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Universality of TMD distribution functions of definite rank

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Content

- Introducing TMD correlators

- Moment analysis
  - Single weighting
  - Double weighting

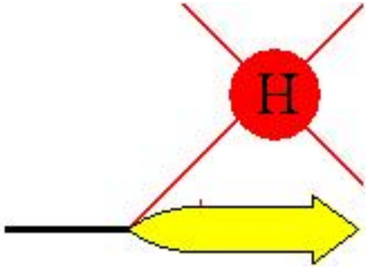
- Universality of TMD correlators and PDFs

- TMDs in experiments (weighted cross sections vs convolutions)

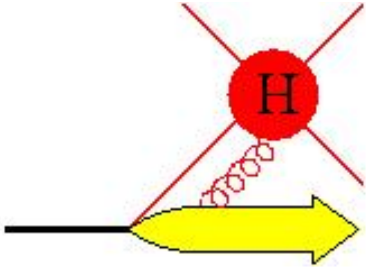
Hadron correlators

- Hadronic correlators establish the diagrammatic link between hadrons and partonic hard scattering amplitude
- Quark, quark + gluon, gluon, …

$$\left\langle 0 \middle| \psi_i\,(\xi) \middle| p,s \right\rangle = u_i(p,s)\, e^{-ip.\xi}$$

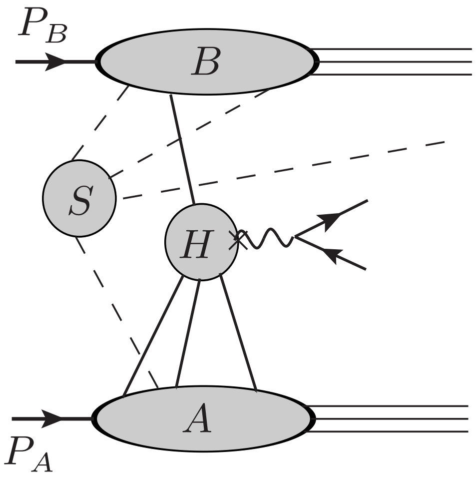


$$\left\langle X \middle| \psi_i\,(\xi) \middle| P \right\rangle e^{+ip.\xi}$$



$$\left\langle X \middle| \psi_i\,(\xi) A^\mu(\eta) \middle| P \right\rangle e^{+i(p-p_1).\xi+ip_1.\eta}$$

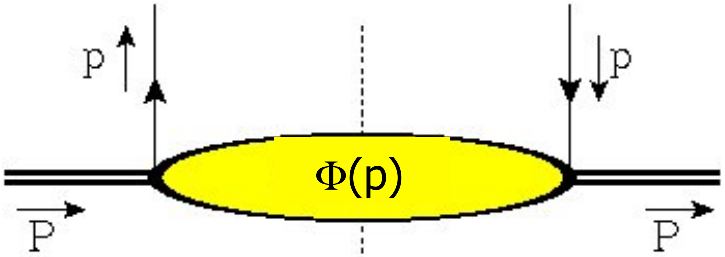
- Disentangling a hard process into parts involving hadrons, hard scattering amplitude and soft part is non-trivial



J.C. Collins, Foundations of Perturbative QCD, Cambridge Univ. Press 2011

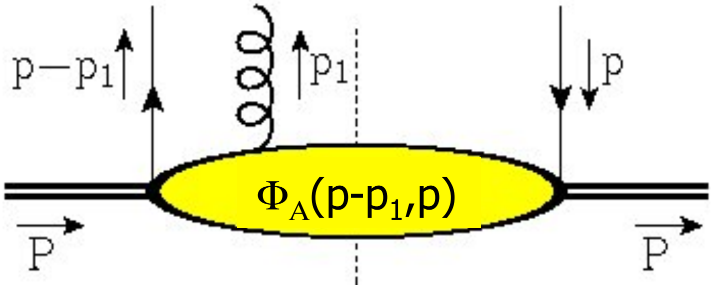
Hadron correlators

- At high energies soft parts combine amplitudes into forward matrix elements of parton fields to account for distributions and fragmentation



$$\Phi_{ij}(p;P) = \Phi_{ij}(p\,|\,p) = \int \frac{d^4\xi}{(2\pi)^4}\, e^{i\,p.\xi} \left\langle P \middle| \bar{\psi}_j(0)\,\psi_i(\xi) \middle| P \right\rangle$$

