Top pair and single top differential cross sections

Nikolaos Kidonakis

(Kennesaw State University)

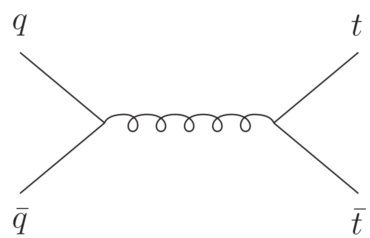
- $t\bar{t}$ **and single top production channels**
- **Higher-order two-loop corrections**
- $t\bar{t}$ **cross section at LHC and Tevatron**
- **Top** p_T **and rapidity distributions**
- t**-channel and** s**-channel production**
- tW^-, tH^-, **and FCNC processes**
- W **production at large** p_T

Top production partonic processes at LO

Top-antitop pair production

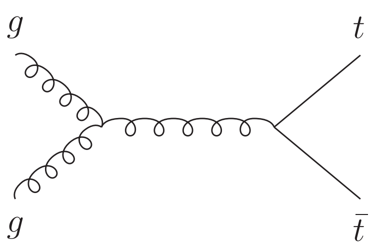
- $q\bar{q} \longrightarrow t\bar{t}$

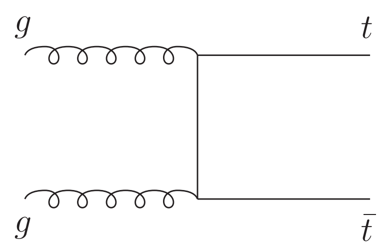
dominant at Tevatron

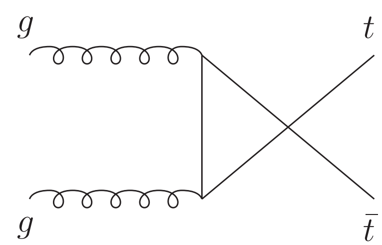


- $gg \longrightarrow t\bar{t}$

dominant at LHC



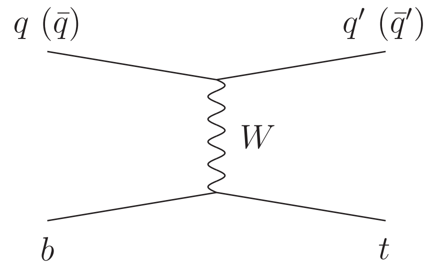




Single top quark production

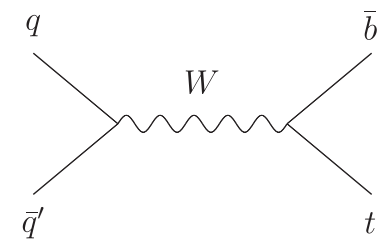
• t **channel:** $qb \to q't$ **and** $\bar{q}b \to \bar{q}'t$

dominant at Tevatron and LHC



• s **channel:** $q\bar{q}' \to \bar{b}t$

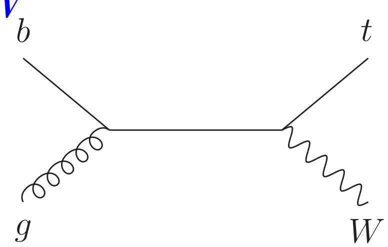
small at Tevatron and LHC

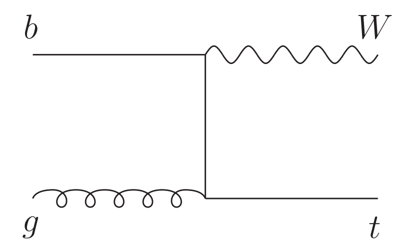


• **associated** tW **production:** $bg \to tW^-$

very small at Tevatron, significant at LHC

Related process: $bg \to tH^-$





Higher-order corrections

QCD corrections significant for top pair and single top quark production

Soft-gluon corrections from emission of soft (low-energy) gluons

Soft corrections: $\left[\frac{\ln^k(s_4/m^2)}{s_4}\right]_+$ with $k \le 2n-1$, s_4 distance from threshold

Soft-gluon corrections are dominant near threshold

Resum these soft corrections - factorization and RGE

Complete results at NNLL–two-loop soft anomalous dimension

NK, PRD 82, 114030 (2010); PRD 84, 011504 (2011) ($t\bar{t}$)

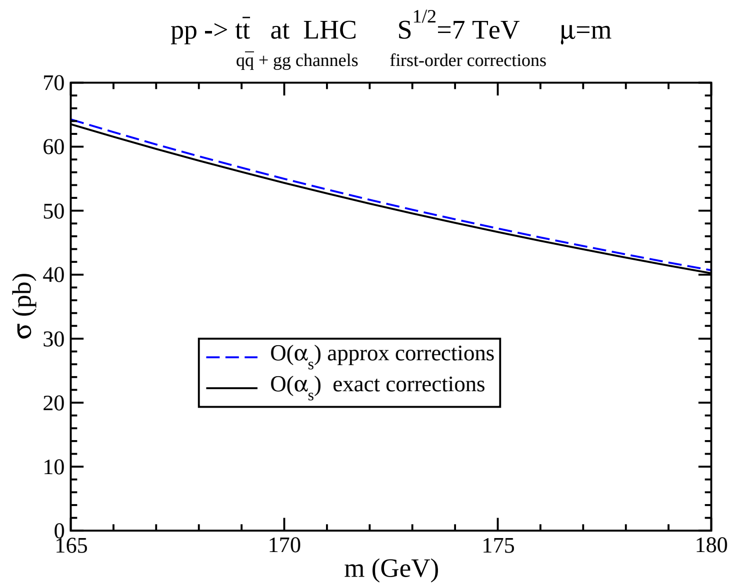
NK, PRD81, 054028 (2010); PRD 82, 054018 (2010); PRD 83,091503 (2011) (single top)

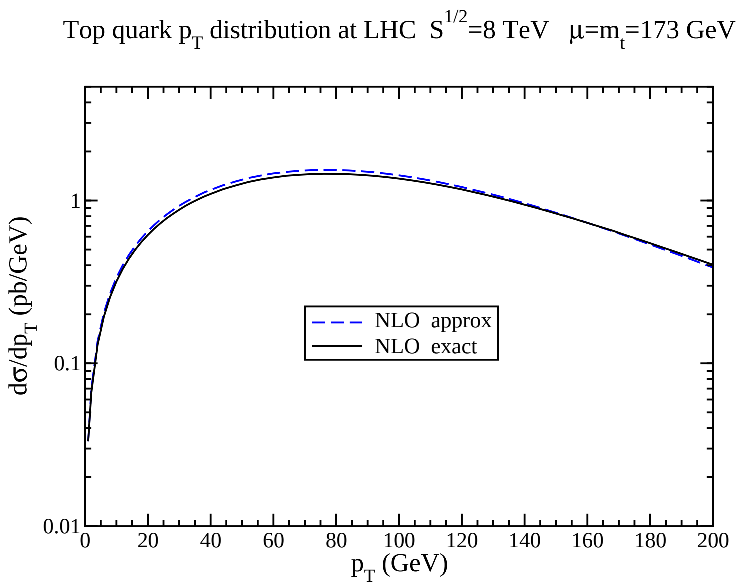
Approximate NNLO cross section from expansion of resummed cross section

This is the only calculation for partonic threshold at the double differential cross section level using the standard moment-space resummation in pQCD

Threshold approximation

Approximation works very well not only for Tevatron but also for LHC energies because partonic threshold is still important





only 1% difference between first-order approximate and exact corrections
→ less than 1% difference between NLO approximate and exact cross sections
Also true for differential distributions

For best prediction add NNLO approximate corrections to exact NLO cross section

Factorization and Resummation

Resummation follows from factorization properties of the cross section - performed in moment space

$\sigma = (\prod \psi)\, H_{IL}\, S_{LI}\, (\prod J)$ H: **hard-scattering function**

S: **soft-gluon function**

Use RGE to evolve soft-gluon function

$$\left(\mu \frac{\partial}{\partial \mu} + \beta(g_s)\frac{\partial}{\partial g_s}\right) S_{LI} = -(\Gamma_S^\dagger)_{LB} S_{BI} - S_{LA}(\Gamma_S)_{AI}$$

Γ_S **is the soft anomalous dimension - a matrix in color space and a function of kinematical invariants** s, t, u

Resummed cross section

$$\hat{\sigma}^{res}(N) = \exp\left[\sum_i E_i(N_i)\right] \exp\left[\sum_j E'_j(N')\right] \exp\left[\sum_{i=1,2} 2\int_{\mu_F}^{\sqrt{s}} \frac{d\mu}{\mu}\, \gamma_{i/i}\left(\tilde{N}_i, \alpha_s(\mu)\right)\right]$$
$$\times \operatorname{tr}\left\{H\left(\alpha_s\right) \exp\left[\int_{\sqrt{s}}^{\sqrt{s}/\tilde{N}'} \frac{d\mu}{\mu}\, \Gamma_S^\dagger\left(\alpha_s(\mu)\right)\right] S\left(\alpha_s\left(\frac{\sqrt{s}}{\tilde{N}'}\right)\right) \exp\left[\int_{\sqrt{s}}^{\sqrt{s}/\tilde{N}'} \frac{d\mu}{\mu}\, \Gamma_S\left(\alpha_s(\mu)\right)\right]\right\}$$

collinear and soft radiation from incoming partons

$$E_i(N_i)=\int_0^1 dz \frac{z^{N_i-1}-1}{1-z}\left\{\int_1^{(1-z)^2}\frac{d\lambda}{\lambda}A_i\left(\alpha_s(\lambda s)\right)+D_i\left[\alpha_s((1-z)^2 s)\right]\right\}$$

purely collinear: replace $\frac{z^{N-1}-1}{1-z}$ **by** $-z^{N-1}$

factorization scale μ_F **dependence controlled by**
$\gamma_{i/i}=-A_i \ln \tilde{N}_i+\gamma_i$

Noncollinear soft gluon emission controlled by the soft anomalous dimension Γ_S

determine Γ_S **from coefficients of ultraviolet poles in dimensionally regularized eikonal diagrams**

Γ_S **is process-dependent; calculated at two loops**

We are resumming $\ln^k N$ **- we can expand to fixed order and invert to get** $\ln^k(s_4/m_t^2)/s_4$

Eikonal approximation

Feynman rules for soft gluon emission simplify

$$\bar{u}(p)\,(-ig_s T_F^c)\,\gamma^\mu \frac{i(\not p + \not k + m)}{(p+k)^2 - m^2 + i\epsilon} \rightarrow \bar{u}(p)\,g_s T_F^c\,\gamma^\mu \frac{\not p + m}{2p \cdot k + i\epsilon} = \bar{u}(p)\,g_s T_F^c\,\frac{v^\mu}{v \cdot k + i\epsilon}$$

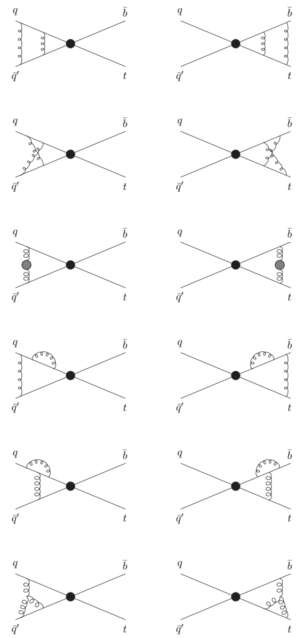
Perform calculations in momentum space and Feynman gauge

