which breaks E_8 to the GUT group.

For SU(5) GUT group, we need an SU(5) bundle and the correspondence between the representations $\rho(V)$ and and those of the unbroken $SU(5)_{GUT}$ is:

$V \leftrightarrow 10$ (V is the 5 of vector bundle SU(5))

$\wedge^2 V^* \leftrightarrow 5$ ($\wedge^2 V$ is the 10 of vector bundle SU(5))

For SO(10), we need an SU(4) bundle and the correspondence between the representations $\rho(V)$ and and those of the unbroken $SO(10)_{GUT}$ is:

$V \leftrightarrow 16$

$\wedge^2 V^* \leftrightarrow 10$

There is no correspondent for 126 so it is not clear what 126_H would be in the heterotic or F-theory picture for these models.

Concentrate on the minimal $SO(10)$ model with only the $f_{ij} 16_i 16_j 10_H$ coupling.

F-theory Picture

Compactify F-theory on an elliptic fibered Calabi-Yau 4-fold

$\pi : X \to B_3$.

We assume that X is given by Weierstrass equation:

$y^2 = x^3 \ + \ x \ f + \ g$

The gauge groups obtained on branes are A_n, D_n, E_n groups.

A_n groups are obtained with stacks of multiple D7 branes of the same (p,q) type.

D_n groups are obtained with stacks of D7 branes and lifts of O7 planes.

E_n groups can only be obtained by overlaps of different (p,q) 7-branes.

Local F-theory phenomenology approach (Beasley, Heckmann, Vafa;Donagi-Weijnholt

Focus on effective 4D $N = 1$ Super-Yang-Mills theory on divisor S

Many aspects of gauge theory associated with the discriminant locus S, only on the geometry of X around S.

The matter multiplets only sees the geometry along the codimension-2 loci of B_3

The Yukawa couplings are obtained from codimension-3 loci of B_3.

Let us considered the SU(5) model (A_4 singularity) and go to the codimension 3 singularities.

This is supposed to be related to an "E_6 singularity" (at local level)

Unbroken SU(5) gauge symmetry - elliptic fibered manifold

$$y^2 = x^3 + a_5 yx + a_4 zx^2 + a_3 z^2 y + (a_2 z^3 + f_0 z^4)x + (a_0 z^5 + g_0 z^6), ,$$

where (x, y) are coordinates for the elliptic fiber of the elliptic fibration

z is the coordinate normal to the discriminant locus S of the GUT gauge group.

$a_{0,2,3,4,5}$ are functions of local coordinates of S.

For SU(5) GUT models - The discriminant of this elliptic fibration is given by

$$\Delta \propto z^5 \left(\frac{1}{16} a_5^4 P^{(5)} + \frac{z}{16} a_5^2 \left(12 a_4 P^{(5)} - a_5^2 R^{(5)} \right) + z^2 \left(a_3^2 a_4^3 + \mathcal{O}(a_5) \right) + z^3 \left(\frac{27}{16} a_3^4 + \mathcal{O}(a_5) \right) + \mathcal{O}(z^4) \right) (1)$$

$z = 0$ is the locus of $SU(5)$ GUT gauge fields (codimension-1 singularity in a base 3-fold)

Two matter curves (codimension-2 singularities):

$a_5 = 0$: $SU(5)$-$\mathbf{10} + \overline{\mathbf{10}}$ representations are localized

$P^{(5)} = a_0 a_5^2 - a_2 a_5 a_3 + a_4 a_3^2 = 0$: $\mathbf{5} + \bar{\mathbf{5}}$ representations are localized.

$a_5 = 0$: singularity in (x, y, z)-surface is "enhanced" from A_4 to "D_5"

"D_5" means that it could be the genuine D_5 or something else

$P^{(5)} = 0$:singularity in (x, y, z)-surface is "enhanced" from A_4 to "A_5"

"A_5" means that it could be the genuine A_5 or something else

$\Delta \propto z^7$ when $a_5 = 0$ (as in D_5 singularity)

$\Delta \propto z^6$ for $P^{(5)} = 0$ (as in A_5 singularity).

There are isolated codimension-3 singularities along the matter curves.

On a_5 curve:

– type (a): common zero of a_5 and a_4,

– type (d): common zero of a_5 and a_3.

