Supersymmetric unification and R symmetries

TECHNISCHE UNIVERSITÄT MÜNCHEN

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Excellence Cluster Universe

Bad Honnef, October 5, 2012

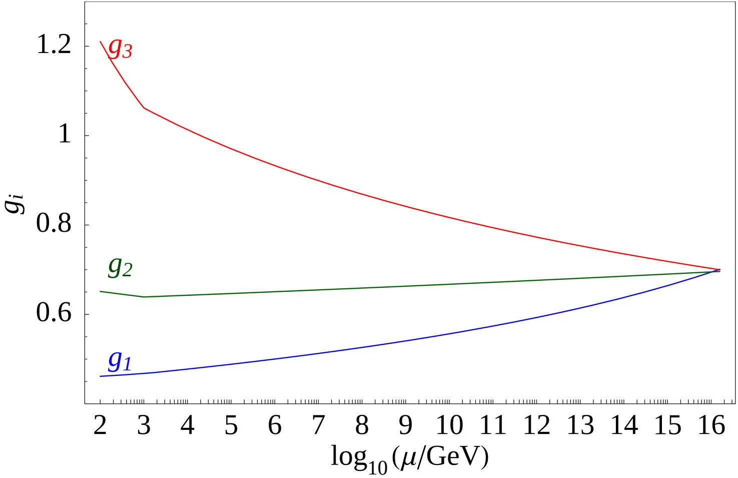
Based on:

- H.M. Lee, S. Raby, G. Ross, M.R., R. Schieren, K. Schmidt–Hoberg & P. Vaudrevange, Phys. Lett. **B** 694, 491-495 (2011) & Nucl. Phys. **B** 850, 1-30 (2011)
- R. Kappl, B. Petersen, S. Raby, M.R., R. Schieren & P. Vaudrevange, Nucl. Phys. **B** 847, 325-349 (2011)
- M. Fallbacher, M.R. & P. Vaudrevange, Phys. Lett. **B** 705, 503-506 (2011)
- M.–C. Chen, M.R., C. Staudt & P. Vaudrevange, Nucl. Phys. **B** 866, 157–176 (2013)
- M. Fischer, M.R., J. Torrado & P. Vaudrevange, arXiv:1209.3906

Gauge coupling unification in the MSSM

- Running couplings in the (minimal) supersymmetric standard model (MSSM)

Dimopoulos et al. (1981)



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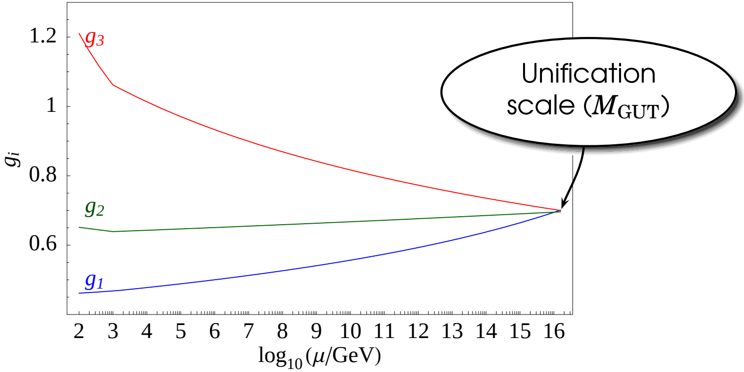

- Gauge coupling unification might be a consequence of $G_{\mathrm{SM}} = \mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1) \subset \mathrm{SU}(5)$

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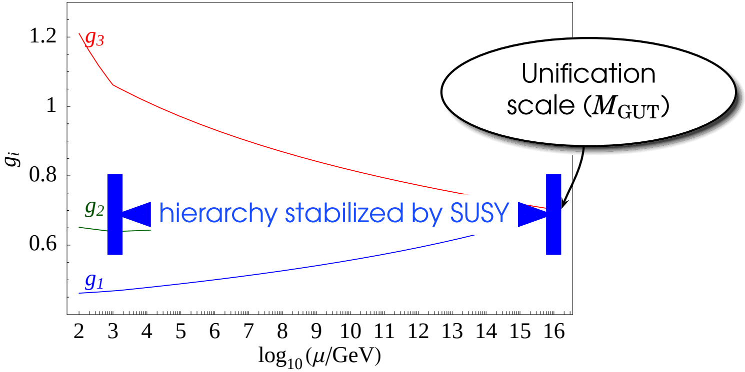

- Gauge coupling unification might be a consequence of $G_{\mathrm{SM}} = SU(3) \times SU(2) \times U(1) \subset SU(5)$

SU(5) and SO(10)

$\mathrm{SU}(5)$ grand unified theories (GUTs) ...

- ☞ explain charge quantization
- ☞ simplify matter content

SM generation $=$ $\mathbf{10}+\overline{\mathbf{5}}$

further simplification of matter sector

Fritzsch & Minkowski (1975)

$$\begin{aligned} \mathrm{SO}(10) & \supset \mathrm{SU}(5) \\ \mathbf{16} & = \mathbf{10}\oplus\overline{\mathbf{5}}\oplus\mathbf{1} \\ & = \text{SM generation with `right–handed' neutrino} \end{aligned}$$

- ☞ One of the main assumptions in this talk: this is not an accident

Outline

1. Introduction & Motivation ✓
2. Anomaly–free discrete symmetries & unification
   - anomaly cancellation
   - consistency with unification
   - unique $\mathbb{Z}_4^R$ symmetry
   - no–go theorems in 4D
3. String model(s)
4. Summary

Anomaly–free

discrete symmetries

and

grand unification

- anomaly cancellation
- consistency with unification
- unique $\mathbb{Z}_4^R$ symmetry
- no–go theorems in 4D

Prejudices and assumptions

Assumptions:

Chen et al. (2012)

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Important ingredient :

☞ Green–Schwarz anomaly cancellation

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Anomaly constraints in models with non–accidental gauge coupling unification :

Anomaly freedom
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$$\left.\begin{array}{c}\text{Anomaly freedom}\\ +\\ \text{Grand unification}\\ +\\ \text{Green–Schwarz}\\ \text{anomaly cancellation}\end{array}\right\} \to \text{“Anomaly universality”}$$

Anomaly freedom

- ☞ Universality condition $A_{G_i-G_i-\mathrm{U}(1)_{\mathrm{anom}}} = \rho$

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- However, a may have different couplings c_i to different field strengths of the SM gauge group

$$\mathscr{L}_\mathrm{axion} \supset \sum_i c_i \frac{a}{8} F_i^b \widetilde{F}_i^b$$

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however:

- different c_i are inconsistent with an underlying GUT symmetry
- a non–trivial VEV of the scalar partner of a will destroy gauge coupling unification

Anomaly universality for discrete symmetries

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- Example: anomaly coefficients for $\mathbb{Z}_N$ symmetry

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\begin{aligned}
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sum over all representations of G

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sum over all fermions

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Dynkin index

discrete charges

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traditional anomaly freedom:

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$$\eta := \begin{cases} N & \text{for } N \text{ odd} \\ N/2 & \text{for } N \text{ even} \end{cases}$$

Ibáñez and Ross (1991)
Banks and Dine (1992)

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Anomaly–free symmetries, μ and unification

- ☞ Working assumptions:
  - (i) anomaly freedom (allow for GS anomaly cancellation)

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1. assuming (i) & $\mathrm{SU}(5)$ relations:
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3. R symmetries are not available in 4D GUTs

