State-of-the-art variance reduction methods for Monte Carlo radiative transfer in atmospheric remote sensing

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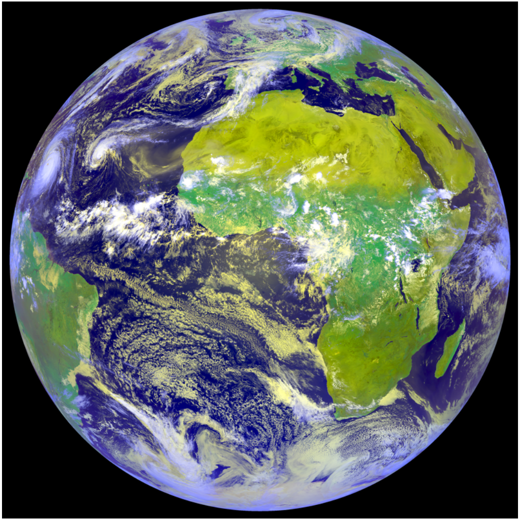
Motivation

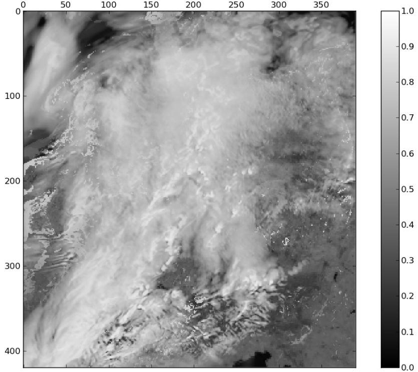
Why do we need a Monte Carlo radiative transfer solver?

False color composite of Metesat images



Why do we need a Monte Carlo radiative transfer solver?

Satellite (Meteosat) image simulation using clouds from numerical weather prediction model: Comparison 1D/3D



Why do we need a Monte Carlo radiative transfer solver?

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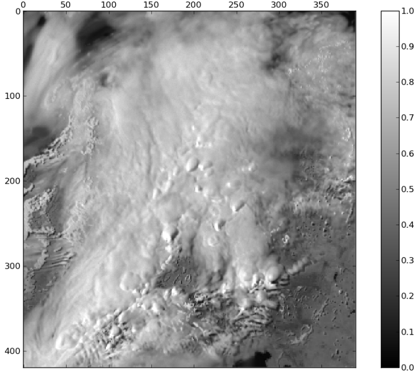
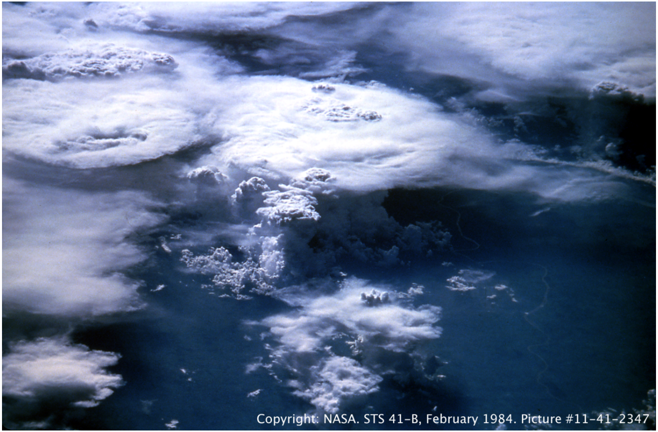


Why do we need a Monte Carlo radiative transfer solver?

Image taken from ISS



- Measurements & Cloud shadows: 1D satellite imager retrievals fail

Why do we need a Monte Carlo radiative transfer solver?

- For simulating remote sensing in the atmosphere, we need to solve the time-independent radiative transfer (RT) equation.
- Analytic 1D solutions to the RT equation exist (e.g. discrete ordinates), but for many applications this can lead to unacceptable errors:
  - Measurements & Cloud shadows: 1D satellite imager retrievals fail
  - Numerical Weather Prediction & Cloud shadows: spatially shifted heating
  - Lidar/Radar: multiple scattering contribution missing
  - Circum-solar and cloud-/rainbow radiances

⇒ In order to get “exact” results, use 3D Monte Carlo!

