FLAG - the PDG for the lattice

Lattice Practices 2012

Desy Zeuthen

10/2012

Andreas Jüttner

UNIVERSITY OF Southampton

Two talks

FLAG

a_μ

Very strong claims are made based on lattice QCD results:

- "We find a (2-3)σ tension in the unitarity triangle"
  Laiho, Lunghi, Van de Water, PRD 81 (2010) 034503
- "...confirming CKM unitarity at the permille level"
  FLAG Eur.Phys.J. C71 (2011) 1695
- "...we find evidence of new physics in both B_d and B_s systems ..."
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  Lunghi, Soni, PLB 697, 323-328 (2011)

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How to deal with lattice results?

Are these statements water-proof?

Phenomenological impact

- new experimental results from CERN experiments, B-factories, ..., putting even more pressure on lattice QCD to produce
- hadronic effects often limiting: form factors, bag paramteres, $g-2$, ϵ'/ϵ,...
- lattice some-times only sound theoretical tool (low recoil B-decay form factors), complemented by alternatives like QCD sum-rules, chiral perturbation theory, dispersion relations

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so phenomenologists need lattice QCD input but it's hard for them to scrutinise quality

- therefore FLAG takes over task to compile and judge lattice QCD results (similar to PDG, HFAG)

Lattice phenomenology

Experiment	=	params.	×	PT SM	×	NP SM
$\Gamma_{\pi\to l\nu}$	=	$\lvert V_{ud}\rvert^2$	×	weak, elm.	×	f_π^2
				perturbation theory		lattice QCD sum rules

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Other NP SM observables or parameters:

- quark masses, α_s
- meson and baryon spectra
- matrix elements relevant for phenomenology:

- decay constants	$(f_\pi, f_K, f_{D_{(s)}}, f_{B_{(s)}})$
- form factors	$(f^{\pi\pi}, f^{K\pi}, f^{D\to K}, f^{B\to\pi}, \ldots)$
- mixing matrix elements	$(B, B_{B_{(s)}}, \ldots)$
- hadronic K-decays	(A_0, A_2)

FLAG-1

→ Flavia Net Lattice Averaging Group (**FLAG**) was founded late 2007 to allow also to an outsider to judge the quality and 'state-of-the-art'-fulnes of lattice results relevant to flavor physics (↔ PDG, HFAG)

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→ quantities: $m_{u,d}$, m_s, $f_+^{K\pi}(0)$, f_K/f_π, B_K, NLO LECs, potentially more in the future

→ criteria:
- publication status
- chiral extrapolation
- continuum extrapolation
- finite volume errors
- renormalisation
- renormalisation scale running

→ averages where sensible

→ lattice dictionary, simulation parameters

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Other effort: Laiho, Lunghi, Van de Water: *PRD81 (2010) 034503*

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some original contributions:

- discussion and parameterisation of electromagnetic contributions to meson masses
- some new χPT formulae
- SM consistency check (CKM-unitarity)

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- chiral extrapolation
  - ★ $m_{\pi,\mathrm{min}} < 250\mathrm{MeV}$
  - ● $250\mathrm{MeV} < m_{\pi,\mathrm{min}} < 400\mathrm{MeV}$
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  - ■ otherwise
- renormalisation where applicable
  - ★ non-perturbative
  - ● 2-loop perturbation theory
  - ■ otherwise

FLAG-1 → FLAG-2

before: FLAG = FLAVIANet Lattice Averaging Group
Eur.Phys.J. C71 (2011) 1695

now: FLAG = Flavour Lattice Averaging Group

has now entered phase 2 and has been extended in various directions

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- quantities to be reviewed
  main extension: light quarks + heavy quarks + α_s
- representatives from
  - world regions Europe, Japan, US
  - collaborations Alpha, BMW, CLS, ETMC, Fermilab, HPQCD, JLQCD, MILC, PACS-CS, QCDSF, RBC/UKQCD, SWME
- number of people: $\approx$ 28
- possibly FLAG-2 public in early 2013???

