Karlsruher Institut für Technologie

Standard Model of Electroweak Interactions

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Course "Higgs Physics"

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Some Thoughts on “Perfect” Theoretical Models

- **Features of *nice* theoretical models**
- **Aesthetics (Symmetries)**

Emmy Noether

Noether-Theorem (1918) :

Symmetries ⇒ conservation laws

Continuous (space-time) symmetries

- **Spatial translation ⇒ momentum conservation**
- **Time translation ⇒ energy conservation**
- **Rotation ⇒ angular momentum conservation**

Continuous (gauge) symmetries (examples)

- **QED (U(1) symmetry) ⇒ conservation of em. charge**
- **QCD (SU(3) symmetry) ⇒ conservation of color charge**

Discrete symmetries (examples)

- **Mirror reflection** $\vec{r} \longrightarrow -\vec{r}$ **⇒ Parity**
- **Time inversion** $t \longrightarrow -t$ **⇒ T-Parity**
- **Particle ⇔ anti-paricle ⇒ Charge-Parity**

Nice Theoretical Models

- **Built in a spirit of “*Ockham's razor*” principle**
  - **Economical and able to account for new phenomena with least possible number of assumptions**

Geocentric planetary model $\Rightarrow$ complex model, involving epicycles to explain motion of planets in sky

Heliocentric model $\Rightarrow$ planets revolve around Sun, no need for epicycles

- **Consistent with our empirical knowledge**
  - **Scientific spirit rules out models which contradict experimental observations**

- **Testable experimentally**
- **Has predictive power**
  - **Example : Standard Model formulated by Salam, Weinberg and predicted existence of heavy vector particles**
    - **W and Z boson discovered at CERN SPS (1981)**

A Path to Relativistic Quantum Field Theory

Situation in physics in 1920-es

Two new big pillars :

Theory of Relativity	Quantum Mechanics

P. Dirac (1928) attempts to find “relativistic” equivalent of Schrödinger Equation which is

1) **Consistent with Klein-Gordon Equation** $\left(\partial^\mu \partial_\mu + m^2\right)\Phi = 0$
2) **Linear in first derivatives**
3) **Incorporates electron spin phenomenology**

Quantum Electrodynamics

$$\Big(i\gamma^\mu \partial_\mu - m\Big)\psi = 0$$

Dirac Equation

marked birth of Quantum Electrodynamics (QED) $\Rightarrow$ first successful (Gauge) Quantum Field Theory (QFT)

- **based on principle of gauge invariance (obeys U(1) symmetry)**
- **describes electromagnetic interactions between charged spin ½ particles (fermions) mediated by massless photons**

Lagrange formalism in QFT :

$$S = \int dx^4 \mathcal{L}(\partial_\mu \Phi, \Phi), \quad \delta S = 0$$ $\Rightarrow$ **Lagrange-Euler Eq.**

$$\frac{\partial \mathcal{L}}{\partial \Phi} - \partial_\mu \left(\frac{\partial \Phi}{\partial(\partial_\mu \Phi)} \right) = 0$$

Lagrange Density of Free Fermion Field in QED

Lagrangian which leads to Dirac Equation

$$\mathcal{L} = i\bar{\psi}(\gamma^\mu \partial_\mu - m)\psi$$

$$\frac{\partial \mathcal{L}}{\partial \bar{\psi}} - \partial_\mu \left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \bar{\psi})} \right) = 0 \Rightarrow (i\gamma^\mu \partial_\mu - m)\psi = 0$$

$$\psi = \left(\begin{array}{c} \psi_1 \\ \psi_2 \\ \psi_3 \\ \psi_4 \end{array} \right)$$

Gamma matrices and their properties:

$$\gamma^0 = \left(\begin{array}{cc} \mathbb{I}_2 & 0 \\ 0 & -\mathbb{I}_2 \end{array} \right) \quad \vec{\gamma} = \left(\begin{array}{cc} 0 & \vec{\sigma} \\ -\vec{\sigma} & 0 \end{array} \right)$$

$$\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}\mathbb{I}_4$$

adjoint Dirac field $\bar{\psi} = \psi^\dagger \gamma^0 = (\psi_1^*, \psi_2^*, -\psi_3^*, -\psi_4^*)$

Solutions of Dirac Equation

Dirac Equation suggests Ansatz

$$\psi(x) = u(\vec{p})e^{-ipx}$$

$\Rightarrow$ momentum space Dirac Equation

$$(\gamma^\mu p_\mu - m)u(\vec{p}) = 0$$

two positive energy solutions

$$u(\vec{p},\sigma) = \mathcal{N}\left(\begin{array}{c} \chi^{(\sigma)} \\ \frac{\vec{\sigma}\cdot\vec{p}}{E+m}\chi^{(\sigma)} \end{array}\right), \quad \chi^{(1)} = \left(\begin{array}{c} 1 \\ 0 \end{array}\right), \quad \chi^{(2)} = \left(\begin{array}{c} 0 \\ 1 \end{array}\right)$$

Solutions of Dirac Equation

Dirac Equation suggests Ansatz

$$\psi(x) = u(\vec{p})e^{-ipx}$$

$\Rightarrow$ momentum space Dirac Equation

$$(\gamma^\mu p_\mu - m)u(\vec{p}) = 0$$

two negative energy solutions

$$v(\vec{p},\sigma) = \mathcal{N} \begin{pmatrix} \frac{\vec{\sigma}\cdot\vec{p}}{E+m} i\sigma^2\chi^{(\sigma)} \\ i\sigma^2\chi^{(\sigma)} \end{pmatrix}, \quad \chi^{(1)} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad \chi^{(2)} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

