Monte Carlo Event Generators

in high-energy and astro-particle physics

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Outline

- Particle Physics
- Event Generators
- Parton Showers
- Monte Carlo activities in Lund

The Standard Model of Particle Physics

All known matter is built up by quarks and leptons.

- Quarks are bound inside hadrons (eg. protons)
- Protons and neutrons are bound together in nuclei.
- Electrons are bound to nuclei and form atoms
- Atoms bind together and form molecules
- ...

There are four fundamental forces known which act on matter:

- Gravity:
  $G_{\mu\nu} + g_{\mu\nu}\Lambda = \frac{8\pi G}{c^4} T_{\mu\nu}$
- Electromagnetism and weak interaction:
  $\mathcal{L}_{\mathrm{EW}} = -\frac{1}{4}W_a^{\mu\nu}W_{\mu\nu}^a - \frac{1}{4}B^{\mu\nu}B_{\mu\nu} + \overline{Q}_i i\not{D} Q_i + \overline{L}_i i\not{D} L_i$
- Strong interaction:
  $\mathcal{L}_{\mathrm{QCD}} = \bar{U}(\delta_\mu - i g_s G_\mu^a T^a)\gamma^\mu U + \bar{D}(\delta_\mu - i g_s G_\mu^a T^a)\gamma^\mu D$

Everything in the microcosm is described by
The standard model.

Quantum Field Theory:
all paricles are fields, all fields are particles.

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s $\bar{s}$	c $\bar{c}$	μ^- μ^+	ν_μ $\bar{\nu}_\mu$
b $\bar{b}$	t $\bar{t}$	τ^- τ^+	ν_τ $\bar{\nu}_\tau$

Force carriers

γ Z^0 $W^\pm$ g

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Accelerators and colliders

To study these particles and forces we collide particles together in huge facilities, such as the Large Hadron Collider at CERN, where they recently found *a higgs-like particle*.

$$\mathcal{L}_H = -\frac{1}{2}[(\delta_\mu - iW_\mu^a t^a - iB_\mu)\phi]^2 - \frac{\mu^2}{2}\phi^\star\phi - \frac{\lambda}{4}(\phi^\star\phi)^2$$

How do you look for a Higgs particle?

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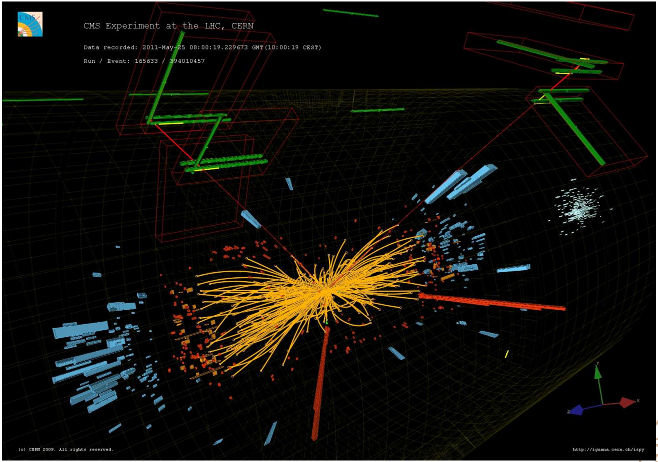
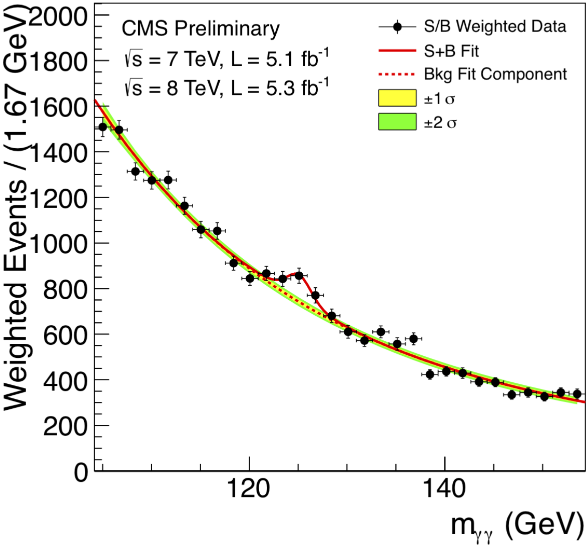

