Higgs Mechanism in Standard Model (I)

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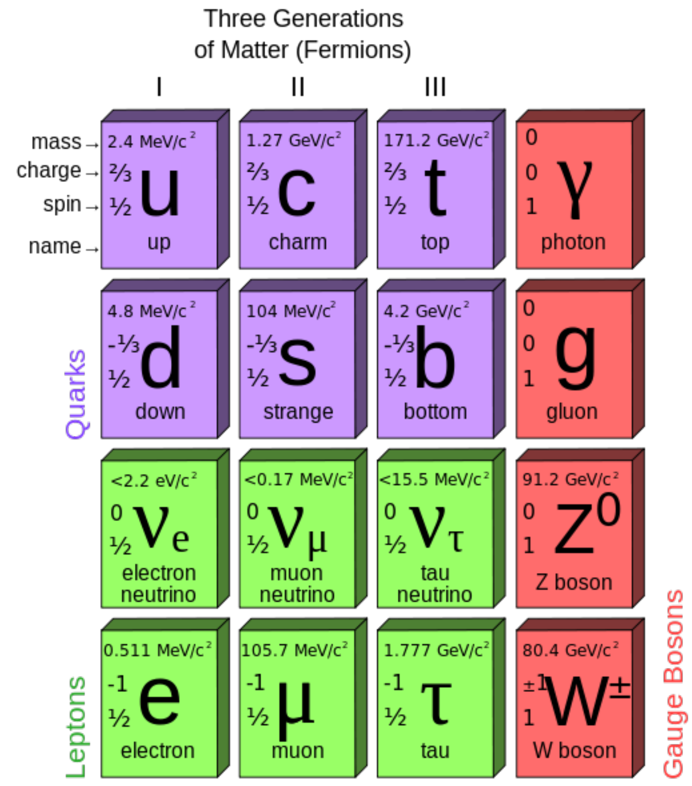
Standard Model (Summary)



- **Building blocks of Standard Model**

  fermions : matter fields

  gauge bosons : force carriers

- **Aesthetics of model :**
  - **classification of fields**
  - **U(1)⊗SU(2)⊗SU(3) symmetry → gauge interactions**

- **simple Lagrange formalism describes this very well but only for massless particles**

- **terms** $m(\bar{f}_R f_L + \bar{f}_L f_R),\quad M^2 V^\mu V_\mu$ **break SU(2) symmetry**

- **model is fairly consistent with experimental data assuming massive fermion and weak boson fields**

⇒ needs gauge invariant mechanism of mass generation

Higgs Mechanism

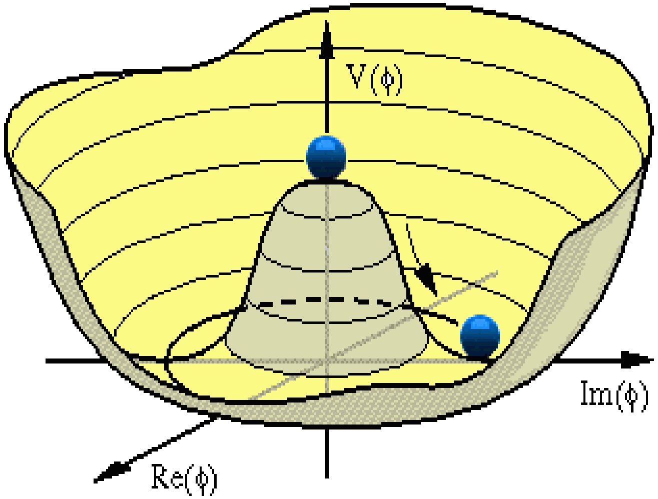
Consider SU(2) doublet of scalar complex fields

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix}$$

	T_3	Y	Q
ϕ^+	$+1/2$	1	1
ϕ^0	$-1/2$	1	0

Higgs Potential

$$V(\Phi^\dagger\Phi) = \lambda\left(\Phi^\dagger\Phi - \frac{v^2}{2}\right)^2$$



4-Dim sphere of minima

$$\phi_1^2 + \phi_2^2 + \phi_3^2 + \phi_4^2 = v^2$$

Ground state (vacuum)

$$\langle\Phi\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix}$$

Higgs Lagrangian

$$\mathcal{L}_H = \left(\partial_\mu \Phi^\dagger\right)\left(\partial^\mu \Phi\right) - V\left(\Phi^\dagger \Phi\right)$$

“Radial” excitations of vacuum:

$$\Phi + \delta\Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H(x) \end{pmatrix}$$

$$\Rightarrow \mathcal{L}_H = \frac{1}{2}\partial_\mu H \partial^\mu H - \lambda v^2 H^2 - \lambda v H^3 - \frac{\lambda}{4} H^4$$