FLAG-1 $\rightarrow$ FLAG-2

- Advisory Board: S. Aoki, C. Bernard, C. Sachrajda
- Editorial Board: G. Colangelo, H. Leutwyler,
  T. Vladikas, U. Wenger
- Working Groups

Quark masses	L. Lellouch, T. Blum, V. Lubicz
V_{us}, V_{ud}	AJ, T. Kaneko, S. Simula
LEC	S. Dürr, H. Fukaya
B_K	H. Wittig, J. Laiho, S. Sharpe
α_s	R. Sommer, T. Onogi, J. Shigemitsu
f_B, B_B	A. El Khadra, Y. Aoki, M. Della Morte
$B \rightarrow Hl\nu$	R. Van de Water, E. Lunghi, C. Pena

FLAG-2 criteria and averages

criteria

- will change as function of time
- α_s and HQ will need modified or additional criteria (HQ-treatment, improvement, matching, matching scale)

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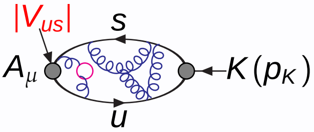
results averaged if

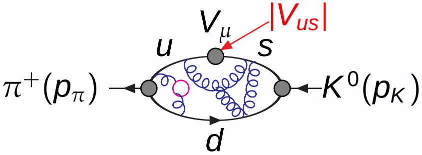
- no red tag
- published (peer-review) or update of published result (within reasonable limits like additional lattice spacings but otherwise unchanged analysis)
- individual averages for each N_f

Outline

- averaging procedure
- f_K/f_π and $f_+^{K\pi}(0)$

A FLAG example - the kaon sector





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$f_+(q^2), f_-(q^2)$

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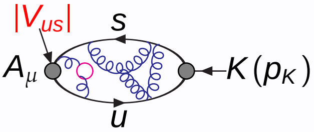

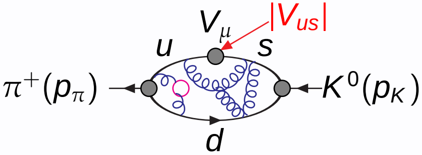

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$$\frac{\Gamma(K \to \mu\bar{\nu}_\mu(\gamma))}{\Gamma(\pi \to \mu\bar{\nu}_\mu(\gamma))} = \frac{|V_{us}|^2}{|V_{ud}|^2}\left(\frac{f_K}{f_\pi}\right)^2 \frac{m_K(1-m_\mu^2/m_K^2)}{m_\pi(1-m_\mu^2/m_\pi^2)} \times 0.9930(35)$$

(Marciano, Phys.Rev.Lett. 2004)

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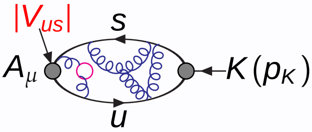

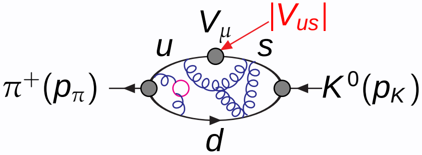

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$$\Gamma_{K\to\pi l\nu} = C_K^2 \frac{G_F^2 m_K^5}{192\pi^2} I\, S_{\rm EW}[1+\Delta_{SU(2)}+\Delta_{\rm EM}] \times |V_{us}|^2 |f_+^{K\pi}(0)|^2$$

Lattice computation

- $\langle 0|A_\mu|K(p)\rangle_{\mathrm{QCD}}$ from $\langle A_\mu P\rangle$ 2pt-function
- $\langle \pi(p')|V_\mu|\pi(p)\rangle_{\mathrm{QCD}}$ from $\langle PV_\mu P\rangle$ 3pt-function
- systematics:
  - scale setting
  - continuum limit
  - chiral extrapolation
  - finite volume limit
  - discretisation specific issues
  - iso-spin breaking and elm. effects
  - ...
- pretty much standard simulations, f_K/f_π published by most collaborations generating ensembles