CMS Experiment at the LHC, CERN
Data recorded: 2011-May-25 08:00:19.229673 GMT(10:00:19 CEST)
Run / Event: 165633 / 394010457
(c) CERN 2009. All rights reserved.
http://iguana.cern.ch/ispy


CMS Preliminary
√s = 7 TeV, L = 5.1 fb-1
√s = 8 TeV, L = 5.3 fb-1
S/B Weighted Data
S+B Fit
Bkg Fit Component
±1 σ
±2 σ
Weighted Events / (1.67 GeV)
0
200
400
600
800
1000
1200
1400
1600
1800
2000
120
140
mγγ (GeV)

- Differences in length scales: 10^{-18}m — 10 m
- Differences in time scales: 10^{-26}s — 10^{-12}s
- Differences in strengths: Strong force $\sim 10^8\times$ weak force.
- Differences in probabilities: A Higgs particle is produced in one out of 10^{10} proton collisions.
- Differences in numbers: The formulae describe what happens to a hand-full of particles — in each collision there are hundreds.

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Monte Carlo Event Generators

Started in the ’80s for simple processes.

Pioneered in Lund, implementing non-perturbative models for *hadronization* (The Lund model).

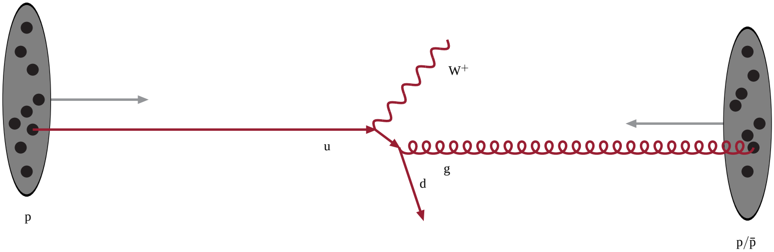
Today large software frameworks

- PYTHIA (Lund model)
- HERWIG
- SHERPA

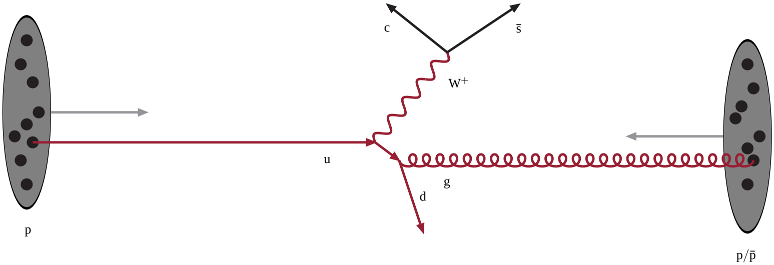
Used in **all** particle physics experiments, also in Astro-particle physics.