On the $P^{(5)} = 0$ curve they are at:

– type (c1): common zero of $P^{(5)}$ and $R^{(5)}$ but $a_5 \neq 0$, with

$$R^{(5)} \equiv \left(a_2 - 2\frac{a_3}{a_5}a_4\right)^2 - a_5^2\left(\left(\frac{a_3}{a_5}\right)^3 + f_0\left(\frac{a_3}{a_5}\right) - g_0\right) \quad (2)$$

In the local picture, when zooming towards the gauge divisors, the deformations of genuine E_6, D_6 and A_6 singularity, respectively, to A_4 are good approximation of local geometry of each of the three types of codimension-3 singularities above.

In this limit, field theory local model with E_6, $SO(12)$ and $SU(7)$ gauge groups can be used to analyze physics localized at these types of codim.-3 singularities.

Type (a) Singularity: $"E_6" \to A_4$

The type (a) codimension-3 singularity is generated the Yukawa couplings of the form

$$\Delta W = \mathbf{10}^{ab}\mathbf{10}^{cd}\mathbf{5}^{e}\epsilon_{abcde} \tag{3}$$

The most generic deformation of E_6 singularity

$$Y^2 = X^3 + X(\epsilon_2 Z^2 + \epsilon_5 Z + \epsilon_8) + \left(\frac{Z^4}{4} + \epsilon_6 Z^2 + \epsilon_9 Z + \epsilon_{12}\right). \tag{4}$$

When $\epsilon_{2,5,8}$ and $\epsilon_{6,9,12}$ are zero - E_6 singularity.

$\epsilon_{2,5,8}$ and $\epsilon_{6,9,12}$ are functions of local coord. u_m $(m = 1, 2)$ on S.

To preserve SU(5) unbroken symmetry, the deformation is parametrized by two complex numbers for a given point on S, a_4 and a_5.

By zooming and rescaling, we could see that the deformed E_6 geometry maps into the A_4 singular geometry.

We can choose E_6 as the gauge group of field theory local model for the geometry around the type (a) codimension-3 singularity,

The deformation can be seen by studying the vev of the Higgs field in the directions orthogonal to the D7 branes.

This works for diagonal Higgs field. Other consideration: non-diagonal Higgs field which leads to T-branes.

Consider now going beyond the local version of the geometries. The question would be is if we get from one type of Kodaira fibre to another one if we go in higher codimension.

There are several way to do this:

a) using only blow-ups like in Sven Krause, Christoph Mayrhofer, Timo Weigand (1109.3454) based on Grimm-Weigand 1006.0226

b) using blow-ups plus small resolutiong with consideration of several patches Esole-Yau 1107.0733

c) using blow-ups and small resolutions with single patch Marsano and Schafer-Nameki 1108.1794

In this talk: b) $+$ c)

Question: does the field theory enhancing of singularity survive in the global model?

Instead of working with a singular CY 4-fold, they resolved the singularities along the gauge divisors and then took to 0 various parameters appearing in the geometry.

To resolve the singularities they used two P^2 blow-ups and 2 small resolutions.

The first two P^2 blow-ups give rise to a smooth geometry in case a_3, a_4, a_5 are all different from zero

In the case some of the a_i coefficents or the corresponding combinations are 0 (as before), the geometry becomes an affine binomial variety

$x\ y = u\ v\ t$

where y and t depend on the coefficients a_i.

This is similar to the well-known conifold singularity $xy = uv$ which is resolved with one P^1 cycle.

In the case of the binomial variety, one needs two P^1 cycles.

The conclusion of 1107.0733: E_6 Kodaira fiber is not recovered.

Nevertheless, this is not unexpected as it was not neccesary to get E_6

Actually there are two ways of making the blow-ups:

1) blow up 4 times (with 2 P^2 and small resolutions) and then take the limits when some of the a_i coefficents or the corresponding combinations are 0.

2) blow up the two P^2 cycles, then take the limits when some of the a_i coefficients or the corresponding combinations are 0 and then perform the small resolutions

For A_4 singularity, going the route 1) or 2) reaches the same result.

Going the route 1), one starts with four 2-cycles: $C_{1\pm}, C_{2\pm}$ when a_3, a_4, a_5 are all different from zero and obtain $C_{3\pm}, C_{4\pm}$ when some a_i are zero.

