PHYSICS AT THE TERA SCALE

Helmholtz Alliance

The Secret Life of Partons

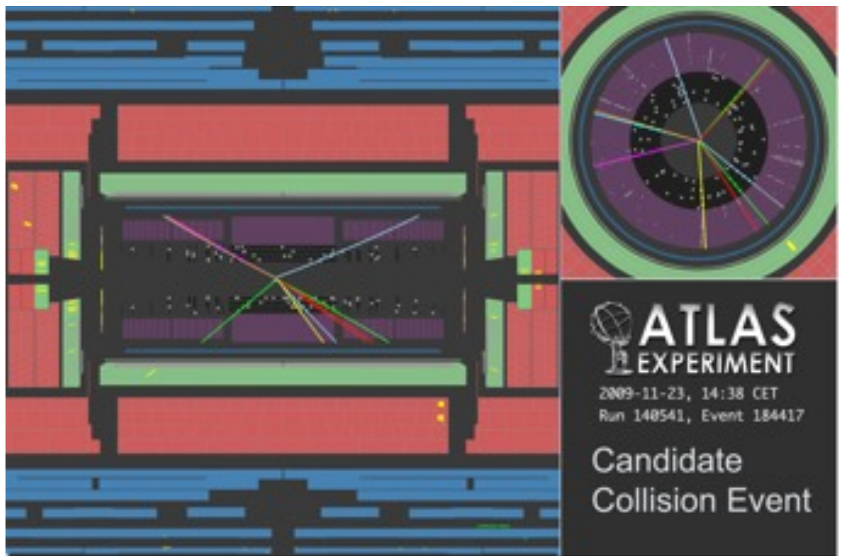
Zoltán Nagy

DESY

in collaboration with Dave Soper

Introduction

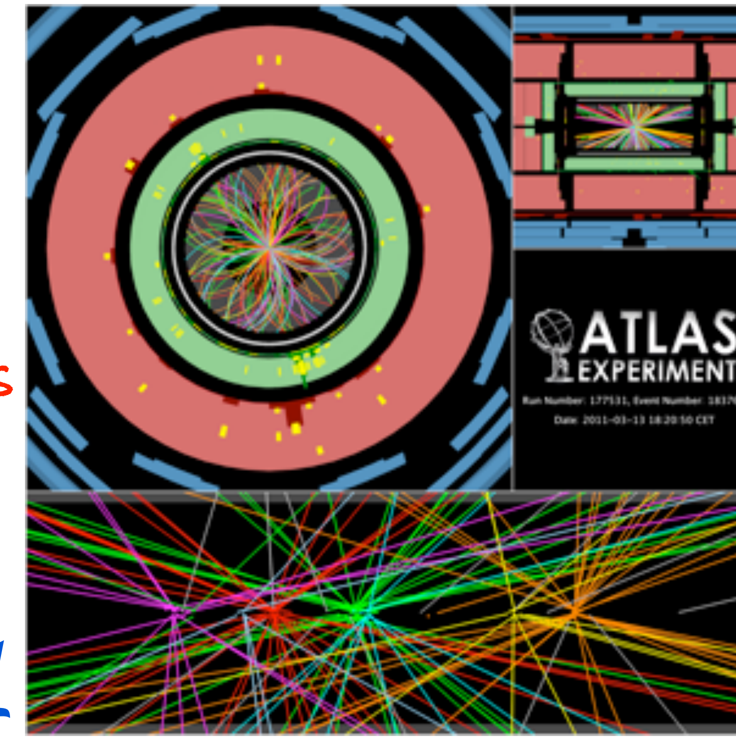
Pile-up events



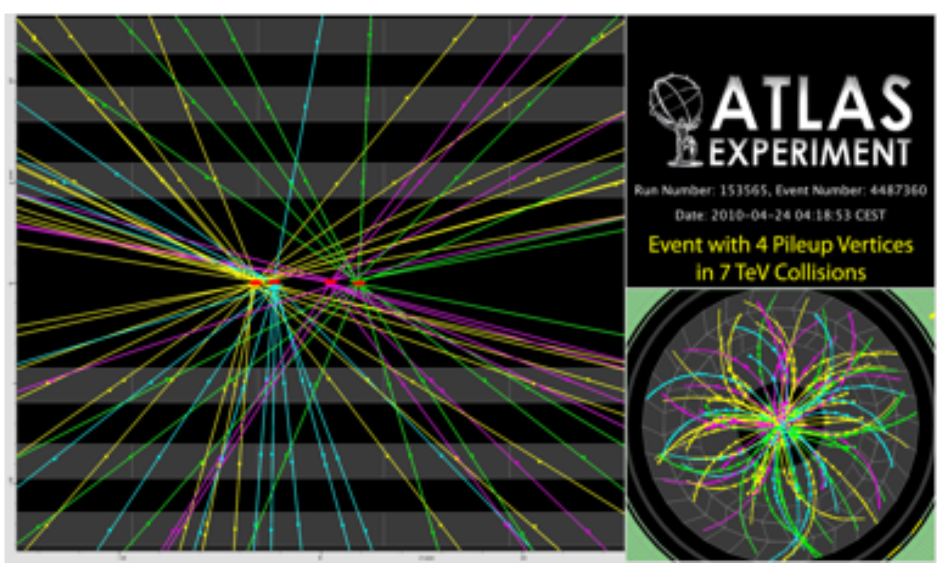
2009

single vertex reconstructed!

