Computation of Physics Processes

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Course "Higgs Physics"

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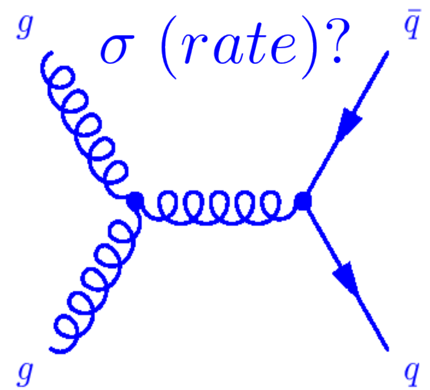
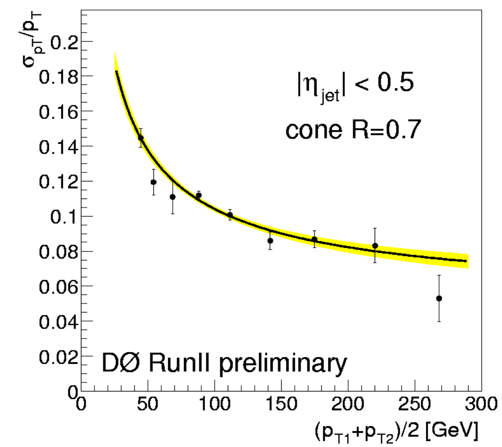
Experimental Measurements

What we measure in experiments?



- * **total rate of certain process (integral cross-section or decay width)**

- * **process rate in dependence of physical quantity, e.g. p_T of produced particles (differential cross-section or differential distributions in decays)**



$$\frac{dN}{dX}$$

Measurements (differential distributions, cross-sections, asymmetries, etc) ⇒ model testing (particle masses and quantum numbers, couplings, interaction vertices structures etc)

Golden Formula of Experimental Particle Physics

Luminosity →

flux of colliding particles / unit area / time interval (experimental characteristics)

$$\frac{dN}{dt} = \mathcal{L} \cdot \sigma$$

Process rate →

 # events / time interval

(results of experimental measurements)

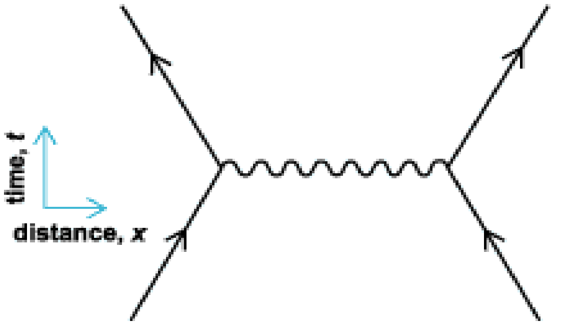
Cross section →

measure of probability of certain process to happen normalized to collision rate

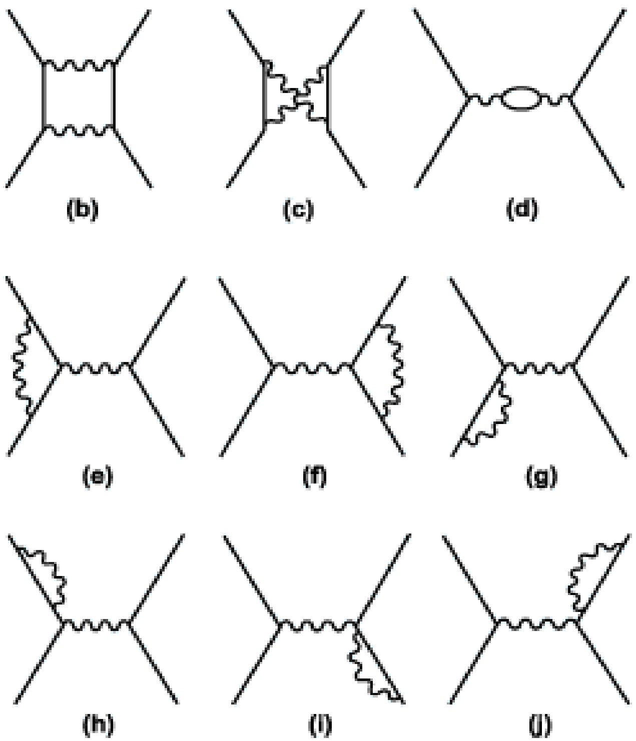
(independent of experimental conditions)