- Also needed are multi-parton correlators



$$\Phi^{\alpha}_{A;ij}(p-p_1,p_1\,|\,p) = \int \frac{d^4\xi\, d^4\eta}{(2\pi)^8}\, e^{i(p-p_1).\xi+ip_1.\eta} \left\langle P \middle| \bar{\psi}_j(0)\,A^{\alpha}(\eta)\,\psi_i(\xi) \middle| P \right\rangle$$

- Correlators usually just will be parametrized (nonperturbative physics)

Hard scale

- In high-energy processes other momenta available, such that P.P′ ~ s with a hard scale s = $Q^2 >> M^2$
- Additional scale accessible through non-collinearities, e.g. in SIDIS γ^*+p is not aligned with produced hadron, or momenta inside a jet
- Employ light-like vectors P and n, such that P.n = 1 (e.g. n = P′/P.P′) to make a Sudakov expansion of parton momentum

$$p = \underset{\overset{\nearrow}{\sim Q}}{xP^\mu} + \underset{\overset{\uparrow}{\sim M}}{p_T^\mu} + \underset{\overset{\nwarrow}{\sim M^2/Q}}{\sigma n^\mu}$$

$$x = p^+ = p.n \sim 1$$

$$\sigma = p.P - xM^2 \sim M^2$$

- Enables importance sampling (twist analysis) for integrated correlators,

$$\Phi(p) = \Phi(x, p_T, p.P) \;\Rightarrow\; \Phi(x, p_T) \;\Rightarrow\; \Phi(x) \;\Rightarrow\; \Phi$$

(Un)integrated correlators

$$\Phi(x,p_T,p.P)=\int\frac{d^4\xi}{(2\pi)^4}\,e^{i\,p.\xi}\left\langle P\middle|\overline{\psi}(0)\,\psi(\xi)\middle|P\right\rangle$$

- unintegrated

$$\Phi(x,p_T;n)=\int\frac{d(\xi.P)d^2\xi_T}{(2\pi)^3}\,e^{i\,p.\xi}\left\langle P\middle|\overline{\psi}(0)\,\psi(\xi)\middle|P\right\rangle\Big/_{\boxed{\xi.n=0}}$$

- TMD (light-front)

- Time-ordering automatic, allowing interpretation as forward anti-parton – target scattering amplitude
- Involves operators of twists starting at a lowest value (which is usually called the 'twist' of a TMD)

$$\Phi(x)=\int\frac{d(\xi.P)}{(2\pi)}\,e^{i\,p.\xi}\left\langle P\middle|\overline{\psi}(0)\,\psi(\xi)\middle|P\right\rangle\Big/_{\boxed{\xi.n=\xi_T=0}\ or\ \boxed{\xi^2=0}}$$

- collinear (light-cone)

- Involves operators of a definite twist. Evolution via splitting functions (moments are anomalous dimensions)

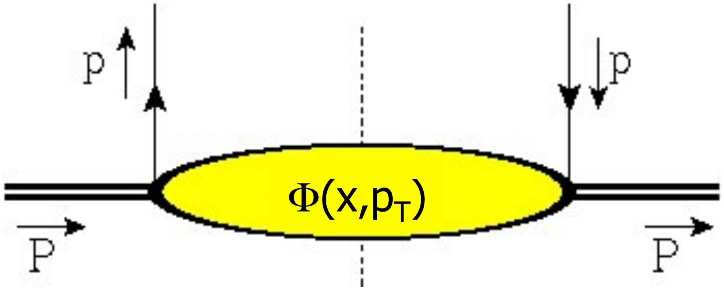
$$\Phi=\left\langle P\middle|\overline{\psi}(0)\,\psi(\xi)\middle|P\right\rangle\Big/_{\boxed{\xi=0}}$$