- Use MC for improvement/validation of faster methods
- Use MC when it IS fastest method

Outline

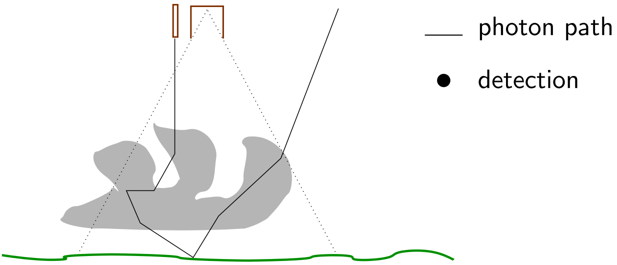
- Short intro to Monte Carlo (MC) in atmospheric RT (source-detector configuration)
- Variance Reduction I: Solving the forward scattering problem. "DDIS" (Importance Sampling), etc.
- Variance Reduction II: Spectral calculations. "ALIS" (Common random numbers)

Monte Carlo

Simple Monte Carlo (MC)

“Photons” follow phyically distributed random paths:

- pathlength chosen randomly according to optical depth τ
- scattering angle ϑ chosen randomly according to scattering phase function $p(\vartheta)$

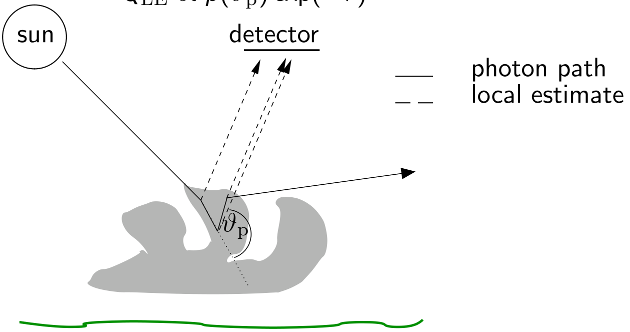


Problem: The detector is tiny compared to the atmosphere, very few photons arrive (first photon arrives after 30 years of computing).

Local estimate (LE): Computation of hit probability

At each scattering, we calculate the probability that the photon scatters towards the detector and is not extinct on its way to the detector (Marchuk '80).

$$Q_{\mathrm{LE}} \propto p(\vartheta_{\mathrm{p}}) \exp(-\tau)$$



Neumann-rule ⇒ Correct radiance is $I = N_{\mathrm{photons}}^{-1} \sum Q_{\mathrm{LE}}$

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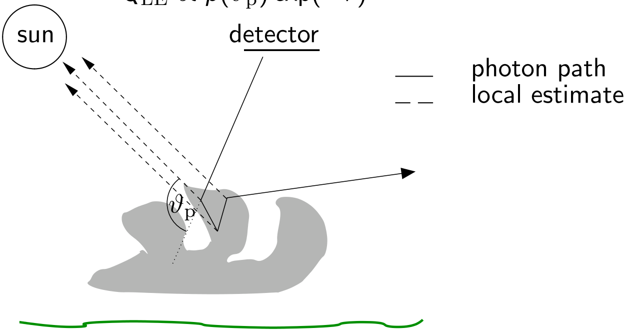
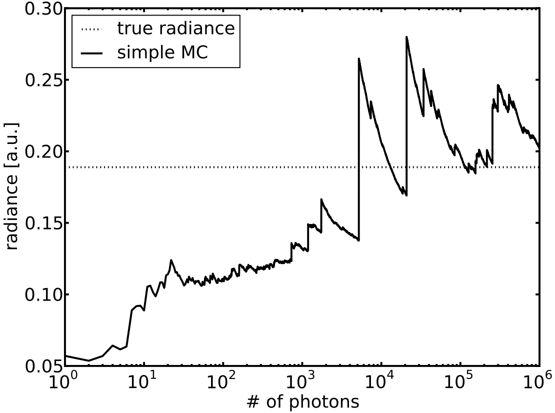


...even better: backward Monte Carlo (Sun is easier to hit!)