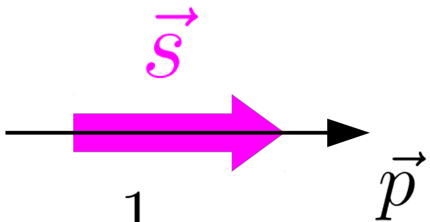
interpreted as positive energy solutions for anti-particle

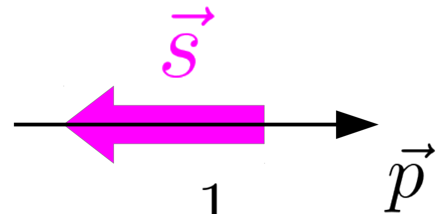
$$(\gamma^\mu p_\mu + m)v(\vec{p}) = 0$$

Helicity and Chirality

Helicity : eigenvalue of helicity operator $\dfrac{\vec{\sigma}\cdot\vec{p}}{2|\vec{p}|}$

$\Rightarrow$ projection of spin on the direction of motion





$$\lambda = +\frac{1}{2},\ \psi_+ \qquad\qquad \lambda = -\frac{1}{2},\ \psi_-$$

Chirality : eigenvalue of projective operators

$$\mathbb{P}_L = \frac{1-\gamma^5}{2}, \qquad \mathbb{P}_R = \frac{1+\gamma^5}{2}$$

$$\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} 0 & \mathbb{I}_2 \\ \mathbb{I}_2 & 0 \end{pmatrix}$$

$$\mathbb{P}_L{}^2 = \mathbb{P}_L, \quad \mathbb{P}_R{}^2 = \mathbb{P}_R$$

$$\mathbb{P}_L\mathbb{P}_R = \mathbb{P}_R\mathbb{P}_L = 0$$

Helicity and Chirality

Dirac field can be represented as superposition of left-handed and right-handed components

$$\psi = \mathbb{P}_L \psi + \mathbb{P}_R \psi = \frac{1-\gamma^5}{2}\psi + \frac{1+\gamma^5}{2}\psi = \psi_L + \psi_R$$

massless fermions

chirality eigenvector = helicity eigenvector

$$\mathbb{P}_L \psi_- = -\psi_-, \quad \mathbb{P}_R \psi_+ = \psi_+$$

massive fermions

v≈c : chirality eigenvector ≈ helicity eigenvector

$$\frac{|\mathbb{P}_L \psi_-|^2 - |\mathbb{P}_L \psi_+|^2}{|\mathbb{P}_L \psi_-|^2 + |\mathbb{P}_L \psi_+|^2} = -\frac{v}{c}$$

Bilinear Covariants and Conserved Current

Name	Form	Parity
scalar	$\bar{\psi}\gamma^0\psi$	$+$
pseudoscalar	$\bar{\psi}\gamma^5\psi$	$-$
vector	$\bar{\psi}\gamma^\mu\psi$	$+$
axial $-$ vector	$\bar{\psi}\gamma^5\gamma^\mu\psi$	$-$

Dirac Equation ⇒ conservation of current

$$J^\mu = \bar{\psi}\gamma^\mu\psi, \qquad \partial_\mu J^\mu = 0$$

$$J^\mu = (\rho, \vec{j}) \Rightarrow \frac{1}{c}\frac{\partial\rho}{\partial c} + \nabla \cdot \vec{j} = 0$$

Principle of Gauge Invariance

Lagrangian $$\mathcal{L} = \bar{\psi}\left(i\gamma^{\mu}\partial_{\mu} - m\right)\psi$$

is invariant under global U(1) symmetry ⇒

phase rotation $\psi' = e^{i\phi}\psi$ **[U(1) transformation]**

leaves Lagrangian invariant

global U(1) symmetry ⇔

phase ϕ **is independent of space-time coordinates**

Let us consider local transformation

$$\psi' = e^{ie\phi(x_{\mu})}\psi, \qquad \bar{\psi}' = e^{-ie\phi(x_{\mu})}\bar{\psi}$$

Principle of Gauge Invariance

Consider derivative

$$\partial_\mu \psi' = \partial_\mu \big(e^{ie\phi(x)}\psi\big) = ie\big(\partial_\mu \phi(x)\big)e^{ie\phi(x)}\psi + e^{ie\phi(x)}\partial_\mu \psi$$

Transformation of Lagrangian

$$\mathcal{L} = \bar{\psi}'\big(i\gamma^\mu \partial_\mu - m\big)\psi' = \bar{\psi}\big(i\gamma^\mu \partial_\mu - m\big)\psi - \boxed{e\bar{\psi}\big(\gamma^\mu \partial_\mu \phi(x)\big)\psi}$$

Non-invariant under local gauge transformation

Remedy : introduce vector field A_μ with transformation rule

$$A'_\mu = A_\mu - \partial_\mu \phi(x)$$

and covariant derivative

$$\partial_\mu \to \mathcal{D}_\mu = \partial_\mu + ieA_\mu$$

Complete QED Lagrangian

$$\mathcal{L} = \bar{\psi}\big(i\gamma^{\mu}\mathcal{D}_{\mu} - m\big) - \frac{1}{4}F^{\mu\nu}F_{\mu\nu}, \quad \mathcal{D}_{\mu} = \partial_{\mu} + ieA_{\mu}$$

$F^{\mu\nu} = \partial_{\mu}A_{\nu} - \partial_{\nu}A_{\mu}$ **: tensor of electromagnetic field**