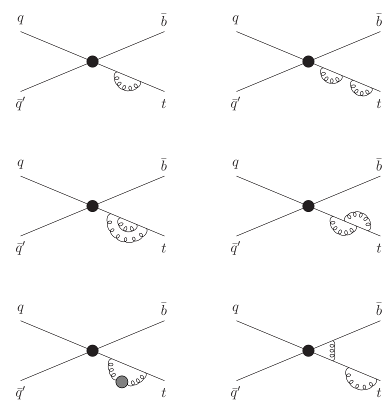
Complete two-loop results for

- **soft (cusp) anomalous dimension for** $e^+e^- \rightarrow t\bar{t}$
- $t\bar{t}$ **hadroproduction**
- t**-channel single top production**
- s**-channel single top production**
- $bg \rightarrow tW^-$ **and** $bg \rightarrow tH^-$
- **direct photon production**
- W **and** Z **production at large** p_T

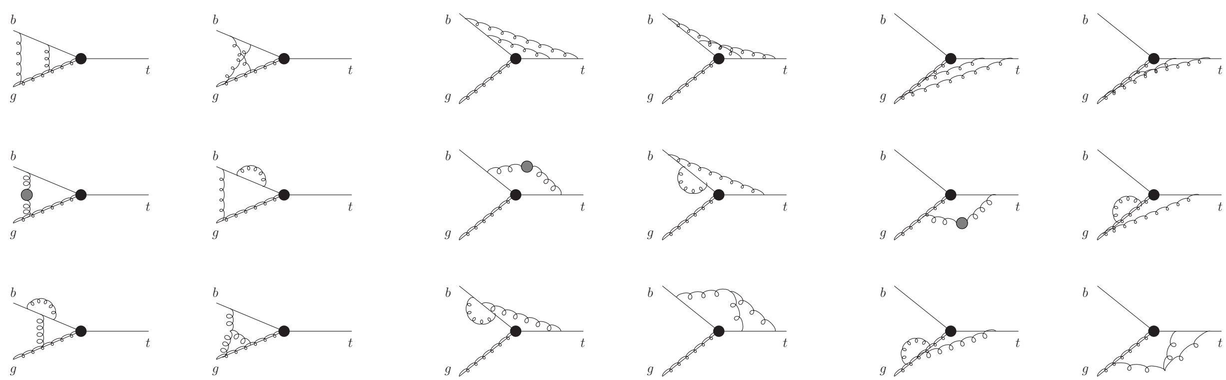
Typical two-loop eikonal diagrams

Single-top s-channel





tW production



NNLO approximate cross sections

NLO expansion

Define $\mathcal{D}_k(s_4) \equiv \left[\frac{\ln^k(s_4/M^2)}{s_4}\right]_+$

$$\hat{\sigma}^{(1)} = \sigma^B \frac{\alpha_s(\mu_R)}{\pi} \{c_3\, \mathcal{D}_1(s_4) + c_2\, \mathcal{D}_0(s_4) + c_1\, \delta(s_4)\} + \frac{\alpha_s^{d_{\alpha_s}+1}(\mu_R^2)}{\pi} [A^c\, \mathcal{D}_0(s_4) + T_1^c\, \delta(s_4)]$$

where $c_3 = \sum_i 2\, A_i^{(1)} - \sum_j A_j^{(1)}$ **and** $c_2 = c_2^\mu + T_2$**, with** $c_2^\mu = -\sum_i A_i^{(1)} \ln\left(\frac{\mu_F^2}{M^2}\right)$

and

$$T_2 = \sum_i \left[-2\, A_i^{(1)} \ln\left(\frac{-t_i}{M^2}\right) + D_i^{(1)} - A_i^{(1)} \ln\left(\frac{M^2}{s}\right)\right] + \sum_j \left[B_j^{(1)} + D_j^{(1)} - A_j^{(1)} \ln\left(\frac{M^2}{s}\right)\right]$$

Also $A^c = \mathrm{tr}\left(H^{(0)}\Gamma_S^{(1)\,\dagger} S^{(0)} + H^{(0)} S^{(0)} \Gamma_S^{(1)}\right)$

$c_1 = c_1^\mu + T_1$**, with**

$$c_1^\mu = \sum_i \left[A_i^{(1)} \ln\left(\frac{-t_i}{M^2}\right) - \gamma_i^{(1)}\right] \ln\left(\frac{\mu_F^2}{M^2}\right) + d_{\alpha_s} \frac{\beta_0}{4} \ln\left(\frac{\mu_R^2}{M^2}\right)$$

NNLO expansion

$$\hat{\sigma}^{(2)} = \sigma^B \frac{\alpha_s^2(\mu_R)}{\pi^2} \left\{ \frac{1}{2} c_3^2 \, \mathcal{D}_3(s_4) + \left[\frac{3}{2} c_3 c_2 - \frac{\beta_0}{4} c_3 + \sum_j \frac{\beta_0}{8} A_j^{(1)} \right] \mathcal{D}_2(s_4) \right.$$

$$+ \left[c_3 c_1 + c_2^2 - \zeta_2 c_3^2 - \frac{\beta_0}{2} T_2 + \frac{\beta_0}{4} c_3 \ln\left(\frac{\mu_R^2}{M^2}\right) + \sum_i 2A_i^{(2)} - \sum_j A_j^{(2)} + \sum_j \frac{\beta_0}{4} B_j^{(1)} \right] \mathcal{D}_1(s_4)$$

$$+ \left[c_2 c_1 - \zeta_2 c_3 c_2 + \zeta_3 c_3^2 + \frac{\beta_0}{4} c_2 \ln\left(\frac{\mu_R^2}{s}\right) - \sum_i \frac{\beta_0}{2} A_i^{(1)} \ln^2\left(\frac{-t_i}{M^2}\right) \right.$$

$$+ \sum_i [\left(-2A_i^{(2)} + \frac{\beta_0}{2} D_i^{(1)}\right) \ln\left(\frac{-t_i}{M^2}\right) + D_i^{(2)} + \frac{\beta_0}{8} A_i^{(1)} \ln^2\left(\frac{\mu_F^2}{s}\right) - A_i^{(2)} \ln\left(\frac{\mu_F^2}{s}\right)]$$

$$\left. \left. + \sum_j [B_j^{(2)} + D_j^{(2)} - \left(A_j^{(2)} + \frac{\beta_0}{4}(B_j^{(1)} + 2D_j^{(1)})\right) \ln\left(\frac{M^2}{s}\right) + \frac{3\beta_0}{8} A_j^{(1)} \ln^2\left(\frac{M^2}{s}\right)] \right] \mathcal{D}_0(s_4) \right\}$$

$$+ \frac{\alpha_s^{d_{\alpha_s}+2}(\mu_R)}{\pi^2} \left\{ \frac{3}{2} c_3 A^c \, \mathcal{D}_2(s_4) + \left[\left(2c_2 - \frac{\beta_0}{2}\right) A^c + c_3 T_1^c + F^c \right] \mathcal{D}_1(s_4) \right.$$

$$\left. + \left[\left(c_1 - \zeta_2 c_3 + \frac{\beta_0}{4} \ln\left(\frac{\mu_R^2}{s}\right)\right) A^c + c_2 T_1^c + F^c \ln\left(\frac{M^2}{s}\right) + G^c \right] \mathcal{D}_0(s_4) \right\}$$

where

$$F^c = \mathrm{tr}\left[H^{(0)}\left(\Gamma_S^{(1)\,\dagger}\right)^2 S^{(0)} + H^{(0)} S^{(0)} \left(\Gamma_S^{(1)}\right)^2 + 2H^{(0)}\Gamma_S^{(1)\,\dagger} S^{(0)} \Gamma_S^{(1)}\right]$$

$$G^c \quad = \quad \mathrm{tr}\left[H^{(1)}\Gamma_S^{(1)\,\dagger}S^{(0)} + H^{(1)}S^{(0)}\Gamma_S^{(1)} + H^{(0)}\Gamma_S^{(1)\,\dagger}S^{(1)} + H^{(0)}S^{(1)}\Gamma_S^{(1)} \right. \\ \left. + H^{(0)}\Gamma_S^{(2)\,\dagger}S^{(0)} + H^{(0)}S^{(0)}\Gamma_S^{(2)}\right]$$

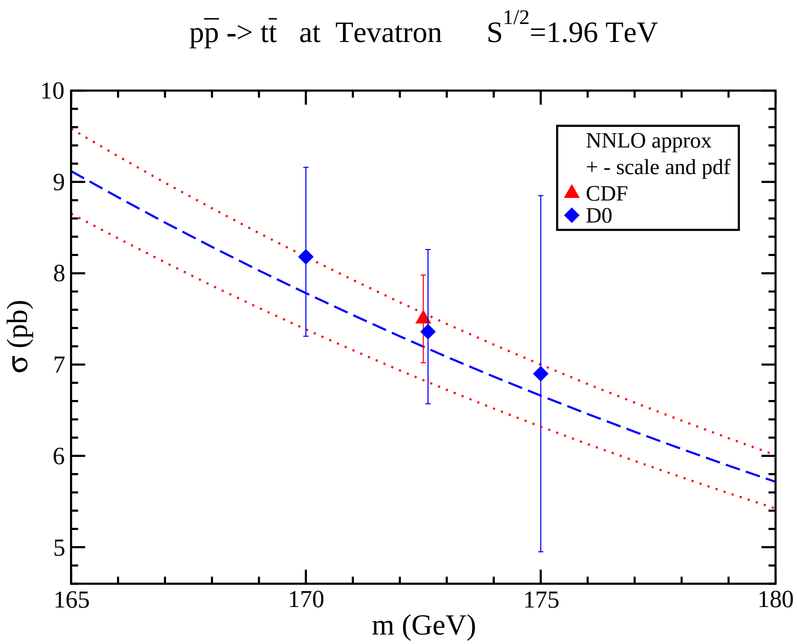
and c_3, c_2, c_1, etc are from the NLO expansion

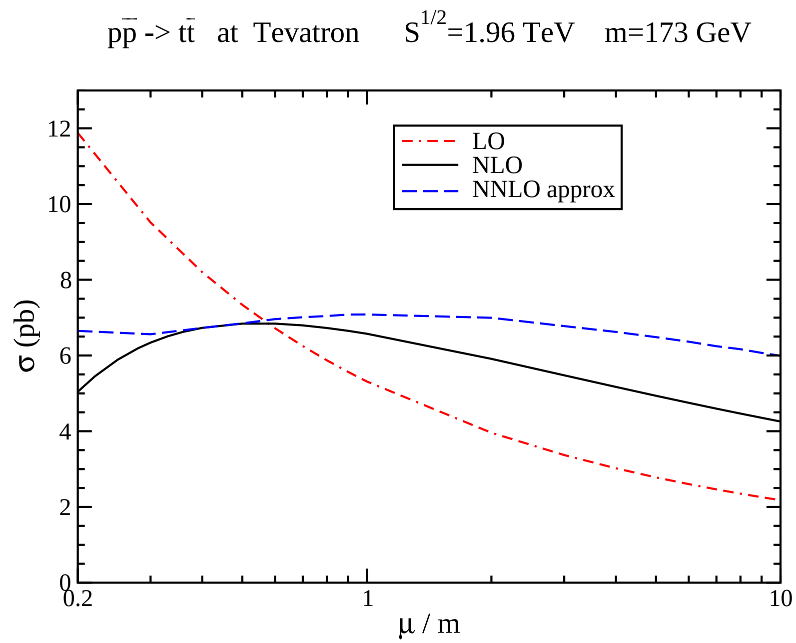
Two-loop universal quantities $A^{(2)}$, $B^{(2)}$, $D^{(2)}$ known

