Measurement of Differential Luminosity using Bhabha-Events

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For precision mass measurements the effect of correlation in beamstrahlung has to be accounted for in differential luminosity [3]. In this note a possible parametrisation for correlation in beamstrahlung is proposed and a qualitative improvement over the circe parametrisation [2] is shown.

1 Introduction

1.1 Differential Luminosity

The differential luminosity $L(x)$ effectively lowers the nominal cross-section $\tilde{\sigma}(\sqrt{s})$.

$$\sigma(\sqrt{s}) = \int L(x)\tilde{\sigma}(x\sqrt{s}) \tag{1}$$

Differential Luminosity consists of initial state radiation, which can be calculated by QED, beam-spread, which can be measured upstream of the interaction point and beamstrahlung, which strongly depends on beam parameters, some of which can not be measured. Only beamstrahlung will be considered here. The Beamstrahlung spectra are generated with GuineaPig(GP).[4]

1.2 Circe Parametrisation

The energy distribution of electrons or positrons after beamstrahlung is described by the circe parametrisation(2)[2].

$$f(x) = a_0\delta(1-x) + (1-a_0)a_{norm}x^{a_2}(1-x)^{a_3} \tag{2}$$

$$1 = \int_0^1 a_{norm}x^{a_2}(1-x)^{a_3}dx \qquad \text{Normalisation} \tag{3}$$

The spectra for e^+e^- pairs after beamstrahlung is the product

$$L(x_e, x_p) = f(x_e)f(x_p) \tag{4}$$

The parameters can be found by a direct fit to the GP-spectra (See Table 1).

For technical reasons a variable transformation is done.

$$x \to t = (1-x)^{1/5} \tag{5}$$

This leads to a separation between the peak and the continuum of the spectra, due to the limited numerical precision in GP. For the two dimensional spectrum these will be called: Peak, where neither particle radiated, Arm1 and Arm2, where only one particle radiated and Body, where both particles radiated.

Table 1: Circe Parameters found by direct fit and by measurement

	Direct	Measured	Fit-Error
a_0	0.580	0.575	0.003
a_2	12.6	12.5	0.1
a_3	-0.657	-0.671	0.004

1.3 Measuring the parameters

In order to measure the parameters the reweighting technique is used. From the acollinearity of the Bhabha-Events the center of mass energy $\sqrt{s_{rec}}$ can be reconstructed, if the nominal energy $\sqrt{s}$ is known.[3]

$$\frac{\sqrt{s_{\mathrm{rec}}}}{\sqrt{s}} = \sqrt{1 - 2\frac{\sin\left(\Theta_1 + \Theta_2\right)}{\sin\left(\Theta_1 + \Theta_2\right) - \sin\Theta_1 - \sin\Theta_2}} \tag{6}$$

The reconstructed energy spectrum can then be compared with a spectrum given by monte-carlo events and trough the reweighing technique the parameters can be found. The measured parameters are within 2σ of the real parameters found by a direct fit (Table 1).

1.4 Correlation

Beamstrahlung depends on the field seen by the radiating particle and the field depends on the position in the bunch. And because particles only interact, if they have a similar longitudinal and transverse position in the bunch, the beamstrahlung is correlated. Because the circe parametrisation factorizes for electrons and positrons it can not contain correlation.

2 Correlated Parametrisation

Comparison between the Circe and GuineaPig spectra shows (Table 2), that parameter a_0 is not enough to describe the distribution for the different regions. It also shows, that the shape of the distribution is slightly different and therefore the parameters a_2 and a_3 might be different for the arms and the body.

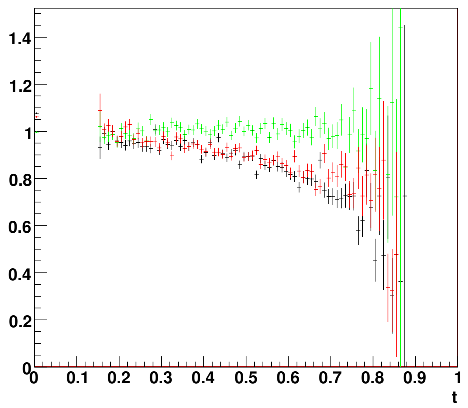
Table 2: Distribution of e^+e^- pairs after the beamstrahlung for the different regions, normalized to GP.