7 vertices



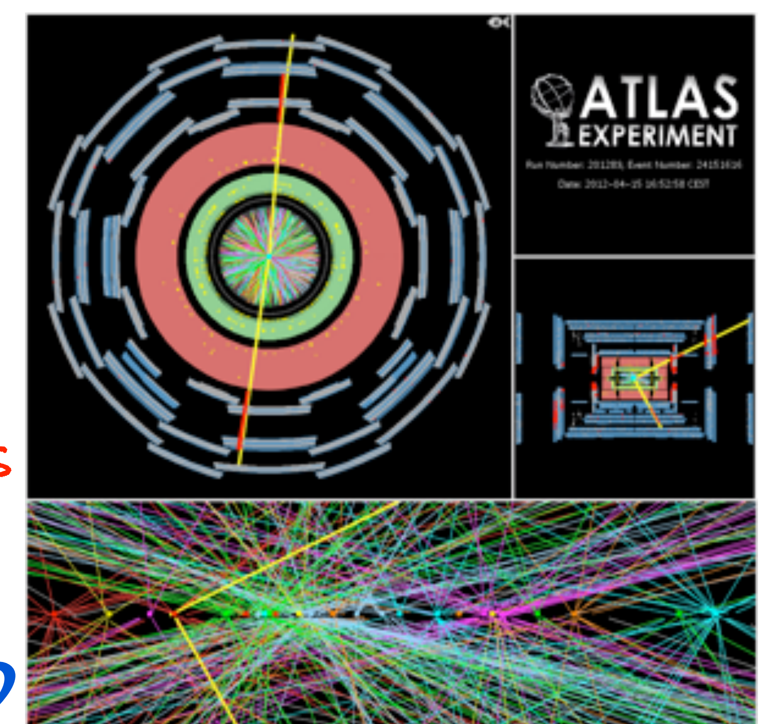
2011



4 vertices

2010

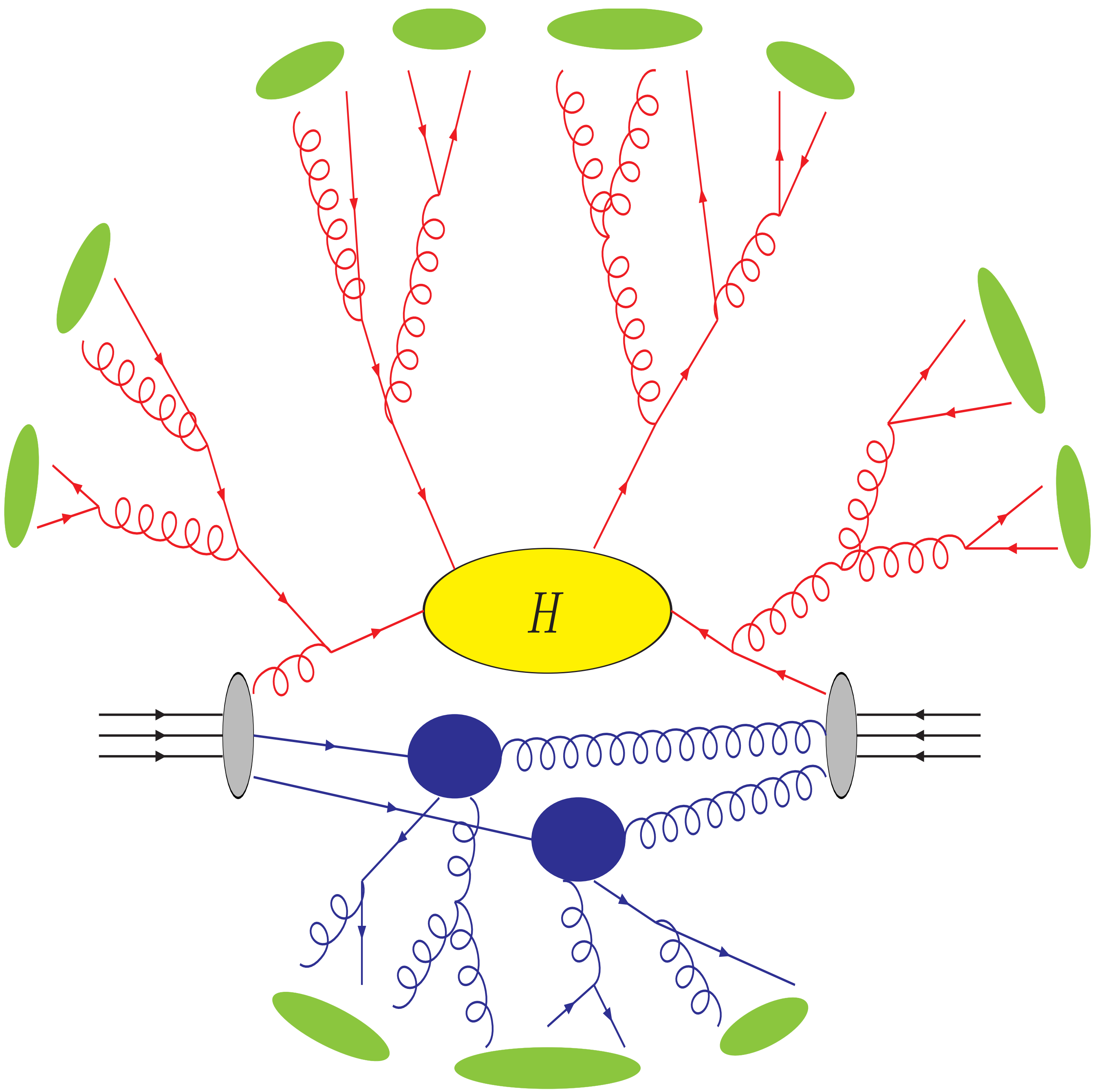
25 vertices



2012

Introduction

From theory point of view an event at the LHC looks very complicated



1. Incoming hadron (gray bubbles)
   - Parton distribution function
   - Multi parton distribution functions
2. Hard part of the process (yellow bubble)
   - Matrix element calculation, cross sections at LO, NLO, NNLO level
3. Radiation (red graphs)
   - Parton shower calculation
   - Partonic decay
   - Matching to NLO, NNLO
4. Underlying event (blue graphs)
   - Models based on multiple interaction
   - Diffraction
5. Hardonization (green bubbles)
   - Universal models
   - Hadronic decay
   - ....

What do we want?

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A general purpose parton shower program must generate partonic final states ready for hadronization

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- in a *FULLY exclusive way* (momentum, flavor, spin and color are fully resolved)

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- in a *FULLY exclusive way* (momentum, flavor, spin and color are fully resolved)
- as *precisely* as possible (*e.g.*: sums up large logarithms at NLL level).

How to Design Parton Showers?

Mandatory design principles

1. Shower generates events and calculates cross sections approximately using the soft and collinear factorization of the QCD amplitudes (tree and 1-loop level).
2. The emissions are strongly ordered.
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Some technical choices

6. Everything that makes the implementation simpler
   - leading color approximation
   - spin averaging
   - angular ordering (loosing full exclusiveness of the event)
   - Catani-Seymour momentum mapping
   - ....

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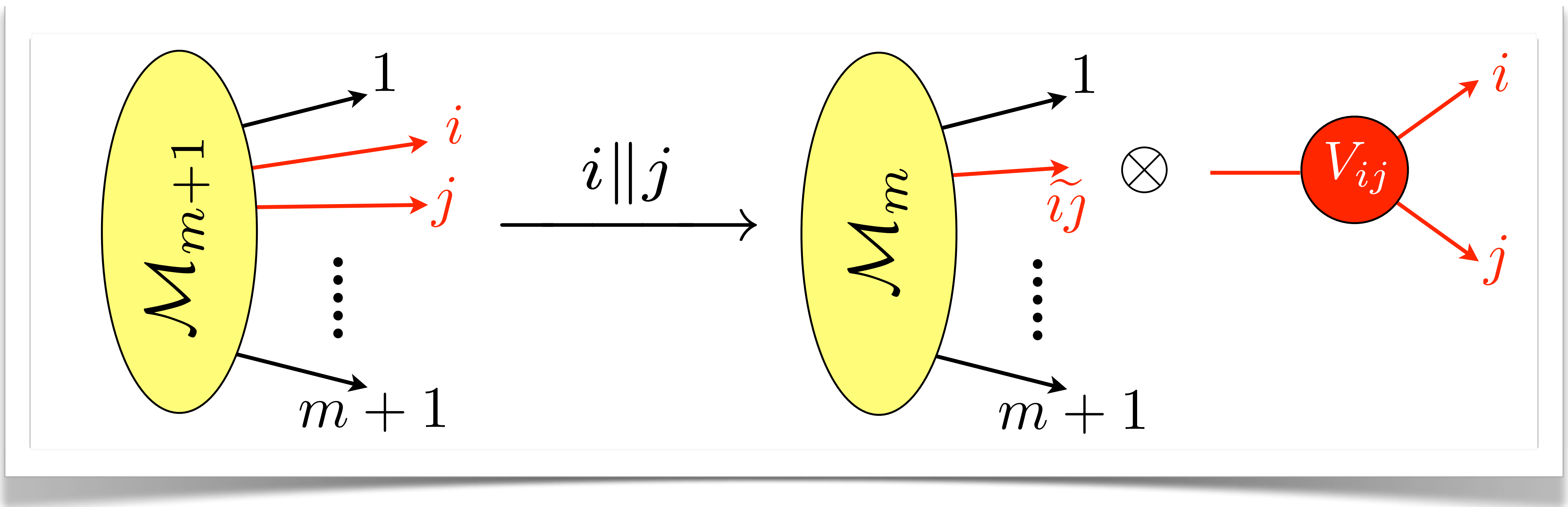
Factorization: Collinear limit

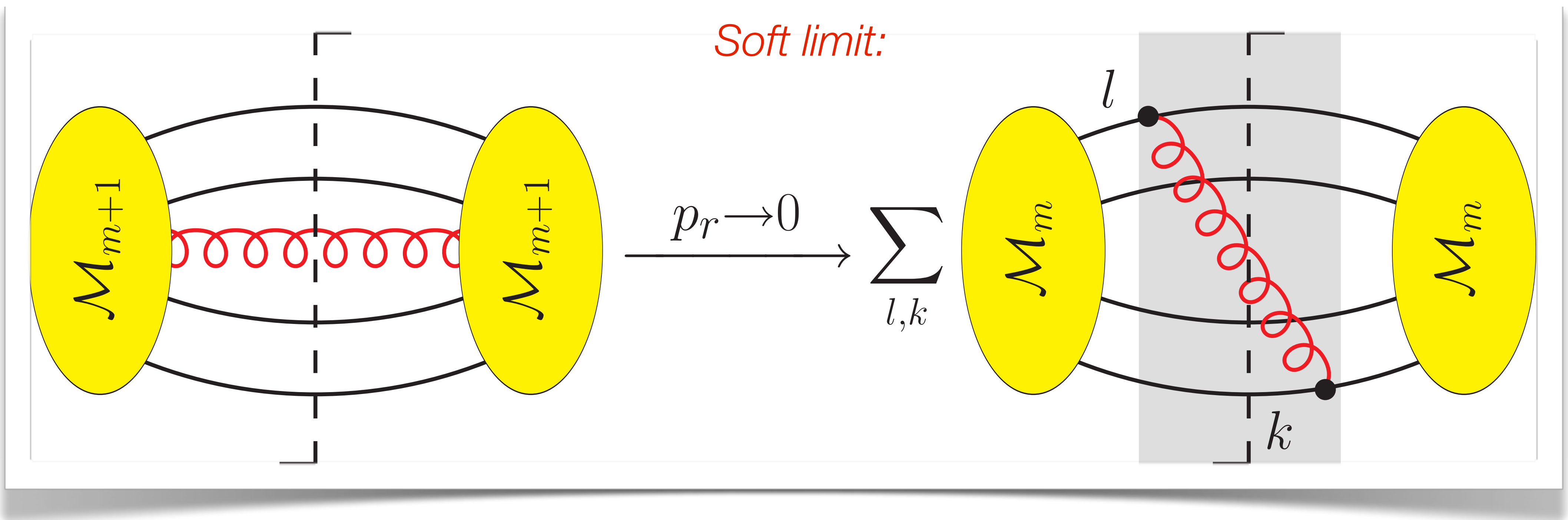
The QCD matrix elements have universal factorization property when two external partons become collinear