Perturbation series and Matrix Elements

Born level (LO)



(a)



$$\sigma = \frac{|\mathcal{M}|^2 \cdot [\text{Phase space}]}{[\text{Colliding particle flux}]}$$

$|\mathcal{M}|^2$ **: Matrix Element**

Describes dynamics of the given process

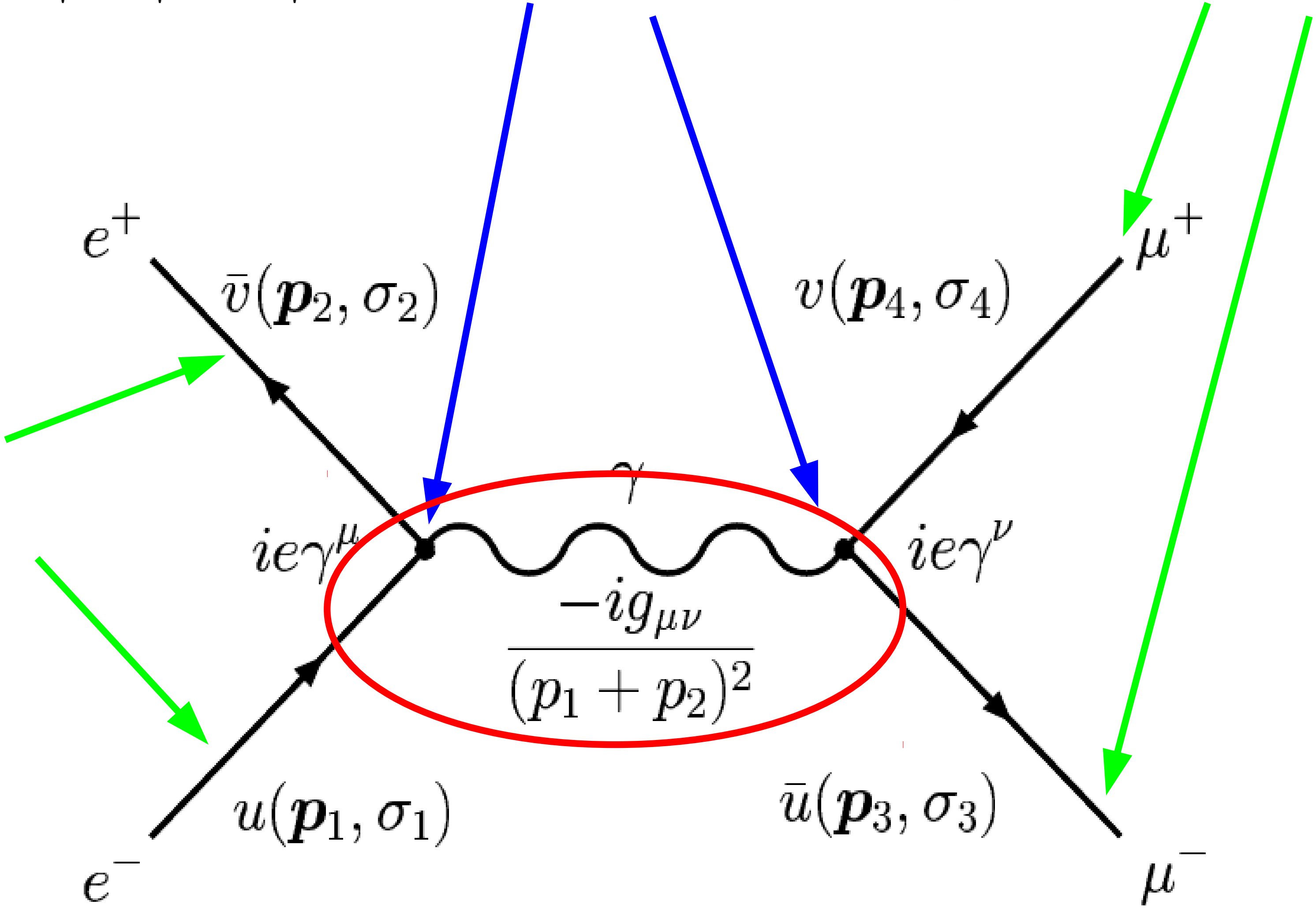
Perturbation series

$$\mathcal{M} = \mathcal{M}_a + \mathcal{M}_b + \mathcal{M}_c + \ldots$$

Makes sense only if involved coupling strength is small $|g| < 1$

Computation of Matrix Elements

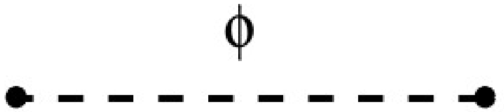
$$|\mathcal{M}|^2 = |\text{Vertex factors} \cdot \text{Propagator} \cdot \text{External lines}|^2$$



Propagator → Inverse Equation of Motion

Scalar Field

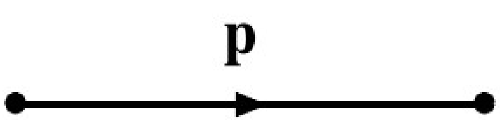
$$(p^2 - m^2)\phi = 0 \quad \Rightarrow$$



$$= \frac{i}{p^2 - m^2}$$