Non–R symmetries do not do the job

- Anomaly coefficients for non–R symmetry with SU(5) relations for matter charges

$$A_{\mathrm{SU}(3)^2-\mathbb{Z}_N} = \frac{1}{2}\sum_{g=1}^{3}\left(3q^g_{\mathbf{10}} + q^g_{\overline{\mathbf{5}}}\right)$$

$$A_{\mathrm{SU}(2)^2-\mathbb{Z}_N} = \frac{1}{2}\sum_{g=1}^{3}\left(3q^g_{\mathbf{10}} + q^g_{\overline{\mathbf{5}}}\right) + \frac{1}{2}\left(q_{H_u} + q_{H_d}\right)$$

charge of g^{th} $\mathbf{10}$–plet

charge of g^{th} $\overline{\mathbf{5}}$–plet

Higgs charges

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- Anomaly universality: $A_{\mathrm{SU}(2)^2-\mathbb{Z}_N} - A_{\mathrm{SU}(3)^2-\mathbb{Z}_N} = 0$

$$\curvearrowright \quad \frac{1}{2}\left(q_{H_u} + q_{H_d}\right) = 0 \mod \begin{cases} N & \text{for } N \text{ odd} \\ N/2 & \text{for } N \text{ even} \end{cases}$$

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bottom–line:

non–R $\mathbb{Z}_N$ symmetry cannot forbid μ term

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- Obvious: if anomaly–free discrete non–R symmetries cannot forbid the μ term, this also applies to continuous non–R symmetries

Only discrete R symmetries may do the job

- ☞ Obvious: if anomaly–free discrete non–R symmetries cannot forbid the μ term, this also applies to continuous non–R symmetries
- ☞ There are no anomaly–free continuous R symmetries in the MSSM

Chamseddine and Dreiner (1996)

Only discrete R symmetries may do the job

- Obvious: if anomaly–free discrete non–R symmetries cannot forbid the μ term, this also applies to continuous non–R symmetries

- There are no anomaly–free continuous R symmetries in the MSSM

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- Only remaining option: discrete R symmetries

't Hooft anomaly matching for R symmetries

Chen et al. (2012)

☞ Powerful tool: anomaly matching

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- Powerful tool: anomaly matching
- At the $\mathrm{SU}(5)$ level: one anomaly coefficient

$$A_{\mathrm{SU}(5)^2-\mathbb{Z}_M^R} = \underbrace{A^{\text{matter}}_{\mathrm{SU}(5)^2-\mathbb{Z}_M^R}}_{\text{matter}} + \underbrace{A^{\text{extra}}_{\mathrm{SU}(5)^2-\mathbb{Z}_M^R}}_{\text{extra}} + \underbrace{5q_\theta}_{\text{gauginos}}$$

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universal

SM gauginos

extra gauginos from X, Y bosons

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➥ Extra stuff must be non–universal (split multiplets)

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bottom–line:

't Hooft anomaly matching for (discrete) R symmetries implies the presence of split multiplets below the GUT scale!

SO(10) implies unique symmetry

Lee et al. (2011) ; Chen et al. (2012)

☞ Consider $\mathbb{Z}_M^R$ symmetry which commutes with $\mathrm{SO}(10)$
i.e. quarks and leptons have universal charge q

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- Consider $\mathbb{Z}_M^R$ symmetry which commutes with $\mathrm{SO}(10)$ i.e. quarks and leptons have universal charge q
- existence of u- and d–type Yukawas requires that

$$2q + q_{H_u} = 2q_\theta \mod M \quad \text{and} \quad 2q + q_{H_d} = 2q_\theta \mod M$$

R charge of superspace coordinate θ

superpotential has R charge $2q_\theta$

$$\int \mathrm{d}^2\theta\, \mathscr{W} \subset \mathscr{L}$$

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bottom–line:

$$q_{H_u} = q_{H_d} = 0 \mod M \;\&\; q = q_\theta \mod M$$

Unique $\mathbb{Z}_4^R$ symmetry

Lee et al. (2011) ; Chen et al. (2012)

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- We know already that $\begin{cases} \bullet\ q = q_\theta \\ \bullet\ q_{H_u} = q_{H_d} = 0 \mod M \end{cases}$
- Simplest possibility: $M = 4$ & $q = q_\theta = 1 \curvearrowright \mathbb{Z}_4^R$ symmetry

  $M = 2$ does not work since this is not an R symmetry

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- Alternatives: $\mathbb{Z}_{4m}^R$ symmetry with $q = q_\theta = m$ & $m \in \mathbb{N}$
- However: these are only trivial extensions (as far as the MSSM is concerned)

Unique $\mathbb{Z}_4^R$ symmetry

Lee et al. (2011) ; Chen et al. (2012)

- We know already that $\left\{ \begin{array}{l} \bullet\ q = q_\theta \\ \bullet\ q_{H_u} = q_{H_d} = 0 \mod M \end{array} \right.$
- Simplest possibility: $M = 4$ & $q = q_\theta = 1 \curvearrowright \mathbb{Z}_4^R$ symmetry
- Alternatives: $\mathbb{Z}_{4m}^R$ symmetry with $q = q_\theta = m$ & $m \in \mathbb{N}$
- However: these are only trivial extensions (as far as the MSSM is concerned)

bottom–line:

unique symmetry : $\mathbb{Z}_4^R$ w/ $q = q_\theta = 1$ & $q_{H_u} = q_{H_d} = 0$

first discussed in Babu et al. (2003)

Unique $\mathbb{Z}_4^R$ symmetry & GS anomaly cancellation

☞ Anomaly coefficients

$$\begin{aligned}
A_{\mathrm{SU}(3)^2-\mathbb{Z}_4^R} &= 6q - 3q_\theta \;=\; 1q_\theta \mod 4/2 \\
A_{\mathrm{SU}(2)^2-\mathbb{Z}_4^R} &= 6q + \frac{1}{2}\left(q_{H_u} + q_{H_d}\right) - 5q_\theta \;=\; 1q_\theta \mod 4/2
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bottom-line:

$\mathbb{Z}_4^R$ is anomaly–free via non–trivial GS mechanism

Implication of GS anomaly cancellation

- ☞ GS axion a contained in superfield S (w/ $S|_{\theta=0} = s + \mathrm{i}\,a$)

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- Main example

  $\mu\,H_u\,H_d$ forbidden

but

$B\,\mathrm{e}^{-b\,S}\,H_u\,H_d$ allowed (for appropriate b)

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bottom–line:

holomorphic $\mathrm{e}^{-b\,S}$ terms appear to violate $\mathbb{Z}_M^R$ symmetry

Interpretation

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$$\mathscr{L} \supset \int \mathrm{d}^2\theta \, f_S \, S \, W_\alpha W^\alpha$$

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bottom–line:

$$\left.\begin{array}{l}\bullet \text{ compatibility w/ } \mathrm{SO}(10)\\ \bullet \text{ anomaly freedom}\end{array}\right\} \curvearrowright \left\{\begin{array}{c}\mu \text{ term appears}\\ \text{non–perturbatively}\end{array}\right.$$

Implications of $\mathbb{Z}_4^R$

☞ Gauge invariant superpotential terms up to order 4

$$
\begin{aligned}
\mathscr{W} \;=\; & \mu\, H_d H_u + \kappa_i\, L_i H_u \\
& + Y_e^{ij}\, L_i H_d \overline{E}_j + Y_d^{ij}\, Q_i H_d \overline{D}_j + Y_u^{ij}\, Q_i H_u \overline{U}_j \\
& + \lambda_{ijk}\, L_i L_j \overline{E}_k + \lambda'_{ijk}\, L_i Q_j \overline{D}_k + \lambda''_{ijk}\, \overline{U}_i \overline{D}_j \overline{D}_k \\
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forbidden at the perturbative level