Collinear limit:



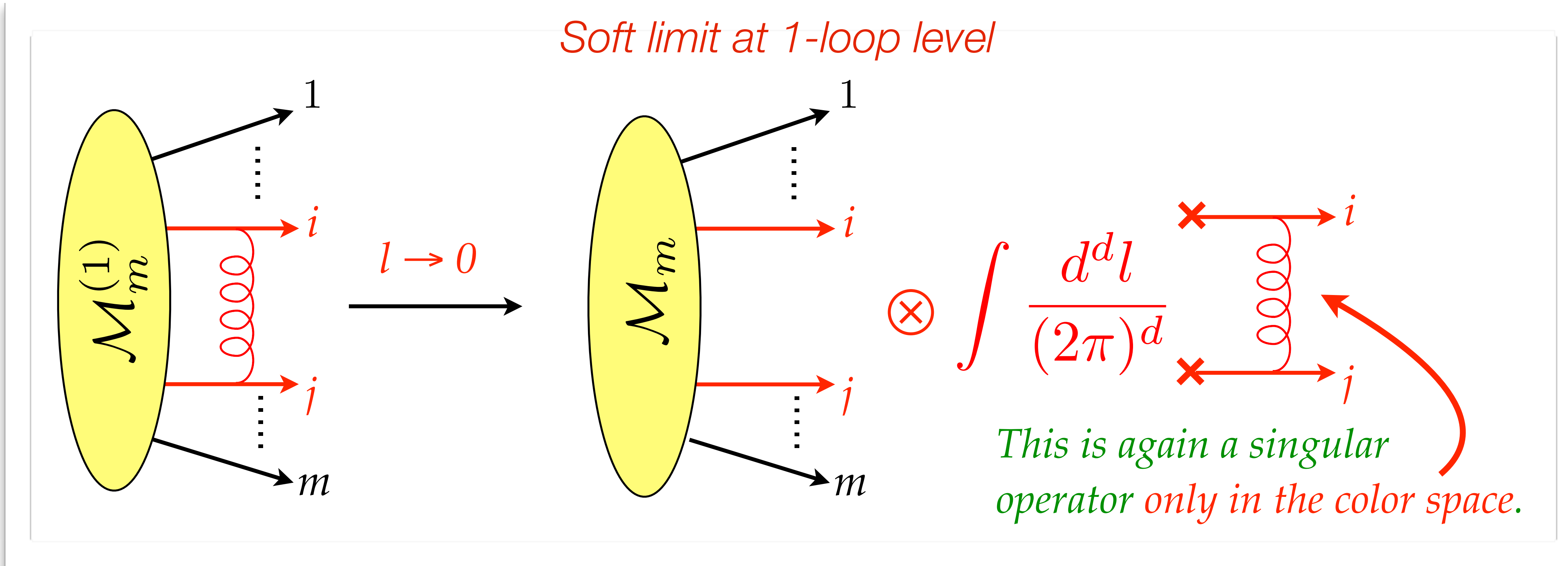
Factorization: Soft limit

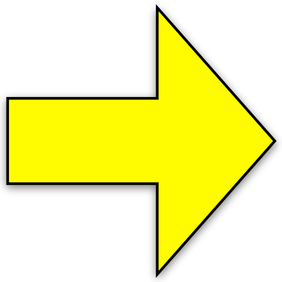
The QCD matrix elements have universal factorization property when an external gluon becomes soft



Factorization: Soft limit (1-loop)

There is another type of the unresolvable radiation, *the virtual (loop graph) contributions.* We have *universal factorization properties* for the loop graphs. E.g.: in the soft limit, when the loop momenta become soft we have





The splitting operators can be obtained from these factorization rules.

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Approx. of the Density Operator

Real radiation

$$|\rho^{\mathrm{R}}_{\infty}) \approx \int_{t}^{\infty} d\tau\, \mathcal{H}_I(\tau)|\rho(t))$$

Virtual radiation

$$|\rho^{\mathrm{V}}_{\infty}) \approx -\int_{t}^{\infty} d\tau\, \mathcal{V}_I^{(\epsilon)}(\tau)\,|\rho(t))$$

Here we impose strong ordering. Only the softer or more collinear radiation are allowed.

Some of the real emissions are not resolvable. Having a snapshot of the system at shower time t'

$$|\rho^{\mathrm{R}}_{\infty}) \approx \underbrace{\int_{t}^{t'} d\tau\, \mathcal{H}_I(\tau)\,|\rho(t))}_{\text{Resolved emissions}} + \underbrace{\int_{t'}^{\infty} d\tau\, \mathcal{V}_I^{(\epsilon)}(\tau)\,|\rho(t))}_{\text{Unresolved emissions}}$$

Resolved emissions

Unresolved emissions
This is a singular contribution

Combining the real and virtual contribution we have got

$$|\rho^{\mathrm{R}}_{\infty}) + |\rho^{\mathrm{V}}_{\infty}) = \int_{t}^{t'} d\tau\, [\mathcal{H}_I(\tau) - \mathcal{V}_I(\tau)]|\rho(t))$$

This operator dresses up the physical state with one real and virtual emissions those *are softer or more collinear than the hard state*. Thus the emissions are ordered.

Shower Operator

Now we can use this to build up physical states by considering all the possible way to go from t to t'.