Lagrangian invariant under local gauge transformations

$$\psi' = e^{ie\phi(x)}\psi, \qquad A'_{\mu} = A_{\mu} - \partial_{\mu}\phi(x)$$

$\mathcal{L} = i\bar{\psi}\gamma^{\mu}\partial_{\mu}\psi$ **fermion kinetic term**

$-m\bar{\psi}\psi$ **fermion mass term**

$-e\bar{\psi}\gamma^{\mu}\psi A_{\mu}$ **interaction (fermion–boson)**

$-\frac{1}{4}F^{\mu\nu}F_{\mu\nu}$ **kinetic term of gauge field**

Interactions not Explained within QED

Beta-decay (charged-current interactions) :

$$n \rightarrow p^{+} + e^{-} + \bar{\nu}_e$$

$$\begin{pmatrix} Z \\ A \end{pmatrix} \rightarrow \begin{pmatrix} Z+1 \\ A \end{pmatrix} + e^{-} + \bar{\nu}_e$$

$$\begin{pmatrix} Z \\ A \end{pmatrix} \rightarrow \begin{pmatrix} Z-1 \\ A \end{pmatrix} + e^{+} + \nu_e$$

Neutral current interactions : $\nu_e + e^{-} \rightarrow \nu_e + e^{-}$

Discovered with large volume heavy-liquid bubble chamber (Gargamelle) experiment at CERN (1973)

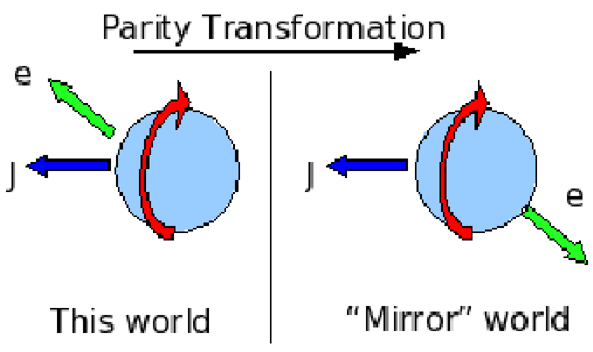
Features of “non-electromagnetic” interactions

these interactions are much weaker compared to electromagnetic ones

$$\sigma(\nu e^- \to \nu e^-) \ll \sigma(\gamma e^- \to \gamma e^-)$$

$$\tau(n \to p^+ e^- + \bar{\nu}_e) \approx 15 min \quad \text{vs.} \quad \tau(\pi^0 \to \gamma\gamma) \approx 10^{-16} sec$$

violate parity!



C.S. Wu et al. 1957:

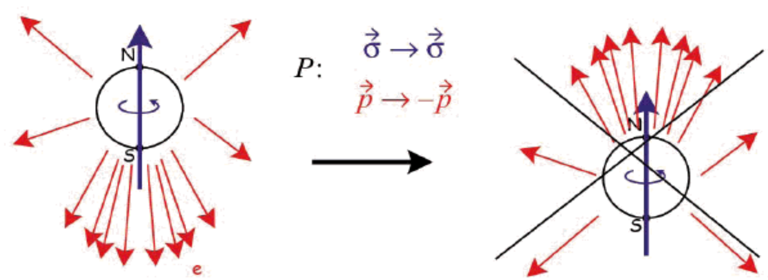
- Ausrichtung von Kobalt 60 Kernen (Spin) in einem Magnetfeld (bei T=1° K)
- Messung der Emissionsrichtung der Elektronen im β-Zerfall $^{60}Co \to {}^{60}Ni^* + e^- + \bar{\nu}_e$
- Ergebnis: Elektronen werden bevorzugt entgegen der Spinrichtung emittiert.

Intensitätsverteilung: $I(\theta_e) = 1 + \alpha\frac{\vec{\sigma}_{Co}\vec{p}_e}{E_e} = 1 + \alpha\beta\cos\theta_e$

- $\vec{\sigma}\cdot\vec{p}$ ist Pseudoskalar und verletzt die Parität

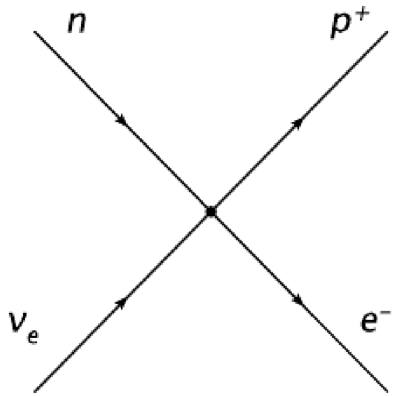
Wu Experiment (1957)
maximal parity violation in decays

$$^{60}\mathrm{Co} \to {}^{60}\mathrm{Ni} + e^- + \bar{\nu}_e$$



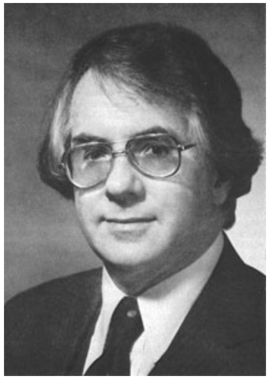
Path towards SM of electroweak interactions

Fermi theory (four fermion interactions)

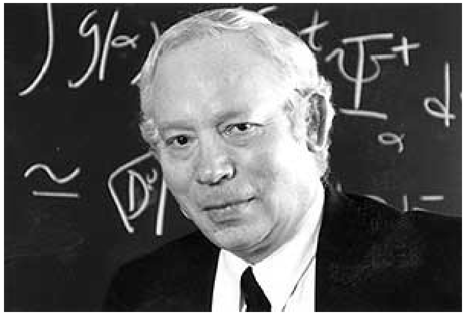


$$\mathcal{M} = \frac{G_F}{\sqrt{2}} \left(\bar{u}_P \gamma^\mu (1-\gamma_5) u_N\right) \left(\bar{u}_e \gamma_\mu (1-\gamma_5) u_\nu\right)$$

phenomenologically successful but...
non-renormalizable theory



1960: Sheldon Glashow discovers a way to combine the electromagnetic and weak interactions