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appear at non–perturbative level

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also forbidden at
non–perturbative level by
“non–anomalous” $\mathbb{Z}_2$ subgroup
which is equivalent
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μ term from $\begin{cases} \text{Giudice–Masiero mechanism (optional)} \\ \text{holomorphic `non–perturbative' term} \end{cases}$

- order parameter for R symmetry breaking: superpotential VEV $\langle \mathscr{W} \rangle$

➥ $\mu \sim m_{3/2} \simeq \langle \mathscr{W} \rangle / M_\mathrm{P}^2$

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$$

non–perturbatively generated terms harmless
$\kappa_{ijk\ell}^{(1,2)} \sim m_{3/2}/M_\mathrm{P}^2 \ll 10^{-8}/M_\mathrm{P}$

R symmetries vs. 4D GUTs

☞ We have seen that only R symmetries can forbid the μ term

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  2. residual R symmetries

The basic argument

- ☞ Consider $\mathrm{SU}(5)$ model with an (arbitrary) R symmetry and a $\mathbf{24}$–plet breaking $\mathrm{SU}(5) \to G_{\mathrm{SM}}$

$$\mathbf{24} \;\to\; (\mathbf{8},\mathbf{1})_0 \oplus (\mathbf{1},\mathbf{3})_0 \oplus (\mathbf{3},\mathbf{2})_{-5/6} \oplus (\overline{\mathbf{3}},\mathbf{2})_{5/6} \oplus (\mathbf{1},\mathbf{1})_0$$

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- Loophole for infinitely many $\mathbf{24}$–plets

cf. Goodman and Witten (1986)

Generalizing the basic argument

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- singlet extensions of the MSSM

for details see Fallbacher et al. (2011)

Discussion

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➥ Need to go to extra dimensions/strings

String model(s)

- evading the no–go theorem
- origin of $\mathbb{Z}_4^R$
- higher–dimensional operators (effective μ term etc.)

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- KK towers provide us with infinitely many states and allow us to evade the no–go theorem
- Even more, R symmetries have a clear geometric interpretation in terms of the Lorentz symmetry of compact dimensions

Discrete R symmetries from orbifolds

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  and are arguably on the same footing as the fundamental symmetries C, P and T

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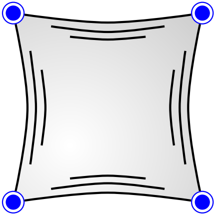

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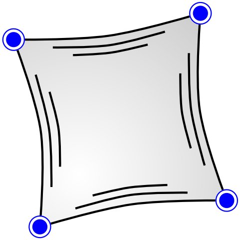

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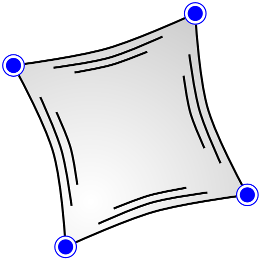

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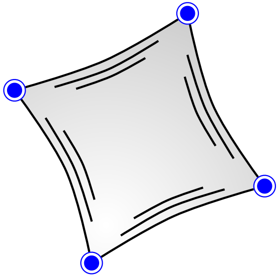

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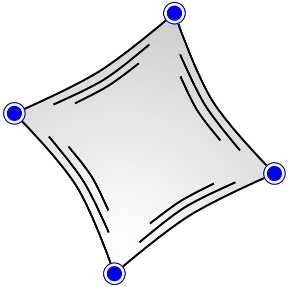

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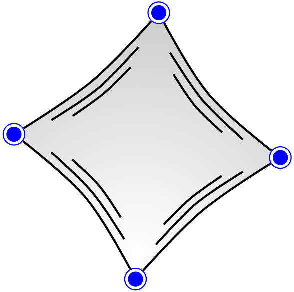

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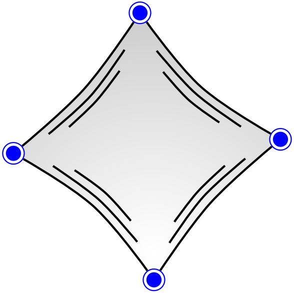

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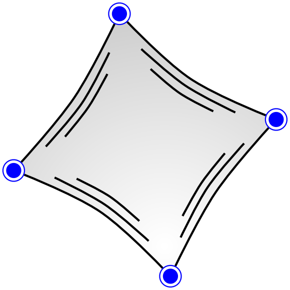

Discrete R symmetries from orbifolds

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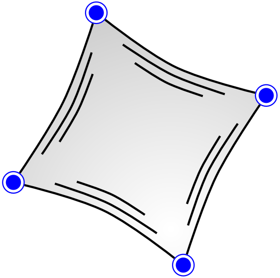

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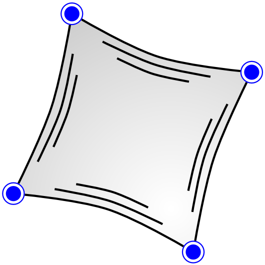

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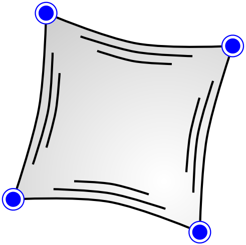

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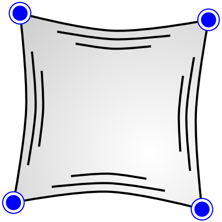

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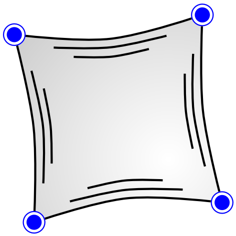

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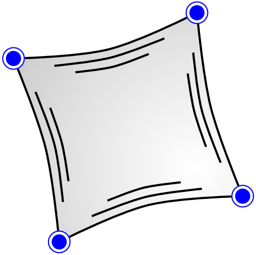

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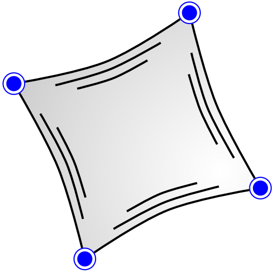

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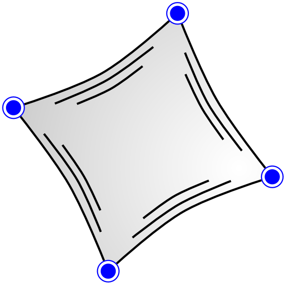

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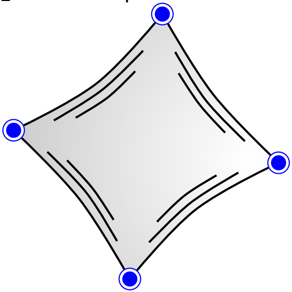

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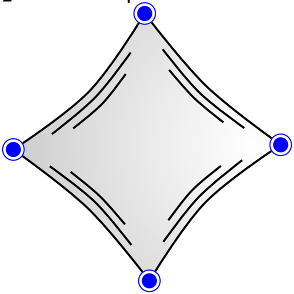

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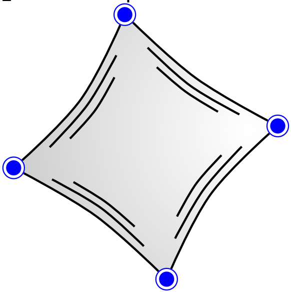