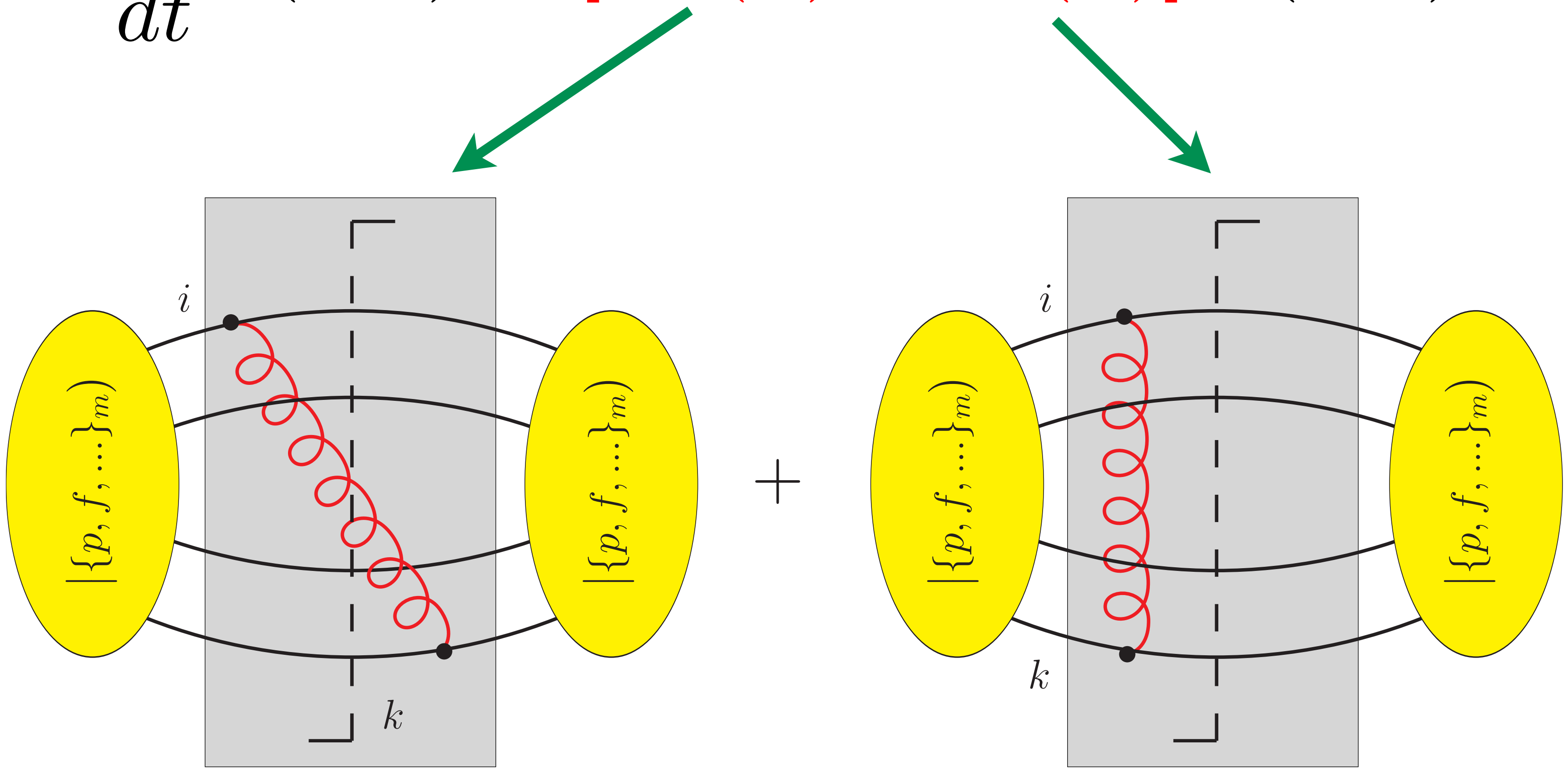
$$
\begin{aligned}
\big|\rho(t')\big) = &\ \big|\rho(t)\big) \\
&+ \int_t^{t'} d\tau\, [\mathcal{H}_I(\tau) - \mathcal{V}_I(\tau)]\big|\rho(t)\big) \\
&+ \int_t^{t'} d\tau_2\, [\mathcal{H}_I(\tau_2) - \mathcal{V}_I(\tau_2)] \int_t^{\tau_2} d\tau_1\, [\mathcal{H}_I(\tau_1) - \mathcal{V}_I(\tau_1)]\big|\rho(t)\big) \\
&+ \cdots \\
= &\ \underbrace{\mathbb{T}\exp\left\{\int_t^{t'} d\tau\, [\mathcal{H}_I(\tau) - \mathcal{V}_I(\tau)]\right\}}_{\mathcal{U}(t',t)\ \textit{shower evolution operator}}\big|\rho(t)\big)
\end{aligned}
$$

$$\big|\rho(t')\big) = \mathcal{U}(t',t)\big|\rho(t)\big)$$

Evolution Equation

The evolution operator obeys the following equation

$$\frac{d}{dt}\mathcal{U}(t',t) = [\mathcal{H}_I(t') - \mathcal{V}_I(t')]\,\mathcal{U}(t',t)$$



resolved radiations *unresolved radiation*

Evolution Equation

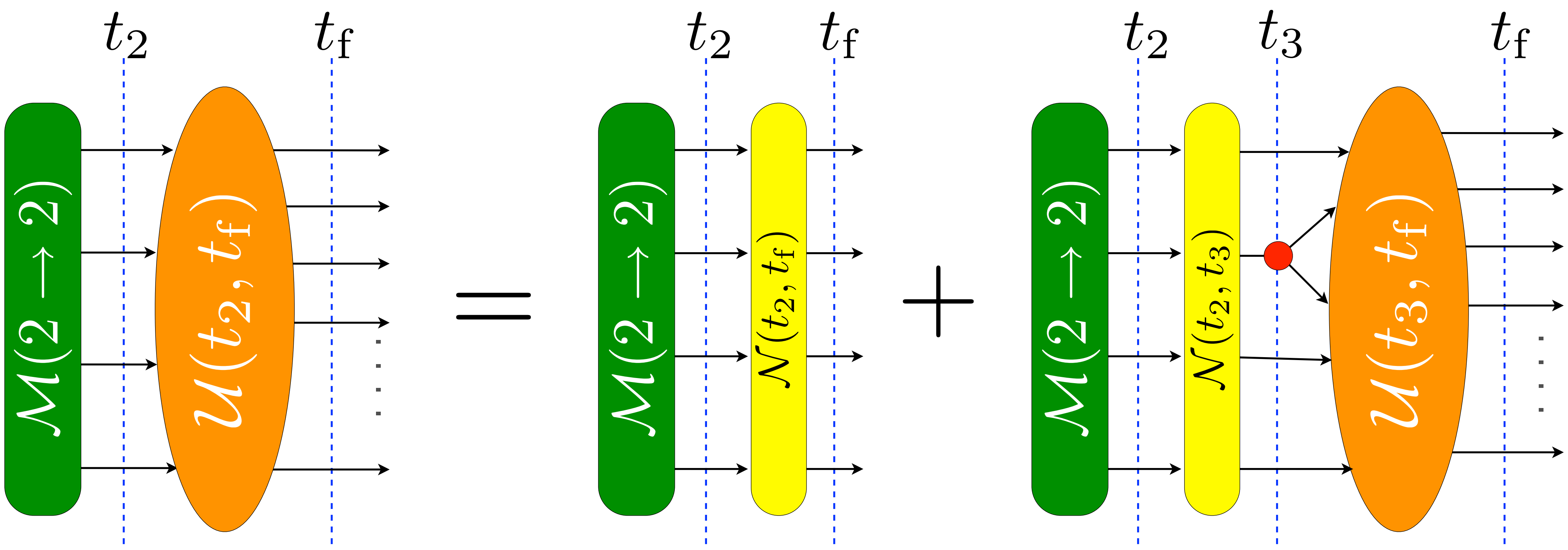
We can write the evolution equation in an integral equation form

$$\mathcal{U}(t_{\rm f},t_2) = \underbrace{\mathcal{N}(t_{\rm f},t_2)}_{\text{“Nothing happens”}} + \overbrace{\int_{t_2}^{t_{\rm f}} dt_3\, \mathcal{U}(t_{\rm f},t_3)\mathcal{H}_I(t_3)\mathcal{N}(t_3,t_2)}^{\text{“Something happens”}}$$

where the non-splitting operator is

$$\mathcal{N}(t',t) = \mathbb{T}\exp\left\{-\int_t^{t'} d\tau\, \mathcal{V}_I(\tau)\right\}$$

Sudakov operator



Splitting Operator

Very general splitting operator (*no spin correlation*) is

$$\big(\{\hat{p},\hat{f},\hat{c}',\hat{c}\}_{m+1}\big|\mathcal{H}(t)\big|\{p,f,c',c\}_m\big)$$
$$= \sum_{l=\mathrm{a},\mathrm{b},1,\dots,m} \delta\Big(t - T_l\big(\{\hat{p},\hat{f}\}_{m+1}\big)\Big)\,\big(\{\hat{p},\hat{f}\}_{m+1}\big|\mathcal{P}_l\big|\{p,f\}_m\big)\,\frac{m+1}{2}$$
$$\times \frac{n_\mathrm{c}(a)n_\mathrm{c}(b)\,\eta_\mathrm{a}\eta_\mathrm{b}}{n_\mathrm{c}(\hat{a})n_\mathrm{c}(\hat{b})\,\hat{\eta}_\mathrm{a}\hat{\eta}_\mathrm{b}}\,\frac{f_{\hat{a}/A}(\hat{\eta}_\mathrm{a},\mu_F^2)f_{\hat{b}/B}(\hat{\eta}_\mathrm{b},\mu_F^2)}{f_{a/A}(\eta_\mathrm{a},\mu_F^2)f_{b/B}(\eta_\mathrm{b},\mu_F^2)}\sum_k \Psi_{lk}(\{\hat{f},\hat{p}\}_{m+1})$$
$$\times \sum_{\beta=L,R}(-1)^{1+\delta_{lk}}\big(\{\hat{c}',\hat{c}\}_{m+1}\big|\mathcal{G}_\beta(l,k)\big|\{c',c\}_m\big)$$

Momentum and flavor mapping

PDF factor

Color dependence

Important: $A_{lk} + A_{kl} = 1$

Arbitrary function, helps to distribute the soft gluon along the collinear directions.