A FLAG example - the kaon sector

Collaboration	N_f	publication status	chiral extrapolation	continuum extrapolation	finite volume errors	f_K/f_π
MILC 11	2+1+1	C	●	●	●	$1.1872(42)^{\dagger}_{\rm stat.}$
ETM 10E	2+1+1	C	●	●	●	$1.224(13)_{\rm stat}$
Laiho 11	2+1	C	●	●	●	$1.202(11)_{\rm stat}(9)_{\chi\rm PT}(2)_{\rm scale}(5)_{m_q}{}^{\dagger\dagger}$
MILC 10	2+1	C	●	★	★	$1.197(2)(^{+3}_{-7})^*$
JLQCD/TWQCD 10	2+1	C	●	■	★	1.230(19)
RBC/UKQCD 10A	2+1	A	●	●	★	1.204(7)(25)
PACS-CS 09	2+1	A	★	■	■	1.333(72)
BMW 10	2+1	A	★	★	★	1.192(7)(6)
JLQCD/TWQCD 09A	2+1	C	●	■	■	$1.210(12)_{\rm stat}$
MILC 09A	2+1	C	●	★	★	$1.198(2)(^{+6}_{-8})^*$
MILC 09	2+1	A	●	★	★	$1.197(3)(^{+6}_{-13})^*$
Aubin 08	2+1	C	●	●	●	1.191(16)(17)
PACS-CS 08, 08A	2+1	A	★	■	■	1.189(20)
RBC/UKQCD 08	2+1	A	●	■	★	1.205(18)(62)
HPQCD/UKQCD 07	2+1	A	●	★	●	1.189(2)(7)
NPLQCD 06	2+1	A	●	■	■	$1.218(2)(^{+11}_{-24})$
MILC 04	2+1	A	●	●	●	$1.210(4)(13)^*$
BGR 11	2	A	★	■	■	1.215(41)
ETM 10D	2	C	●	★	●	$1.190(8)_{stat}$
ETM 09	2	A	●	★	●	1.210(6)(15)(9)
QCDSF/UKQCD 07	2	C	●	●	★	1.21(3)

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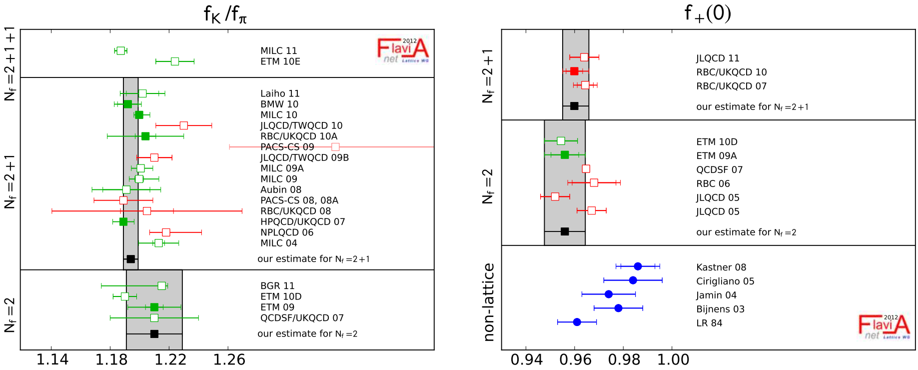

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  - uncorrelated $\chi^2(a) = \sum_i \frac{(x_i - a)^2}{\sigma_i^2}$

    (sum of the squares of independent standard normal random variables)

    x_i ith result

    σ_i error on ith result

    a average

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  - minmise: $\chi^2(a) = \chi^2_{\rm min}$
    find: σ such that $\chi^2(a+\sigma) = \chi^2_{\rm min} + 1 \rightarrow 1\sigma$-error

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- not the final result:
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  - above example systematics dominated: error on average result smaller than smallest individual systematic error!
    fishy, FLAG thinks one needs to inflate error

