Karlsruher Institut für Technologie

Higgs Mechanism in Standard Model (II)

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Course "Higgs Physics"

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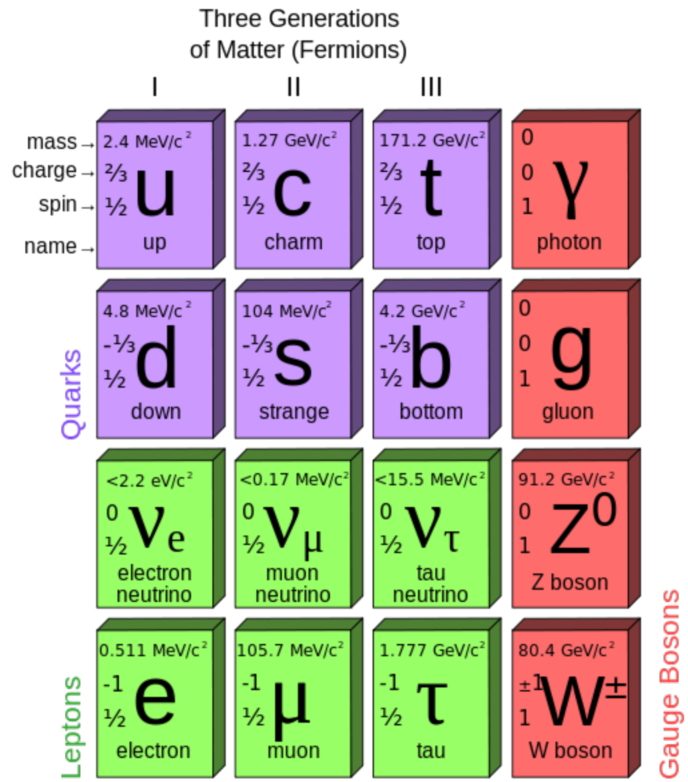
Standard Model (Summary)



- **Building blocks of Standard Model**

  fermions : matter fields

  gauge bosons : force carriers

- **Aesthetics of model :**
  - **classification of fields**
  - **U(1)⊗SU(2) symmetry → gauge interactions**
- **simple Lagrange formalism describes this very well but only for massless particles**
- **fermion and gauge boson terms break SU(2) symmetry**
- **model is fairly consistent with experimental data assuming massive fermion and weak boson fields**

⇒ needs gauge invariant mechanism of mass generation

Higgs Mechanism

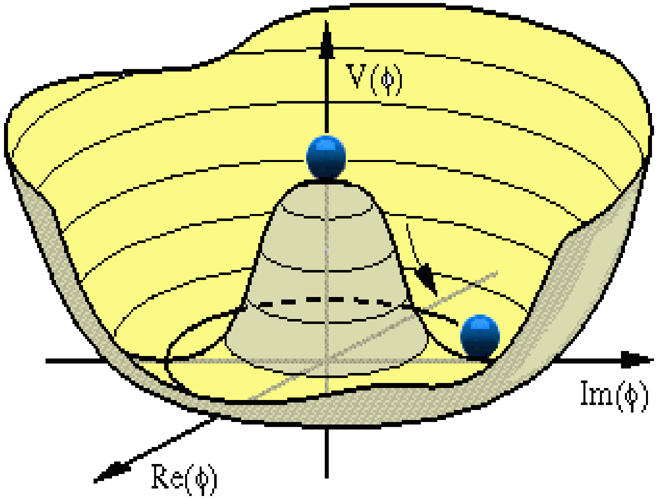
Consider SU(2) doublet of scalar complex fields