Two-loop process-dependent $\Gamma_S^{(2)}$ recently calculated for several processes

Additional purely collinear corrections of the form $(\ln^k N)/N$ provide very small contributions and do not improve the threshold approximation

$t\bar{t}$ cross section at the Tevatron





$$\sigma_{t\bar{t}}^{\rm NNLOapprox}(m_t = 173\,{\rm GeV},\, 1.96\,{\rm TeV}) \quad = \quad 7.08^{+0.00}_{-0.24}{}^{+0.36}_{-0.27}\ {\rm pb}$$

scale **pdf**

NNLO approx: 7.8% enhancement over NLO
scale dependence greatly reduced

used MSTW 2008 NNLO pdf

Comparison of various resummation/NNLO approx approaches

Tevatron 1.96 TeV, scale uncertainty included;
pdf uncertainty not shown - same for all if same assumptions are used
use $m_t = 173$ **GeV unless otherwise indicated**

NLO $6.74^{+0.36}_{-0.76}$

Kidonakis, PRD 82, 114030 (2010) $7.08^{+0.20}_{-0.24}$

Aliev *et al*, **CPC 182, 1034 (2011)** $7.13^{+0.31}_{-0.39}$

Ahrens *et al*, **PLB 703, 135 (2011)** $6.65^{+0.08}_{-0.41}$

Beneke *et al*, **NPB 855, 695 (2012)** ($m_t = 173.3$) $7.22^{+0.31}_{-0.47} \to 7.29$ **at** $m_t = 173$

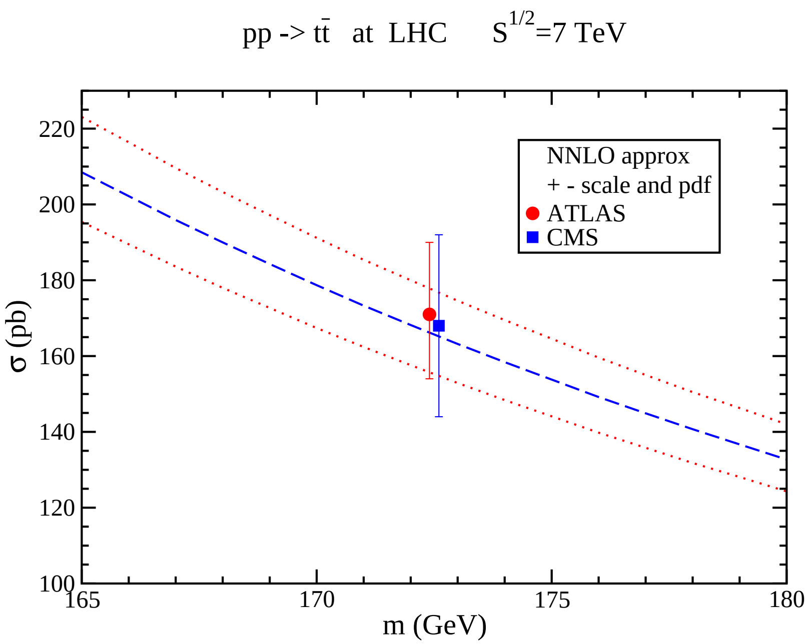
Cacciari *et al*, **PLB 710, 612 (2012)** ($m_t = 173.3$) $6.72^{+0.24}_{-0.41} \to 6.78$ **at** $m_t = 173$

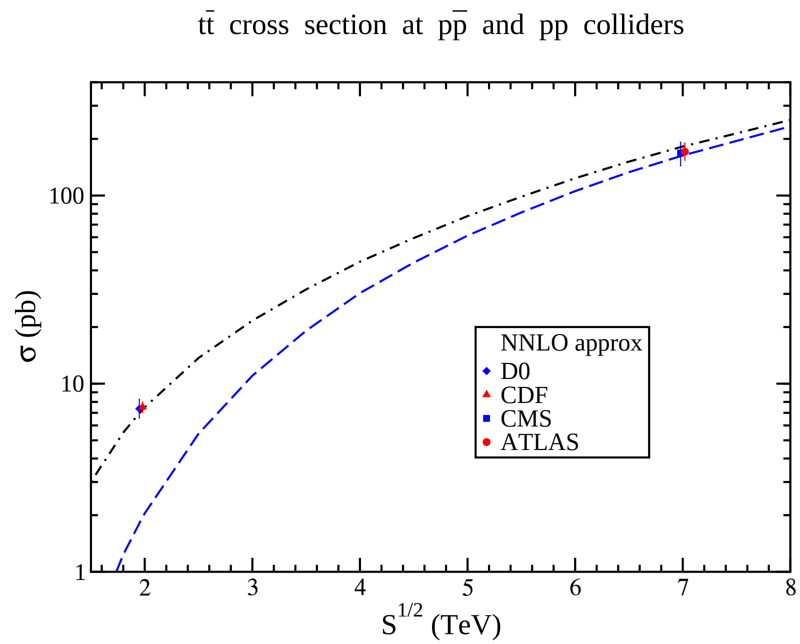
[See also
Moch *et al* (2012) $7.27^{+0.41}_{-0.46}$ threshold + high-energy terms
Brodsky & Wu (2012) ($m_t = 172.9$) $7.626 \to 7.602$ at $m_t = 173$ PMC]

partly exact NNLO (exact for $q\bar{q}$ **plus approx for** gg**)**
Barnreuther *et al* **(2012)** ($m_t = 173.3$) $7.005^{+0.202}_{-0.310} \to 7.07$ **at** $m_t = 173$

The PRD 82 result is very close to the partly exact NNLO:
7.08 **vs** 7.07 **with similar scale uncertainty**

$t\bar{t}$ cross section at the LHC





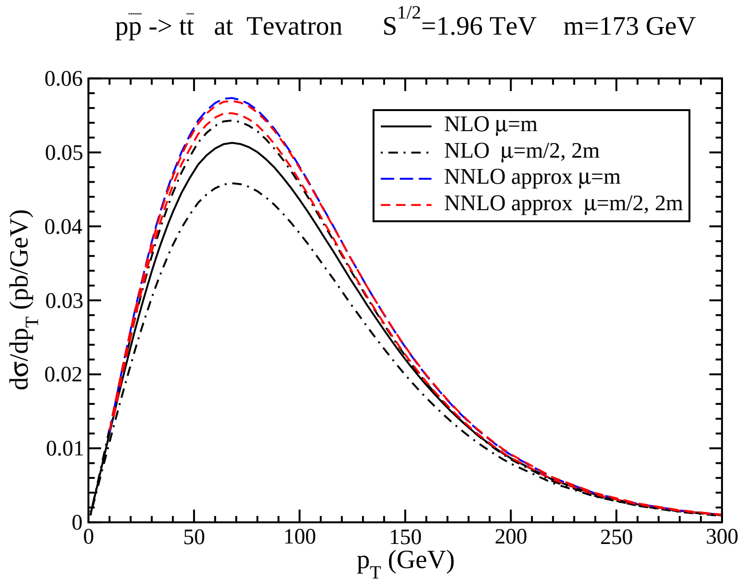
$$\sigma_{t\bar{t}}^{\rm NNLOapprox}(m_t = 173\,{\rm GeV},\, 7\,{\rm TeV}) \quad = \quad 163^{+7}_{-5} \pm 9 \text{ pb}$$

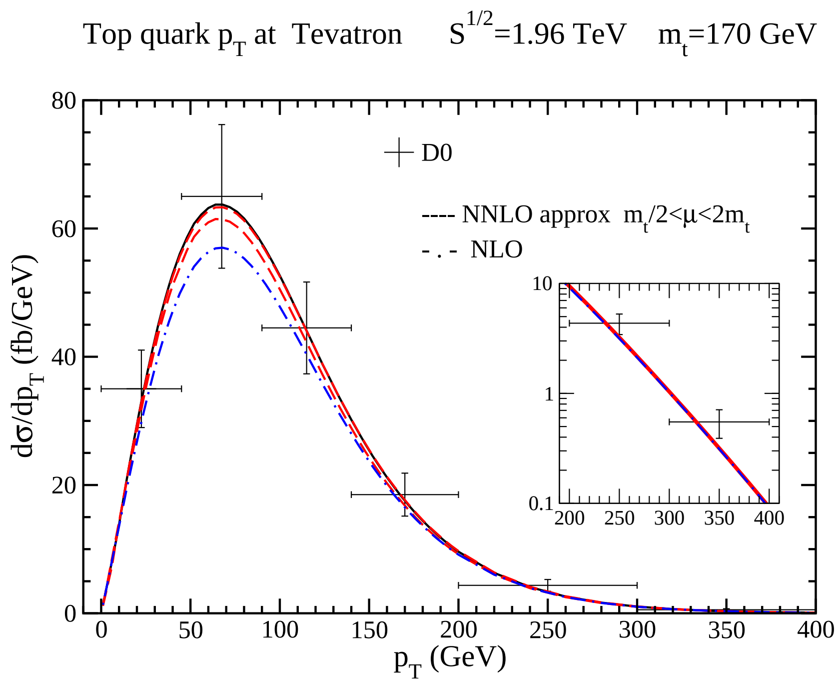
$$\sigma_{t\bar{t}}^{\rm NNLOapprox}(m_t = 173\,{\rm GeV},\, 8\,{\rm TeV}) \quad = \quad 234^{+10}_{-7} \pm 12 \text{ pb}$$

$$\sigma_{t\bar{t}}^{\rm NNLOapprox}(m_t = 173\,{\rm GeV}, 14\,{\rm TeV}) \quad = \quad 920^{+50}_{-39}{}^{+33}_{-35} \text{ pb}$$

NNLO approx: enhancement over NLO (same pdf) is 7.6% at 7 TeV; 7.8% at 8 TeV; 8.0% at 14 TeV