Fermions $\Big(i\gamma^\mu \partial_\mu - m\Big)\psi = 0 \Rightarrow (\gamma^\mu p_\mu - m)\psi = 0$

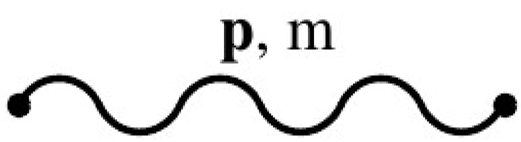
$$\frac{1}{\gamma^\mu p_\mu - m} = \frac{\gamma^\mu p_\mu + m}{(\gamma^\mu p_\mu - m)(\gamma^\mu p_\mu + m)} = \frac{\gamma^\mu p_\mu + m}{p^2 - m^2}$$



$$= \frac{i \sum_\sigma u(\boldsymbol{p}, \sigma)\bar{u}(\boldsymbol{p}, \sigma)}{p^2 - m^2}$$

Propagators

Gauge (vector) fields

massive: 

$$= \frac{-i\left(g^{\mu\nu} - \frac{p^{\mu}p^{\nu}}{m^2}\right)}{p^2 - m^2},$$

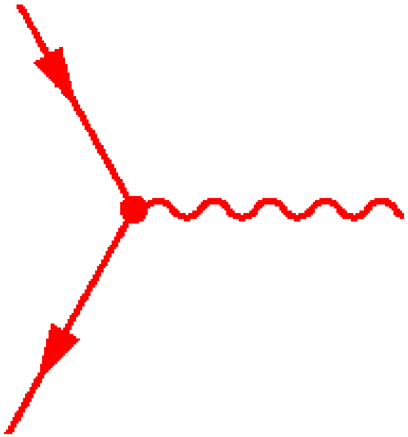
massless: 

$$= \frac{-ig^{\mu\nu}}{p^2}.$$

Vertex Factors

Vertex factor → interaction term in the Lagrange density with all fields removed

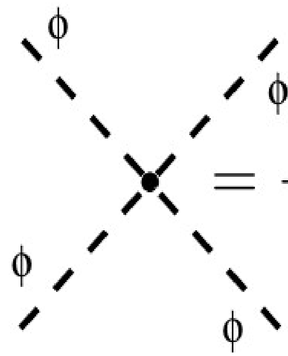
Example : Electromagnetic interaction



$$i\mathcal{L}_{\text{int}} = -ie\bar{\psi}\gamma^{\mu}\psi A_{\mu} \Rightarrow -ie\gamma^{\mu}$$