The relation between $C_{1\pm}, C_{2\pm}$ and $C_{3\pm}, C_{4\pm}$ show us how the cycles are redistributed.

Going the route 2), one needs to keep trak of cycles who shrink and then perform new resolutions and study the intersections between the old and new cycles.

For D_5, going the route 1) is clear, going the route 2) is less clear. For higher groups, going the route 2) becomes harder.

The problem: higher Dynkin diagrams?

Possible explanation: a_3, a_4, a_5 are all complex deformations of the geometry. Deforming singularities by taking combinations of a_3, a_4, a_5 to zero and then resolving might not be the same as doing the other way around.

Similar situation in the resolutions of $N = 2$ singularities and then $N = 2 \to N = 1$ by a superpotential for the adjoint fields.

$W = a_{ij} \Phi_i^j$

Even when considering the case 1) for the blow-up, the disadvantage of Esole-Yau method is that one needs to consider 3 patches at every step of the resolution.

This can become cumbersome for higher singularities when trying to identify the recombination of cycles or their intersection ($SO(10), E_6$)

Marsano and Schafer-Nameki (1108.1794): consider a global picture where one looks at a single patch for every step of the resolution.

Look at the vanishing of various section inside some CY 4-fold.

The method clearer to follow.

The success of this method consisted in showing that one does not need to think of enhacing the singularity but one should look at the weights of representations.

This is why one needs to use "E_6" or "E_7".

- Consider the roots of the affine A_4: $\alpha_i, i = 1, \cdots, 5$

- Codimension 2 singularity - over the 10 matter locus $a_5 = 0$

$$-\alpha_2 \to (\mu_{10} + \alpha_1 + \alpha_2 + \alpha_3) + (\mu_{10} - \alpha_1 - 2\alpha_2 - \alpha_3)$$

$$-\alpha_4 \to (\mu_{10} + \alpha_1 + \alpha_2 + \alpha_3) + (\mu_{10} - \alpha_2 - \alpha_3 - \alpha_4) + (-\alpha_1)$$

To each α_i one associates a divisor $D_{-\alpha_i}$ related to vanishing of some section in the global geometry.

Their intersection gives rise to the extended SU(5) Dynkin diagram

After the splitting, one gets representations of $SU(5)$.

How to apply to $SO(10)$ - D_5 singularity?

We expect 16 from "E_6" enhancement, 10 from "D_6" enhancement and 16 16 10 from "E_7" enhacement (again, these are field theory indications)

We construct the resolution in the auxiliary 5-fold X_5 - P^2 bundle

σ is inherited from the hyperplane of the P^2 fiber.

The Tate form for an $SO(10)$ singularity at $z = 0$ is

$$y^2 w + b_1 zxyw + b_3 z^2 yw^2 = x^3 + b_2 zx^2 w + b_4 z^3 xw^2 + b_6 z^5 w^3 \,. \quad (5)$$

The resolution of the singularity is obtain with with three blowups and two small resolutions:

$$x = \zeta x_1 \,, \qquad y = \zeta y_1 \,, \qquad z = \zeta z_1$$

where $\zeta = 0$ gives rise to an exceptional divisor E_1.

$$x_1 = x_2\alpha\,, \qquad y_1 = y_2\alpha\,, \qquad \zeta = \zeta_2\alpha\,.$$

The section $\alpha = 0$ gives rise to an exceptional divisor E_2.

Third blow up along $y_2 = \zeta_2 = \alpha = 0$, which we do by setting

$$y_2 = y_3\beta\,, \qquad \zeta_2 = \zeta_3\beta\,, \qquad \alpha = \alpha_3\beta\,.$$

The section $\beta = 0$ gives rise to a new exceptional divisor E_3.

The fourth blow up is along $y_3 = \zeta_3 = 0$ and we do this by setting $y_3 = y_4\delta_4, \qquad \zeta_3 = \zeta_4\delta_4$

The fifth blow up is along $y_4 = \alpha_3 = 0$ and is $y_4 = y_5\delta_5, \alpha_3 = \alpha_5\delta_5$

The sections $\delta_4 = 0$ and $\delta_5 = 0$ give rise to new divisors E_4 and E_5 .