The structure of a proton collision

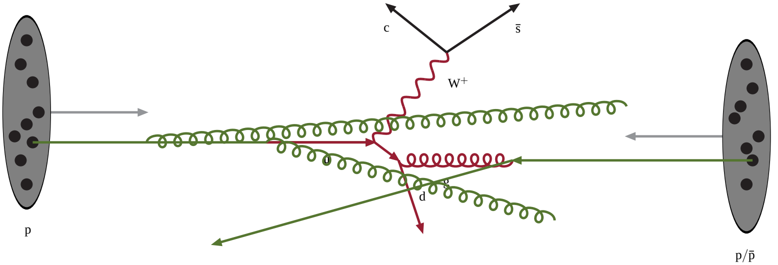
The hard/primary scattering



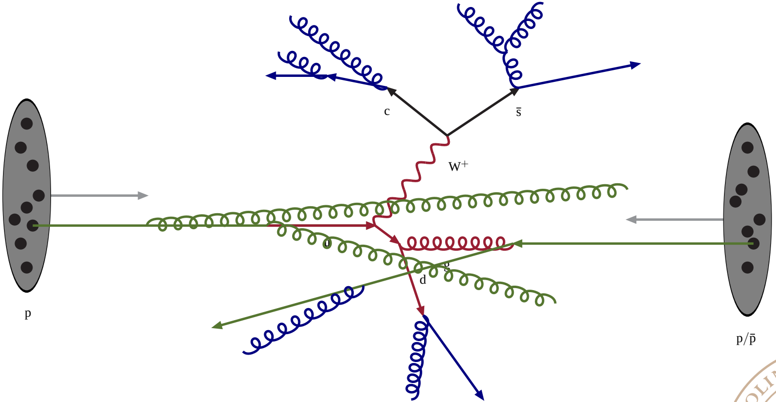
Immediate decay of unstable elementary particles



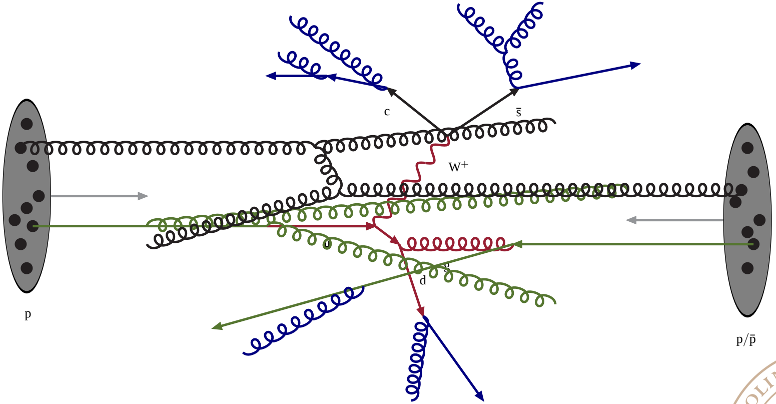
Radiation from particles before primary interaction