another Example : Higgs quartic self-coupling

$$i\mathcal{L}_{\text{I}} = -i\frac{g}{4!}\phi^{4} \overset{\text{removing fields}}{\Rightarrow}$$



$$= -i\frac{g}{4!}$$

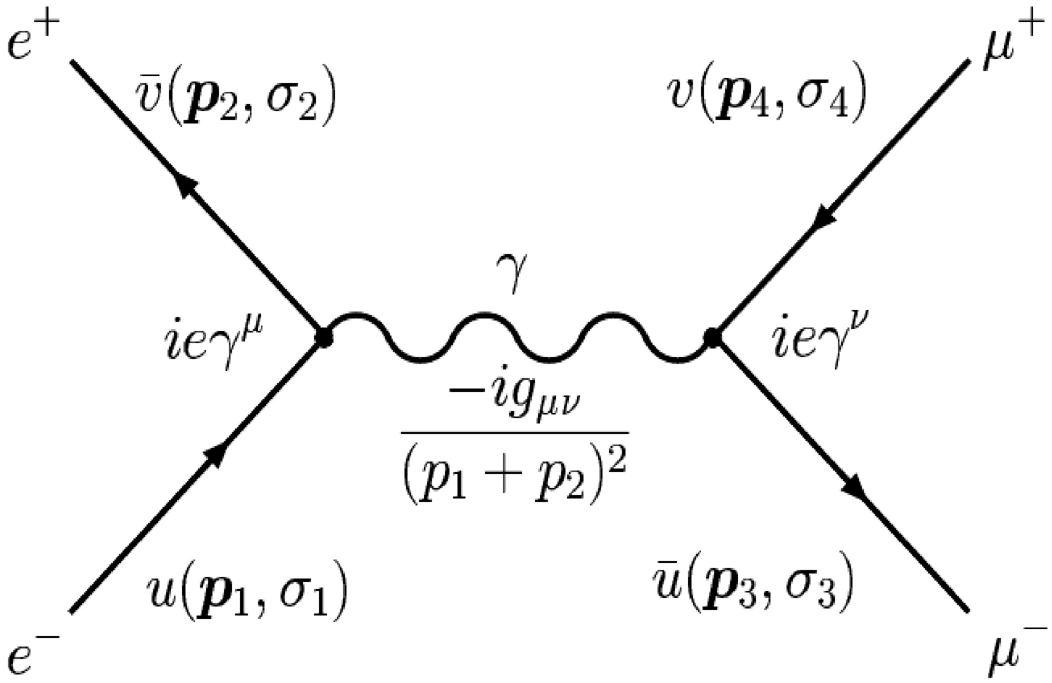
External Lines

External lines → wave functions of involved particles

External lines are represented by appropriate polarization vectors

particle	Feynman rule
ingoing fermion	u
outgoing fermion	$\bar{u}$
ingoing antifermion	$\bar{v}$
outgoing antifermion	v
ingoing photon	ϵ^{μ}
outgoing photon	$\epsilon^{\mu *}$
ingoing scalar	1
outgoing scalar	1

Combining Factors Together



$$-i\mathcal{M} = [\bar{u}(\boldsymbol{p}_3, \sigma_3)(ie\gamma^\nu)v(\boldsymbol{p}_4, \sigma_4)]\,\frac{-ig_{\mu\nu}}{(p_1+p_2)^2}\,[\bar{v}(\boldsymbol{p}_2, \sigma_2)(ie\gamma^\mu)u(\boldsymbol{p}_1, \sigma_1)]$$

or, introducing abbreviation $u_1 \equiv u(\boldsymbol{p}_1, \sigma_1)$,

$$\mathcal{M} = \frac{e^2}{(p_1+p_2)^2}[\bar{u}_3\gamma_\mu v_4][\bar{v}_2\gamma^\mu u_1]$$

From Matrix Elements to Observables

Golden Feynman Rules

Partial Decay width:

$$d\Gamma = \frac{1}{2E_1}\overline{|\mathcal{M}_{\beta\alpha}|^2}\,(2\pi)^4\delta^4(p_1 - p_1' - \cdots - p_n')\prod_{i=1}^{n}\frac{d^3p_i'}{(2\pi)^3\,2E_i'}$$