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    FLAG therefore quotes $f_K/f_\pi = 1.193(5)$
- there is no first-principles approach to proceed and discussions about what to do can be endless
- it's important to explain the adopted procedure and then stick to it

What to do with all these data? FLAG-2 average

shortcomings:

- HPQCD and MILC use same set of MILC lattices, statistical correlation
- extrapolation models are often the same, correlation in systematic

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- don't know the correlations, so need to estimate them somehow from measurements and their total errors
- assuming Gaussian errors, correlations are described by the covariance matrix

$$C_{i,j} = \langle (x_i - \bar{x}_i)(x_j - \bar{x}_j) \rangle$$

  - $\langle\rangle$ is over MC ensemble
  - x_i e.g. MILC and HPQCD measurements of an observable
  - $\bar{x}_i$ e.g. MILC and HPQCD mean value

then $C_{i,j}$ tells us how MILC and HPQCD results are correlated (in this example C is 2×2-matrix)

What to do with all these data? FLAG-2 average

assuming for a minute we had full knowledge of $C_{i,j}$

$$\chi^2(a) = \sum_{i,j=1}^{m} (\bar{x}_i - a)(C^{-1})_{i,j}(\bar{x}_j - a)$$

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Solution for minimum (check this):

$$\begin{aligned} a &= \left(\sum_{i,j=1}^{m} (C^{-1}))_{i,j}\right)^{-1} \left(\sum_{i,j=1}^{m} (C^{-1})_{i,j}\bar{x}_j\right) \\ \sigma^2(a) &= \left(\sum_{i,j=1}^{m} (C^{-1}))_{i,j}\right)^{-1} \end{aligned}$$

unbiased estimate of mean with smallest possible error

What to do with all these data? FLAG-2 average

one often has large correlations making the inversion of the covariance matrix difficult/unstable

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- let's abandon the requirement of smallest error for the average:
  any linear combination

$$a = \sum_{i=1}^{m} w_i x_i \quad \text{with} \quad \sum_{i=1}^{m} w_i = 1$$

  of the individual measurements is an unbiased estimate of its true value

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- for the FLAG-2 average value take the weighted average (no correlations):

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- uncorrelated data → smallest possible error
- correlated data → error not optimal but fit stable also if large correlations

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in practice in our example MILC and HPQCD have of course not done a joint analysis and $C_{i,j}$ is therefore not known!
What can we learn from the value of $\chi^2(a)$?

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- if $\chi^2(a)/\mathrm{dof} > 1$ inflate $\sigma(a)$ with PDG factor $\sqrt{\chi^2(a)/dof}$

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in practice in our example MILC and HPQCD have of course not done a joint analysis and $C_{i,j}$ is therefore not known!
What can we learn from the value of $\chi^2(a)$?

- if $\chi^2(a)/\mathrm{dof} > 1$ inflate $\sigma(a)$ with PDG factor $\sqrt{\chi^2(a)/dof}$
- if $\chi^2(a)/\mathrm{dof} < 1$
  - errors over-estimated $\rightarrow$ $\sigma(a)$ over-estimated
  - there is positive correlation $\rightarrow$ $\sigma(a)$ under-estimated

  if positive correlation can be assumed, χ^2 can be used to estimate size of correlations and to reevaluate the esimtate for the error estimate

FLAG-2 average

- positive correlations → estimate them from χ^2:

$$C_{ij} = f \cdot C_{i,j}^{\text{max}} \quad \text{where} \quad C_{i,j}^{\text{max}} = \sqrt{C_{ii}}\sqrt{C_{jj}}$$

$f = 0 \rightarrow$ no correlation, $f = 1 \rightarrow$ 100% correlation

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- can in principle adjust f such that

$$\chi^2(f) = \sum_{i,j}^{m}(x_i - a)(C(f)^{-1})_{i,j}(x_j - a) = m - 1$$

FLAG-2 average

- in case of positive correlations, FLAG assumes them to be 100%, e.g.
  - two collaborations using the same set of gauge configurations (e.g. HPQCD and MILC for f_K/f_π)
  - two observables computed on the same gauge ensemble (e.g. ETM for f_K/f_π and $f_+^{K\pi}(0)$)
  - ...