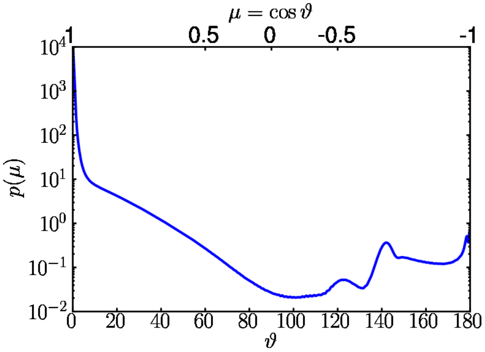
Spikes: Why local estimate and clouds don't work together



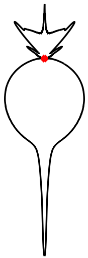
. . . What is going on here??

Spikes: Why local estimate and clouds don't work together

Problem: Scattering phase functions of clouds and aerosols

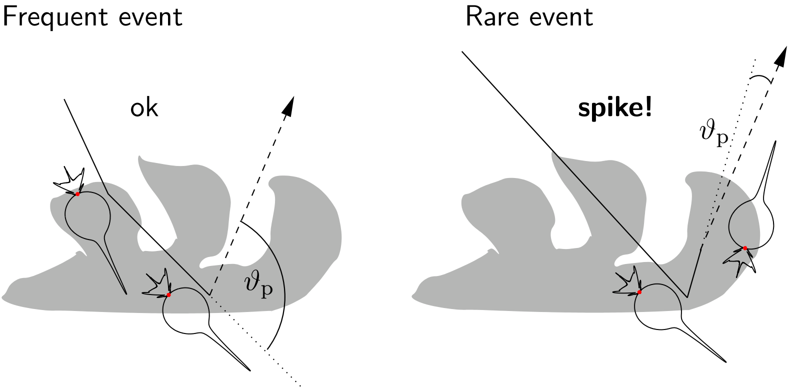


In log-polar coordinates:



Example of water cloud phase function for $r_{\text{eff}} = 10\mu$m (wavelength $\lambda \simeq 500$nm)

Spikes: Why local estimate and clouds don't work together

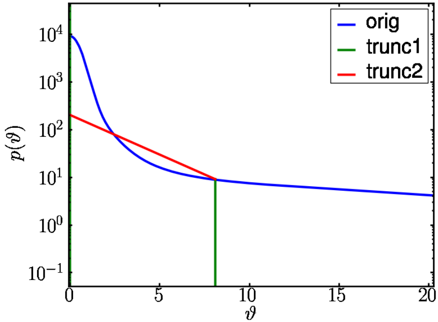


Very few photons contribute very much to the result $\Rightarrow$ slow convergence (up to 100 CPU hours to reach 1% precision)!

Bad Idea!

What most people would do now...

Truncation / δ-scaling



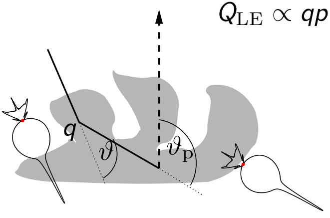
This alters physics! (bias)
But: Why use expensive Monte Carlo if it's not correct anyway???

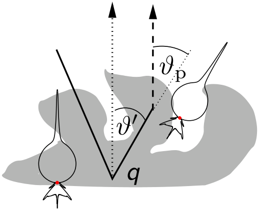
Variance Reduction I: Solving the forward scattering problem

Solution: Detector directional importance sampling (DDIS)

For half of the scattering photons the direction is turned toward the detector before “scattering” (i.e. applying the phase function). The bias introduced hereby is corrected with a photon weight “q”.

$$Q_{\mathrm{LE}} \propto q p(\vartheta_{\mathrm{p}}) \exp(-\tau)$$





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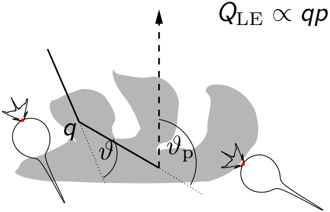