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix}$$

	T_3	Y	Q
ϕ^+	$+1/2$	1	1
ϕ^0	$-1/2$	1	0

Higgs Potential

$$V(\Phi^\dagger\Phi) = \lambda\left(\Phi^\dagger\Phi - \frac{v^2}{2}\right)^2$$



4-Dimensional sphere of minima

$$\phi_1^2 + \phi_2^2 + \phi_3^2 + \phi_4^2 = v^2$$

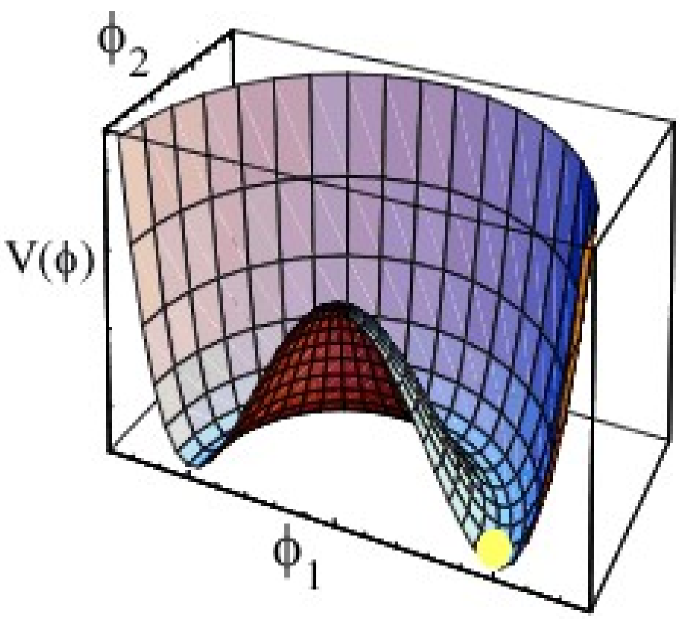
Ground state (vacuum)

$$\langle\Phi\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix}$$

Higgs Lagrangian

$$\mathcal{L}_H = \left(\partial_\mu \Phi^\dagger\right)\left(\partial^\mu \Phi\right) - V(\Phi^\dagger \Phi)$$

“Radial” excitations of vacuum:



$$\langle \Phi \rangle + \delta\Phi = \frac{1}{\sqrt{2}} \left(\begin{array}{c} 0 \\ v + H(x) \end{array} \right)$$

$$\Rightarrow \mathcal{L}_H = \frac{1}{2}\partial_\mu H \partial^\mu H - \lambda v^2 H^2 - \lambda v H^3 - \frac{\lambda}{4} H^4$$

$H(x)$ **- physical state**

with mass $\boxed{m_H = \sqrt{2\lambda} v}$

Higgs mechanism

Impose local gauge invariance

$$\left(\partial_\mu \Phi^\dagger\right)\left(\partial^\mu \Phi\right) \Rightarrow \left(\mathcal{D}_\mu \Phi^\dagger\right)\left(\mathcal{D}^\mu \Phi\right)$$

$$\mathcal{D}_\mu = \partial_\mu + i g_1 \frac{Y}{2} B_\mu + i g_2 \frac{1}{2} \vec{\sigma} \cdot \vec{W}$$

Consider ground state (vacuum)

$$\langle \Phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$$

Generation of Gauge Boson Masses

The kinetic term in the Higgs Lagrangian

$$\left(\mathcal{D}^\mu \langle\Phi\rangle^\dagger\right)\left(\mathcal{D}_\mu \langle\Phi\rangle\right)$$

Recall :

$$A_\mu = \frac{g_2 B_\mu + g_1 W^3_\mu}{\sqrt{g_1^2 + g_2^2}}, \qquad Z_\mu = \frac{-g_1 B_\mu + g_2 W^3_\mu}{\sqrt{g_1^2 + g_2^2}}$$

$$W^\pm = \frac{1}{\sqrt{2}}\left(W^1 \pm iW^2\right)$$

$$\Rightarrow \boxed{\frac{1}{4} g_2^2 v^2 W^+_\mu W^{-,\mu} + \frac{1}{8}(g_1^2 + g_2^2) v^2 Z_\mu Z^\mu}$$

Generation of Weak Boson Masses

$$\frac{1}{4}g_2^2v^2W_\mu^+W^{-,\mu} + \frac{1}{8}(g_1^2 + g_2^2)v^2 Z_\mu Z^\mu$$

These are W and Z boson mass terms!

$$m_W = \frac{g_2 v}{2} \qquad m_Z = \frac{v}{2}\sqrt{g_1^2 + g_2^2}$$

field A_μ **remains massless** $m_\gamma = 0$

W and Z bosons acquire masses through interaction with the Higgs ground state (vacuum)!