1967/1968: Abdus Salam, Steven Weinberg incorporated Higgs mechanism into Glashow's electroweak theory

Building Standard Model

Building unified gauge theory of electromagnetic and weak interactions ⇒

model should incorporate three generation of matter particles (fermion fields)

interacting via vector particles (gauge fields)

local gauge invariance ⇒ gauge interactions

should explain peculiarities of observed interactions

Electromagnetic : long-range (massless gauge field), parity-conserving

Weak : short-range (massive gauge fields), parity-violating

⇒ only left-handed fermions (right handed anti-fermions) participate in CC interactions

⇒ V-A structure of NC interactions

Building Gauge Theory: Universal Prescription

Gauge Theory :

fermions (matter fields) + gauge bosons (force carriers)

local gauge invariance associated with group $\mathcal{G}$

representation $\mathcal{R}$ **in fermion fields realized in vector space** $\mathbb{C}^n$

$\hat{t}^a_{\mathcal{R}}$ **– generators of group obeying commutation relations**

$$[\hat{t}^a_{\mathcal{R}}, \hat{t}^b_{\mathcal{R}}] = i f^{abc} \hat{t}^c_{\mathcal{R}}$$

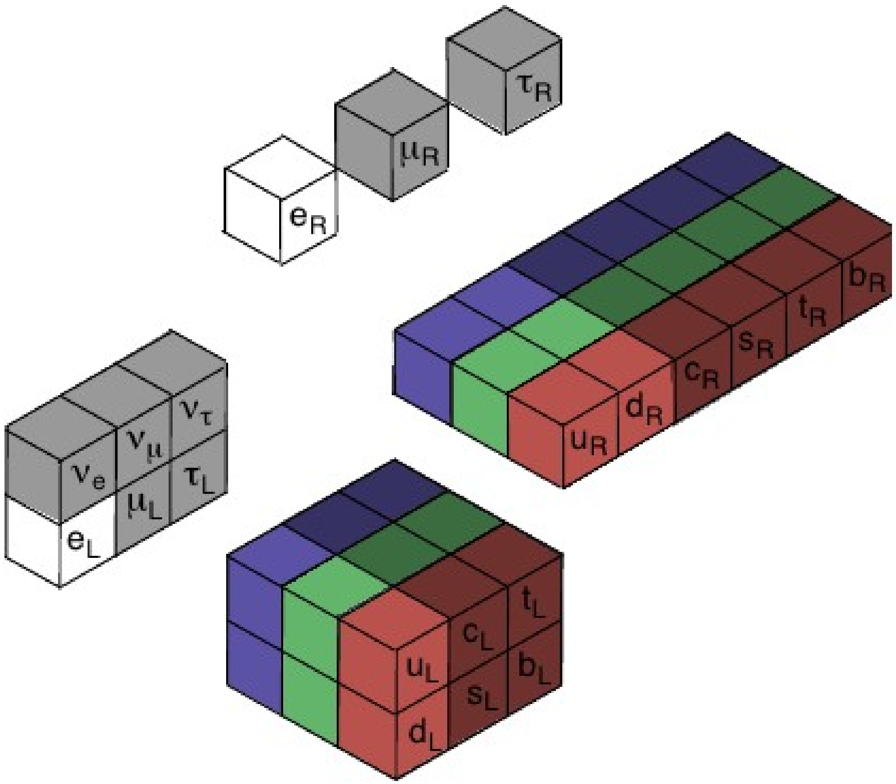
$$\mathcal{L}_F = \bar{F}_{\mathcal{R}}(i\gamma^\mu \mathcal{D}_\mu - M_{\mathcal{R}})F_{\mathcal{R}} \qquad F_{\mathcal{R}} = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \ldots \\ \psi_{n_{\mathcal{R}}} \end{pmatrix}$$

$$\mathcal{D}_\mu = \partial_\mu - ig\hat{t}^a_{\mathcal{R}} G^a_\mu$$

$$\mathcal{L}_G = -\frac{1}{4} G^{\mu\nu,a} G^a_{\mu\nu}, \quad G^a_{\mu\nu} = \partial_\mu G^a_\nu - \partial_\nu G^a_\mu - g f^{abc} G^b_\mu G^c_\nu$$

Our Picture of Matter

Pointlike $(r \leq 10^{-18}\mathrm{m})$ **quarks and leptons**



Electroweak sector of SM (let's put QCD aside) : $U(1)_Y \otimes SU(2)_L$

$\Rightarrow$ **four gauge fields :** $\gamma \ (m_\gamma = 0), \qquad Z,\ W^\pm \ (m_Z, m_W \neq 0)$

$U(1)_Y \otimes SU(2)_L$ Group of SM

$\mathbf{U(1)_Y}$ gauge interactions generated by hypercharge $Y/2$

$\mathbf{SU(2)_L}$ gauge interactions generated by isospin $\vec{T} = \frac{1}{2}\vec{\sigma}$