Splitting kernel is

It is only LO!

$$\Psi_{lk} = \frac{\alpha_\mathrm{s}}{2\pi}\frac{1}{\hat{p}_l\cdot\hat{p}_{m+1}}\underbrace{\left[A_{lk}\frac{2\hat{p}_l\cdot\hat{p}_k}{\hat{p}_k\cdot\hat{p}_{m+1}} + H_{ll}^{\mathrm{coll}}(\{\hat{f},\hat{p}\}_{m+1})\right]} + \cdots$$

Finite pieces

Altarelli-Parisi splitting function in a very general form

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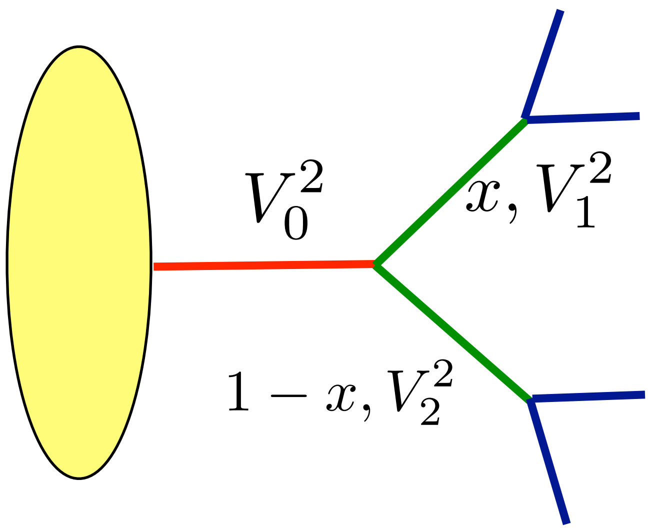
Shower Time

Let us consider a jet with two subsequent emission:



But in the shower algorithms the daughter partons were generated with zero virtuality and this lead to a different virtuality for the mother parton.

In order for the approximation of neglecting the virtualities of the daughter partons to be valid we need:

$$\frac{V_1^2}{x} \ll V_0^2$$

$$\frac{V_2^2}{1-x} \ll V_0^2$$

The shower time has to be

$$t = -\log \frac{(p_1 + p_2)^2 - m^2}{p \cdot Q_0}$$

The evolution variable has to be the *virtuality of the splitting divided by the mother parton energy*.

(Let's discuss this a little bit latter in more detail!)

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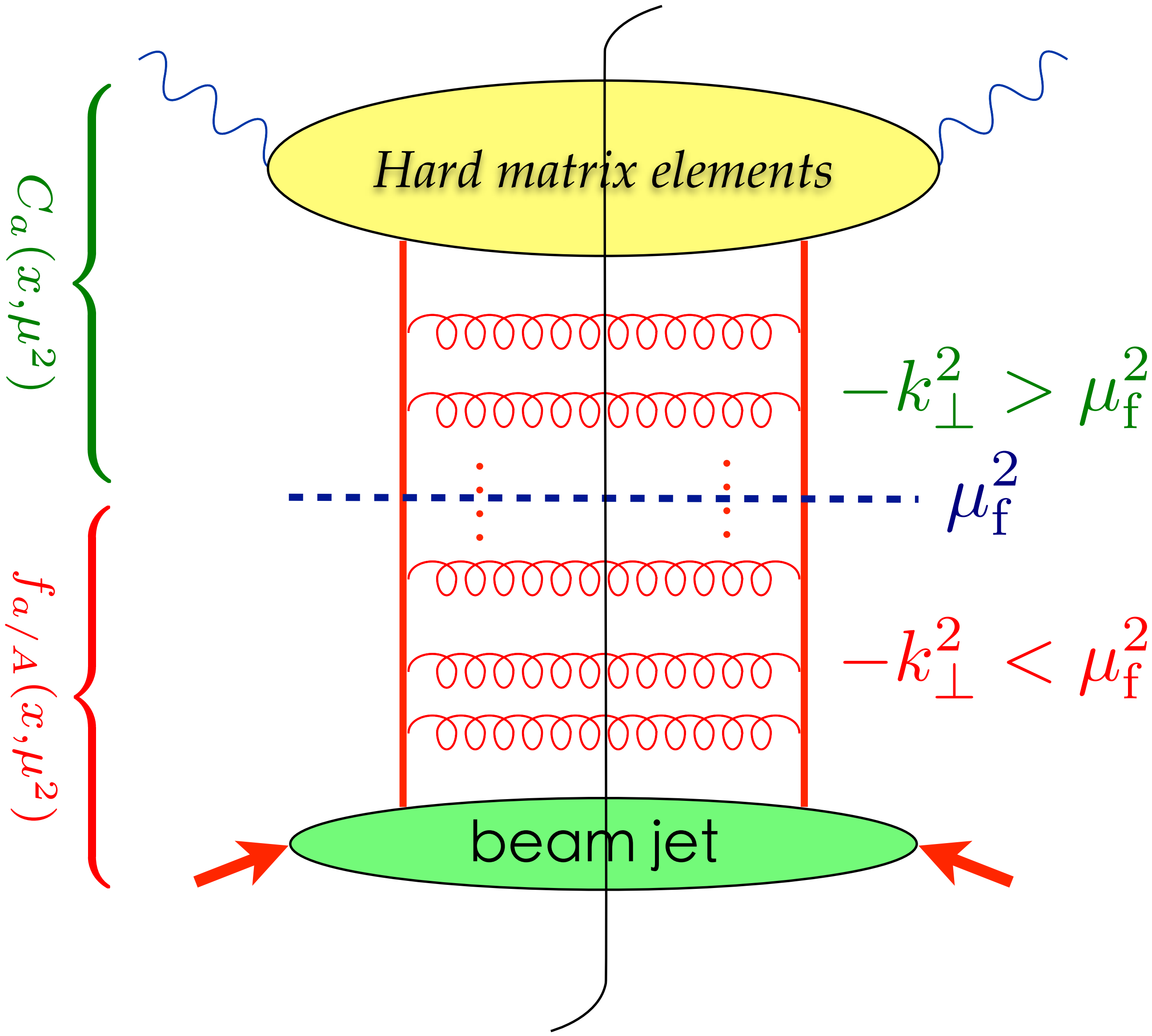
DGLAP Evolution of PDFs



Perturbative part (what we calculate)
Completely independent of the PDFs

$$|\rho(t_\mathrm{f})) = \underbrace{\mathcal{F}(t_\mathrm{f})}\,\overbrace{|\rho_\mathrm{pert}(t_\mathrm{f}))}$$

PDFs: The non-perturbative physics is only here

It MUST BE independent of the PDF, otherwise the perturbative and non-perturbative physics are mixed.

Non-trivial PDF dependence

$$\mathbb{T}\exp\left\{\overbrace{-\int_t^{t'} d\tau\,\mathcal{V}_I(\tau)}\right\} = \mathcal{N}(t',t) = \mathcal{F}(t')\mathcal{N}_\mathrm{pert}(t',t)\mathcal{F}^{-1}(t) = \mathcal{F}(t')\,\mathbb{T}\exp\left\{\overbrace{-\int_t^{t'} d\tau\,\mathcal{V}_I^\mathrm{pert}(\tau)}\right\}\mathcal{F}^{-1}(t)$$

Leads to the evolution equation of the parton distribution functions.