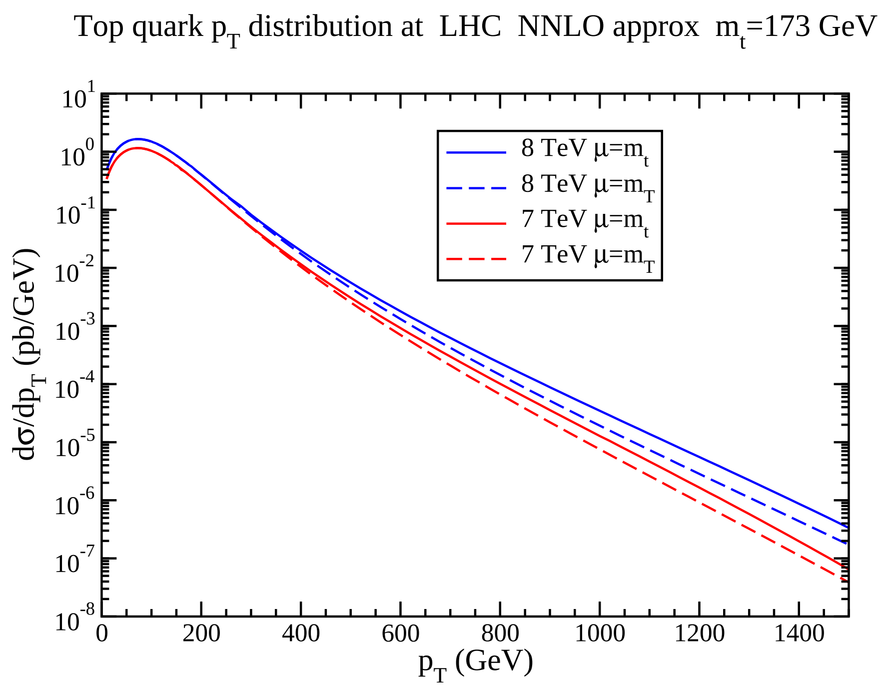
Top quark p_T distribution at Tevatron

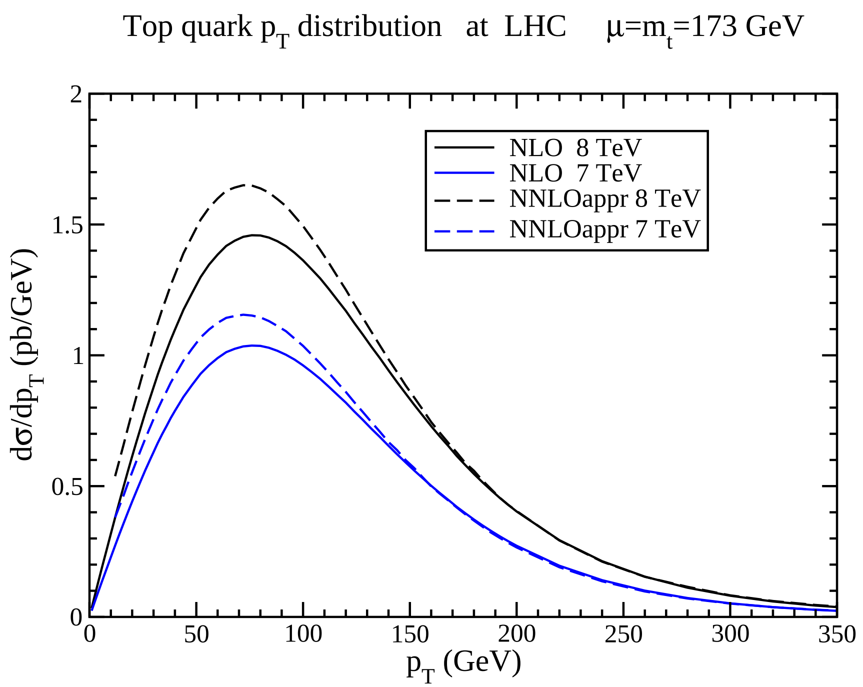




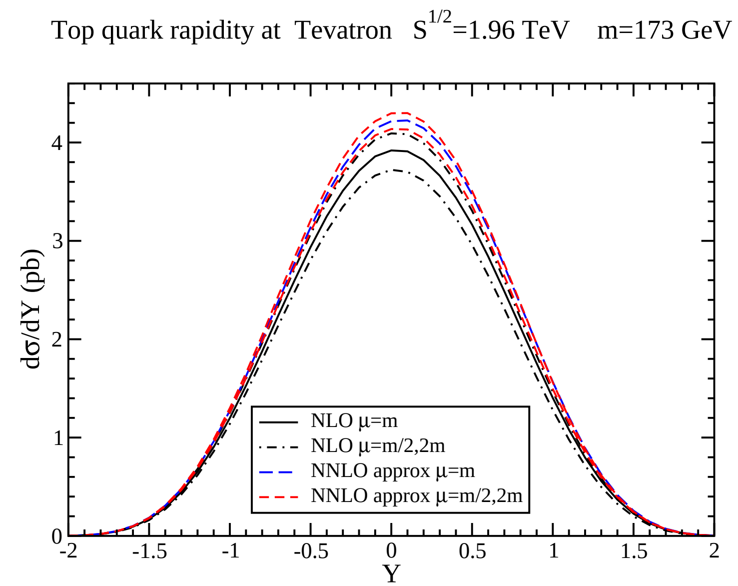
Excellent agreement of NNLO approx results with D0 data

Top quark p_T distribution at the LHC





Top quark rapidity distribution at Tevatron



Top Forward-backward asymmetry

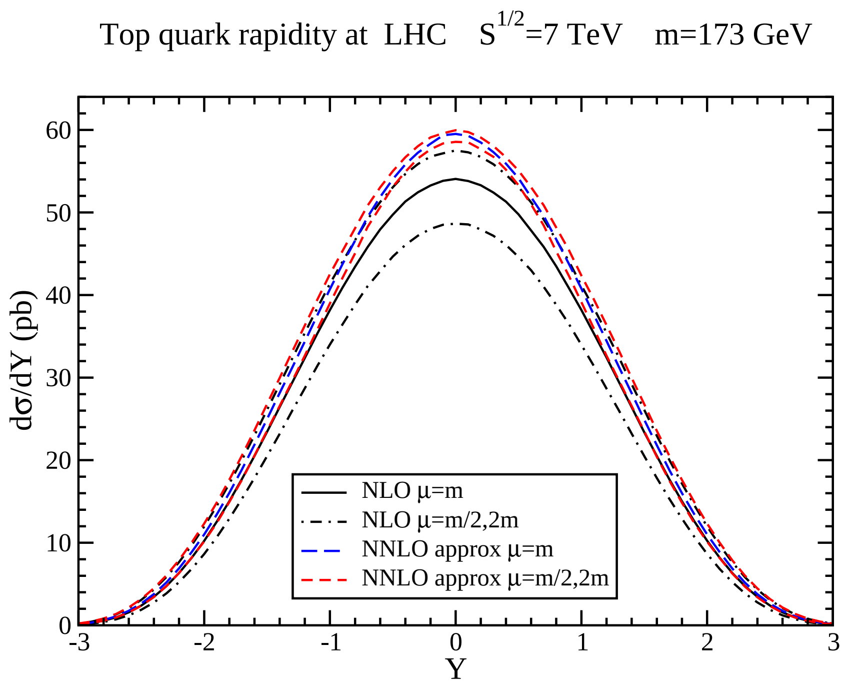
$$A_{\rm FB} = \frac{\sigma(Y>0) - \sigma(Y<0)}{\sigma(Y>0) + \sigma(Y<0)}$$