	Circe	Correlated
Peak	0.975	1.005
Arm1	1.044	1.002
Arm2	1.044	1.007
Body	0.937	0.980

This leads to the correlated parametrisation of beamstrahlung:

$$\begin{aligned}
f(x_e, e_p) &= a_{peak}\delta(1-x_e)\delta(1-x_p) \\
&+ a_{arm}\delta(1-x_e)a_{norm_1}x_p^{a_2}(1-x_p^{a_3}) \\
&+ a_{arm}a_{norm_1}x_e^{a_2}(1-x_e^{a_3})\delta(1-x_p) \\
&+ a_{body}a_{norm_2}x_p^{a_4}(1-x_p^{a_5})a_{norm_2}x_e^{a_4}(1-x_e^{a_5}) \qquad (7) \\
a_{\mathrm{body}} &= 1 - a_{\mathrm{peak}} - 2a_{\mathrm{arm}} \qquad \text{Normalisation} \qquad (8)
\end{aligned}$$

The parameters are found by a two dimensional fit onto the GP-Spectrum. The a_{norm} factors fulfill the same condition as (3).



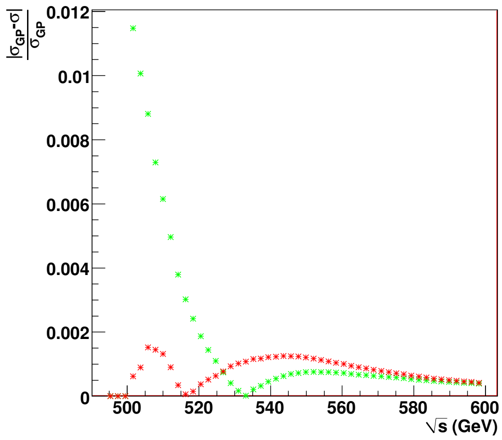


Figure 1: On the Left: The Ratios between the x_1-Distribution for $x_2 = 1.0$ and the x_1-distribution for $x_2 < 1.0$. GP in Black, Circe in green, Correlated in Red. On the Right: The relative differences between GP and the circe parametrisation (green) and the correlated parametrisation (red) for a simple threshold scan. The Fit for the correlated parametrisation has been improved since the workshop.

3 Results

Table 2 and Figure (1, left) show that the beamstrahlung spectrum is better described by the correlated parametrisation. This can also be shown by a threshold scan for a toy particle with a mass of 250 GeV. A simple β dependency (9) for the cross-section in(1) is assumed.

$$\tilde{\sigma}(\sqrt{s}) = \frac{C}{s}\sqrt{1 - \frac{4m}{s}} \qquad (9)$$

The correlated parametrisation clearly improves the threshold scan (See: 1, right) near the threshold at $\sqrt{s} = 500$GeV.

4 Conclusions and Outlook

The new parametrisation is able to describe the correlation in the beamstrahlung spectrum. It remains to be shown, that it is also possible to measure the parameters for the correlated parametrisation in the same way as it was done for the circe parametrisation. It might also be possible to include the beam spread into the parametrisation and accommodate differences between the electron and positron beam.

References

[1] Slides: http://indico.desy.de/contributionDisplay.py?contribId=24&sessionId=8&confId=585

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[3] K. Moenig "Measurement of Differential Luminosity using Bhabha events in the Forward-Tracking region at TESLA", LC-PHSM-2000-60-TESLA, December 29, 2000

[4] D. Schulte, "Beam-Beam Simulation with GUINEA-PIG", Proc. of ICAP 1998. http://citeseer.ist.psu.edu/321371.html