DGLAP Evolution

In general the incoming parton can be massive, this leads to a slightly modified DGLAP evolution. That is

$$\mu^2 \frac{d}{d\mu^2} f_{a/A}(\eta, \mu^2) = \sum_{\hat{a}} \int_0^1 \frac{dz}{z} \, \frac{\alpha_s(\mu^2)}{2\pi} P_{a\hat{a}}\left(z, z\frac{m^2}{\mu^2}\right) f_{\hat{a}/A}(\eta/z, \mu^2)$$

with the modified evolution kernels:

$$P_{\mathrm{qq}}(z,\lambda) = C_{\mathrm{F}} \left[\left(\frac{2}{1-z} - (1+z) - 2\lambda \right) \theta\left(\frac{1}{1-z} > 1 + \lambda \right) \right]_+ \quad ,$$

$$P_{\mathrm{gg}}(z,\lambda) = 2C_{\mathrm{A}} \left[\frac{1}{(1-z)_+} - 1 + \frac{1-z}{z} + z(1-z) \right] + \gamma_{\mathrm{g}}(\lambda)\, \delta(1-z) \quad ,$$

$$P_{\mathrm{qg}}(z,\lambda) = T_{\mathrm{R}} \left[1 - 2\, z\, (1-z) + 2\lambda \right] \theta(z(1-z) > \lambda) \quad ,$$

$$P_{\mathrm{gq}}(z,\lambda) = C_{\mathrm{F}} \left[\frac{1+(1-z)^2}{z} - 2\lambda \right] \theta\left(\frac{1}{z} > 1 + \lambda \right) \quad .$$

DGLAP Evolution

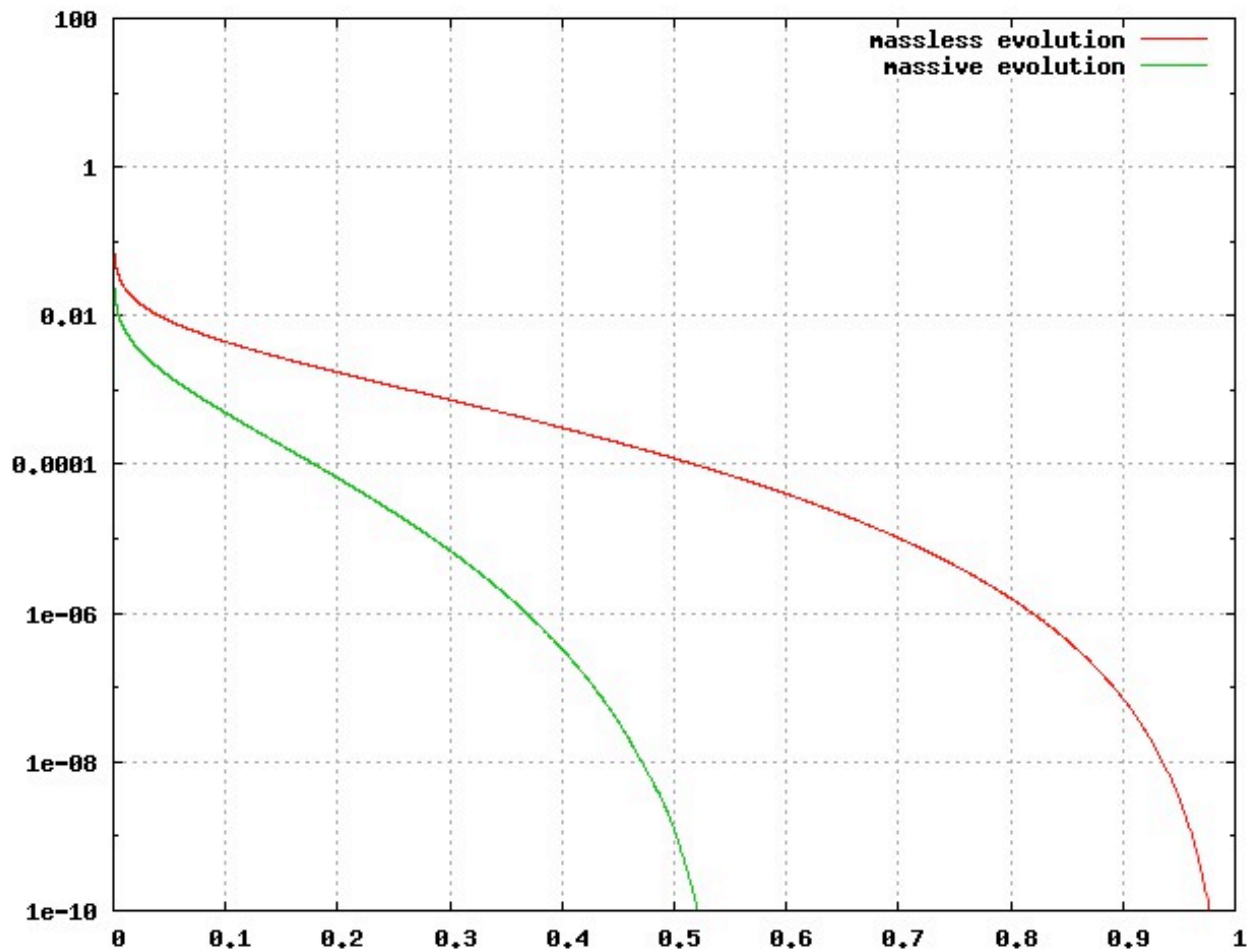
In general the … evolution.
That is

$$\mu^2 \frac{d}{d\mu^2} \; \ldots \; (\eta/z, \mu^2)$$

with the moc…

$$P_{qq}(z, \ldots \quad ,$$
$$+$$
$$P_{gg}(z, \ldots \quad (1-z) \quad ,$$
$$P_{qg}(z, \ldots$$
$$P_{gq}(z, \ldots$$

b-quark distribution at q^2=50 GeV^2



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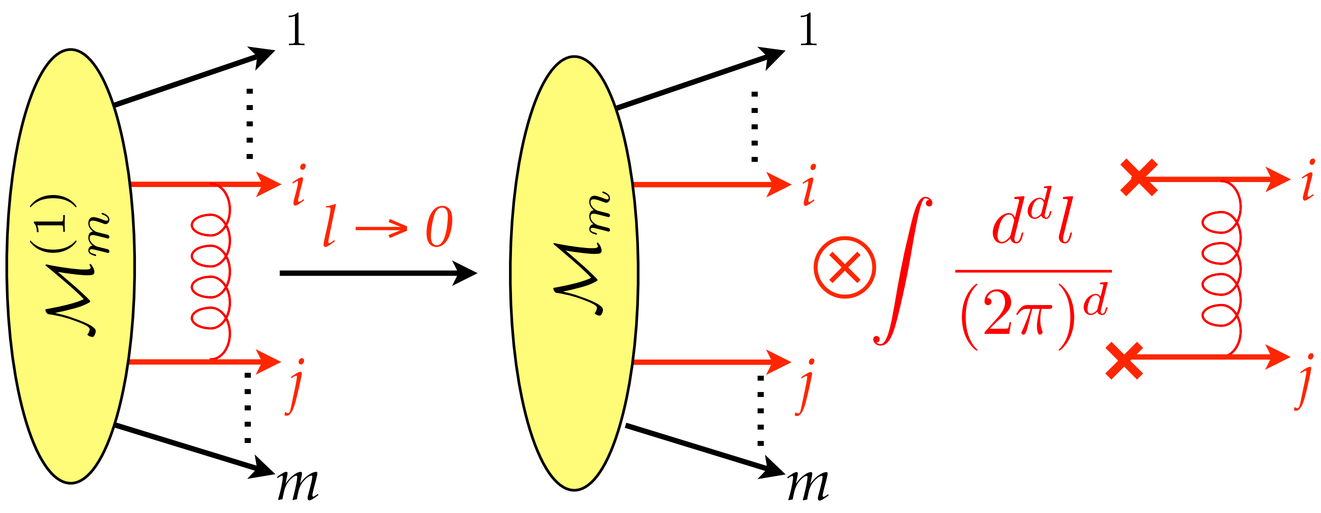
Unitarity Condition



$$\mathcal{V}_I(t) = \mathcal{V}(t) + \underbrace{\mathrm{i}\pi\widetilde{\mathcal{V}}(t)}$$

Coulomb gluon

$$(1|\widetilde{\mathcal{V}}(t) = 0$$

The singularities must be cancelled in the soft and collinear limits between the real and virtual emissions

$$(1|\,[\mathcal{H}_I(t) - \mathcal{V}_I(t)] = \mathrm{Finite(t)} \xrightarrow{t\to\infty} 0$$

In parton shower implementation we always choose

$$\mathrm{Finite(t)} = 0 \qquad \text{for every } t$$

Unitarity condition

The shower evolution *doesn't change the normalization.*

Unitarity condition is not God given, not derived from first principles. It is only a convenient choice!!! In some cases it is rather an unpleasant limitation....

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- in a *FULLY exclusive way* (momentum, flavor, spin and color are fully resolved)
- as *precisely* as possible (*e.g.*: sums up large logarithms at NLL level).