$H(x)$ **– physical state with mass** $m_H = \sqrt{2\lambda} v$

Impose local gauge invariance

$$\left(\partial_\mu \Phi^\dagger\right)\left(\partial_\mu \Phi\right) \Rightarrow \left(\mathcal{D}_\mu \Phi^\dagger\right)\left(\mathcal{D}_\mu \Phi\right)$$

$$\mathcal{D}_\mu = \partial_\mu + i g_1 \frac{Y}{2} B_\mu + i g_2 \frac{1}{2} \vec{\sigma} \cdot \vec{W}$$

Consider ground state (vacuum)

$$\langle \Phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$$

Generation of Gauge Boson Masses

The kinetic term in the Higgs Lagrangian

$$\left(\mathcal{D}^{\mu}\langle\Phi\rangle^{\dagger}\right)\left(\mathcal{D}_{\mu}\langle\Phi\rangle\right)$$

Recall :

$$A_{\mu} = \frac{g_2 B_{\mu} + g_1 W_{\mu}^3}{\sqrt{g_1^2 + g_2^2}}, \qquad Z_{\mu} = \frac{-g_1 B_{\mu} + g_2 W_{\mu}^3}{\sqrt{g_1^2 + g_2^2}}$$

$$W^{\pm} = \frac{1}{\sqrt{2}}\left(W^1 \pm i W^2\right)$$

$$\Rightarrow \boxed{\frac{1}{8} g_2^2 v^2 W_{\mu}^{+} W^{-,\mu} + \frac{1}{8}(g_1^2 + g_2^2) v^2 Z_{\mu} Z^{\mu}}$$

Generation of Weak Boson Masses

$$\frac{1}{8} g_2^2 v^2 W_\mu^+ W^{-,\mu} + \frac{1}{8}(g_1^2 + g_2^2) v^2 Z_\mu Z^\mu$$

These are W and Z boson mass terms!

$$m_W = \frac{g_2 v}{2} \qquad m_Z = \frac{v}{2}\sqrt{g_1^2 + g_2^2}$$

field A_μ **remains massless** $m_\gamma = 0$

W and Z bosons acquire masses through interaction with the Higgs ground state (vacuum)!

Generation of Fermion Masses

Fermion mass terms

$$m(\bar{f}_R f_L + \bar{f}_L f_R)$$

break SU(2) symmetry!

Right-handed fermions transform as singlets

$$f'_R = f_R$$

Left-handed fermions transform as doublets

$$\begin{pmatrix} \nu'_L \\ \ell'_L \end{pmatrix} = \exp\left(i\frac{g_2}{2}\vec{\beta}\cdot\vec{\sigma}\right)\begin{pmatrix} \nu_L \\ \ell_L \end{pmatrix}$$

Yukawa Interactions

Yukawa interactions ⇒ interactions between fermions and Higgs doublet

corresponding gauge invariant Lagrangian (lepton sector) :

$$\mathcal{L}_{Y,\ell} = \mathcal{G}_{Y,\ell}\left(\bar{\ell}_R \Phi^\dagger L_L + \bar{L}_L \Phi \ell_R\right)$$

$$L_L = \left(\begin{array}{c} \nu_L \\ \ell_L \end{array}\right) \qquad \bar{L}_L = \left(\bar{\nu}_L,\ \bar{\ell}_L\right)$$

$$\Phi = \frac{1}{\sqrt{2}}\left(\begin{array}{c} \phi^+ \\ \phi^0 \end{array}\right) \qquad \Phi^\dagger = \frac{1}{\sqrt{2}}\left(\phi^-, \ \ \phi^{0*}\right)$$

$$\phi^- = \left(\phi^+\right)^*$$

Generation of Fermion Masses

Consider Yukawa interactions between ground Higgs state (vacuum) and leptons

$$\mathcal{L}_{Y,\ell} = \mathcal{G}_{Y,\ell}\left(\bar{\ell}_R \langle\Phi\rangle^\dagger L_L + \bar{L}_L \langle\Phi\rangle \ell_R\right)$$

$$\Rightarrow \frac{v}{\sqrt{2}}\mathcal{G}_{Y,\ell}\left(\bar{\ell}_R \ell_L + \bar{\ell}_L \ell_R\right)$$

Mass terms generated!

Charged lepton mass is given by

$$m_\ell = \frac{v}{\sqrt{2}}\mathcal{G}_{Y,\ell}$$

Generation of Fermion Masses

$g_{Y,\ell}$ – **Yukawa coupling of lepton ℓ to Higgs field**

Masses of down-type quarks are generated in similar way

Masses of up-type quarks are generated via Yukawa interactions with charge conjugate Higgs doublet

$$\Phi^C = -i\sigma_2\Phi^* = \begin{pmatrix} -\phi^{0*} \\ \phi^- \end{pmatrix}$$