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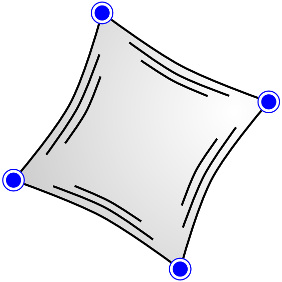

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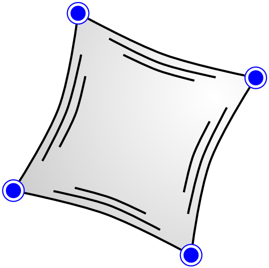

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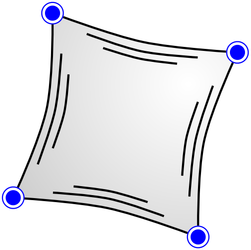

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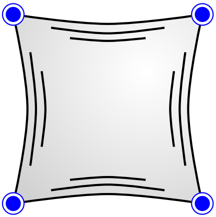

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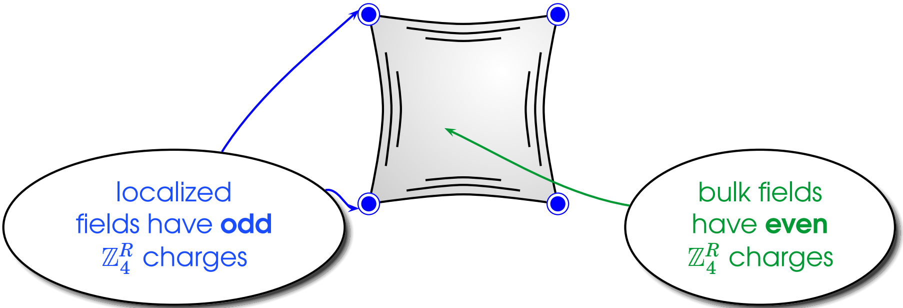
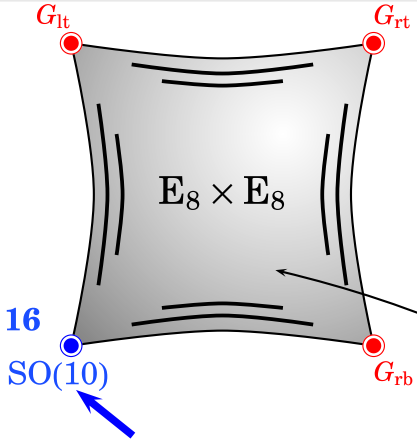


Local grand unification & $\mathbb{Z}_4^R$



$\mathbb{Z}_4^R$ (bottom–up)

	$\mathbb{Z}_4^R$ charge
matter	1
Higgs	0

SM generation(s):
localized in region with SO(10) symmetry

Higgs doublets:
live in the 'bulk'

$\mathbb{Z}_4^R$ from a $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold model

Blaszczyk et al. (2010) ; Kappl et al. (2011)

- We constructed models with the exact MSSM spectrum based on $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifolds

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'vacuum'
means a field
configuration
that preserves
supersymmetry
perturbatively

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- ☺ Various good features
  - ✓ non–local GUT breaking

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- However:
  - $\mathrm{SU}(5)$ Yukawa relations also for light generations
  - hidden sector gauge group only $\mathrm{SU}(3)$

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bottom–line:

Successful string embedding of $\mathbb{Z}_4^R$ possible!

Main features

1. GUT symmetry breaking non-local
   $\curvearrowright$ (almost) no 'logarithmic running above the GUT scale'

Hebecker and Trapletti (2005) ; Anandakrishnan and Raby (2012)

Main features

1. GUT symmetry breaking non-local
2. No localized flux in hypercharge direction
   $\curvearrowright$ complete blow–up without breaking SM gauge symmetry in principle possible

Main features

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2. No localized flux in hypercharge direction
3. 4D gauge group:
   $\mathsf{SU}(3)_C \times \mathsf{SU}(2)_\mathrm{L} \times \mathsf{U}(1)_Y \times [\mathsf{SU}(3) \times \mathsf{SU}(2)^2 \times \mathsf{U}(1)^8]$

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4. massless spectrum

#	representation	label
3	$(\mathbf{3},\mathbf{2};\mathbf{1},\mathbf{1},\mathbf{1})_{1/6}$	Q
3	$(\overline{\mathbf{3}},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{1})_{1/3}$	$\overline{D}$
3	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{1})_{1}$	$\overline{E}$
6	$(\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1},\mathbf{1})_{-1/2}$	h
3	$(\overline{\mathbf{3}},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{1})_{1/3}$	$\overline{\delta}$
3	$(\mathbf{1},\mathbf{1};\mathbf{3},\mathbf{1},\mathbf{1})_{0}$	x
6	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{2})_{0}$	y

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3	$(\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1},\mathbf{1})_{-\frac{1}{2}}$	L
37	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{1})_{0}$	s
6	$(\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1},\mathbf{1})_{1/2}$	$\overline{h}$
3	$(\mathbf{3},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{1})_{-1/3}$	δ
5	$(\mathbf{1},\mathbf{1};\overline{\mathbf{3}},\mathbf{1},\mathbf{1})_{0}$	$\overline{x}$
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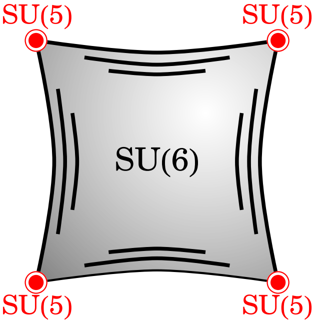
   spectrum $=$ $3 \times$ generation + vector–like

Local vs. non–local GUT breaking

Blaszczyk et al. (2010) ; Kappl et al. (2011)



1. construct $\mathbb{T}^2/\mathbb{Z}_2$ orbifold which breaks $\mathrm{SU}(6)$ locally to $\mathrm{SU}(5)$

$$\mathbb{Z}_2 \quad : \quad (x_5, x_6) \quad \rightarrow \quad (-x_5, -x_6)$$

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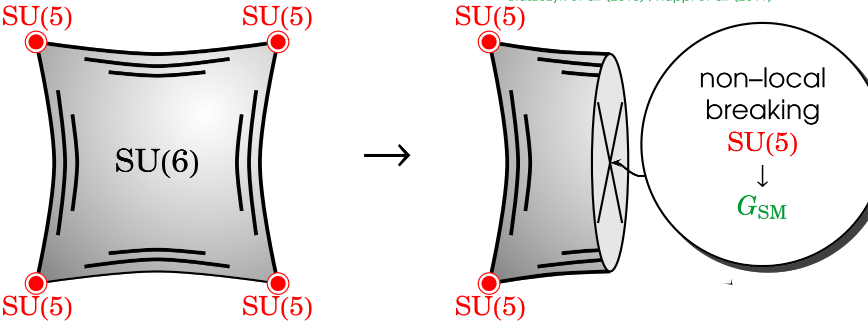

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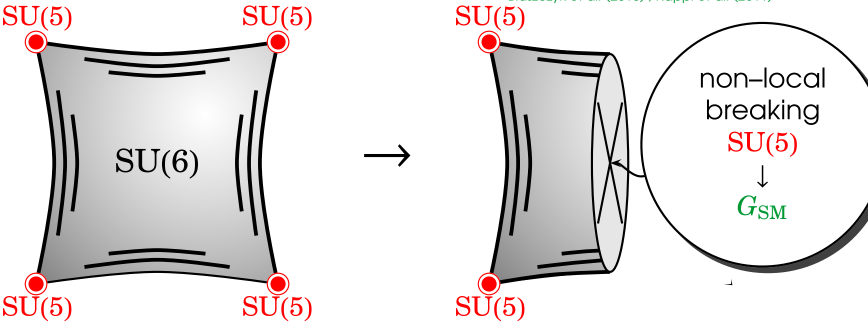