Cross section :

$$d\sigma = \frac{1}{u_\alpha}\frac{1}{2E_1}\frac{1}{2E_2}\overline{|\mathcal{M}_{\beta\alpha}|^2}\,(2\pi)^4\delta^4(p_1 + p_2 - p_1' - \cdots - p_n')\prod_{i=1}^{n}\frac{d^3p_i'}{(2\pi)^3\,2E_i'}$$

$$u_\alpha = \frac{\sqrt{(p_1\cdot p_2)^2 - m_1^2 m_2^2}}{E_1E_2}$$

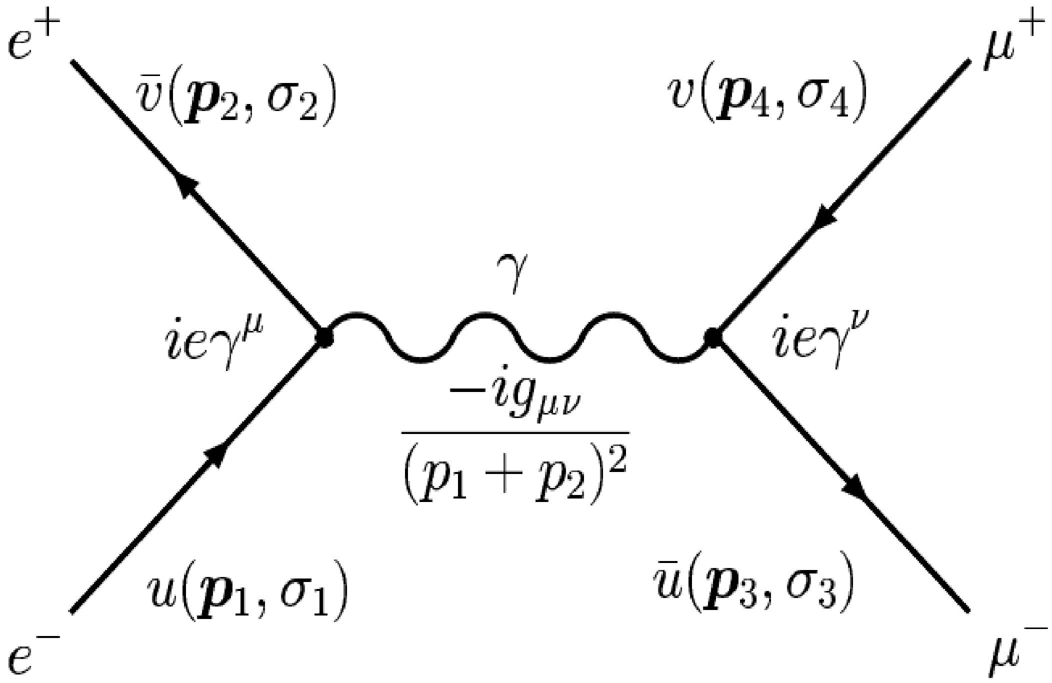
Relative velocity of particles 1 & 2

Summing up over polarization states

One usually averages over polarization states of initial state particles and sums over polarization states of final state particles

$$|\mathcal{M}|^2 \to \overline{|\mathcal{M}|^2} = \underbrace{\frac{1}{2}\,\frac{1}{2}\sum_{\sigma_1\sigma_2}}_{\text{avg. over initial pol.}} \overbrace{\sum_{\sigma_3\sigma_4}}^{\text{sum over final pol.}} |\mathcal{M}|^2$$

Summing over polarization states



$$\mathcal{M} = \frac{e^2}{(p_1+p_2)^2}[\bar{u}_3\gamma_\mu v_4][\bar{v}_2\gamma^\mu u_1]$$

$$\Rightarrow \quad \overline{|\mathcal{M}|^2} = \frac{e^4}{4(p_1+p_2)^4}\sum_{\sigma_{1,2,3,4}}[\bar{v}_4\gamma_\mu u_3][\bar{u}_1\gamma^\mu v_2][\bar{u}_3\gamma_\nu v_4][\bar{v}_2\gamma^\nu u_1]$$