Yukawa Interactions

Yukawa interactions ⇒ interactions between fermions and Higgs doublet

corresponding gauge invariant Lagrangian (lepton sector) :

$$\mathcal{L}_{Y,\ell} = \mathcal{G}_{Y,\ell}\left(\bar{\ell}_R \Phi^\dagger L_L + \bar{L}_L \Phi \ell_R\right)$$

$$L_L = \begin{pmatrix} \nu_L \\ \ell_L \end{pmatrix} \qquad \bar{L}_L = \left(\bar{\nu}_L,\ \bar{\ell}_L\right)$$

$$\Phi = \frac{1}{\sqrt{2}}\begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \qquad \Phi^\dagger = \frac{1}{\sqrt{2}}\left(\phi^-,\ \ \phi^{0*}\right)$$

$$\phi^- = \left(\phi^+\right)^*$$

Generation of Fermion Masses

Consider Yukawa interactions between ground Higgs state (vacuum) and leptons

$$\mathcal{L}_{Y,\ell} = \mathcal{G}_{Y,\ell}\left(\bar{\ell}_R \langle\Phi\rangle^\dagger L_L + \bar{L}_L \langle\Phi\rangle \ell_R\right)$$

$$\Rightarrow \frac{v}{\sqrt{2}}\mathcal{G}_{Y,\ell}\left(\bar{\ell}_R \ell_L + \bar{\ell}_L \ell_R\right)$$

Mass terms generated!

Masses of charged leptons are given by

$$\boxed{m_\ell = \frac{v}{\sqrt{2}}\mathcal{G}_{Y,\ell}}$$

Generation of Fermion Masses

$\mathcal{G}_{Y,\ell}$ **– Yukawa coupling of lepton ℓ to Higgs field**

Masses of down-type quarks are generated in similar way

Masses of up-type quarks are generated via Yukawa interactions with charge conjugate Higgs doublet

$$\Phi_c = -i\sigma_2\Phi^* = \begin{pmatrix} -\phi^{0*} \\ \phi^- \end{pmatrix}$$

Destiny of Physical State (Higgs boson)

And what happens to physical state H (Higgs boson)?

Let's take look at kinetic term of $\mathcal{L}_H$

$$\mathcal{D}^\mu\Big(\langle\Phi\rangle + \delta\Phi\Big)^\dagger \mathcal{D}_\mu\Big(\langle\Phi\rangle + \delta\Phi\Big)$$

$$\langle\Phi\rangle + \delta\Phi = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v + H(x) \end{pmatrix}$$

$$\mathcal{D}_\mu = \partial_\mu + ig_1\frac{Y}{2}B_\mu + ig_2\frac{1}{2}\vec{\sigma}\cdot\vec{W}$$

Higgs boson Interactions with W and Z

This generates additional terms :

Tri-linear Higgs – gauge boson interactions

$$\frac{1}{2}g_2^2 v W^{+,\mu} W_\mu^- H = \frac{2m_W^2}{v} W^{+,\mu} W_\mu^- H$$

$$\frac{1}{4}(g_1^2 + g_2^2) v Z_\mu Z^\mu H = \frac{m_Z^2}{v} Z_\mu Z^\mu H$$

Quartic Higgs – gauge boson interactions

$$\frac{1}{4}g_2^2 (W^{+,\mu} W_\mu^-) H^2 = \frac{m_W^2}{v^2} (W^{+,\mu} W_\mu^-) H^2$$

$$\frac{1}{8}(g_1^2 + g_2^2) Z_\mu Z^\mu H^2 = \frac{m_Z^2}{2v^2} Z_\mu Z^\mu H^2$$

Higgs boson Interactions with Fermions

Gauge invariant Yukawa interactions

$$\mathcal{L}_{Y,\ell} = \mathcal{G}_{Y,\ell}\Big(\bar{\ell}_R(\langle\Phi\rangle + \delta\Phi)^\dagger L_L + \bar{L}_L(\langle\Phi\rangle + \delta\Phi)\ell_R\Big)$$

$$\langle\Phi\rangle + \delta\Phi = \frac{1}{\sqrt{2}}\left(\begin{array}{c} 0 \\ v + H(x) \end{array}\right)$$

generate not only fermion masses
but also interactions between Higgs boson H
and fermions

$$\mathcal{L}_{H\ell\ell} = \frac{\mathcal{G}_{Y,\ell}}{\sqrt{2}} H\left(\bar{\ell}_R\ell_L + \bar{\ell}_L\ell_R\right) = \frac{m_\ell}{v} H\bar{\ell}\ell$$

same relations hold for quarks!

Prominent Feature of Higgs Mechanism

Let us inspect couplings of Higgs boson to SM particles

$$g_{HWW} \sim m_W^2$$

$$g_{HZZ} \sim m_Z^2$$

$$g_{Hff} \sim m_f$$

Higgs boson couples stronger to heavier particles

$\Rightarrow$ decays more often into heavier accessible particles

Higgs Boson Decay Branching Ratios