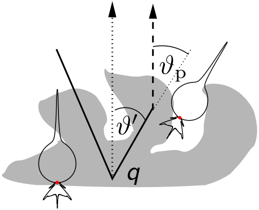

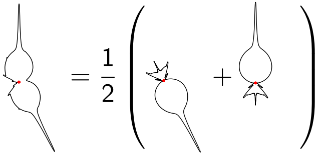

$$q = \frac{p(\vartheta)}{0.5[p(\vartheta) + p(\vartheta')]} \qquad (1)$$

Note: $p(\vartheta_{\mathrm{p}}) \simeq p(\vartheta')$

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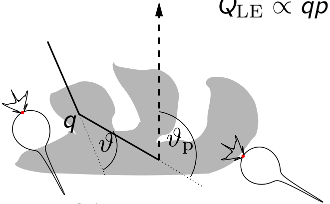
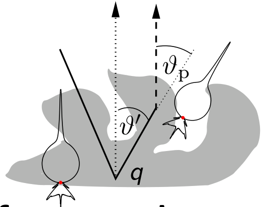
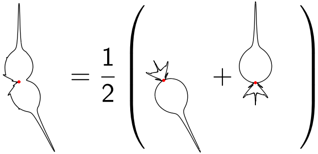




1% error after few seconds of computation.



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The advantage of DDIS

- Monte Carlo with “Local Estimate” und “Detector Directional Importance Sampling” leads **without any approximation** to tolerable computational times.
- For Lidar/Radar, this is the solution!
- For passive remote sensing (e.g. satellite imaging) spikes still occur...

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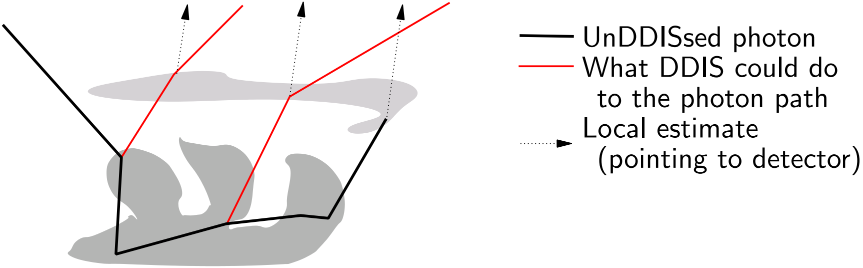
- For passive remote sensing (e.g. satellite imaging) spikes still occur. . .

. . . now comes the nasty part . . .

The problem with DDIS . . .

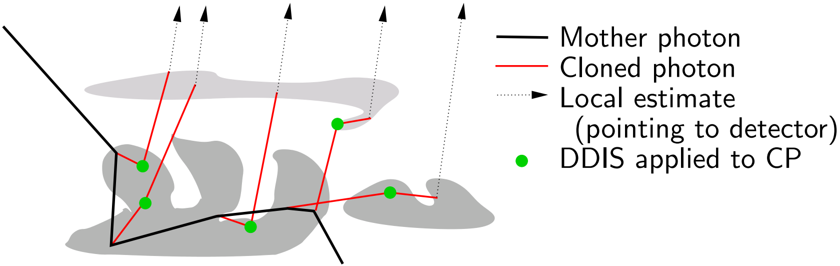
For large scatter numbers n, e.g. in thick clouds, DDIS creates a statistical problem:

the chance that a photon has never “DDIS”sed is $0.5^n \Rightarrow$ Bad statistics!



The solution: n-tupel local estimate (nLE)

The photons (“mother photons”, MP) are not DDISsed, no LE. At each scatter, a “clone photon” (CP) is created, which scatters n times, then performs a local estimate. The CP are DDISsed.

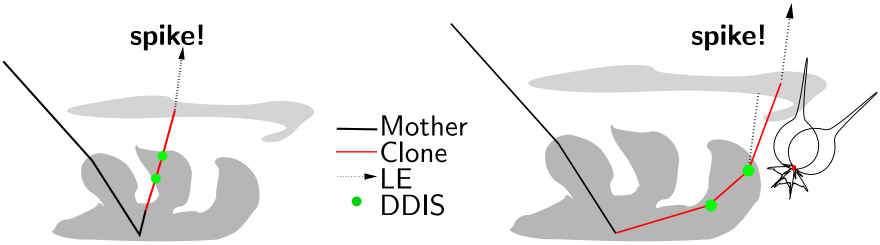


Next problem. . .