$\vec{\sigma}$ – Pauli matrices

Fermion representations of SM group

	Family 1	2	3	T_3	Y/2	Q
Leptons	$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L$	$\begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L$	$\begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L$	$1/2$ $-1/2$	$-1/2$ $-1/2$	0 -1
	e_R	μ_R	τ_R	0	-1	-1
Quarks	$\begin{pmatrix} u \\ d' \end{pmatrix}_L$	$\begin{pmatrix} c \\ s' \end{pmatrix}_L$	$\begin{pmatrix} t \\ b' \end{pmatrix}_L$	$1/2$ $-1/2$	$1/6$ $1/6$	$2/3$ $-1/3$
	u_R	c_R	t_R	0	$2/3$	$2/3$
	d_R	s_R	b_R	0	$-1/3$	$-1/3$

$$Q = T_3 + Y/2$$ ***Gell-Mann Nishijima Relation***

Gauge Transformations in SM

Local gauge transformation associated with $SU(2)_L$

$$\begin{pmatrix} \nu \\ \ell \end{pmatrix}_L' = \exp\left(i\frac{g_2}{2}\vec{\sigma}\cdot\vec{\beta}(x)\right)\begin{pmatrix} \nu \\ \ell \end{pmatrix}_L$$

Local gauge transformation associated with $U(1)_Y$

$$\begin{pmatrix} \nu \\ \ell \end{pmatrix}_L' = \exp\left(ig_1\frac{Y}{2}\theta(x)\right)\begin{pmatrix} \nu \\ \ell \end{pmatrix}_L$$

$$\nu_R' = \exp\left(ig_1\frac{Y}{2}\theta(x)\right)\nu_R \qquad \ell_R' = \exp\left(ig_1\frac{Y}{2}\theta(x)\right)\ell_R$$

Same relations hold for quarks

Gauge Fields in SM

Local gauge invariance ⇒ gauge fields

$$\mathbf{U(1)_Y : \quad one \;\; field} \Rightarrow$$

$$B'_\mu = B_\mu - \partial_\mu \theta(x)$$

$$\mathbf{SU(2)_L : \quad three \;\; fields} \Rightarrow$$

$$\vec{W}_\mu = (W^1_\mu, W^2_\mu, W^3_\mu)$$

$$\vec{W}'_\mu = W_\mu - \partial_\mu \vec{\beta}(x) + g_2 \vec{\beta} \times \vec{W}_\mu$$

Covariant Derivatives ⇒ Gauge Interactions

Covariant derivative of fermion fields in SM takes form

$$\mathcal{D}_\mu = \partial_\mu + ig_1 \frac{Y}{2} B_\mu + ig_2 \vec{T} \cdot \vec{W}$$

$$i \sum_f \bar{\psi}_f \gamma^\mu \mathcal{D}_\mu \psi_f = i \sum_f \bar{\psi}_f \gamma^\mu \partial_\mu \psi_f$$ **free fermion field**

$$-g_1 \sum_f Y_f \bar{\psi}_f \gamma^\mu \psi_f B_\mu$$ **gauge interactions** $\left(\mathbf{U(1)_Y}\right)$

$$-g_2 \sum_f \bar{\psi}_{fL} \gamma^\mu \vec{T} \cdot \vec{W}_\mu \psi_{fL}$$ **gauge interactions** $\left(\mathbf{SU(2)_L}\right)$

Gauge Sector of the SM

free gauge fields are described by Lagrangian

$$\mathcal{L}_G = -\frac{1}{4} B^{\mu\nu} B_{\mu\nu} - \frac{1}{4} \vec{W}^{\mu\nu} \cdot \vec{W}_{\mu\nu}$$

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$$

$$\vec{W}_{\mu\nu} = \partial_\mu \vec{W}_\nu - \partial_\nu \vec{W}_\mu + g_2 \vec{W}_\mu \times \vec{W}_\nu$$

Relations between fundamental fields and physical states:

$$W^{\pm} = \frac{1}{\sqrt{2}} (W_1 \pm i W_2)$$

Gauge Sector of SM

Recall : photon field couples only to charged particles (quarks , charged leptons : electron, muon, tau)

does not couple to neutrinos

$A^\mu(\gamma)$ **is linear combination of fields** W_3^μ, B^μ

orthogonal to field $Z^\mu(Z^0)$

$$A^\mu = c_B B^\mu + c_W W_3^\mu$$

photon coupling to neutrinos ~ $\frac{1}{2} c_W g_2 - \frac{1}{2} c_B g_1 = 0$

$$A^\mu = +\cos\theta_W B^\mu + \sin\theta_W W_3^\mu \qquad \sin\theta_W = \frac{g_1}{\sqrt{g_1^2 + g_2^2}}$$

$$Z^\mu = -\sin\theta_W B^\mu + \cos\theta_W W_3^\mu \qquad \sin^2\theta_W \approx 0.231$$

Weinberg (mixing) angle

Interactions in Terms of Physical States

$$\mathcal{L}_I = -eQ_f\bar{\psi}\gamma^\mu\psi_f A_\mu$$ **EM interactions**

$$-\frac{g_2}{2\cos\theta_W}\bar{\psi}_f\gamma^\mu(g_f^V - g_f^A\gamma^5)\psi_f Z_\mu$$ **Weak NC interactions**

$$-\frac{g_2}{2\sqrt{2}}\bar{F}_L\gamma^\mu(\sigma^+W_\mu^+ + \sigma^-W_\mu^-)F_L$$ **Weak CC interactions**

$$F_L = \begin{pmatrix} \nu_L \\ \ell_L \end{pmatrix} \ \textbf{or} \ \begin{pmatrix} u_L \\ d_L \end{pmatrix}$$

$$\psi_L = \frac{1-\gamma^5}{2}\psi, \qquad \bar{\psi}_L = \bar{\psi}\frac{1+\gamma^5}{2}$$