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☞ More detailed discussion in preparation

Anandakrishnan et al. (2012)

Non–local GUT breaking: comments

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  ↷ almost no ‘logarithmic running above the GUT scale’

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- running does not really stop
- high precision unification

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☞ In heterotic orbifolds: several geometries with non–trivial fundamental group:
31 affine classes of space groups, based on the $\mathbb{Q}$–classes $\mathbb{Z}_2 \times \mathbb{Z}_2$, $\mathbb{Z}_2 \times \mathbb{Z}_4$ and $\mathbb{Z}_3 \times \mathbb{Z}_3$

cf. the complete classification Fischer et al. (2012)

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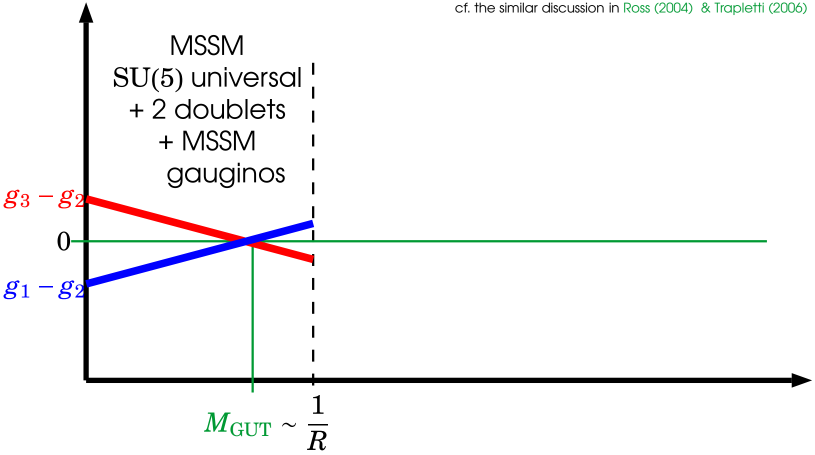
- Precision gauge unification works well in schemes with comparatively light colored states

  Raby et al. (2010)

Gauge unification: non–local GUT breaking

Anandakrishnan et al. (2012)

cf. the similar discussion in Ross (2004) & Trapletti (2006)



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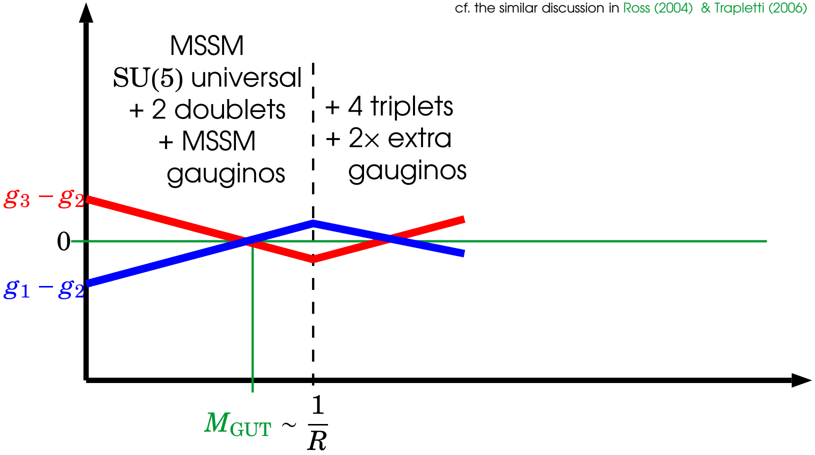

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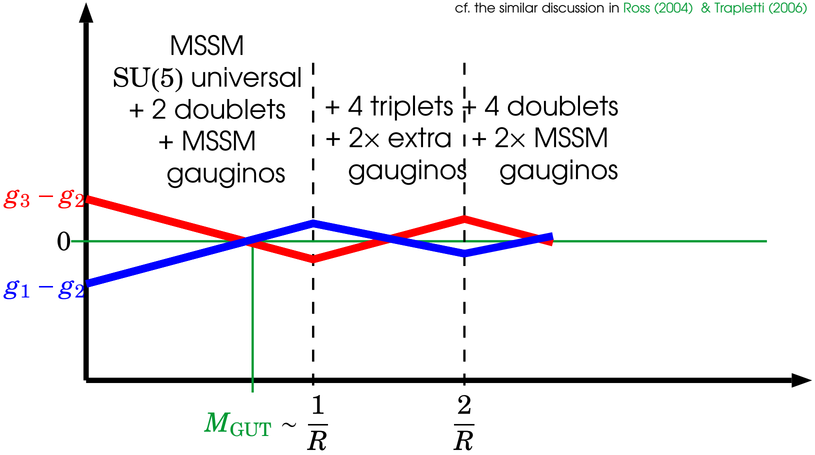

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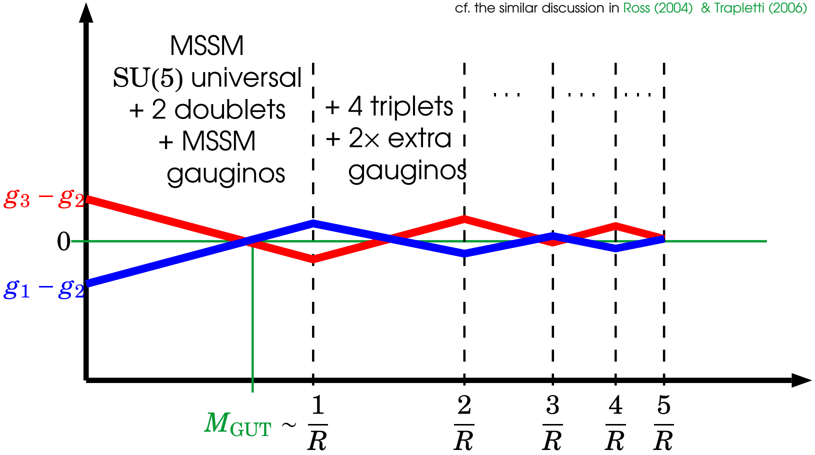

➥ Slight discrepancy between GUT and compactification scales

SUSY vacua with $\mathbb{Z}_4^R$

- Recall: situation for gauge theories with generic superpotential

e.g. Luty and Taylor (1996)

solutions of D–equations $\cap$ solutions of F–equations $=$ non–trivial

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  solutions of D–equations $\cap$ solutions of F–equations $=$ non–trivial

- However: $\langle \mathscr{W} \rangle \neq 0$ generically
- Vacua with residual $\mathbb{Z}_4^R$ are slightly different

SUSY vacua with $\mathbb{Z}_4^R$

- Recall: situation for gauge theories with generic superpotential
e.g. Luty and Taylor (1996)

solutions of D–equations $\cap$ solutions of F–equations $=$ non–trivial

- However: $\langle \mathscr{W} \rangle \neq 0$ generically
- Vacua with residual $\mathbb{Z}_4^R$ are slightly different
- Example: consider one field ϕ_0 with R-charge 0 and one field ϕ_2 with R-charge 2

$$\mathscr{W} \;=\; \phi_2 \cdot f(\phi_0) + \mathcal{O}({\phi_2}^3) \qquad \text{with} \quad \langle \mathscr{W} \rangle \;=\; 0 \text{ automatic}$$

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SUSY vacua with $\mathbb{Z}_4^R$ (cont'd)

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- ➥ expect solutions for $N \geq M$
- ➥ M non–trivial mass terms (also for T- and Z–moduli!)