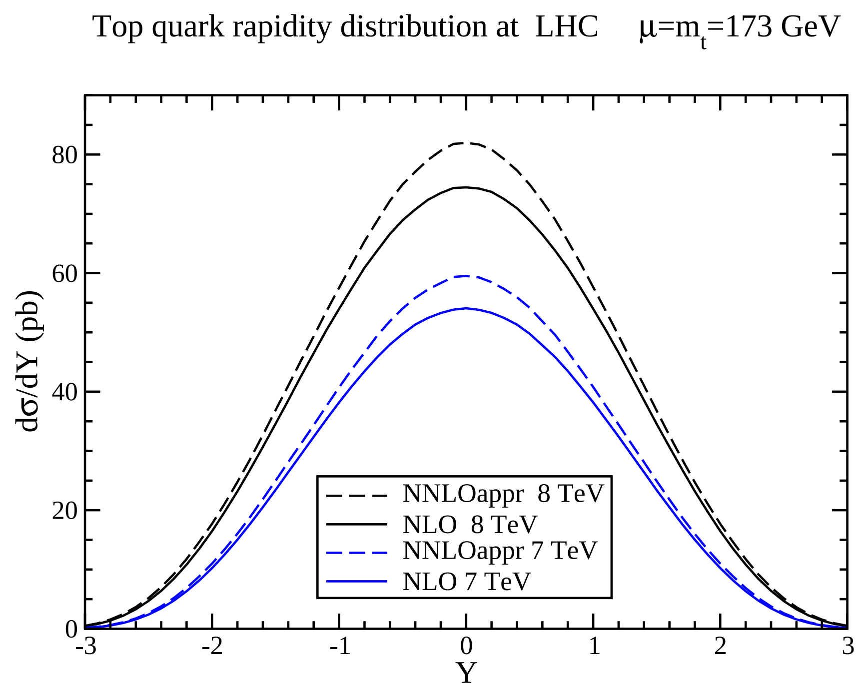
Asymmetry significant at the Tevatron

Theoretical result at Tevatron: $A_{\rm FB} = 0.052^{+0.000}_{-0.006}$

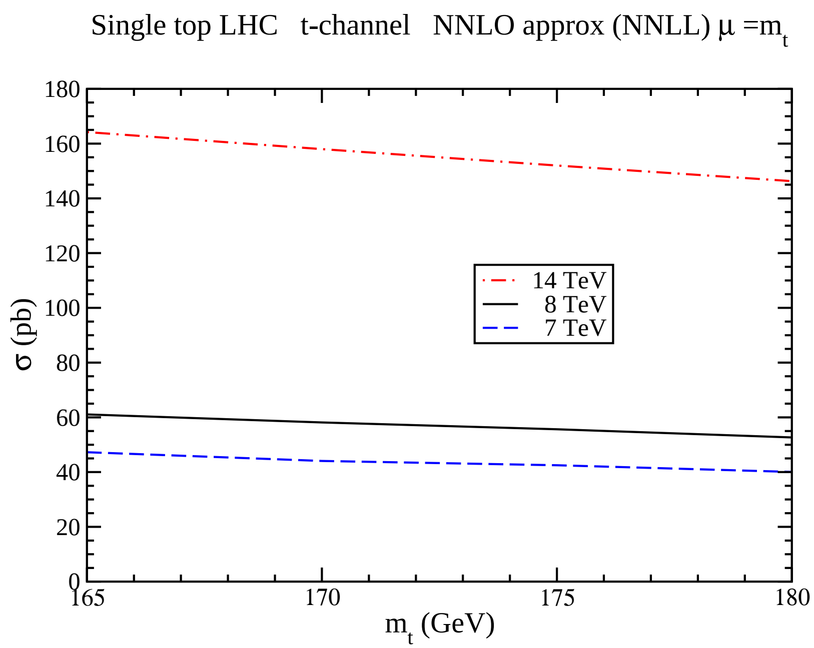
smaller than observed values

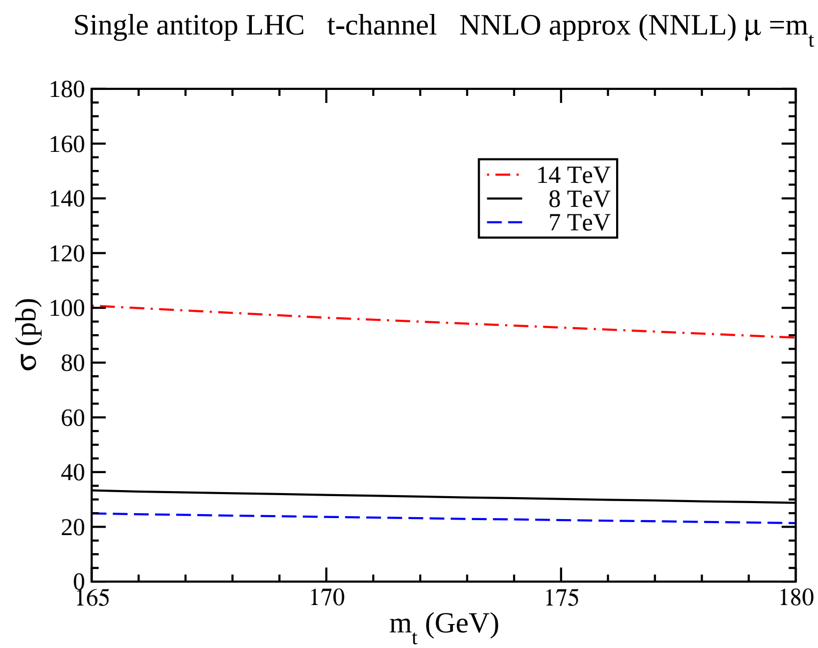
Top quark rapidity distribution at LHC





t-channel cross sections at LHC

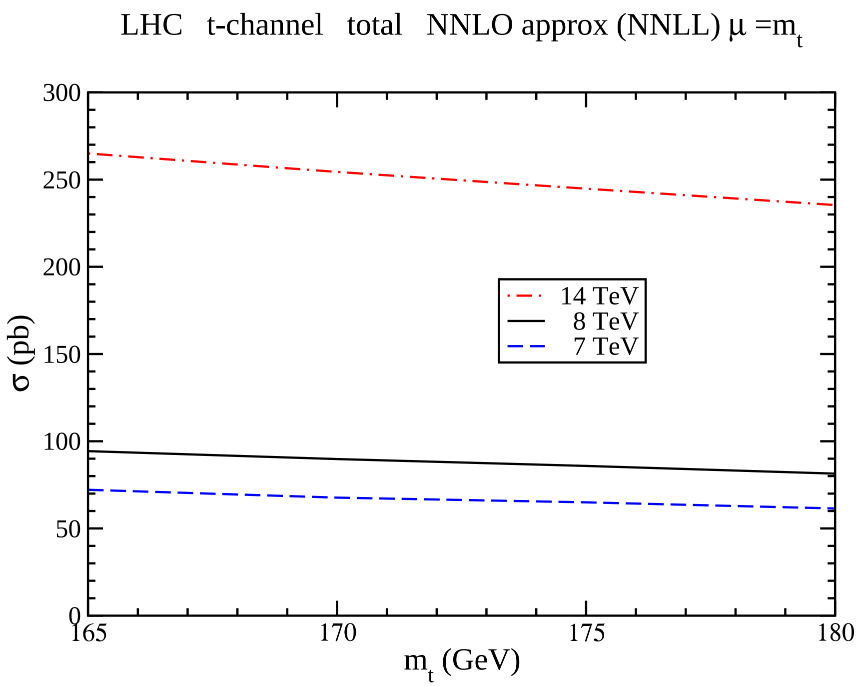


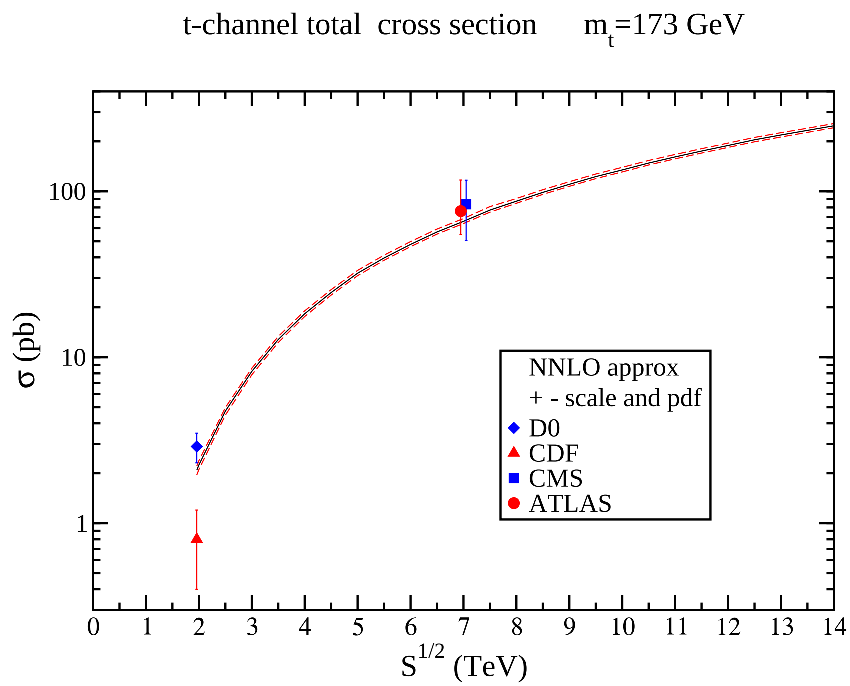


For $m_t = 173$ GeV

LHC	t	$\bar{t}$
7 TeV	$43.0^{+1.6}_{-0.2} \pm 0.8$	$22.9 \pm 0.5^{+0.7}_{-0.9}$
8 TeV	$56.4^{+2.1}_{-0.3} \pm 1.1$	$30.7 \pm 0.7^{+0.9}_{-1.1}$
14 TeV	$154^{+4}_{-1} \pm 3$	$94^{+2}_{-1}{}^{+2}_{-3}$

t-channel total cross section at LHC





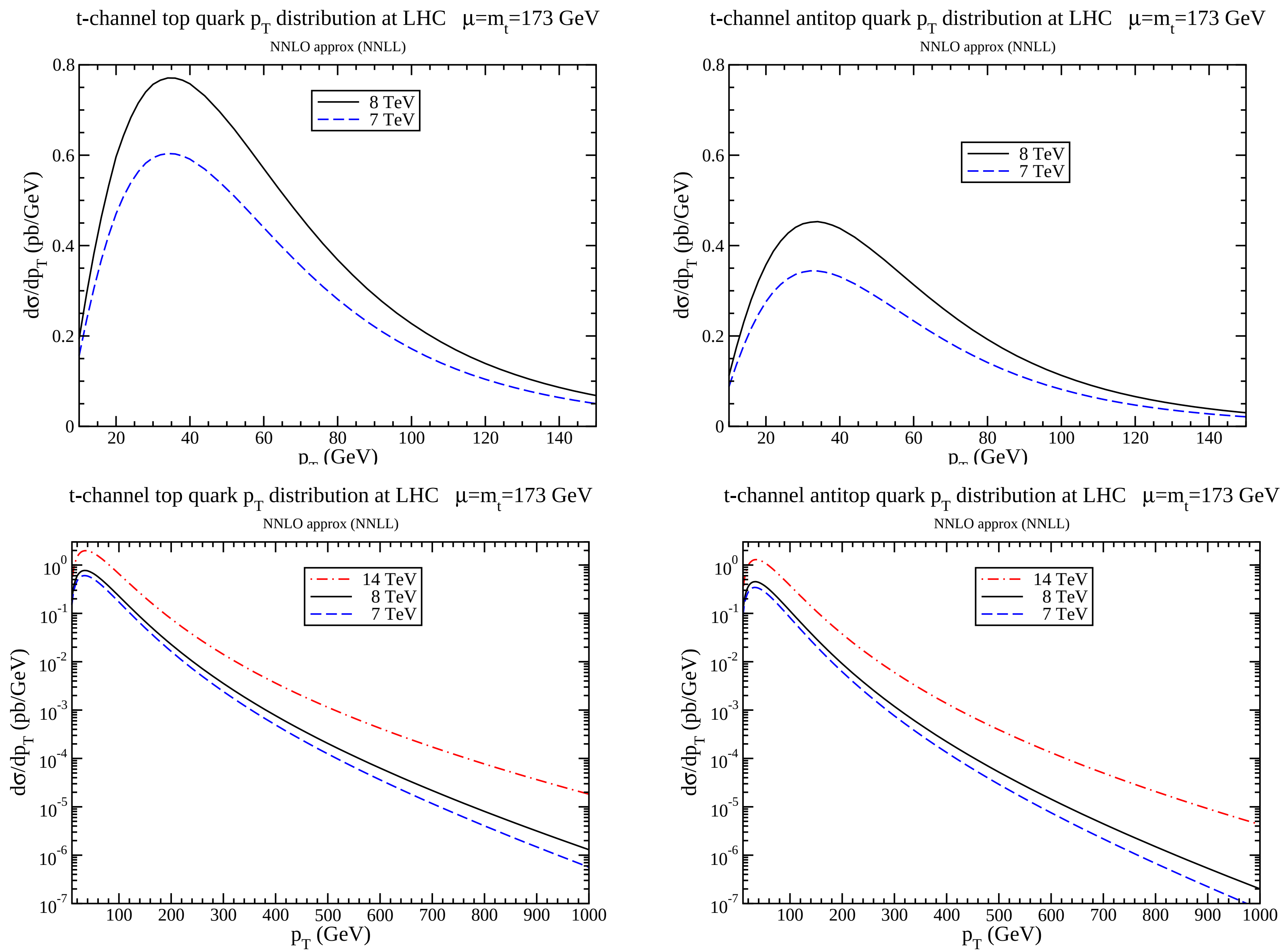
$$\sigma^{\rm NNLOapprox,\,total}_{t-{\rm channel}}(m_t = 173\,{\rm GeV},\,7\,{\rm TeV}) = 65.9^{+2.1\,+1.5}_{-0.7\,-1.7}\ {\rm pb}$$

$$\sigma^{\rm NNLOapprox,\,total}_{t-{\rm channel}}(m_t = 173\,{\rm GeV},\,8\,{\rm TeV}) = 87.2^{+2.8\,+2.0}_{-1.0\,-2.2}\ {\rm pb}$$

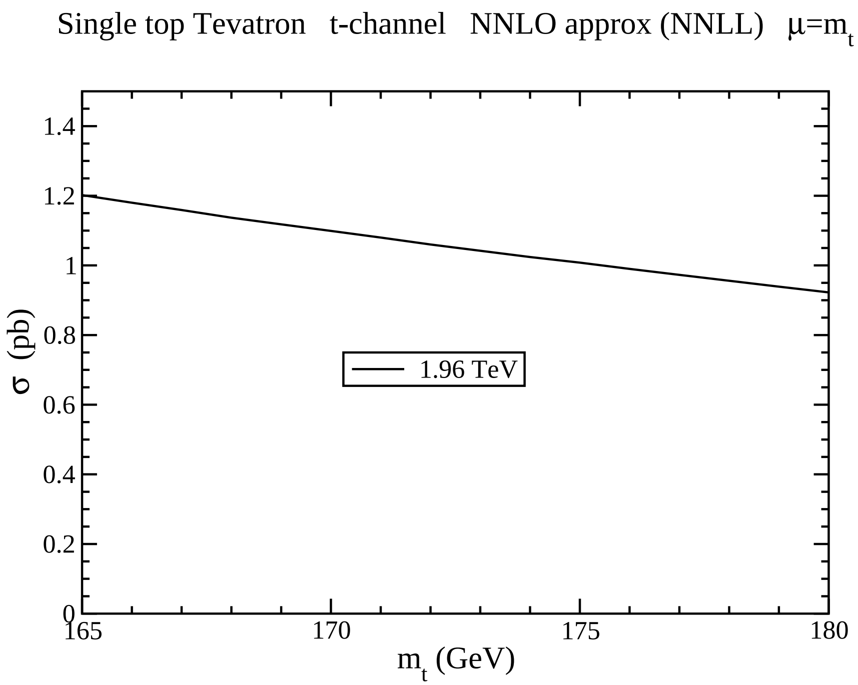
$$\sigma^{\rm NNLOapprox,\,total}_{t-{\rm channel}}(m_t = 173\,{\rm GeV},\,14\,{\rm TeV}) = 248^{+6\,+5}_{-2\,-6}\ {\rm pb}$$