The section $z = 0$, where the D_5 singularity is located, splits as

$$z = z_1\zeta_4\alpha_5\beta^2\delta_4\delta_5 = 0$$

The Cartan divisors are these six factors restricted to the resolved 4-fold $\tilde{Y}_4$, and are given by

$$
\begin{array}{c|c|c}
\text{Cartan Divisor} & \text{Component} & \text{Class in } Y_4 \\
\hline
\mathcal{D}_{-\alpha_0} & (z_1 = 0)|_{Y_4} & S_2 - E_1 \\
\mathcal{D}_{-\alpha_1} & (\delta_4 = 0)|_{Y_4, \zeta_4 \neq 0} & -E_1 + E_2 + E_3 + 2E_4 \\
\mathcal{D}_{-\alpha_2} & (\zeta_4 = 0)|_{Y_4} & E_1 - E_2 - E_3 - E_4 \\
\mathcal{D}_{-\alpha_3} & (\beta = 0)|_{Y_4} & E_3 \\
\mathcal{D}_{-\alpha_4} & (\delta_5 = 0)|_{Y_4} & E_5 \\
\mathcal{D}_{-\alpha_5} & (\alpha_5 = 0)|_{Y_4} & (E_2 - E_3 - E_5)
\end{array} \tag{6}
$$

whose intersection of the Cartan divisors is exactly the Cartan matrix of the extended $SO(10)$.

Therefore the codimension 1 discussion involving gauge fields is correct.

Now move to matter and Yukawas couplings.

The discriminant of the $SO(10)$ singularity has an expansion

$$\Delta = -16 z^7 b_2^3 b_3^2 r + \Big(-27 b_3^4 - 8 b_1^2 b_2^2 b_3^2 + 72 b_2 b_4 b_3^2 + 4 b_1 b_2$$
$$(9 b_3^2 + 4 b_2 b_4 b_3 + 16 b_2^2 \Big(b_4^2 - 4 b_2 b_6)) \Big) z^8 + O(z^9)\,.$$

In summary, we would expect the following enhancements:

$''D_6'':\qquad b_3 = 0,''\,E_6:''\qquad b_2 = 0$

$''E_7:''\qquad b_2 = b_3 = 0,''\,D_7:''\qquad b_3 = b_4^2 - 4b_2b_6 = 0$

• We expect to get matter in the 16 of $SO(10)$ along $z = b_2 = 0$.

Look at one specific component of $z = 0$, namely $\beta = 0$.

$\beta = b_2 = 0 \to y_5 \delta_4 (y_5 \delta_5 + b_3 \zeta_4) = 0$

so it reduces to three components

$$[\beta] \cdot [b_2] = [\beta] \cdot [y_5] + [\beta] \cdot ([\delta_4] - [\zeta_4]) + [\beta] \cdot ([b_2] - [y_5] - [\delta_4] + [\zeta_4])$$

Second component is $[\beta]\cdot([\delta_4]-[\zeta_4])$ since the first Cartan divisor restricted to b_2 gives $\beta = 0$.

$z = 0$ splits into 7 components along $b_2 = 0$

The splitting of the weight associated to the third root is

$$-\alpha_3 = (0, 1, -2, 1, 1) \to (-2, 1, 0, 0, 0)+(1, 0, -1, 1, 0)+(1, 0, -1, 0, 1)$$

(1,0,-1,1,0) corresponds to $-(\mu_{\mathbf{16}} - \alpha_1 - \alpha_2 - \alpha_3 - \alpha_4 - \alpha_5)$

(1,0,-1,0,1) corresponds to $\mu_{\mathbf{16}} - \alpha_2 - 2\alpha_3 - \alpha_4 - \alpha_5$,

Result: instead of having the E_6 enhacement, we obtain a splitting involving weights of the representation $\mathbf{16}$ for SO(10).

- Matter in the 10 representation

We expect to get matter in the $\mathbf{10}$ along $z = b_3 = 0$.

Look at another specific component of $z = 0$, namely $\delta_5 = 0$ which splits into 7 components, as one would expect from a "D_6" enhancement.

The $\delta_5 = 0$ root splits as

$$(0,0,1,-2,0) \to (0,0,1,0,-2) + (0,0,0,-1,1) + (0,0,0,-1,1)$$

The first component is a Cartan divisor, but the other two are both given by $\mu_{\mathbf{10}} - \alpha_1 - \alpha_2 - \alpha_3 - \alpha_4$

Result: instead of E_6 enhacement, split involving weights of the representation $\mathbf{10}$ for SO(10).