$$\sigma^\pm = \frac{1}{2}(\sigma_1 \pm i\sigma_2)$$

Weak Interactions

Weak NC interactions

$$\mathcal{L}_{NC} = -\frac{g_2}{2\cos\theta_W}\bar{\psi}_f\gamma^\mu(g_f^V - g_f^A\gamma^5)\psi_f Z_\mu$$

$$g_f^V = T_3 + 2Q\sin^2\theta_W, \qquad g_f^A = T_3$$

Weak CC interactions (lepton sector)

$$\mathcal{L}_{CC} = -\frac{g_2}{2\sqrt{2}}(\bar{\nu}_L\ \bar{e}_L)\,\gamma^\mu(\sigma^+W_\mu^+ + \sigma^-W_\mu^-)\begin{pmatrix}\nu_L \\ e_L\end{pmatrix}$$

$$\sigma^+ = \begin{pmatrix}0 & 1 \\ 0 & 0\end{pmatrix} \qquad \sigma^- = \begin{pmatrix}0 & 0 \\ 1 & 0\end{pmatrix}$$

Weak CC Interactions

Weak CC interactions (lepton sector)

$$\mathcal{L}_{CC} = -\frac{g_2}{2\sqrt{2}}\bar{\ell}\frac{1+\gamma^5}{2}\gamma^\mu W_\mu^- \frac{1-\gamma^5}{2}\nu$$
$$-\frac{g_2}{2\sqrt{2}}\bar{\nu}\frac{1+\gamma^5}{2}\gamma^\mu W_\mu^+ \frac{1-\gamma^5}{2}\ell$$

Using identity $\gamma^\mu\gamma^5 = -\gamma^5\gamma^\mu$

$$\mathcal{L}_{CC} = -\frac{g_2}{2\sqrt{2}}\left(J^{+,\mu}W_\mu^- + J^{-,\mu}W_\mu^+\right)$$

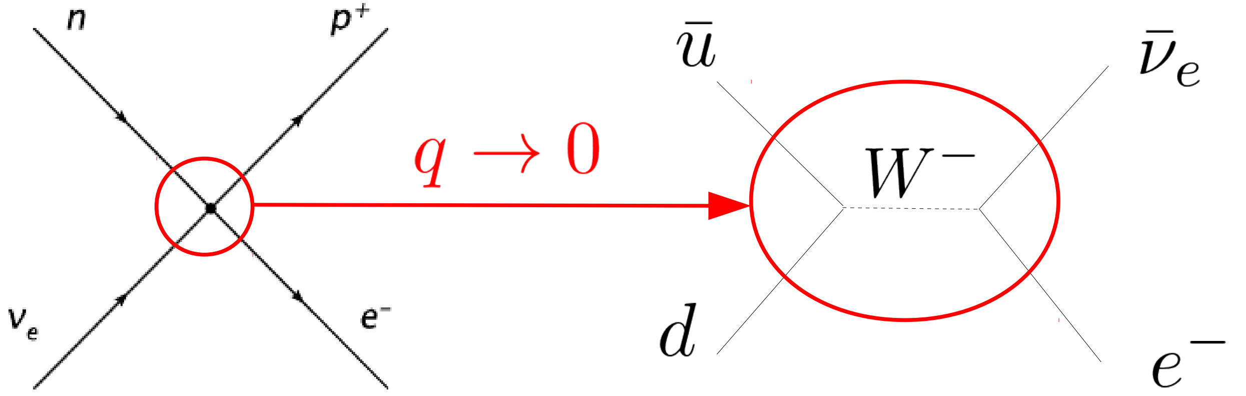
$$J^{+,\mu} = \bar{\ell}\gamma^\mu\frac{1-\gamma^5}{2}\nu, \quad J^{-,\mu} = \bar{\nu}\gamma^\mu\frac{1-\gamma^5}{2}\ell$$

The same structure of CC interactions holds for quarks

Why we need massive Z and W bosons

Fermi Theory of four-fermion interactions

$\Rightarrow$ **approximation of the gauge theory (SM) in the limit of small momentum-transfer**



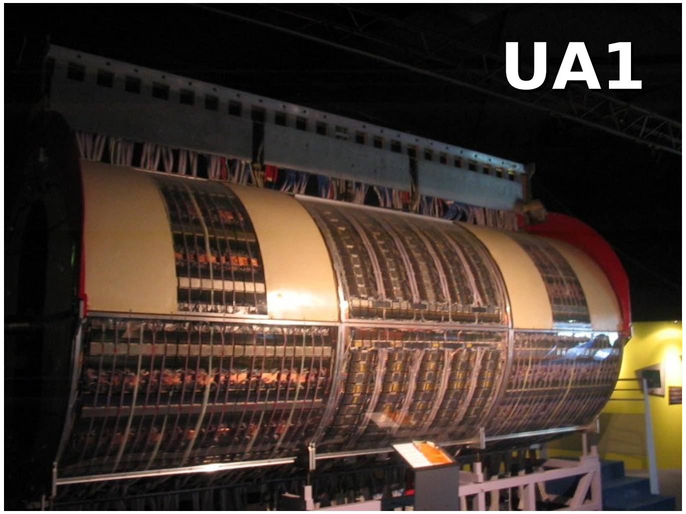
$$\mathcal{M} = \frac{g_2^2}{8} J_\mu(d\bar{u}) \frac{g^{\mu\nu} - \frac{q^\mu q^\nu}{M_W^2}}{q^2 - m_W^2} J_\nu(e^- \bar{\nu}_e)$$

$$\Rightarrow \mathcal{M} = \frac{G_F}{\sqrt{2}} J_\mu^+(pn) J^{-,\mu}(e\bar{\nu}) \Rightarrow \frac{G_F}{\sqrt{2}} = \frac{g_2^2}{8 m_W^2}$$