Photons can still manage to point towards the detector without having a reduced photon weight. Two examples:

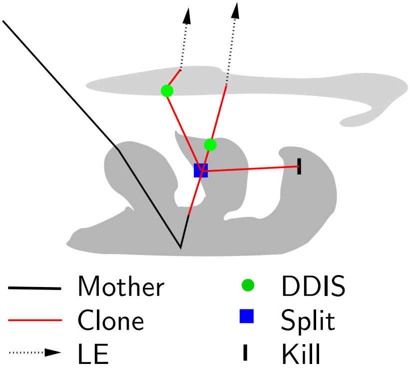
Clone photon starts pointing towards detector:

Clone photon sneaks around the corner:



. . . and its fix: Prediction-based splitting (PBS)

. . . and prediction-based russian roulette (PBRR)



Splitting:
Potentially dangerous photons are split into n_{split} photons.
Reduced weight $q{*}{=}n_{\text{split}}^{-1}$.
Russian roulette:
Photons with potentially very small contribution and weight $q \ll 1$ are killed with probability $1 - q_{\text{RR}}$.
If they survive, weight enhanced by $q{*}{=}q_{\text{RR}}$.

Yet another problem. . .

When looking into the sun, photons will split a lot!

⇒ Suppress splitting in case large contributions are to be expected!

. . . and another one. . .

Looking into the sun through thin Cirrus ⇒ Few photons scatter and contribute a lot to the signal!

⇒ "Virtual Importance Sampling" is applied:

Optical depth enhanced by a factor f. On scatter photon performs a LE with reduced weight f^{-1}. With probability $1 - f^{-1}$ the photon moves on without scattering.

⇒ Photon path is not distorted, rate of LE in Cirrus is enhanced.

The complete solution . . .

- DDIS (detector directional importance sampling)
- n-tupel local estimate
- prediction-based splitting, with circum-solar suppression
- prediction-based russian roulette
- circum-solar splitting-suppressed thin cloud virtual importance sampling
- extinction-dependent RR of local estimator photon

$\Rightarrow$ **No problems left! No spikes!! And still no bias!!!**

Speedup by several orders of magnitude.

The Name:

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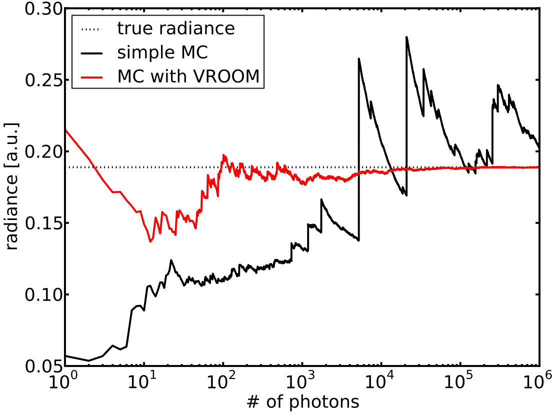
The Name:

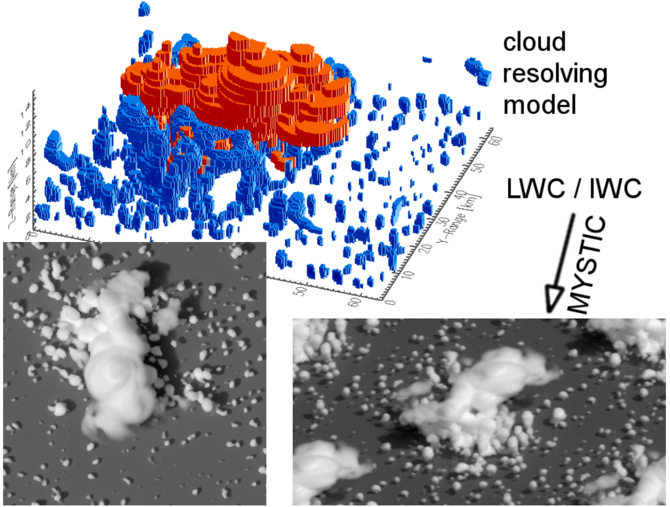
Variance Reduction "Optimal Options" Method (VROOM)

Spikes: No spikes with VROOM



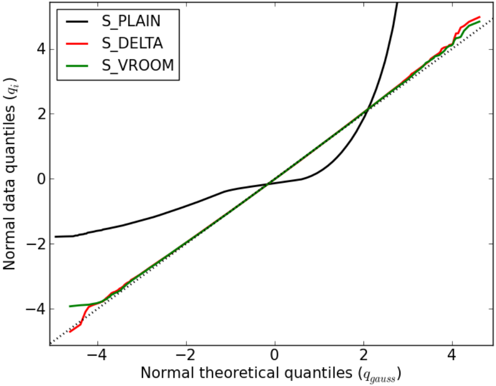
. . . Result converges more than 1000 times faster.
Compared to biasing methods, only factor 5 slower.