- local

- Local operators with calculable anomalous dimension

Large p_T

- p_T-dependence of TMDs



$$\int^{\mu} d^2 p_T \, \Phi(x,p_T) = \Phi(x;\mu^2)$$

Fictitious measurement

Large μ^2 dependence governed by anomalous dim (i.e. splitting functions)

- $$\Phi(x,p_T) \xrightarrow[p_T^2 > \mu^2]{} \frac{1}{\pi p_T^2} \frac{\alpha_s(p_T^2)}{2\pi} \int_x^1 \frac{dy}{y} P\left(\frac{x}{y}\right) \Phi(y;p_T^2)$$

- Consistent matching to collinear situation: CSS formalism

JC Collins, DE Soper and GF Sterman, NP B 250 (1985) 199

Twist analysis

- Dimensional analysis to determine importance in an expansion in inverse hard scale
- Maximize contractions with n

$$\dim[\overline{\psi}(0)\not{n}\,\psi(\xi)] = 2$$

$$\dim[F^{n\alpha}(0)F^{n\beta}(\xi)] = 2$$

$$\dim[\overline{\psi}(0)\not{n}\,A_T^\alpha(\eta)\,\psi(\xi)] = 3$$

- … or maximize # of P's in parametrization of Φ

$$\boxed{\Phi^q(x) = f_1^q(x)\frac{\not{P}}{2} \Leftrightarrow f_1^q(x) = \int\frac{d\lambda}{(2\pi)}e^{ix\lambda}\left\langle P\middle|\overline{\psi}(0)\not{n}\,\psi(\lambda n)\middle|P\right\rangle}$$

- In addition any number of collinear n.A(ξ) = $A^n(x)$ fields (dimension zero!), but of course in color gauge invariant combinations

dim 0: $i\partial^n \longrightarrow iD^n = i\partial^n + gA^n$

dim 1: $i\partial_T^\alpha \longrightarrow iD_T^\alpha = i\partial_T^\alpha + gA_T^\alpha$

Color gauge invariance

- Gauge invariance in a nonlocal situation requires a gauge link U(0,ξ)

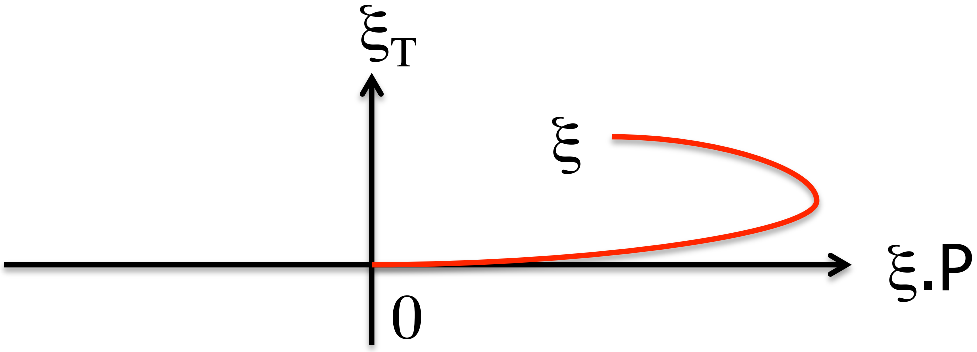
$$\bar{\psi}(0)\psi(\xi) = \sum_n \frac{1}{n!}\xi^{\mu_1}\ldots\xi^{\mu_N}\bar{\psi}(0)\,\partial_{\mu_1}\ldots\partial_{\mu_N}\psi(0)$$

$$U(0,\xi) = \mathcal{P}\ \exp\left(-ig\int_0^\xi ds^\mu A_\mu\right)$$

$$\bar{\psi}(0)U(0,\xi)\psi(\xi) = \sum_n \frac{1}{n!}\xi^{\mu_1}\ldots\xi^{\mu_N}\bar{\psi}(0)\,D_{\mu_1}\ldots D_{\mu_N}\psi(0)$$

- Introduces path dependence for $\Phi(x,p_T)$

$$\Phi^{[U]}(x,p_T) \;\Rightarrow\; \Phi(x)$$



Which gauge links?