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- ↪ expect solutions for $N \geq M$
- ↪ M non–trivial mass terms (also for T- and Z–moduli!)
- Have identified configurations with $N \geq M$ in our $\mathbb{Z}_2 \times \mathbb{Z}_2$ model(s)

Summary

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☞ Assumptions:

(i) anomaly universality ($\leftrightarrow$ gauge coupling unification)

(ii) μ term forbidden at perturbative level

(iii) Yukawa couplings and Weinberg neutrino mass operator allowed

(iv) $\mathrm{SU}(5)$ or $\mathrm{SO}(10)$ GUT relations for quarks and leptons

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☞ Have shown:

1. assuming (i) & $\mathrm{SU}(5)$ relations:
   ↷ only R symmetries can forbid the μ term
2. assuming (i)–(iii) & $\mathrm{SO}(10)$ relations:
   ↷ unique $\mathbb{Z}_4^R$ symmetry
3. R symmetries are not available in 4D GUTs
   ↷ no ‘natural’ solution to doublet–triplet splitting in 4D!

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☞ A simple anomaly–free $\mathbb{Z}_4^R$ symmetry can

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$$\mathbb{Z}_4^R \curvearrowright \left\{\begin{array}{l}\text{dim. 4 proton decay operators completely forbidden}\\ \text{dim. 5 proton decay operators highly suppressed}\\ \mu \text{ appears non–perturbatively}\end{array}\right.$$

Summary

☞ In string models $\mathbb{Z}_4^R$ can arise as a discrete remnant of Lorentz symmetry in extra dimensions

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- In string models $\mathbb{Z}_4^R$ can arise as a discrete remnant of Lorentz symmetry in extra dimensions
- Guided by the (unique) $\mathbb{Z}_4^R$ symmetry we have constructed a globally consistent string model with:
  - exact MSSM spectrum
  - non–local line GUT breaking $\curvearrowright$ precision gauge unification
  - non–trivial full–rank Yukawa couplings
  - exact matter parity
  - no ‘fractionally charged exotics’
  - $\mu \sim m_{3/2}$
  - dimension five proton decay operators sufficiently suppressed

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  - dimension five proton decay operators sufficiently suppressed
- Arguments for supersymmetric Minkowski vacua (@ perturbative level) where most moduli attain large supersymmetric masses

Vielen

Dank!

Backup slides

- (Discrete) Green–Schwarz anomaly cancellation
- Anomaly universality
- Blaszczyk model

Green–Schwarz anomaly cancellation

- Under 'anomalous' $\mathrm{U}(1)$ symmetry transformation of the fermions $\psi^{(f)} \to \mathrm{e}^{\mathrm{i}\,\alpha\,Q^{(f)}_{\mathrm{anom}}}\,\psi^{(f)}$ the path integral measure exhibits non–trivial transformation

Fujikawa (1979) ; Fujikawa (1980)

$$\mathcal{D}\Psi\,\mathcal{D}\overline{\Psi} \;\to\; J(\alpha)\,\mathcal{D}\Psi\,\mathcal{D}\overline{\Psi} \qquad \text{with non–trivial } J(\alpha)$$

path integral measure | Jacobian | (infinitesimal) parameter

Green–Schwarz anomaly cancellation

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- One can absorb the change of the path integral measure in a change of Lagrangean

$$\Delta\mathscr{L}_{\text{anomaly}} = \frac{\alpha}{32\pi^2} F^a \widetilde{F}^a \, A_{G\text{–}G\text{–}\mathrm{U}(1)_{\mathrm{anom}}}$$

anomaly coefficients | e.g. $\mathrm{SU}(3) \subset G_{\mathrm{SM}}$ | 'anomalous' $\mathrm{U}(1)$

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- Provided the Lagrangean also includes axion couplings

$$\mathscr{L} \supset -\frac{a}{8} F^a \widetilde{F}^a$$

$\Delta\mathscr{L}_{\text{anomaly}}$ can be compensated by a shift of the axion a

Green and Schwarz (1984)

Discrete GS anomaly cancellation in SUSY

- Analysis applies also for discrete symmetries
- Specifically for a $\mathbb{Z}_N$ transformation

$$\Phi^{(f)} \;\to\; \mathrm{e}^{-\mathrm{i}\,\frac{2\pi}{N}\,q^{(f)}}\,\Phi^{(f)}$$

the dilaton (containing the axion) has to transform as

$$S \;\to\; S + \frac{\mathrm{i}}{2}\Delta_{\mathrm{GS}}$$

where

$$\pi N\,\Delta_{\mathrm{GS}} \;\equiv\; A_{G-G-\mathbb{Z}_N} \mod \eta \qquad \text{where } \eta = \begin{cases} N & \text{if } N \text{ odd} \\ N/2 & \text{if } N \text{ even} \end{cases}$$

- If $\mathrm{SU}(3)\times\mathrm{SU}(2)\times\mathrm{U}(1)\subset\mathrm{SU}(5)$ the anomaly coefficients need to be universal

Comments on discrete GS mechanism

1. Although the GS mechanism plays a prominent role in string theory, it does not rely on strings.
2. Unlike in the continuous case, for discrete symmetries the transformation of the axion is only fixed modulo η.
3. In the continuous case, the axion has to be massless for the shift symmetry to be a symmetry of the Lagrangean. That is, the axion potential needs to be flat. By contrast, in the discrete case the potential is only required to be periodic, i.e. invariant under the discrete shift. Therefore the axion may have a non–trivial mass prior to the breakdown of the symmetry.

Spectrum and $\mathbb{Z}_4^R$

#	representation	label
3	$(\mathbf{3},\mathbf{2};\mathbf{1},\mathbf{1},\mathbf{1})_{1/6}$	Q
3	$(\overline{\mathbf{3}},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{1})_{1/3}$	$\overline{D}$
3	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{1})_{1}$	$\overline{E}$
6	$(\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1},\mathbf{1})_{-1/2}$	h
3	$(\overline{\mathbf{3}},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{1})_{1/3}$	$\overline{\delta}$
5	$(\mathbf{1},\mathbf{1};\mathbf{3},\mathbf{1},\mathbf{1})_{0}$	x
6	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{2})_{0}$	y

#	representation	label
3	$(\overline{\mathbf{3}},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{1})_{-2/3}$	$\overline{U}$
3	$(\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1},\mathbf{1})_{-1/2}$	L
37	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{1})_{0}$	s
6	$(\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1},\mathbf{1})_{1/2}$	$\overline{h}$
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$\mathbb{Z}_4^R$: discriminate between

matter with $\mathbb{Z}_4^R$ charge 1

and

Higgs/exotics with $\mathbb{Z}_4^R$ charge 0 or 2

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☞ Many other good features:

- no fractionally charged exotics (all SM charged fields come in $\mathrm{SU}(5)$ multiplets)
- non–trivial full–rank Yukawa couplings
- gauge–top unification
- SU(5) relation $y_\tau \simeq y_b$ (but also for light generations)

$\mathbb{Z}_4^R$ charges

- SM fields

	Q_i	$\overline{U}_i$	$\overline{D}_i$	L_i	$\overline{E}_i$
$\mathbb{Z}_4^R$	1	1	1	1	1