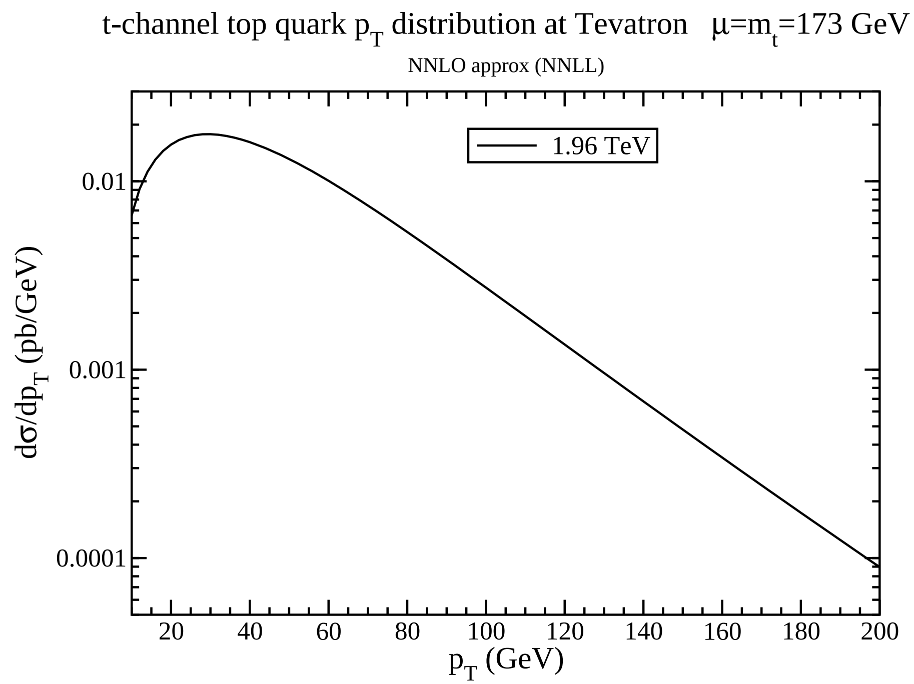
Small $\mathcal{O}(1\%)$ corrections over NLO

t-channel top and antitop p_T distributions at LHC



t-channel top quark production at Tevatron

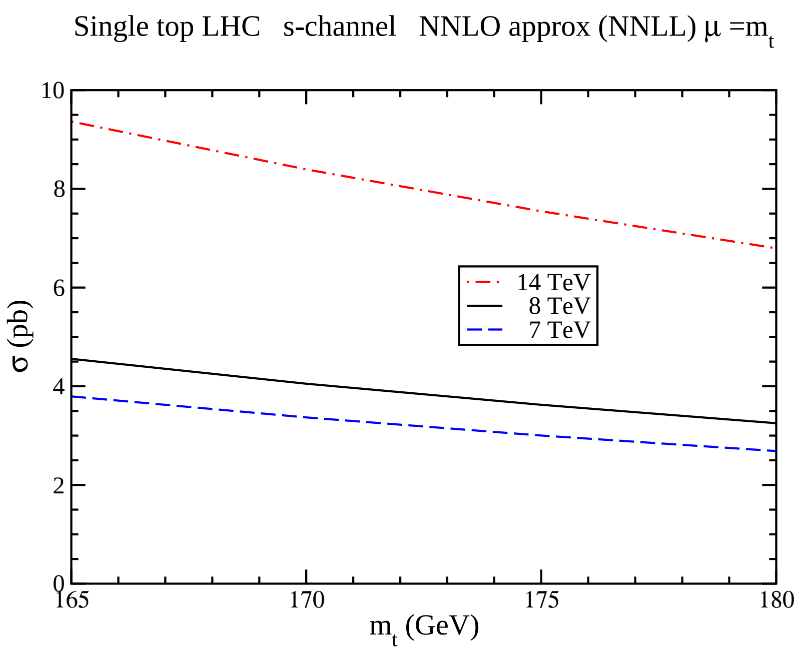


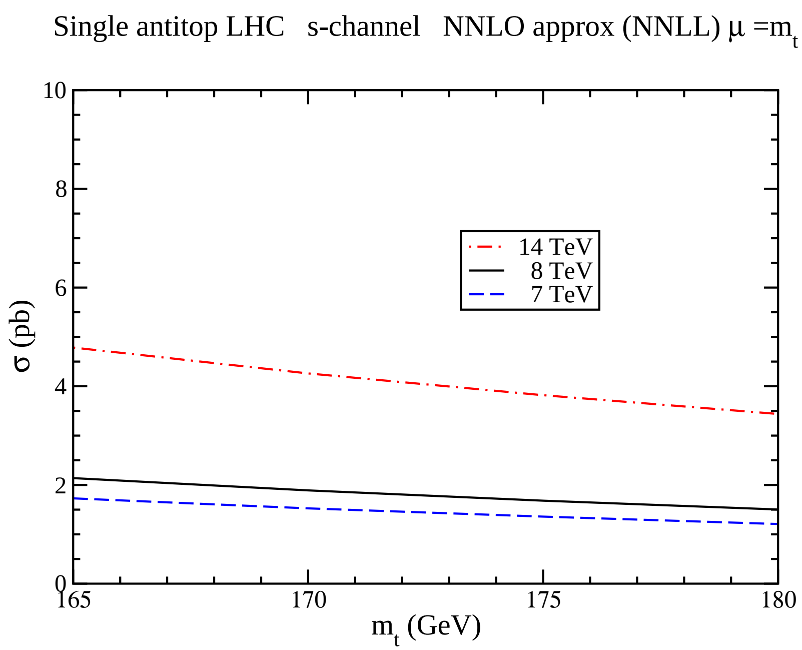


$$\sigma^{\rm NNLOapprox,\, top}_{t-{\rm channel}}(m_t = 173\,{\rm GeV},\, 1.96\,{\rm TeV}) \quad = \quad 1.04^{+0.00}_{-0.02} \pm 0.06 \text{ pb}$$

Cross section for antitop t-channel production at Tevatron is identical

s-channel cross sections



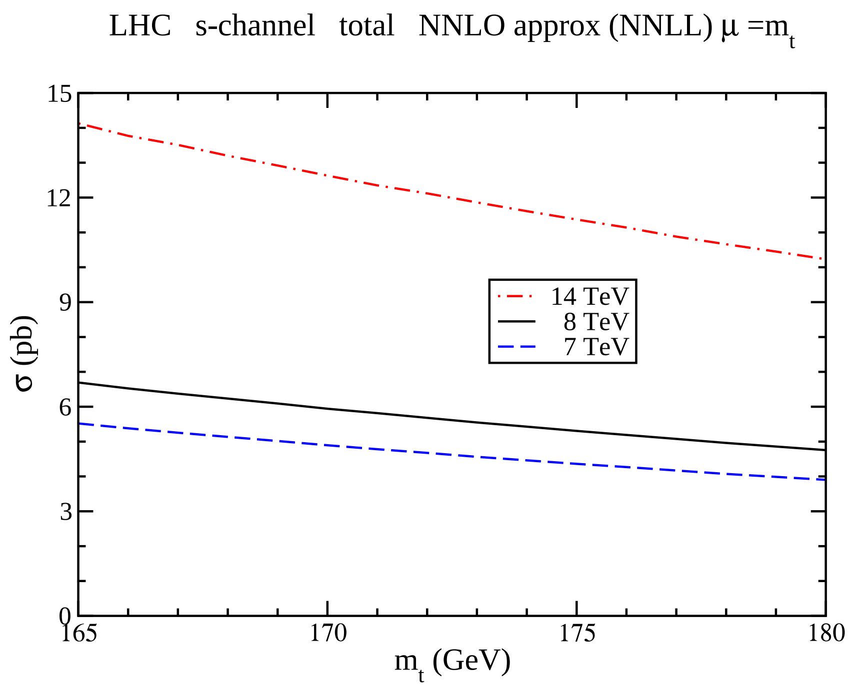


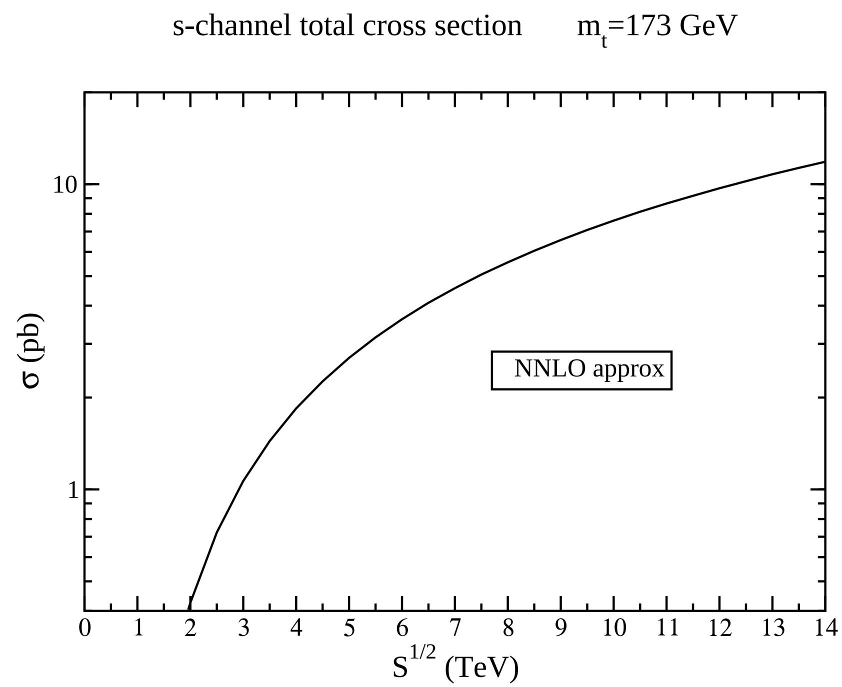
For $m_t = 173$ GeV

LHC	t	$\bar{t}$
7 TeV	$3.14 \pm 0.06^{+0.12}_{-0.10}$	$1.42 \pm 0.01^{+0.06}_{-0.07}$
8 TeV	$3.79 \pm 0.07 \pm 0.13$	$1.76 \pm 0.01 \pm 0.08$
14 TeV	$7.87 \pm 0.14^{+0.31}_{-0.28}$	$3.99 \pm 0.05^{+0.14}_{-0.21}$