FLAG-2 average for f_K/f_π

Collaboration	N_f	publication status	chiral extrapolation	continuum extrapolation	finite volume errors	f_K/f_π
MILC 10	2+1	C	●	★	★	$1.197(2)(^{+3}_{-7})^*$
RBC/UKQCD 10A	2+1	A	●	●	★	1.204(7)(25)
BMW 10	2+1	A	★	★	★	1.192(7)(6)
HPQCD/UKQCD 07	2+1	A	●	★	●	1.189(2)(7)

for the data in the order BMW 10, RBC/UKQCD 10A, MILC 10 and HPQCD/UKQCD 07:

$$C = \begin{pmatrix} 0.85 & 0. & 0. & 0. \\ 0. & 6.74 & 0. & 0. \\ 0. & 0. & 0.29131 & 0.0400903 \\ 0. & 0. & 0.0400903 & 0.53 \end{pmatrix} \cdot 10^{-4}$$

where MILC 10 and HPQCD/UKQCD 07 100% correlated

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- weigthed average with correlated error: $f_K/f_\pi = 1.194(4)$

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Result:

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- error smaller than smallest systematic uncertainty of input

$$\rightarrow f_K/f_\pi = 1.194(5)$$ **PRELIMINARY**

FLAG-2 average for f_K/f_π

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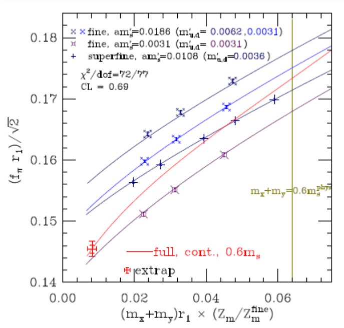
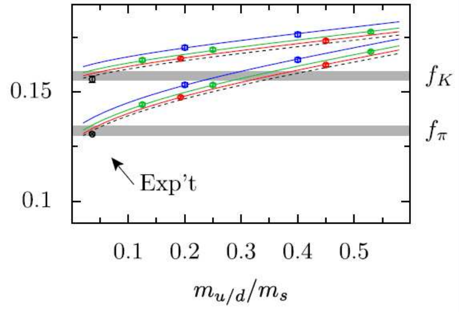
$$\to f_K/f_\pi = 1.194(5)$$ **PRELIMINARY**

- one could also consider correlations amongst systematics

FLAG-2 average for f_K/f_π



MILC



HPQCD 07

systematics correlated?

- scale setting
- finite volume errors
- chiral extrapolation
- ...

Kaon semi-leptonic decay

Collaboration	N_f	publication status	chiral extrapolation	continuum extrapolation	finite volume errors	$f_+(0)$
JLQCD 11	2+1	C	●	■	★	0.964(6)
RBC/UKQCD 10	2+1	A	●	■	★	0.9599(34)($^{+31}_{-47}$)(14)
RBC/UKQCD 07	2+1	A	●	■	★	0.9644(33)(34)(14)
ETM 10D	2	C	●	★	●	0.9544(68)$_{stat}$
ETM 09A	2	A	●	●	●	0.9560(57)(62)
QCDSF 07	2	C	■	■	★	0.9647(15)$_{stat}$
RBC 06	2	A	■	■	★	0.968(9)(6)
JLQCD 05	2	C	■	■	★	0.967(6), 0.952(6)

	f_K/f_π	$f_+^{K\pi}(0)$
$N_f = 2+1$	1.194(5)	0.9591(34)(41)
$N_f = 2$	1.210(6)(17)	0.9460(57)(62)

$N_f = 2+1$: $f_+^{K\pi}(0)$ by RBC/UKQCD 10

f_K/f_π FLAG-average over RBC/UKQCD 10A, HPQCD/UKQCD 07, BMW 10, MILC 10

$N_f = 2$: $f_+^{K\pi}(0)$ by ETM 09A, f_K/f_π ETM 09

Tests of the flavor sector: The CKM-matrix

$$\overline{D}_L^m \gamma_\mu \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} U_L^m$$