- Exotics

	h_1	h_2	h_3	h_4	h_5	h_6	$\overline{h}_1$	$\overline{h}_2$	$\overline{h}_3$	$\overline{h}_4$	$\overline{h}_5$	$\overline{h}_6$
$\mathbb{Z}_4^R$	0	2	0	2	0	0	0	2	0	0	2	2

	δ_1	δ_2	δ_3	$\overline{\delta}_1$	$\overline{\delta}_2$	$\overline{\delta}_3$
$\mathbb{Z}_4^R$	0	2	2	2	0	0

Higgs candidate mass matrix

	h_1	h_2	h_3	h_4	h_5	h_6	$\overline{h}_1$	$\overline{h}_2$	$\overline{h}_3$	$\overline{h}_4$	$\overline{h}_5$	$\overline{h}_6$
$\mathbb{Z}_4^R$	0	2	0	2	0	0	0	2	0	0	2	2

- Mass matrix for exotic doublets h_i and $\overline{h}_j$

$$\mathcal{M}_{h\overline{h}} = \begin{pmatrix} 0 & \phi_6 & 0 & \phi_4 & 0 & 0 \\ \phi_7 & 0 & \phi_2 & 0 & \phi_{13} & \phi_{14} \\ 0 & \phi_1 & 0 & \widetilde{\phi}^3 & 0 & 0 \\ 0 & \widetilde{\phi}^3 & 0 & \widetilde{\phi}^5 & 0 & 0 \\ \widetilde{\phi}^3 & 0 & \phi_{11} & 0 & \phi_8 & \widetilde{\phi}^3 \\ \widetilde{\phi}^3 & 0 & \phi_{12} & 0 & \widetilde{\phi}^3 & \phi_8 \end{pmatrix}$$

SM singlets with zero $\mathbb{Z}_4^R$ charge

Higgs candidate mass matrix

	h_1	h_2	h_3	h_4	h_5	h_6	$\overline{h}_1$	$\overline{h}_2$	$\overline{h}_3$	$\overline{h}_4$	$\overline{h}_5$	$\overline{h}_6$
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☞ One massless linear combination (= Higgs pair)

$$\begin{aligned} H_u &= a_1 \overline{h}_1 + a_2 \overline{h}_3 + a_3 \overline{h}_4 \\ H_d &= b_1 h_1 + b_2 h_3 + b_3 h_5 + b_4 h_6 \end{aligned}$$

Triplet mass matrix

$$\begin{array}{c|cccccc} & \delta_1 & \delta_2 & \delta_3 & \overline{\delta}_1 & \overline{\delta}_2 & \overline{\delta}_3 \\ \mathbb{Z}_4^R & 0 & 2 & 2 & 2 & 0 & 0 \end{array}$$

- Mass matrix for exotic color triplet

$$\mathcal{M}_\delta = \begin{pmatrix} \widetilde{\phi}^5 & 0 & 0 \\ 0 & \phi_8 & \widetilde{\phi}^3 \\ 0 & \widetilde{\phi}^3 & \phi_8 \end{pmatrix}$$

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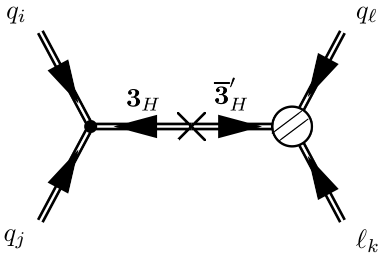
- **Note**: exotic triplets cannot mediate proton decay



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- **Note**: exotic triplets cannot mediate proton decay
- The fact that the numbers of massless doublet and triplet pairs differ is not an accident but already follows from anomaly matching

Effective Yukawa couplings

☞ Effective superpotential

$$\begin{aligned} \mathscr{W}_{\mathrm{Y}} &= \sum_{i=1,3,4} \Big[\big(Y_u^{(i)}\big)^{fg} \, Q_f \, \overline{U}_g \, \bar{h}_i \Big] \\ &+ \sum_{i=1,3,5,6} \Big[\Big(Y_d^{(i)}\Big)^{fg} \, Q_f \, \overline{D}_g \, h_i + \big(Y_e^{(i)}\big)^{fg} \, L_f \, \overline{E}_g \, h_i \Big] \end{aligned}$$

Effective Yukawa couplings

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☞ Effective Yukawa matrices (examples)

$$Y_u^{(1)} = \begin{pmatrix} \widetilde{\phi}^2 & \widetilde{\phi}^4 & \widetilde{\phi}^6 \\ \widetilde{\phi}^4 & \widetilde{\phi}^2 & \widetilde{\phi}^6 \\ \widetilde{\phi}^6 & \widetilde{\phi}^6 & 1 \end{pmatrix}$$

$$Y_u^{(3)} = \begin{pmatrix} 1 & \widetilde{\phi}^6 & \widetilde{\phi}^4 \\ \widetilde{\phi}^6 & 1 & \widetilde{\phi}^4 \\ \widetilde{\phi}^4 & \widetilde{\phi}^4 & \widetilde{\phi}^2 \end{pmatrix}$$

Effective Yukawa couplings

- Effective superpotential

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- Effective Yukawa matrices (examples)

$$Y_e^{(5)} = \left(Y_d^{(5)}\right)^T = \begin{pmatrix} \widetilde{\phi}^6 & \widetilde{\phi}^6 & \widetilde{\phi}^6 \\ \widetilde{\phi}^6 & \widetilde{\phi}^6 & 1 \\ \widetilde{\phi}^6 & 1 & \widetilde{\phi}^4 \end{pmatrix}$$

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Non–local breaking in 6D

Anandakrishnan and Raby (2012)

☞ Eigenstates and parity operations

$$\begin{aligned}
\mathbb{Z}_2 \quad &: \quad \phi_{\pm\widehat{\pm}}(x_\mu, -x_5, -x_6) \;=\; \pm\, \phi_{\pm\widehat{\pm}}(x_\mu, x_5, x_6) \\
\mathbb{Z}_2' \quad &: \quad \phi_{\pm\widehat{\pm}}(x_\mu, -x_5 + \pi R_5, x_6 + \pi R_6) \;=\; \widehat{\pm}\, \phi_{\pm\widehat{\pm}}(x_\mu, x_5, x_6)
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'local'

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\end{aligned}$$

'local'

'non–local'

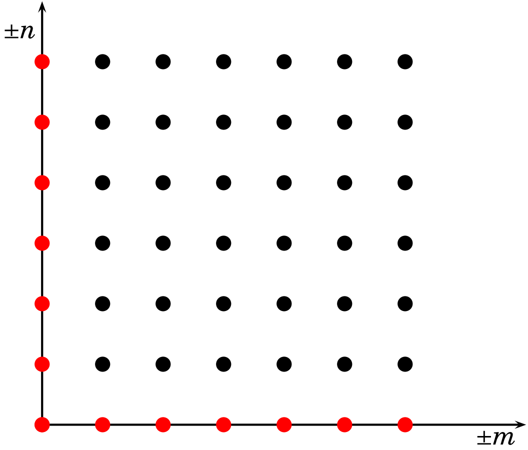
- General mode expansion

$$\begin{aligned}
\phi_{\pm\widehat{\pm}}(x, x_5, x_6) \;=\; & \frac{1}{4\,\sqrt{2R_5\,R_6}} \\
& \cdot \sum_{m,n} \left[\left(\phi^{(m,n)} \pm \phi^{(-m,-n)}\right) \widehat{\pm} (-1)^{m-n} \left(\phi^{(-m,n)} \pm \phi^{(m,-n)}\right)\right] \\
& \cdot \exp\left[\mathrm{i}\left(\frac{m}{R_5}\, x_5 + \frac{n}{R_6}\, x_6\right)\right]
\end{aligned}$$