At Tevatron $\sqrt{S} = 1.96$ TeV: $0.523^{+0.001\,+0.030}_{-0.005\,-0.028}$ **pb for top; same for antitop**

s-channel total cross section at LHC





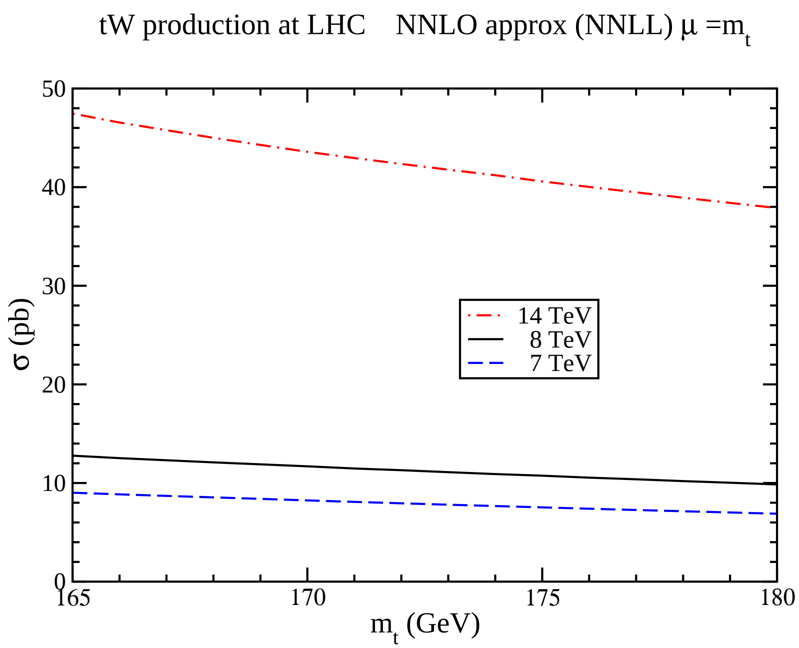
$$\sigma^{\rm NNLOapprox,\,total}_{s-{\rm channel}}(m_t=173\,{\rm GeV},\,7\,{\rm TeV}) \quad = \quad 4.56 \pm 0.07^{+0.18}_{-0.17}\ {\rm pb}$$

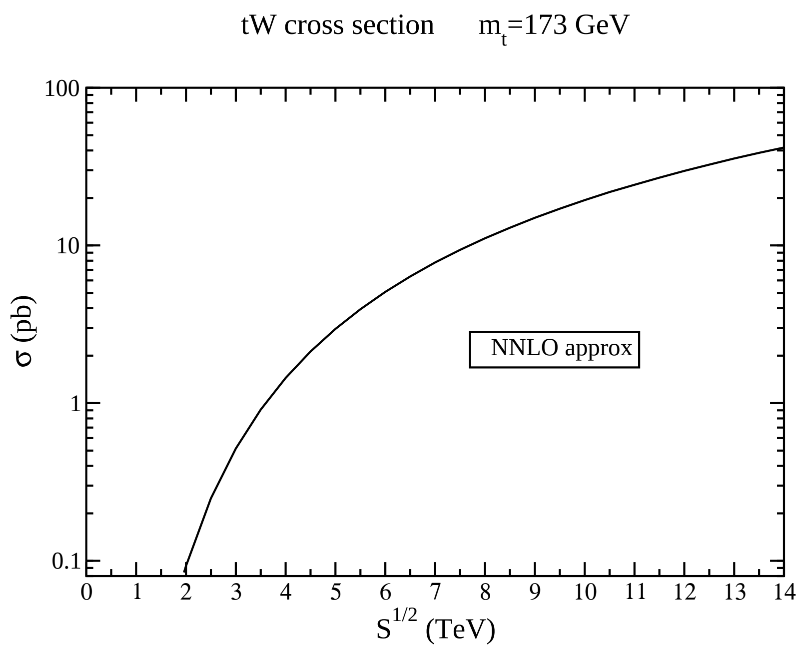
$$\sigma^{\rm NNLOapprox,\,total}_{s-{\rm channel}}(m_t=173\,{\rm GeV},\,8\,{\rm TeV}) \quad = \quad 5.55 \pm 0.08 \pm 0.21\ {\rm pb}$$

$$\sigma^{\rm NNLOapprox,\,total}_{s-{\rm channel}}(m_t=173\,{\rm GeV},\,14\,{\rm TeV}) \quad = \quad 11.86 \pm 0.19^{+0.45}_{-0.49}\ {\rm pb}$$

NNLO approx: enhancement over NLO is $\sim 10\%$

Associated tW^- production at the LHC





$$\sigma_{tW}^{\rm NNLOapprox}(m_t = 173\,{\rm GeV},\, 7\,{\rm TeV}) \quad = \quad 7.8 \pm 0.2^{+0.5}_{-0.6}\ {\rm pb}$$

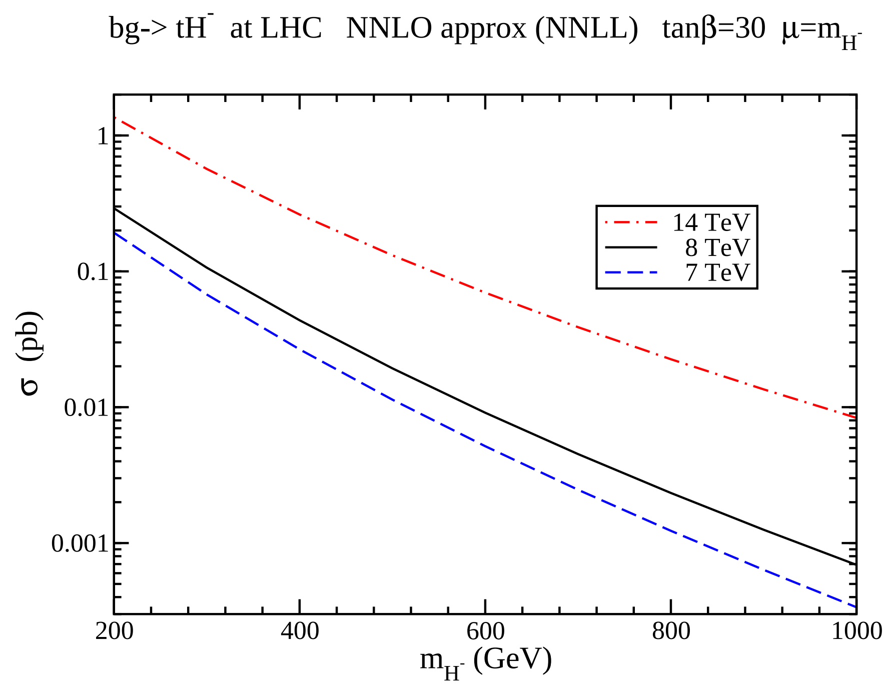
$$\sigma_{tW}^{\rm NNLOapprox}(m_t = 173\,{\rm GeV},\, 8\,{\rm TeV}) \quad = \quad 11.1 \pm 0.3 \pm 0.7\ {\rm pb}$$

$$\sigma_{tW}^{\rm NNLOapprox}(m_t = 173\,{\rm GeV},\, 14\,{\rm TeV}) \quad = \quad 41.8 \pm 1.0^{+1.5}_{-2.4}\ {\rm pb}$$

NNLO approx corrections increase NLO cross section by $\sim 8\%$

Cross section for $\bar{t}W^+$ production is identical

Associated production of a top quark with a charged Higgs



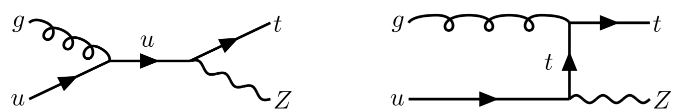
NNLO approx corrections increase NLO cross section by ~ 15 to $\sim 20\%$

FCNC processes

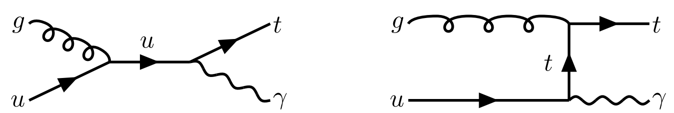
Single-top production via flavor-changing neutral currents
Anomalous couplings in Lagrangian, e.g.

$$\Delta \mathcal{L}^{eff} = \frac{1}{\Lambda}\, \kappa_{tqV}\, e\, \bar{t}\, \sigma_{\mu\nu}\, q\, F_V^{\mu\nu} + h.c.$$

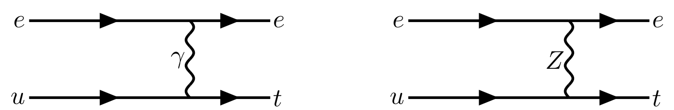
$gu \rightarrow tZ$



$gu \rightarrow t\gamma$



$eu \rightarrow et$



decrease of scale dependence, significant corrections over Born at Tevatron and HERA (e.g. 15 to 20% for $eu \rightarrow et$ at HERA)

Future studies for LHC energies and other couplings

W and Z production at large p_T - parton processes

W and Z hadroproduction useful in testing the SM and in estimates of backgrounds to Higgs production and new physics (new gauge bosons)

p_T distribution falls rapidly as p_T increases

Partonic channels at LO

$q(p_a) + g(p_b) \longrightarrow W(Q) + q(p_c)$

$q(p_a) + \bar{q}(p_b) \longrightarrow W(Q) + g(p_c)$

Define $s = (p_a + p_b)^2$**,** $t = (p_a - Q)^2$**,** $u = (p_b - Q)^2$ **and** $s_4 = s + t + u - Q^2$

At threshold $s_4 \rightarrow 0$

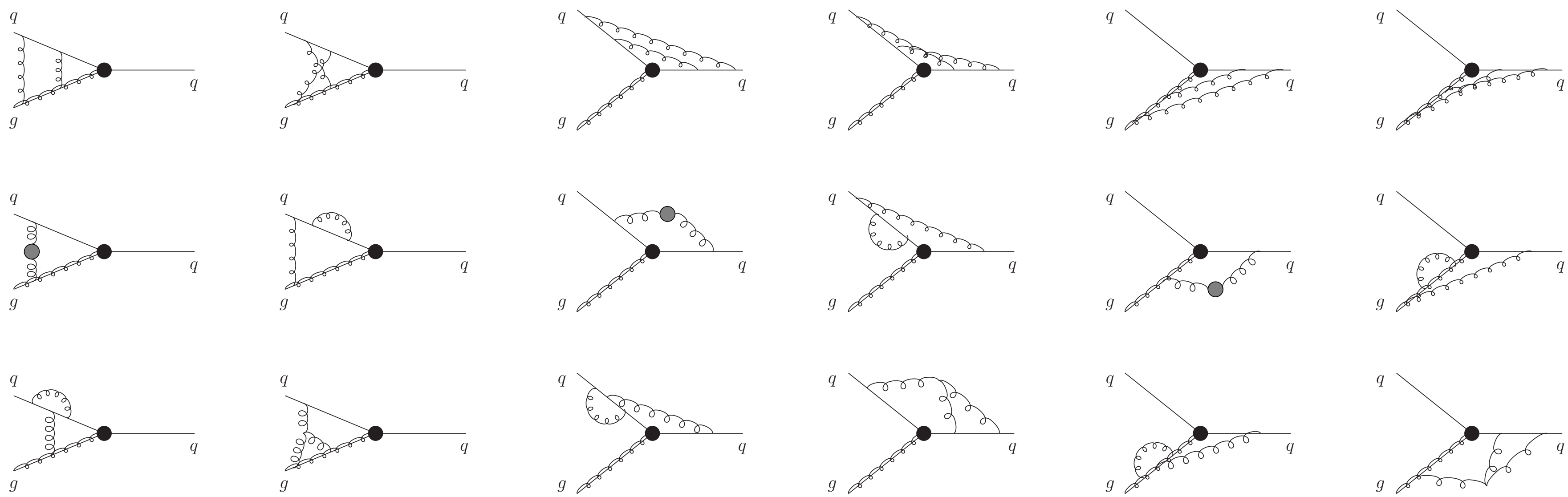
Soft corrections $\left[\frac{\ln^l(s_4/p_T^2)}{s_4}\right]_+$