• Yukawa Couplings

Yukawa interaction - "E_7" enhancement which is given by $b_2 = b_3 = 0$.

This corresponds to the Yukawa coupling $16 \times 16 \times 10$ as can be seen from the splitting of the divisors involving both μ_{10} and μ_{16}

The "E_7" enhancement has only 7 components instead of the expected 8, this is similar to what was shown to happen with E_6 in Esole-Yau

• D7 enhancement

We expect to get a "D7" enhancement at $b_3 = b_4^2 - 4b_2b_6 = 0$.

One sees see that the two previously separate $\mathbf{10}$ matter curves become one, we believe that this corresponds to a $\mathbf{10} \times \mathbf{10} \times \mathbf{1}$ coupling, we do not see a curve for the singlet as it is not part of the GUT divisor.

Now we turn to discuss the G-flux

Remember: the existence of the (2,2) form G-flux is a requirement of the heterotic - F theory duality.

Local geometries - G-flux can be constructed from Heterotic string data in terms of spectral covers.

Global geometries -we use the approach Marsano, Schafer-Nameki et al.

The global approach was successfully applied to the case of A_4 singularities

We follow the same path for the SO(10) group.

Our original 4-fold Y_4 is given by

$$y^2 w + b_1 zxyw + b_3 z^2 yw^2 = x^3 + b_2 zx^2 w + b_4 z^3 xw^2 + b_6 z^5 w^3$$

Tate divisor: $wz(b_2 x^2 + b_4 z^2 xw + b_6 z^4 w^2 - b_1 xy - b_3 zyw)$

it can be rewritten in terms of the $t = y/x$ as $z(b_2 t^4 + b_4 z^2 t^2 + b_6 z^4 -$

$b_1 t^5 - b_3 z t^3$

$s = z/t$ and holding s fixed in the limit $t \to 0$, $z \to 0$:

$$st^5 \left(b_2 + b_4 s^2 + b_6 s^4 - b_3 s \right)$$

For the resolved Calabi-Yau $\tilde{Y}_4$, the total transform of the Tate divisor $wy^2 - x^3 = 0$ is

$$\zeta_4^2 \alpha_5^4 \beta^8 \delta_4^3 \delta_5^6 \left(w y_5^2 \delta_4 - x_2^3 \zeta_4 \alpha_5^2 \beta \right) = 0 \tag{7}$$

$w y_5^2 \delta_4 - x_2^3 \zeta_4 \alpha_5^2 \beta = 0$ is the proper transform of the Tate divisor, which we then restrict to the resolved $\tilde{Y}_4$.

The proper transform of the Tate divisor is reducible, with components given by $\zeta_4 = 0$, $z_1 = 0$, and the remainder.

To see this, set $\zeta_4 = 0$.

We cannot have $w = 0$, $y_5 = 0$, $\alpha_5 = 0$ or $\delta_5 = 0$, so we set these equal 1.

The Tate divisor equation is now $\delta_4 = 0$ and the equation for the resolved $\tilde{Y}_4$ also becomes $\delta_4 = 0$ so Tate divisor equation is automatically satisfied.

Its intersection with the Cartan divisors takes the form:

$$\mathcal{C}_{\mathsf{Tate}} \cdot_{\tilde{Y}_4} \Sigma\alpha_i = (0,0,0,1,0) \times 4\,. \tag{8}$$

Local limit: only $\delta_5 \to 0$ is allowed , giving the required spectral equation $\mathrm{b}_6 y_5^4 + b_4 y_5^2 - b_3 y_5 + b_2 = 0$

The local limit yield the the Higgs bundle description - consistency check.

Quantisation condition :

$G + \frac{1}{2}c_2(\tilde{Y}_4)$ takes values in $H^4(\tilde{Y}_4, Z)$

The condition holds for $SU(5)$ or E_6

Extra constraints on the base B_3 are needed for the $SO(10)$ case. Why?

This was pointed out in Kuntzler, Schafer-Nameki (1205.5688)

Conclusions

- SO(10) GUT needs more study
- Understanding better resolution the singularities for Calaby Yau 4-fold
- Try to introduce Higgs in 126 representation
- Understand the additional contraints impose on the base B_3