Casimir Trick

write $\sum[\bar{u}_1\gamma^\mu v_2][\bar{v}_2\gamma^\nu u_1]$ with explicit spinor indices $\alpha, \beta, \gamma, \delta = 1, 2, 3, 4$:

$$\sum_{\sigma_1\sigma_2} \bar{u}_{1\alpha}\gamma^\mu_{\alpha\beta}v_{2\beta}\;\bar{v}_{2\gamma}\gamma^\nu_{\gamma\delta}u_{1\delta}\;.$$

$$\begin{aligned}\sum_{\sigma_1} u_{1\delta}\,\bar{u}_{1\alpha} &= (\not{p}_1 + m_1)_{\delta\alpha} \\ \sum_{\sigma_2} v_{2\beta}\,\bar{v}_{2\gamma} &= (\not{p}_2 - m_2)_{\beta\gamma}\end{aligned}$$

$$(\not{p}_1 + m_1)_{\delta\alpha}\,\gamma^\mu_{\alpha\beta}\,(\not{p}_2 - m_2)_{\beta\gamma}\,\gamma^\nu_{\gamma\delta} = \mathrm{Tr}[(\not{p}_1 + m_1)\gamma^\mu(\not{p}_2 - m_2)\gamma^\nu]$$

$$\overline{|\mathcal{M}|^2} = \frac{e^4}{4(p_1+p_2)^4}\mathrm{Tr}[(\not{p}_1 + m_1)\gamma^\mu(\not{p}_2 - m_2)\gamma^\nu]\,\mathrm{Tr}[(\not{p}_4 - m_4)\gamma_\mu(\not{p}_3 + m_3)\gamma_\nu]$$

Gamma Matrices and their properties

$$\gamma^0 = \begin{pmatrix} \mathbb{I}_2 & 0 \\ 0 & -\mathbb{I}_2 \end{pmatrix} \qquad \vec{\gamma} = \begin{pmatrix} 0 & \vec{\sigma} \\ -\vec{\sigma} & 0 \end{pmatrix}$$

$$\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}\mathbb{I}_4$$

$$\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} 0 & \mathbb{I}_2 \\ \mathbb{I}_2 & 0 \end{pmatrix}$$

$$\{\gamma^\mu, \gamma^5\} = 0 \qquad \gamma^5\gamma^5 = \mathbb{I}_4$$

Homework (I)

1) Prove identities

$$Tr[\not{p}_1 \not{p}_2] = 4 p_1 \cdot p_2$$

$$Tr[(\not{p}_1 - m)(\not{p}_2 + m)] = 4(p_1 \cdot p_2 - m^2)$$

where

$$\not{p}_1 = \gamma_\mu p_1^\mu \qquad \not{p}_2 = \gamma_\mu p_2^\mu$$

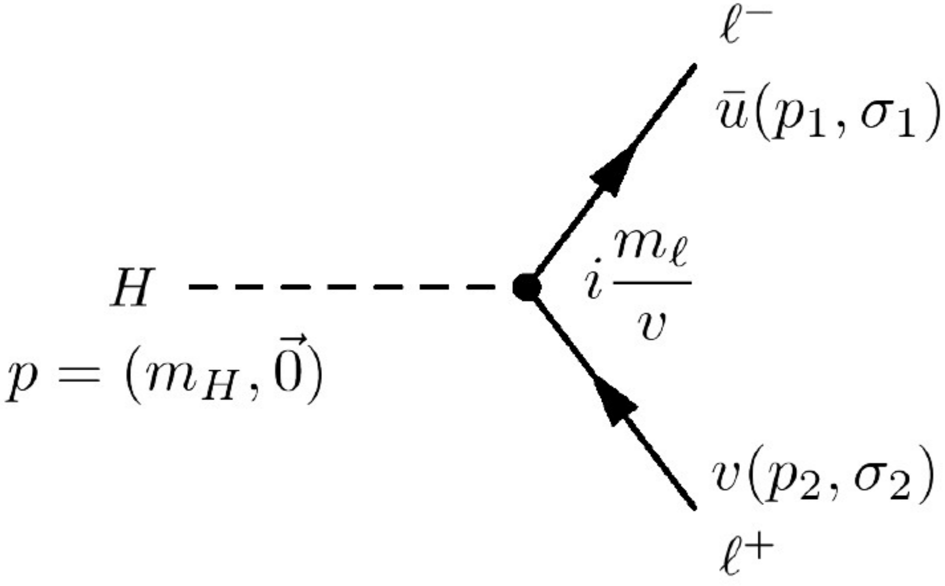
Hint: exploit properties of gamma-matrices and the following identities