Splitting Operator

Most of the component of the parton shower have been fixed

Momentum and flavor mapping

$$
\begin{aligned}
&\left(\{\hat p,\hat f,\hat c',\hat c\}_{m+1}\middle|\mathcal{H}(t)\middle|\{p,f,c',c\}_m\right)\\
&= \sum_{l=\mathrm{a},\mathrm{b},1,\dots,m} \delta\Big(t - T_l\big(\{\hat p,\hat f\}_{m+1}\big)\Big)\left(\{\hat p,\hat f\}_{m+1}\middle|\mathcal{P}_l\middle|\{p,f\}_m\right)\frac{m+1}{2}\\
&\quad\times \frac{n_\mathrm{c}(a)n_\mathrm{c}(b)\,\eta_\mathrm{a}\eta_\mathrm{b}}{n_\mathrm{c}(\hat a)n_\mathrm{c}(\hat b)\,\hat\eta_\mathrm{a}\hat\eta_\mathrm{b}}\,\frac{f_{\hat a/A}(\hat\eta_\mathrm{a},\mu_F^2)f_{\hat b/B}(\hat\eta_\mathrm{b},\mu_F^2)}{f_{a/A}(\eta_\mathrm{a},\mu_F^2)f_{b/B}(\eta_\mathrm{b},\mu_F^2)}\sum_k \Psi_{lk}(\{\hat f,\hat p\}_{m+1})\\
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$$

We still have to say something about the

- *momentum mapping and*
- *soft partitioning function*

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&\big(\{\hat p,\hat f,\hat c',\hat c\}_{m+1}\big|\mathcal{H}(t)\big|\{p,f,c',c\}_m\big) \\
&= \sum_{l=\mathrm{a,b},1,\dots,m} \delta\Big(t - T_l\big(\{\hat p,\hat f\}_{m+1}\big)\Big)\,\big(\{\hat p,\hat f\}_{m+1}\big|\mathcal{P}_l\big|\{p,f\}_m\big)\,\frac{m+1}{2} \\
&\times \frac{n_{\mathrm c}(a)n_{\mathrm c}(b)\,\eta_{\mathrm a}\eta_{\mathrm b}}{n_{\mathrm c}(\hat a)n_{\mathrm c}(\hat b)\,\hat\eta_{\mathrm a}\hat\eta_{\mathrm b}}\,\frac{f_{\hat a/A}(\hat\eta_{\mathrm a},\mu_F^2)f_{\hat b/B}(\hat\eta_{\mathrm b},\mu_F^2)}{f_{a/A}(\eta_{\mathrm a},\mu_F^2)f_{b/B}(\eta_{\mathrm b},\mu_F^2)}\sum_k \Psi_{lk}(\{\hat f,\hat p\}_{m+1}) \\
&\times \sum_{\beta=L,R}(-1)^{1+\delta_{lk}}\big(\{\hat c',\hat c\}_{m+1}\big|\mathcal{G}_\beta(l,k)\big|\{c',c\}_m\big)
\end{aligned}
$$

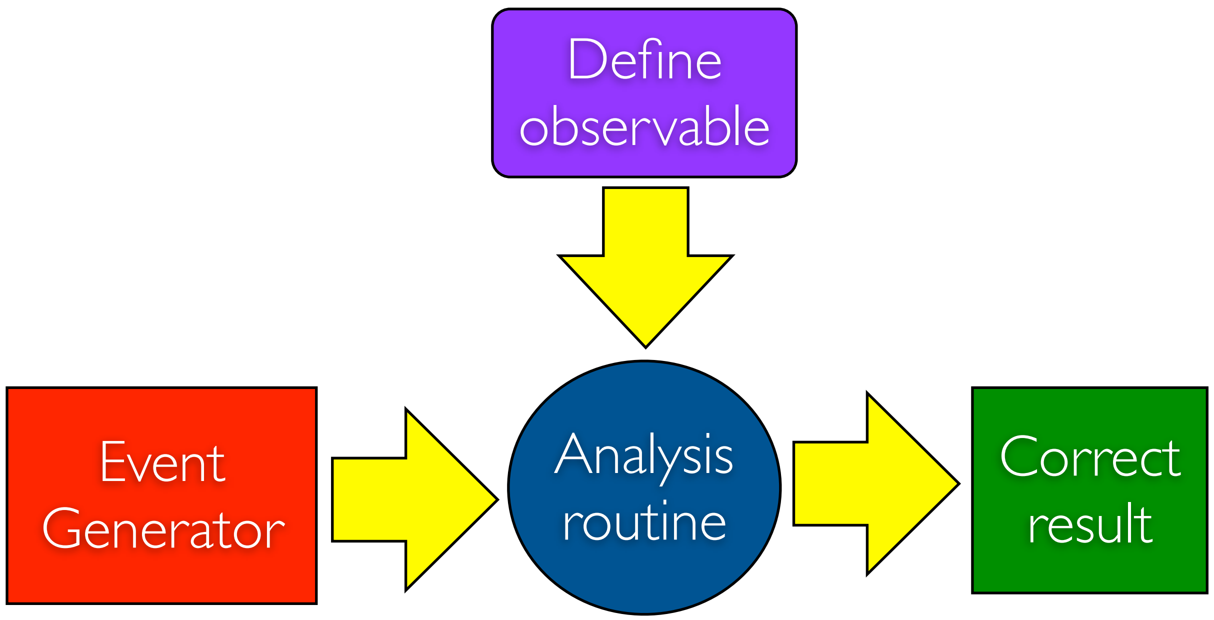
$$
\Psi_{lk} = \frac{\alpha_{\mathrm s}}{2\pi}\,\frac{1}{\hat p_l\cdot\hat p_{m+1}}\left[A_{lk}\,\frac{2\hat p_l\cdot\hat p_k}{\hat p_k\cdot\hat p_{m+1}} + H_{ll}^{\mathrm{coll}}(\{\hat f,\hat p\}_{m+1})\right]
$$

We still have to say something about the

- *momentum mapping and*
- *soft partitioning function*

Is NLL precision inevitable?

One might imagine that because parton splitting functions are correct in the limits of soft and collinear splittings, all large log summations will come out correctly.



- Eye measure doesn't help to validate parton showers against analytical results.
- One has to solve the shower evolution equation analytically and compare the result at NLL level. (e.g.: Drell-Yan pT-distribution, e+e- event shapes) (**JHEP**1003(2010)097)
- "Minor details" are important. Once they are fixed the resummation works.
- It requires more studies to understand what class of observables can be predicted at (N)LL accuracy from parton showers.
- Recent results gives us only some hints about the *soft partitioning function (A_{Ik})* and the *momentum mapping*.