New approximate NNLO from NNLL resummation:

N. Kidonakis and R.J. Gonsalves, arXiv:1201.5265 [hep-ph]

Two-loop soft anomalous dimension

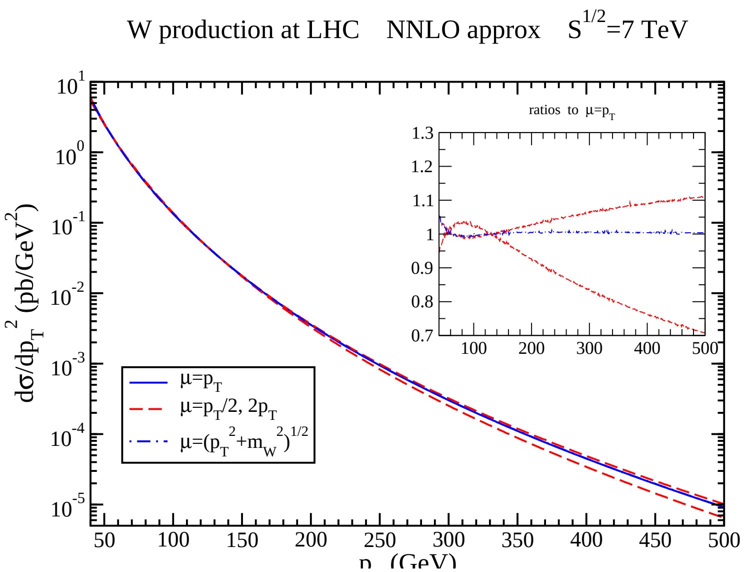
Two-loop eikonal diagrams for $qg \to Wq$

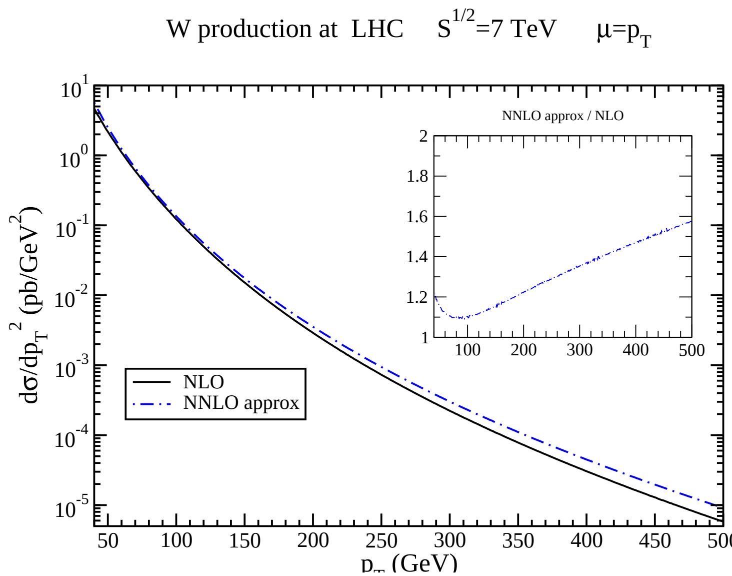


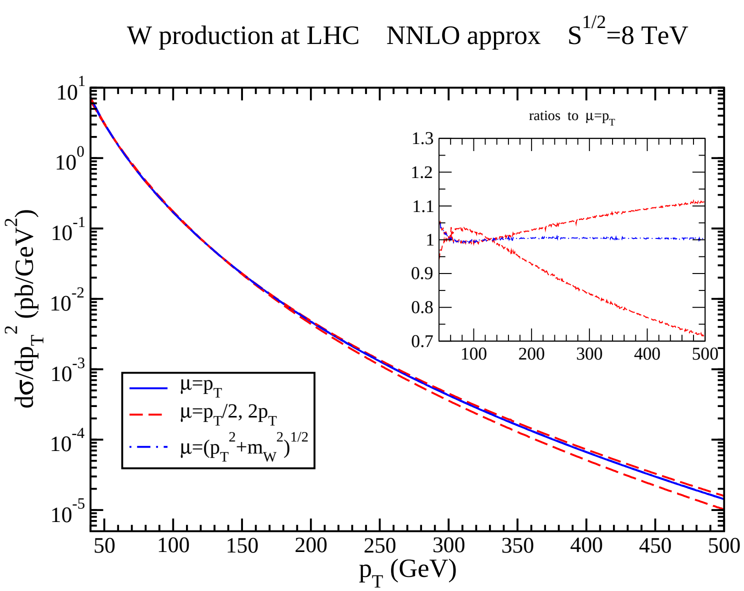
Determine $\Gamma_S^{(2)}$ from UV poles of two-loop dimensionally regularized integrals

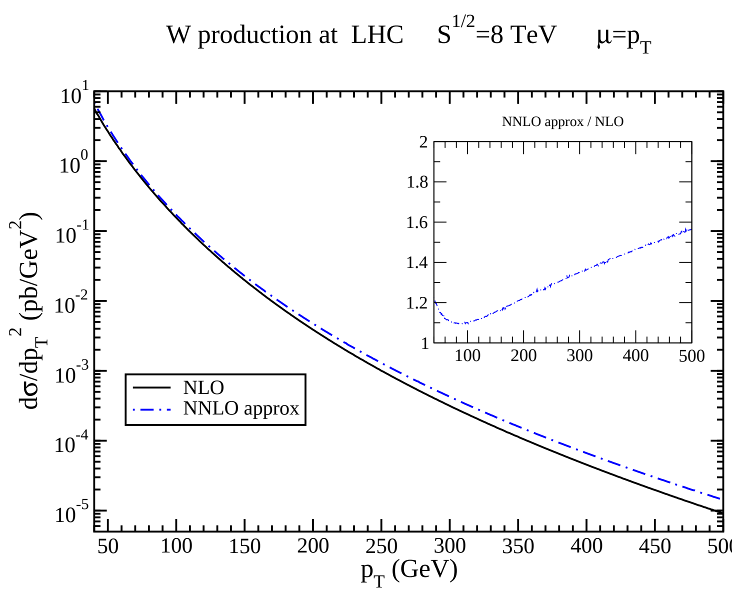
Also for $q\bar{q} \to Wg$

NNLO approx for W production at the LHC

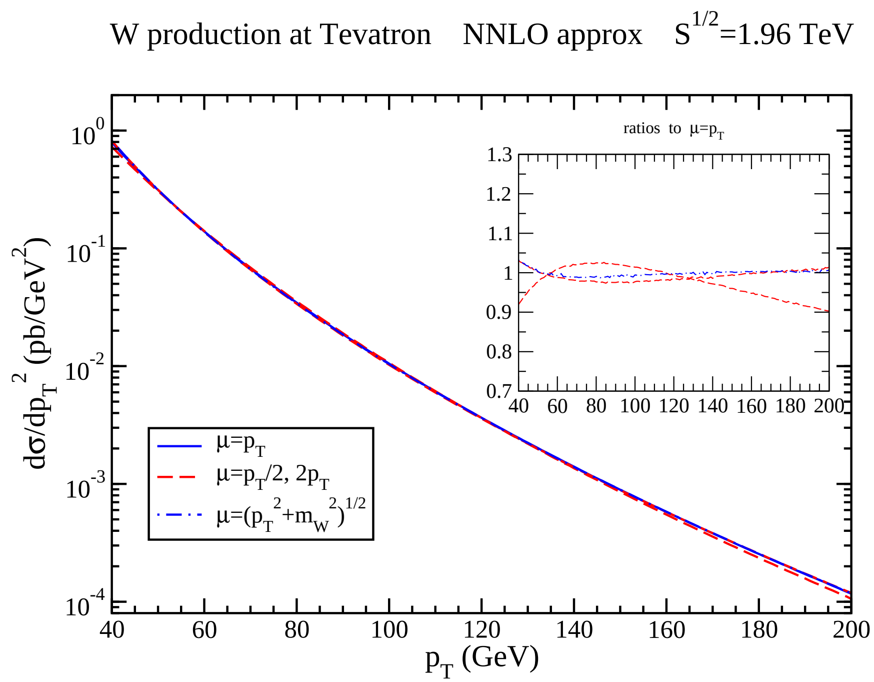


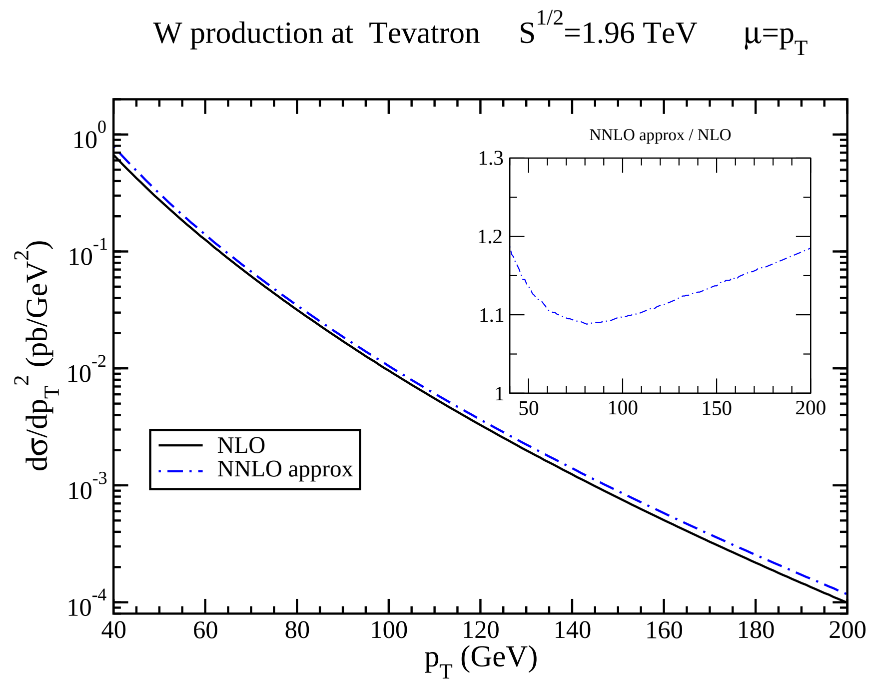






NNLO approx for W production at the Tevatron





Summary

- **NNLL resummation for top quark pair and single top production**
- $t\bar{t}$ **production cross section**
- **top quark** p_T **and rapidity distributions**
- **single top cross sections and** p_T **distributions**
- W **production**
- **NNLO approx corrections are significant at the LHC and the Tevatron**
- **good agreement with LHC and Tevatron data**
- **future work on more differential distributions and on NNNLO soft-gluon corrections**