$$Tr[\gamma^\mu] = Tr[\gamma_\mu] = 0, \qquad Tr[\mathbb{I}_4] = 4$$

Homework (II)

Compute Higgs partial decay widths into charged leptons

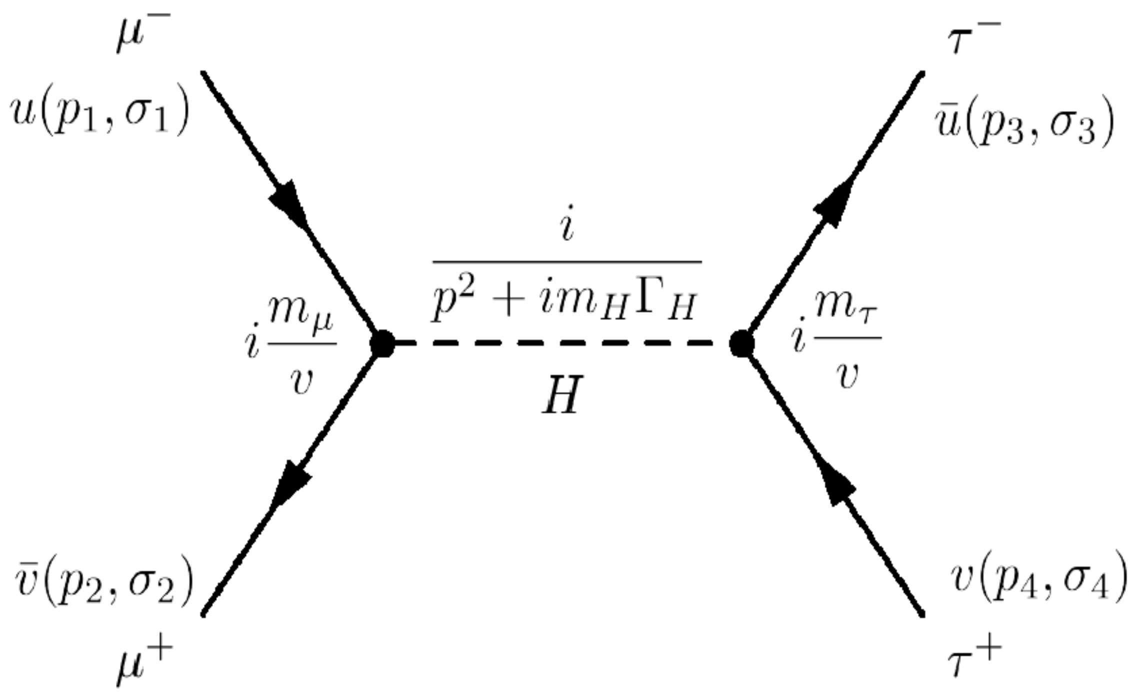
Hint : exploit the following LO Feynman graph



and Feynman golden rule for computation of decay width

Homework (III)

Compute cross section of the resonant scattering



Hint : make use of Feynman golden rule for computation of cross section

Availability of Lecture Notes

All lectures and exercise solution sheets are available at the following web-pages

https://indico.desy.de/categoryDisplay.py?categId=299

http://www-ekp.physik.uni-karlsruhe.de/~quast/