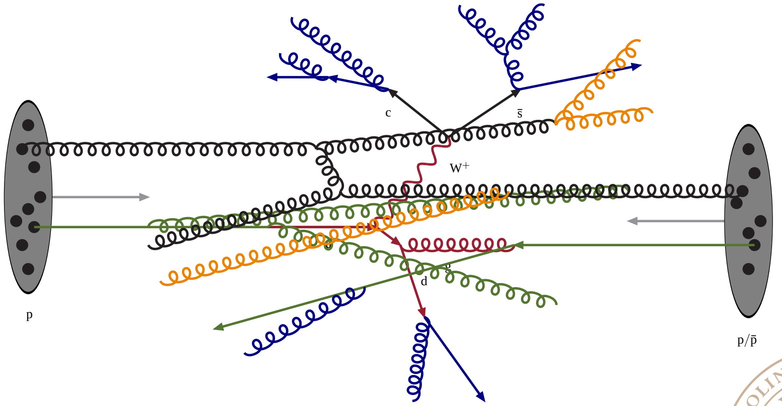
Radiation from produced particles



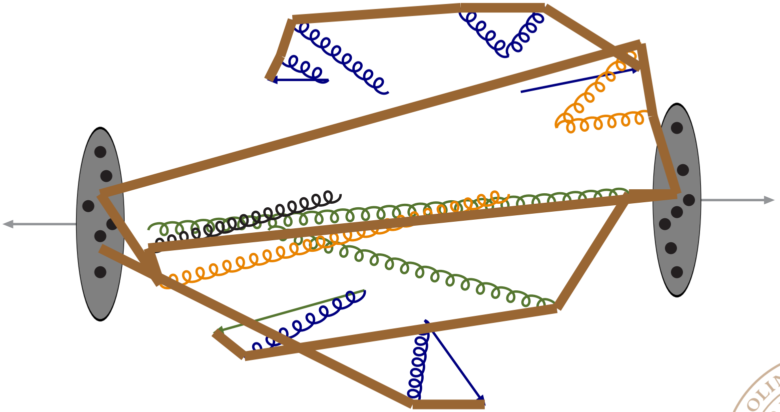
Additional sub-scatterings



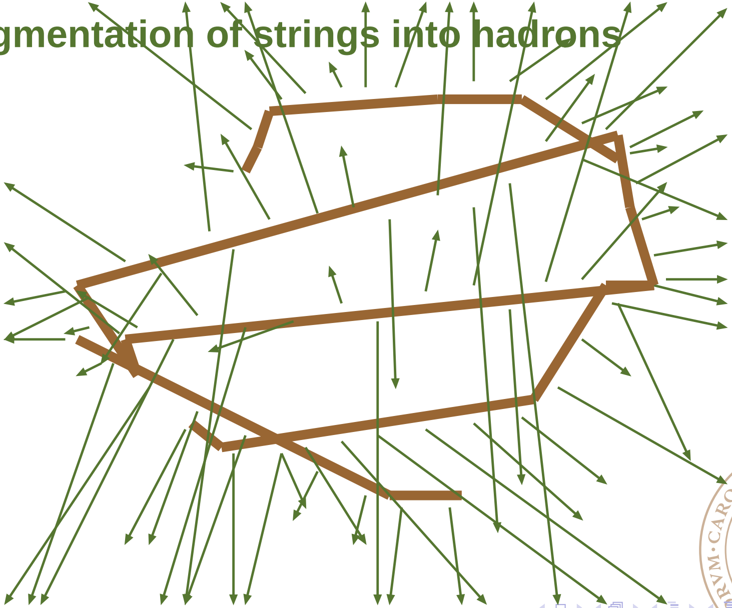
...with accompanying radiation



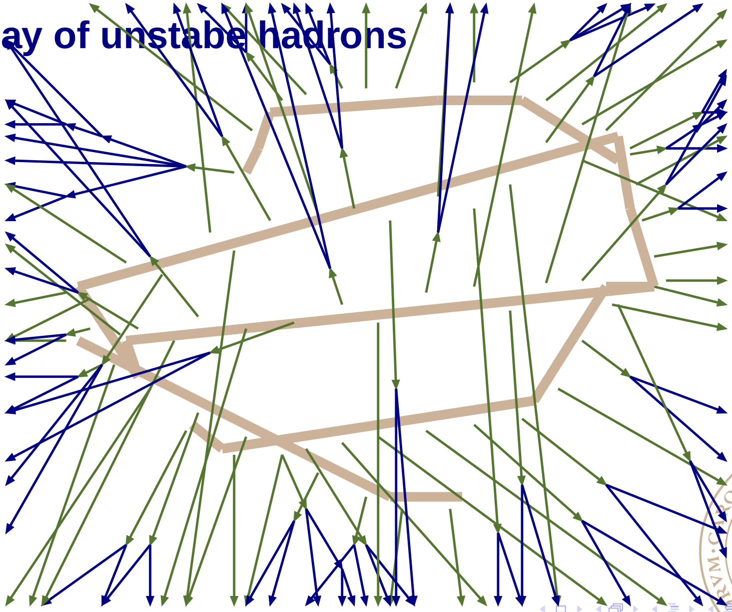
Formation of *colour strings*



Fragmentation of strings into hadrons



Decay of unstabe hadrons



We can easily calculate the probability (cross section) of one *parton* from each proton scattering against eachother, and ending up going in certain directions with some energies

$$d\sigma_{ab} \propto \sum_{ij} \int dx_i dx_j f_i(x_i) f_j(x_j) \left|\mathcal{M}_{ij\to ab}\right|^2 d\Phi_{ab}$$

- $f_i(x_i)$ are (measured) parton densities
- $d\Phi$ is the phase space density
- $\mathcal{M}_{ij\to ab}$ are *matrix elements* calculable for up to $n \lesssim 5$ outgoing partons.