Monte Carlo VROOM simulation



Are there still spikes?

q-q-plot: how close to gaussian/normal distribution?
256.000 simulations of the same measurement:



No spikes!

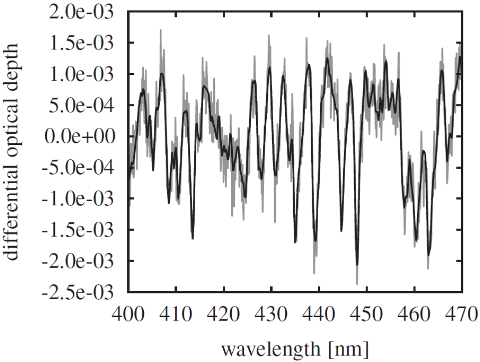
Variance Reduction II: Spectral calculations with Importance Sampling

ALIS – Absorption Lines Importance Sampling

Wanted: Fast calculation of differential optical absorption radiances (DOAS), i.e. for trace gas retrieval

Standard Monte Carlo:

- Each wavelength requires one simulation
- Monte Carlo: Very expensive computation! 10^7 photons times number of wavelengths $\Rightarrow$ 1 CPU day
- Still significant Monte Carlo uncertainty



C.Emde & RB

ALIS – Absorption Lines Importance Sampling

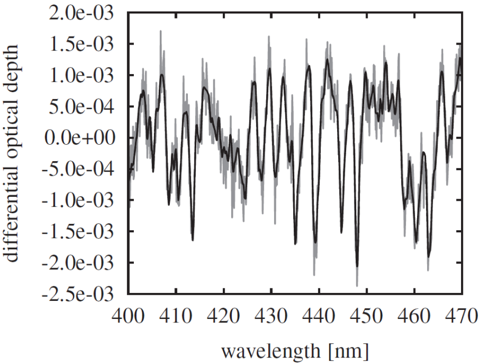
Wanted: Fast calculation of differential optical absorption radiances (DOAS), i.e. for trace gas retrieval

Solution:

- Calculate all wavelength using the same photon paths

⇒ The results for all wavelengths are correlated

⇒ Small **differential** uncertainty (using only 1000 photons)



C.Emde & RB

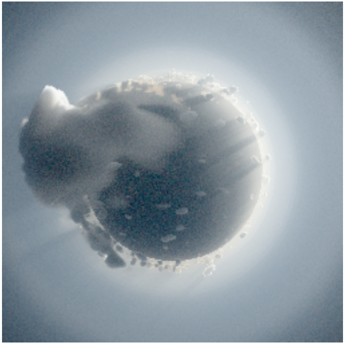
MC+ALIS is even faster than analytic solutions!

Summary I

Results

- Presented two new variance reduction techniques (VRT):
  - DDIS (Detector Directional Importance Sampling)
  - n-tupel Local Estimate (= Cloning)
- These techniques, plus well-known methods (splitting, Russian roulette, importance sampling, etc.) solve Spike problem for cloudy atmospheres.
- With VROOM and ALIS, 3D Monte Carlo RT in the atmosphere has become feasible.
- Computational times reduced by several orders of magnitude.
- For some applications, Monte Carlo is even the fastest solution!

Thank you!



- **VROOM**: RB & B. Mayer 2011: Efficient unbiased variance reduction techniques for Monte Carlo simulations of radiative transfer in cloudy atmospheres: The solution; JQSRT 112 (3) 434
- C. Emde, RB & B. Mayer 2011: **ALIS**: An efficient method to compute high spectral resolution polarized solar radiances using the Monte Carlo approach; JQSRT 112 (10) 1622