✓ The momentum mappings with *global recoil* are *more preferred.*

✓ The soft partitioning function should depend on *only relative angles.*

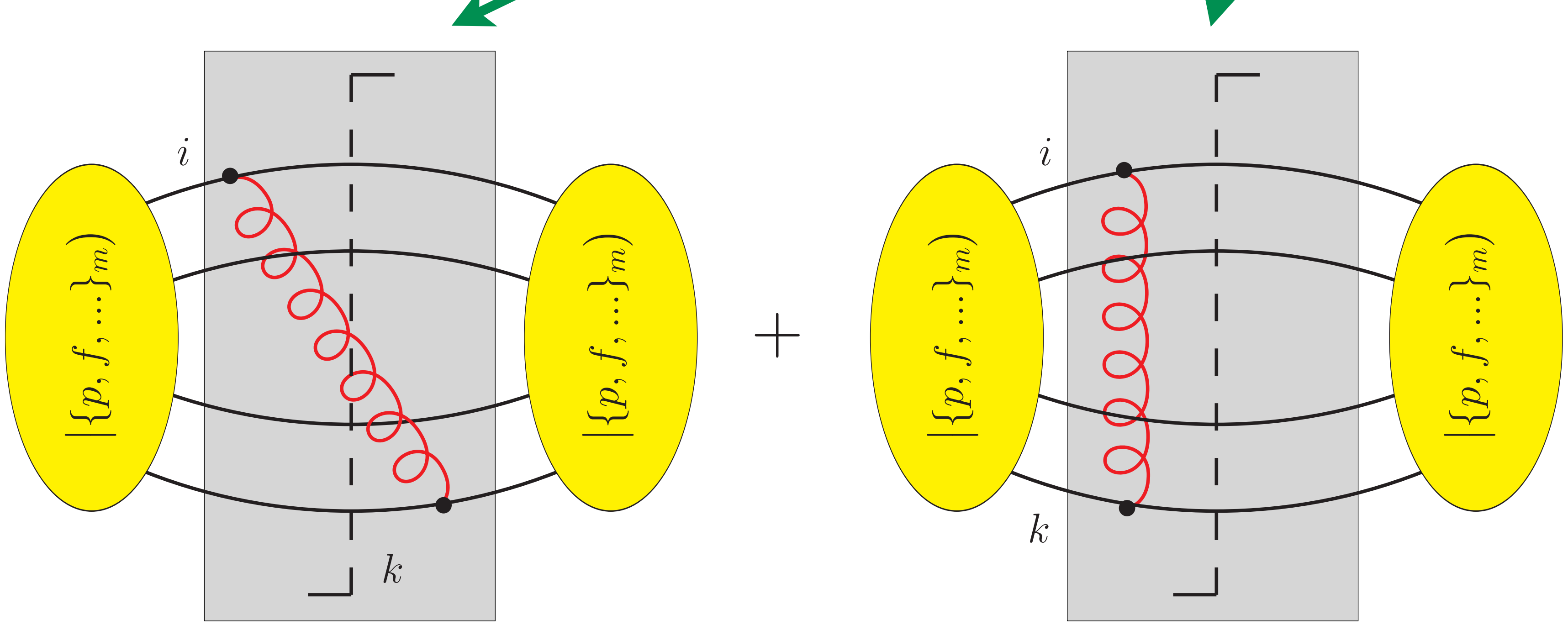
(These are only hints, we don't have solid proof, only some counter examples.)

Matching

The parton shower starts from the simplest *2→2 like process and* generates the QCD density operator approximately. It would be nice to use *exact tree and 1-loop level amplitudes without double counting and destroying the exclusiveness* of the shower events.

$$\big|\rho(t)\big) = \mathcal{U}(t,0)\big|\rho_0\big) = \big|\rho_0\big) + \int_0^t d\tau\, \mathcal{U}(t,\tau)\left[\mathcal{H}_I(\tau)\big|\rho_0\big) - \mathcal{V}_I(\tau)\big|\rho_0\big)\right]$$

Born term



resolved radiations

unresolved radiation

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$$\big|\rho(t)\big) = \mathcal{U}(t,0)\big|\rho_0\big) + \int_0^t d\tau\, \mathcal{U}(t,\tau)\Big\{ \underbrace{[\mathcal{H}_I(\tau) - \mathcal{V}_I(\tau)]\,\big|\rho_0\big)}_{\approx\, |\rho_R(\tau)) + |\rho_V(\tau))} - [\mathcal{H}_I(\tau) + \mathcal{V}_I(\tau)]\,\big|\rho_0\big)\Big\}$$

$\big|\rho_R(\tau)\big)$: The real contribution is based on the Born level *2→3 amplitudes*

$$\big|\rho_V(\tau)\big) = -\mathcal{V}_I^{(\epsilon)}(\tau)\big|\rho_0\big) + \underbrace{\delta(\tau)\big|\tilde{\rho}_V\big)}$$

Finite part of the 1-loop density operator

$$\lim_{t\to\infty}\int_0^t d\tau\big|\rho_V(\tau)\big) \Leftrightarrow \underbrace{\big|M^{(1)}\big\rangle\big\langle M^{(0)}\big| + c.c.}$$

1-loop density operator with the 1/ε and 1/ε² singularities

Matching

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$$\big|\rho(t)\big) = \mathcal{U}(t,0)\,\Big[\big|\rho_0\big) + \big|\tilde{\rho}_V\big)\Big] + \int_0^t d\tau\,\mathcal{U}(t,\tau)\,\Big[\big|\rho_R(\tau)\big) - \mathcal{H}_I(\tau)\big|\rho_0\big)\Big]$$

- ✓ This is NLO level matching.
- ✓ Preserves the precision and exclusiveness of the shower.
- ✓ It is possible because the shower scheme also defines a subtraction scheme to calculate NLO fixed order cross sections.
- ✓ It works only for *2→2 like process.*
- ✗ *For higher multiplicity matching we have to work harder. As far as I know there is no solution in the literature.*

Where is the code???

Where is the code???

Implementation Issues

The implementation of this scheme is not trivial, we have several difficulties

- We have rather *complicated splitting kernels* due to the choice of the soft partitioning function (based on relative angles only) and the momentum mapping (a global mapping).
- We consider massive partons both in the initial and final states. The threshold effects are not trivial. MSbar PDFs cannot be used.
- We *do color evolution*. This is the 30 years old challenge of the parton shower development.

- Simple MC methods are not sufficient enough.
- We need PDF with *massive evolution* that considers kinematical effects. We need extremely *precise PDF tables*.
- The biggest problem is to deal with the Sudakov operator. The solution is still based on approximation but it is *beyond the leading color* approximation and *systematically improvable*.

$$\mathcal{N}(t',t) = \mathbb{T}\exp\left\{-\int_t^{t'} d\tau\, \mathcal{V}_I(\tau)\right\}$$

This is an operator in the color space. For N parton we have to exponentiate an N! x N! non-diagonal matrix.

Implementation

Available features

- The core shower algorithm is implemented
  - ✓ Massive parton in the initial and final states + new DGLAP evolution
  - ✓ Color evolution (**JHEP**06(2012)044)
  - ✓ Hadron-hadron, DIS, e+e- and decay showing
  - ✓ ISR and FSR are NOT independent
- At the moment two processes are available
  - ✓ $e^+e^- \longrightarrow jets$
  - ✓ $pp, p\bar{p} \longrightarrow Z^0/\gamma + jets$

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Testing, testing and more testing

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Testing, testing and more testing

Under development

- More *2→2 like* processes in the core package
  - ➠ These processes will be matched to NLO
- Heavy flavor decays with nested showering
- Coulomb gluon (**JHEP**06(2012)044)
- Spin correlations (**JHEP**0807:025(2008))
- Hadronization from PYTHIA
- Threshold resummation of soft gluons
- More processes from HELAC
  - Multi jet matching at NLO level
- Finding a name (at the moment the package is called to nlojet++-core)

Implementation

We calculate Drell-Yan total cross section at 14TeV with $(0.7\text{ GeV})^2 < Q^2 < (1\text{TeV})^2$

$$\sigma_{\text{tot}} = \sigma_{\text{tot}}\left[0.6588 + \underbrace{\frac{1}{N_c^2}2.097108}_{23\%} + \overbrace{\frac{1}{N_c^4}6.0241887}^{7.5\%} + \underbrace{\frac{1}{N_c^6}19.6786989}_{2.7\%} + \cdots\right]$$

The subleading color contributions are not just 10% what we naively expect.

Comparison

	HERWIG++	PYTHIA	SHERPA (C-S shower)	OUR SHOWER
Shower time	*Emission angle violates exclusiveness*	*Transverse momentum*	*Transverse momentum*	*Virtuality divided by mother parton energy*
Soft partitioning function	*Based on angles*	*Energy dependence*	*Energy dependence*	*Based on angles*
Momentum mapping	*Global mapping (only at the end)*	*Local mapping (FSR) Global mapping (ISR)*	*Local mapping*	*Global mapping*
Color treatment, Wide angle soft gluon	*Full color evolution Wide angle gluon dropped*	*Leading color approx. ??Half dipole (Ini-Fin)*	*Leading color approx.*	*Color evolution*
PDF, initial state massive parton	*MSbar PDF No massive quark*	*MSbar PDF No massive quark*	*MSbar PDF No massive quark*	*Massive PDF modified DGLAP*
Normalization	*Unitarity condition*	*Unitarity condition*	*Unitarity condition*	*Unitarity condition*
Coulomb gluon	*Not possible*	*Not possible*	*Not possible*	*Not yet implemented*

Conclusions

- Building a parton shower is not simple and there are still many things to study in order to be able to consider shower cross sections as predictions.
- We have less freedom than we thought (ordering, soft gluons, splitting kernel, mapping, PDF, matching, higher order effects,...)
- More theory input is required (factorization, PDF definition, resummation, ...)
- But we made some progress and these ideas have been implemented in a general purpose Monte Carlo program.
- More manpower is needed for implementing new features, testing, tuning, ...