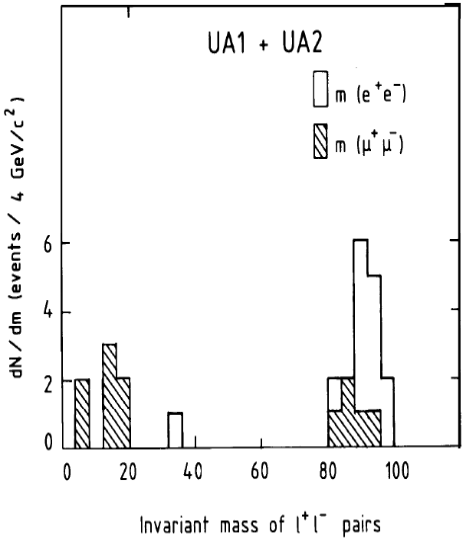
Massive force carriers explain “weakness” of weak interactions

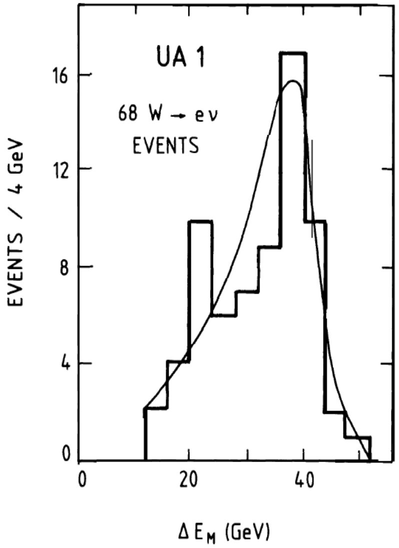
Experimental Tests of Standard Models

Discovery of W and Z bosons at CERN $p\bar{p}$ SPS (1981)
$\Rightarrow$ first triumph of Standard Model



Nobel Prize in Physics (1984)





The Nobel Prize in Physics 1984 was awarded jointly to Carlo Rubbia and Simon van der Meer "for their decisive contributions to the large project, which led to the discovery of the field particles W and Z, communicators of weak interaction"

Experimental Tests of Standard Models

Discovery of W and Z bosons followed by high precision measurements at LEP, SLAC, Tevatron

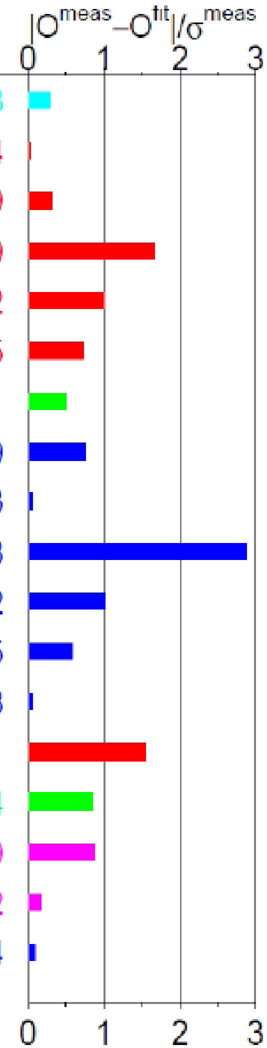
Experimental data

- LEP, SLC
- Tevatron
- Neutrino experiments
- ....

Measurements

- over a thousand individual measurements combined
- very different accelerator and detector setups
- decent agreement with SM

	Measurement	Fit
$\Delta\alpha_{had}^{(5)}(m_Z)$	0.02758 ± 0.00035	0.02768
m_Z [GeV]	91.1875 ± 0.0021	91.1874
Γ_Z [GeV]	2.4952 ± 0.0023	2.4959
σ_{had}^0 [nb]	41.540 ± 0.037	41.479
R_l	20.767 ± 0.025	20.742
$A_{fb}^{0,l}$	0.01714 ± 0.00095	0.01645
$A_l(P_\tau)$	0.1465 ± 0.0032	0.1481
R_b	0.21629 ± 0.00066	0.21579
R_c	0.1721 ± 0.0030	0.1723
$A_{fb}^{0,b}$	0.0992 ± 0.0016	0.1038
$A_{fb}^{0,c}$	0.0707 ± 0.0035	0.0742
A_b	0.923 ± 0.020	0.935
A_c	0.670 ± 0.027	0.668
A_l(SLD)	0.1513 ± 0.0021	0.1481
$\sin^2\theta_{eff}^{lept}(Q_{fb})$	0.2324 ± 0.0012	0.2314
m_W [GeV]	80.399 ± 0.023	80.379
Γ_W [GeV]	2.085 ± 0.042	2.092
m_t [GeV]	173.3 ± 1.1	173.4