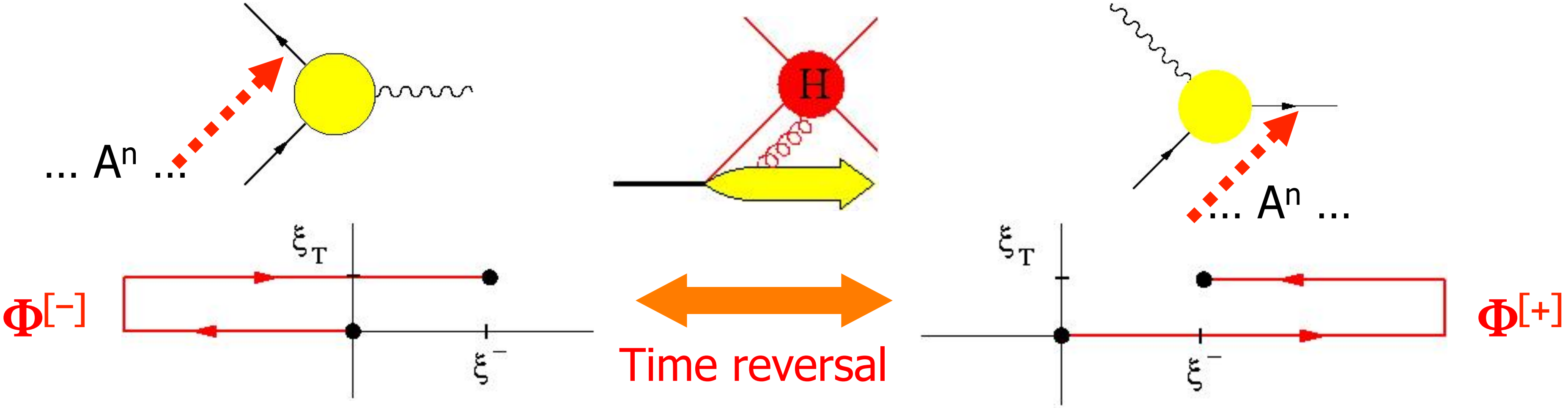
$$\Phi_{ij}^{q[C]}(x,p_T;n) = \int \frac{d(\xi.P)\,d^2\xi_T}{(2\pi)^3}\, e^{i\,p.\xi} \left\langle P\middle|\overline{\psi}_j(0)\,U_{[0,\xi]}^{[C]}\,\psi_i(\xi)\middle|P\right\rangle_{\xi.n=0}$$

TMD

$$\Phi_{ij}^{q}(x;n) = \int \frac{d(\xi.P)}{(2\pi)}\, e^{i\,p.\xi} \left\langle P\middle|\overline{\psi}_j(0)\,U_{[0,\xi]}^{[n]}\,\psi_i(\xi)\middle|P\right\rangle_{\xi.n=\xi_T=0}$$

collinear

- Gauge links for TMD correlators process-dependent with simplest cases



AV Belitsky, X Ji and F Yuan, NP B 656 (2003) 165
D Boer, PJ Mulders and F Pijlman, NP B 667 (2003) 201

Which gauge links?