- relates quarks in the mass basis and in the interaction basis
- as far as we know not determined by symmetries, i.e. free params

Tests of the flavor sector: The CKM-matrix

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \overset{\text{unitarity}}{\longrightarrow} |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 \overset{\text{SM}}{=} 1$$

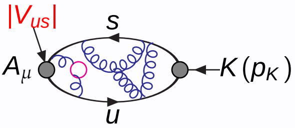
- relates quarks in the mass basis and in the interaction basis
- as far as we know not determined by symmetries, i.e. free params
- unitary in the SM
- source of CP-violation in the SM
- is this matrix also unitary in nature?

Is it really unitary?

To answer this question $\rightarrow$ determine matrix elements

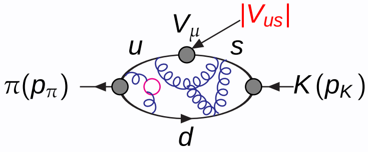
- typical processes e.g.:

K_{l2}-decay



$$\langle 0|A_\mu(0)|K(p_K)\rangle$$

K_{l3}-decay



$$\langle \pi(p_\pi)|V_\mu(0)|K(p_K)\rangle$$

- in practice:
  - measure decay rates $\Gamma(i \rightarrow j)$
  - compute process in SM (FF, RC)
  - $\Gamma(i \rightarrow j) = const. \times G_F^2 |V_{ij}|^2 \times FF \times RC$

Analysis (together with f_K/f_π)

Is the CKM matrix unitary?

- experimental results

$$|V_{us} f_+^{K\pi}(0)| = 0.2163(5) \qquad f_K/f_\pi |V_{us}/V_{ud}| = 0.2758(5)$$

FLAVIA Kaon WG Eur. Phys. J. C 69, 399-424 (2010)

$$|V_{ud}| = 0.97425(22)$$

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  - $f_+^{K\pi}(0)$ and $|V_{ud}|$ from experiment
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  - $f_+^{K\pi}(0)$ and f_K/f_π from lattice

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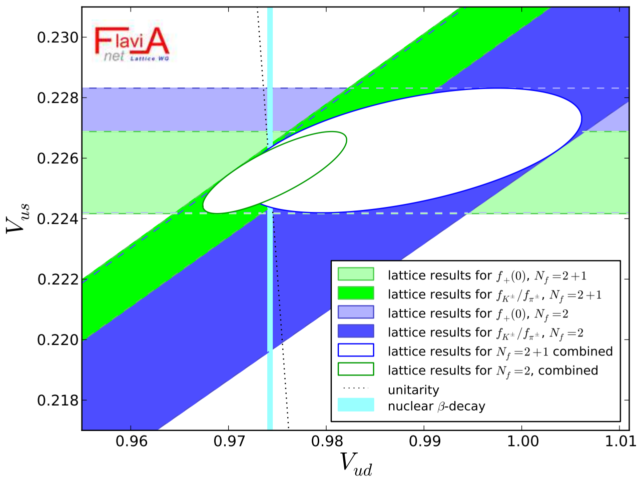
  - $f_+^{K\pi}(0)$ and f_K/f_π from lattice

FLAG Eur.Phys.J. C71 (2011) 1695:

	$f_+^{K\pi}(0)$& $\lvert V_{ud}\rvert_{\rm exp}$	f_K/f_π & $\lvert V_{ud}\rvert_{\rm exp}$	combined$_{\rm no \lvert V_{ud}\rvert_{exp}}$
$N_f = 2+1$	1.0000(7)	0.9999(6)	1.002(15)
$N_f = 2$	1.0004(10)	0.9985(16)	1.037(36)