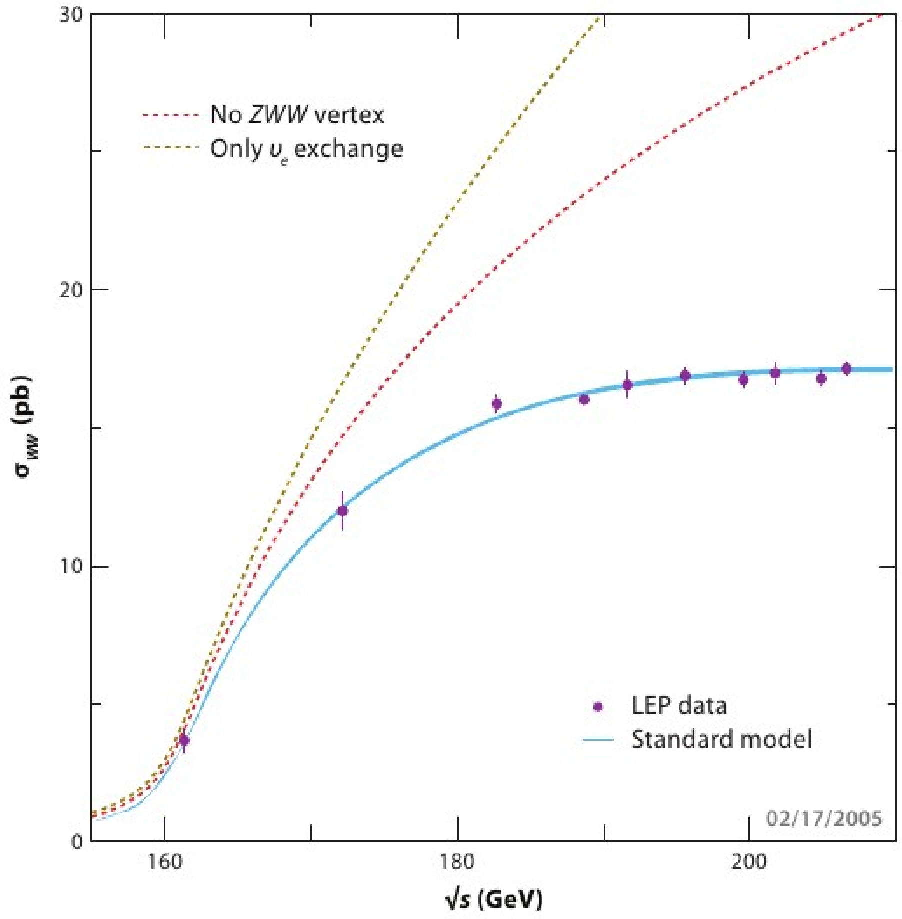


July 2010

Experimental Tests of Standard Model

Gauge symmetry (group-theory structure) tested in

$$e^+e^- \to W^+W^-$$



Problem with Massive Gauge Fields

Massive gauge fields break symmetry of the Standard Model

$\Rightarrow$ **Gauge Boson mass terms :**

$$\mathcal{L}_M = \frac{M_W^2}{2} |W_\mu^\pm|^2 \quad + \quad \frac{M_Z^2}{2} |Z_\mu Z^\mu|$$

are not invariant under gauge transformations

$$B'_\mu = B_\mu - \partial_\mu \theta(x)$$

$$\vec{W}'_\mu = W_\mu - \partial_\mu \vec{\beta}(x) + g_2 \vec{\beta} \times \vec{W}_\mu$$

Problem with Massive Fermions

Fermion mass terms

$$m\bar{f}f = m(\bar{f}_R f_L + \bar{f}_L f_R)$$

break SU(2) symmetry!

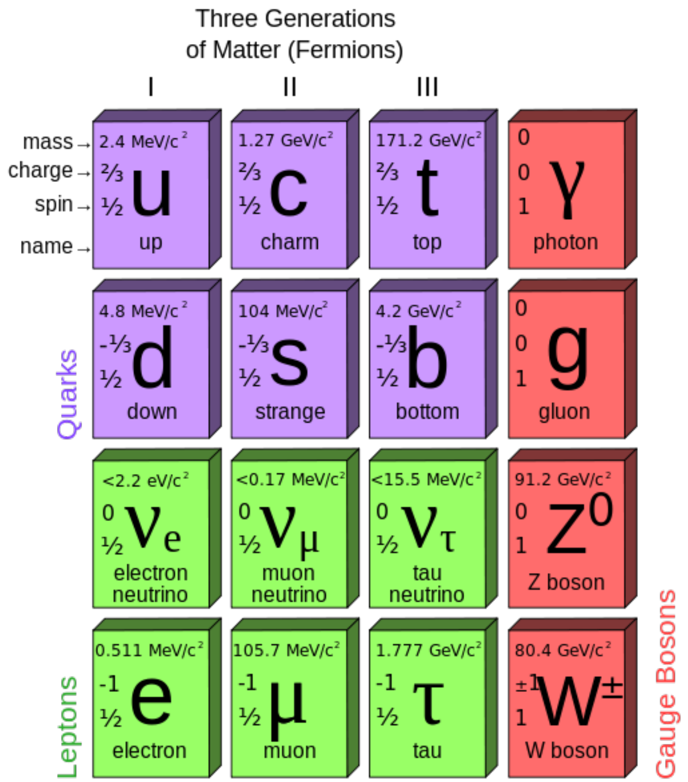
Right-handed fermions transform as singlets

$$f'_R = f_R$$

Left-handed fermions transform as doublets

$$\begin{pmatrix} \nu'_L \\ \ell'_L \end{pmatrix} = \exp\left(i\frac{g_2}{2}\vec{\beta}\cdot\vec{\sigma}\right)\begin{pmatrix} \nu_L \\ \ell_L \end{pmatrix}$$

Standard Model (Summary)



- **Building blocks of Standard Model**

  fermions : matter fields

  gauge bosons : force carriers
- **Aesthetics:**
  - **classification of fields**
  - **U(1)⊗SU(2) symmetry → gauge interactions**
- **simple Lagrange formalism describes this very well but only for massless particles**
- **Gauge boson and fermion mass terms break symmetry**
- **model is fairly consistent with experimental data assuming massive fermion and weak boson fields**

⇒ needs gauge invariant mechanism of mass generation