Parton Showers

The dominant contribution of high-multiplicities of partons comes from the strong interaction (QCD). And we know approximately how to calculate

$$\frac{\left|\mathcal{M}_{ij\to n+1}\right|^2}{\left|\mathcal{M}_{ij\to n}\right|^2} \propto \alpha_{\mathrm{s}} \sum_a P_{a\to bc}(z)\frac{dq^2}{q^2}dzd\phi$$

- $P_{a\to bc}$ are simple splitting functions, some of them proportional to $1/z$ or $1/(1-z)$
- α_{s} is the coupling constant $\sim 1/10$ (cf. $\alpha_{\mathrm{EM}} = 1/137$)
- $z \sim$ the energy sharing between b and c.
  $Q^2 \sim$ their transverse momentum relative to a.
- Allowed splittings: $q \to qg$, $g \to q\bar{q}$, $g \to gg$.

There is a factorization theorem stating that the total cross section is given by $\left|\mathcal{M}_{ij\to 2}\right|^2$, with corrections of $\mathcal{O}(\alpha_s)$.

But the probability of emitting another parton diverges for collinear splittings.

What we see in the detector is a *jet* of hadrons in the (approximate) direction of an energetic quark.

We use jet-algorithms to obtain observables which can be calculated in perturbation theory. These will always depend on a resolution scale.

Jet calculations

Even if the total cross section can be calculated in a convergent perurbative series

$$\sigma_{ab} = C_0 + C_1\alpha_s + C_2\alpha_s^2 + \ldots$$

any jet observable will contain a resolution scale Q_0 and if the transverse momentum of the original parton is Q we have

$$C_n = c_{n,2n} \log\left(\frac{Q}{Q_0}\right)^{2n} + c_{n,2n-1} \log\left(\frac{Q}{Q_0}\right)^{2n-1} + \ldots$$

from the integration of subsequent splittings.

So if Q_0 is small and Q is large, the series does not converge.

We know the coefficients of the leading and next-to-leading logarithms to all orders, so we can *resum* the series, either analytically, or using a **parton shower**.

The latter is needed if we also want to take into account effects of the hadronization process.

The Sudakov Form Factor

When we simulate a parton shower, we organize the splittings so that they are **ordered** in time — or rather in decreasing transverse momentum.

In each step we then generate the *next* splitting. The naive splitting probability then gets modifies by the probability of no other splitting happening before.

$$\alpha_s P_a(z)\frac{dq^2}{q^2}dz \rightarrow \alpha_s P_a(z)\frac{dq^2}{q^2}\Delta_S(q_{prev}^2, q^2)$$

Δ_S is the Sudakov form factor

$$\Delta_S(q_0^2, q_1^2) = \exp\left(-\sum_a \int_{q_1^2}^{q_0^2} \frac{dq^2}{q^2} \int dz\, \alpha_s P_a(z)\right)$$

So, even if the splitting functions are divergent, the probability density for the momentum distribution of the next splitting is finite.

The Sudakov factorizes for all possible splitting possibilities: we can generate one splitting for each possibility and simply select the one with largest q^2.

The integration limits of the z-integral are often non-trivial, and it is not always possible to calculate Δ_S analytically.

The Sudakov veto algorithm

We can use a modification of the standard *hit-and-miss* veto algorithm.

- Start with the previous splitting scale, q_0^2.
- find a simple $\hat{\Gamma}(q^2) \leq \Gamma(q^2) = \frac{\alpha_s}{q^2} \int dz P(z)$
- Generate a q^2 of a next emission using $\hat{\Gamma}(q^2)\hat{\Delta}_S(q_0^2, q^2)$
  - If $\Gamma(q^2) > R \times \hat{\Gamma}(q^2)$, accept the emission
  - if not, set $q_0 = q$ and start over.

The probability of having no emission (not even one that was thrown away)

$$\mathcal{P}_0 = \hat{\Delta}_S(q_0^2, q^2)$$

The probability of having thrown away one emission

$$\begin{aligned}\mathcal{P}_1 &= \int_{q^2}^{q_0^2} dq_1^2 \hat{\Gamma}(q_1^2) \hat{\Delta}_S(q_0^2, q_1^2) \left[1 - \frac{\Gamma(q_1^2)}{\hat{\Gamma}(q_1^2)}\right] \hat{\Delta}_S(q_1^2, q^2) \\ &= \hat{\Delta}_S(q_0^2, q^2) \int_{q^2}^{q_0^2} dq_1^2 \left[\hat{\Gamma}(q_1^2) - \Gamma(q_1^2)\right]\end{aligned}$$