Modes for $\mathbb{T}^2/\mathbb{Z}_2$ (local breaking)

Trapletti (2006)

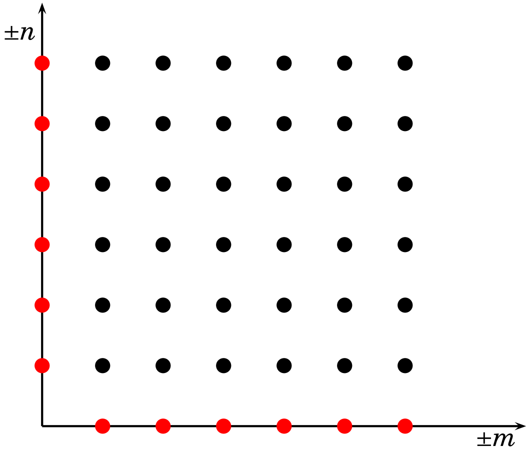
☞ Non–zero $\phi^{(m,n)}$ for + modes



Modes for $\mathbb{T}^2/\mathbb{Z}_2$ (local breaking)

Trapletti (2006)

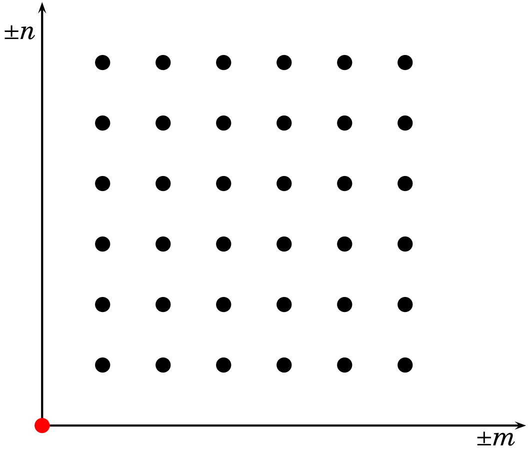
☞ Non–zero $\phi^{(m,n)}$ for – modes



Modes for $\mathbb{T}^2/\mathbb{Z}_2$ (local breaking)

☞ Mismatch

Trapletti (2006)

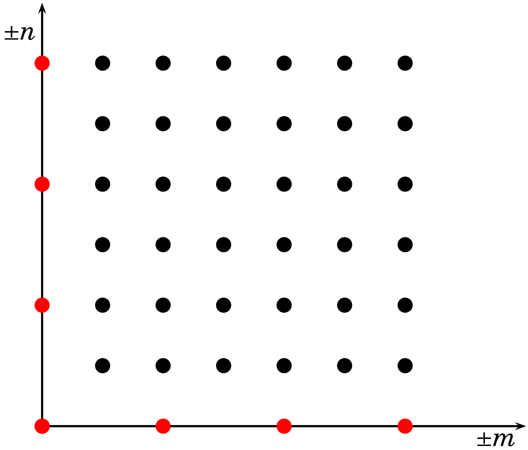


Running does not stop at the compactification scale

Modes

Anandakrishnan and Raby (2012)

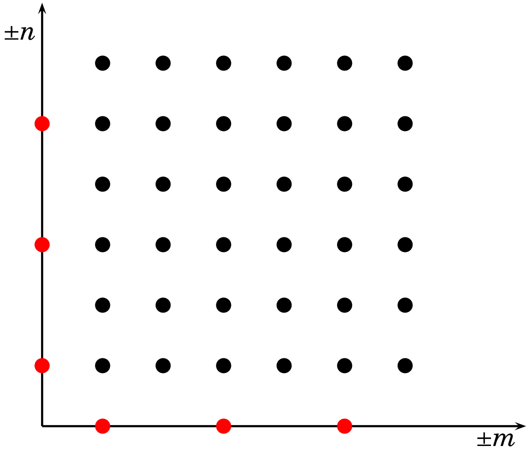
- Non–zero $\phi^{(m,n)}$ for $+\widehat{+}$ modes



Modes

Anandakrishnan and Raby (2012)

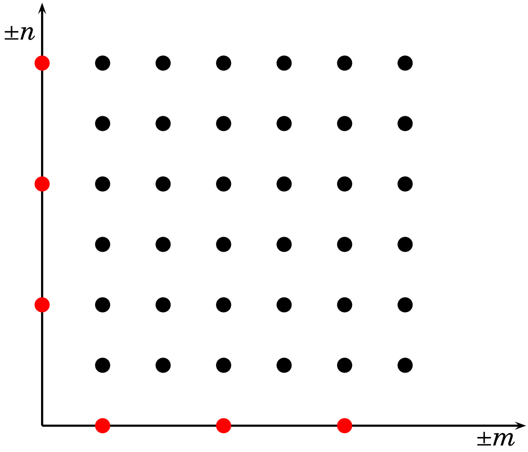
☞ Non–zero $\phi^{(m,n)}$ for $+\widehat{-}$ modes



Modes

Anandakrishnan and Raby (2012)

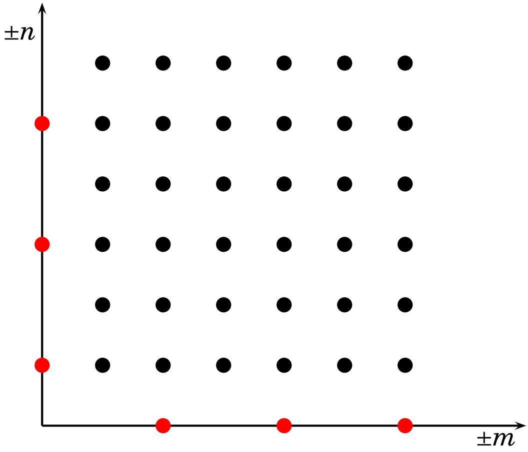
- Non–zero $\phi^{(m,n)}$ for $-\widehat{+}$ modes



Modes

Anandakrishnan and Raby (2012)

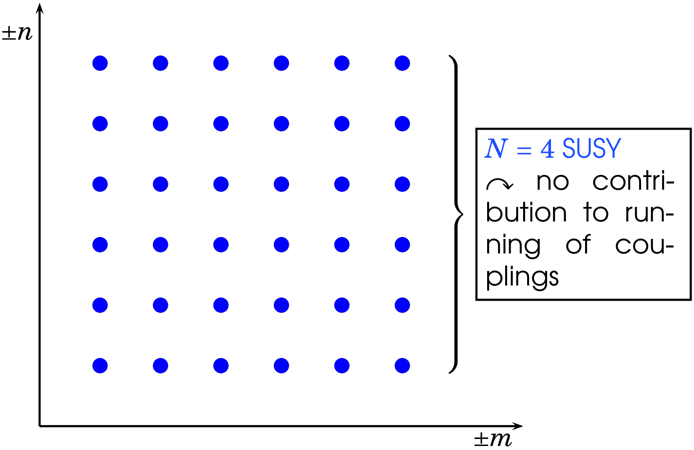
- Non–zero $\phi^{(m,n)}$ for $-\hat{-}$ modes



Modes

Anandakrishnan and Raby (2012)

☞ Non–zero $\phi^{(m,n)}$ for all modes



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