$N_f = 2+1$: $f_+^{K\pi}(0)$ by RBC/UKQCD 10,
f_K/f_π FLAG-average over RBC/UKQCD 10A, HPQCD/UKQCD 07, BMW 10, MILC 10
$N_f = 2$: $f_+^{K\pi}(0)$ by ETM 09A, f_K/f_π ETM 09

Analysis



FlaviA net Lattice WG
0.230
0.228
0.226
0.224
0.222
0.220
0.218
V_{us}
0.96
0.97
0.98
0.99
1.00
1.01
V_{ud}
lattice results for $f_+(0)$, $N_f = 2+1$
lattice results for $f_{K^\pm}/f_{\pi^\pm}$, $N_f = 2+1$
lattice results for $f_+(0)$, $N_f = 2$
lattice results for $f_{K^\pm}/f_{\pi^\pm}$, $N_f = 2$
lattice results for $N_f = 2+1$ combined
lattice results for $N_f = 2$, combined
unitarity
nuclear β-decay

Analysis (together with f_K/f_π)

Use SM-unitarity

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$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$$

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$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$$

- ignore $|V_{ub}| = 4 \cdot 10^{-3}$, then 3 equations, 4 unknowns
- provide either
  - $f_+^{K\pi}(0) \to$ predict $|V_{ud}|$, $|V_{us}|$, f_K/f_π
  - $f_K/f_\pi \to$ predict $|V_{ud}|$, $|V_{us}|$, $f_+^{K\pi}(0)$

Analysis (together with f_K/f_π)

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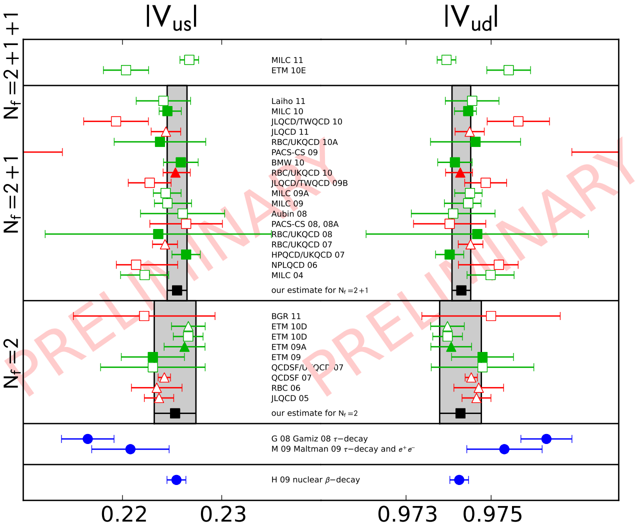
  - $f_K/f_\pi \rightarrow$ predict $|V_{ud}|$, $|V_{us}|$, $f_+^{K\pi}(0)$
- analysis within the SM (relying on the assumption of first-row unitarity) FLAG Eur.Phys.J. C71 (2011) 1695:

	$\lvert V_{us}\rvert$	$\lvert V_{ud}\rvert$	$f_+^{K\pi}(0)$	f_K/f_π
$N_f = 2+1$	0.2253(9)	0.97428(21)	0.9599(38)	1.1927(50)
$N_f = 2$	0.2251(18)	0.97433(42)	0.9604(75)	1.194(10)

$N_f = 2+1$: $f_+^{K\pi}(0)$ by RBC/UKQCD 10,
f_K/f_π FLAG-average over RBC/UKQCD 10A, HPQCD/UKQCD 07, BMW 10, MILC 10

Analysis (together with f_K/f_π)



Summary

- not covered here: light quark masses m_u, m_d, m_s (good read for EM effects and iso-spin splitting), B_K, LECs
- FLAG-1 contains also lattice *dictionary* (appendix) detailed summary of simulation parameters (appendix)
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- it is a responsibility of the lattice community to provide experimentalists and non-lattice theorists with a non-technical review of phenomenologically relevant lattice results
- FLAG has now started its phase 2 with a larger group and broader scope
- we hope that this initiative continues to gain momentum and the support of the whole lattice community
- IMPORTANT: always quote the original literature

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