$$
\begin{aligned}
\mathcal{P}_2 &= \int_{q^2}^{q_0^2} dq_1^2 \hat{\Gamma}(q_1^2) \hat{\Delta}_S(q_0^2, q_1^2) \left[1 - \frac{\Gamma(q_1^2)}{\hat{\Gamma}(q_1^2)} \right] \\
&\quad \times \int_{q^2}^{q_1^2} dq_2^2 \hat{\Gamma}(q_2^2) \hat{\Delta}_S(q_1^2, q_2^2) \left[1 - \frac{\Gamma(q_2^2)}{\hat{\Gamma}(q_2^2)} \right] \hat{\Delta}_S(q_2^2, q^2) \\
&= \hat{\Delta}_S(q_0^2, q^2) \frac{1}{2} \left(\int_{q^2}^{q_0^2} dq_1^2 \left[\hat{\Gamma}(q_1^2) - \Gamma(q_1^2) \right] \right)^2
\end{aligned}
$$

Total probability of not having any emission is

$$\sum_{n=0}^{\infty}\mathcal{P}_n = \hat{\Delta}_S(q_0^2,q^2)\sum_{n=0}^{\infty}\frac{1}{n!}\left(\int_{q^2}^{q_0^2}dq_1^2\left[\hat{\Gamma}(q_1^2)-\Gamma(q_1^2)\right]\right)^n$$

$$= \hat{\Delta}_S(q_0^2,q^2)\exp\left(\int_{q^2}^{q_0^2}dq_1^2\left[\hat{\Gamma}(q_1^2)-\Gamma(q_1^2)\right]\right)$$

$$= \Delta_S(q_0^2,q^2)$$

Matching Parton Showers with Matrix Elements

How can we systematically increase the precision in a Parton Shower?

Matrix Element	Parton Shower
$\lvert\mathcal{M}_n\rvert^2$	$\lvert\mathcal{M}_2\rvert^2 \times P_1 \times \cdots \times P_{n-2}$
exact fixed order	approximate all orders
$n \lesssim 5$	$n \to \infty$
inclusive	exclusive
loops	Sudakov
affects total cross section	unitary

$$
\begin{aligned}
\langle\mathcal{O}\rangle &= 1 \\
&+ \alpha_s L^2 + \alpha_s L \ + \alpha_s \\
&+ \alpha_s^2 L^4 + \alpha_s^2 L^3 + \alpha_s^2 L^2 + \alpha_s^2 L \ + \alpha_s^2 \\
&+ \alpha_s^3 L^6 + \alpha_s^3 L^5 + \alpha_s^3 L^4 + \alpha_s^3 L^3 + \alpha_s^3 L^2 + \alpha_s^3 L + \alpha_s^3 \\
&\quad \vdots \\
&+ \alpha_s^n L^{2n} + \alpha_s^n L^{2n-1} + \cdots \\
&\quad \vdots
\end{aligned}
$$

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\langle \mathcal{O} \rangle \quad = \quad & 1 \\
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+ \quad & \alpha_s^3 L^6 + \alpha_s^3 L^5 + \alpha_s^3 L^4 + \alpha_s^3 L^3 + \alpha_s^3 L^2 + \alpha_s^3 L + \alpha_s^3 \\
& \vdots \\
+ \quad & \alpha_s^n L^{2n} + \alpha_s^n L^{2n-1} + \cdots \\
& \vdots
\end{aligned}
$$

COMPUTE!

PHD-school in the Natural Science Faculty in Lund, bringing together PhD students working on (Monte Carlo) simulations and computations in different areas:

- Astrophysics
- Biochemistry and Structual Biology
- Mathematics
- Computational Biology and Biological Physics
- Experimental Particle Physics
- Mathematical Physics
- Medical Radiation Physics
- Microbial Ecology
- Physical/Theoretical Chemistry
- Physical Geography
- Theoretical High Energy Physics

- Common courses
- Regular Seminars
- Workshops
- . . .

http://cbbp.thep.lu.se/compute

That's all folks!