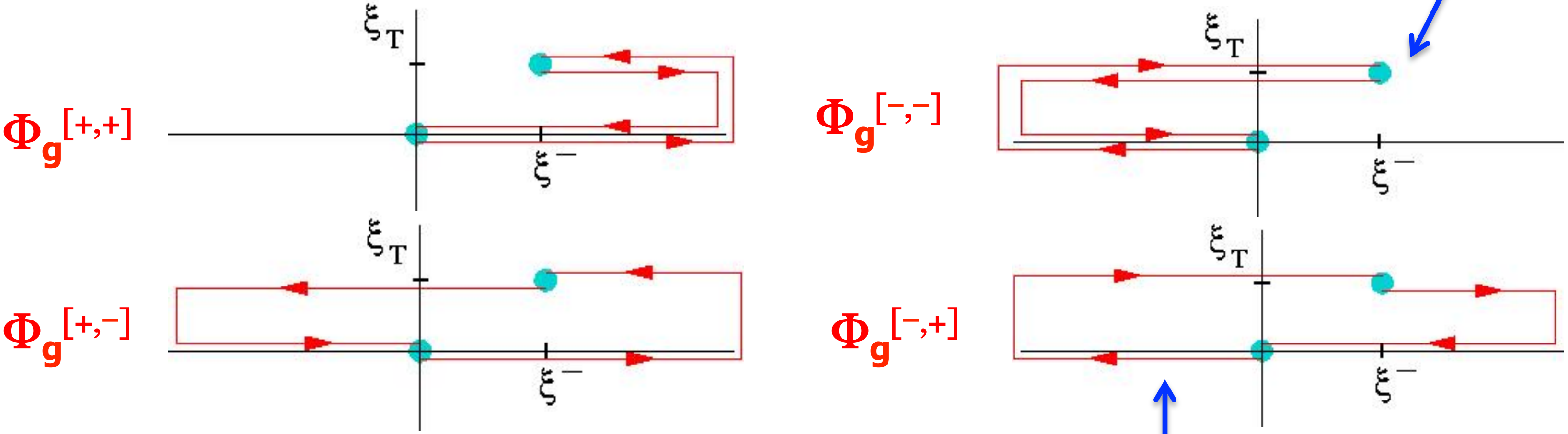
$$\Phi_g^{\alpha\beta[C,C']}(x,p_T;n) = \int \frac{d(\xi.P)d^2\xi_T}{(2\pi)^3}\, e^{i\,p.\xi}\left\langle P\middle|U^{[C]}_{[\xi,0]}F^{n\alpha}(0)U^{[C']}_{[0,\xi]}F^{n\beta}(\xi)\middle|P\right\rangle_{\xi.n=0}$$

- The TMD gluon correlators contain two links, which can have different paths. Note that standard field displacement involves C = C’

$$F^{\alpha\beta}(\xi) \to U^{[C]}_{[\eta,\xi]}F^{\alpha\beta}(\xi)U^{[C]}_{[\xi,\eta]}$$

- Basic (simplest) gauge links for gluon TMD correlators:

gg → H



in gg → QQbar

C Bomhof, PJM, F Pijlman; EPJ C 47 (2006) 147
F Dominguez, B-W Xiao, F Yuan, PRL 106 (2011) 022301

Color gauge invariant correlators

- Matrix elements including multiple possiblities for gauge links
- Quarks:

$$\Phi^{[U]}(x,p_T;n) = \Big\{ f_1^{[U]}(x,p_T^2) - f_{1T}^{\perp[U]}(x,p_T^2)\,\frac{\epsilon_T^{p_T S_T}}{M} + g_{1s}^{[U]}(x,p_T)\gamma_5$$

$$+ h_{1T}^{[U]}(x,p_T^2)\,\gamma_5\,\not{S}_T + h_{1s}^{\perp[U]}(x,p_T)\,\frac{\gamma_5\,\not{p}_T}{M} + i h_1^{\perp[U]}(x,p_T^2)\,\frac{\not{p}_T}{M}\Big\}\,\frac{\not{P}}{2},$$

- Gluons:

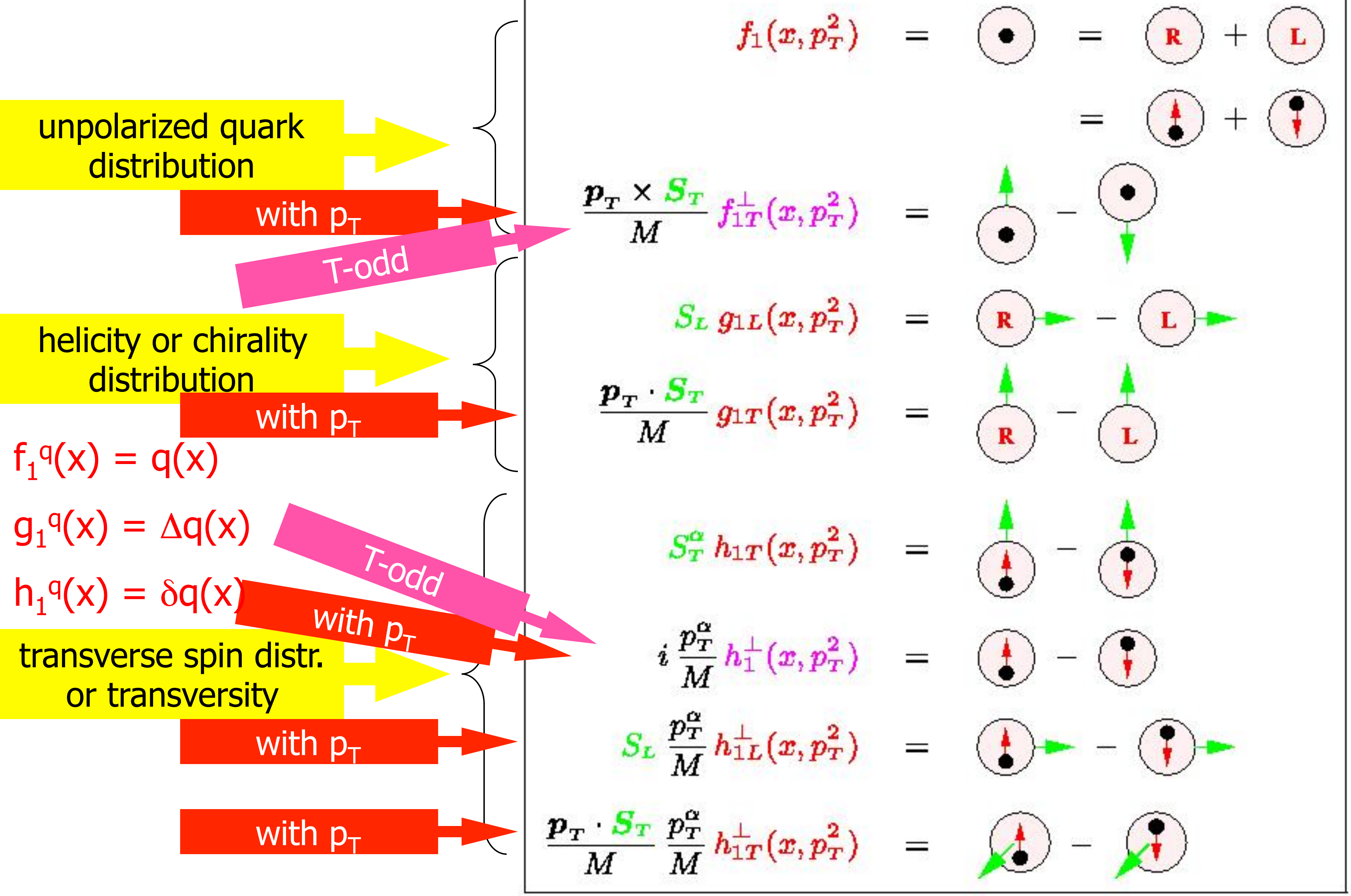
$$2x\,\Gamma^{\mu\nu[U]}(x,p_T) = -g_T^{\mu\nu}\,f_1^{g[U]}(x,p_T^2) + g_T^{\mu\nu}\,\frac{\epsilon_T^{p_T S_T}}{M}\,f_{1T}^{\perp g[U]}(x,p_T^2)$$

$$+ i\epsilon_T^{\mu\nu}\,g_{1s}^{g[U]}(x,p_T) + \left(\frac{p_T^\mu p_T^\nu}{M^2} - g_T^{\mu\nu}\,\frac{p_T^2}{2M^2}\right) h_1^{\perp g[U]}(x,p_T^2)$$

$$- \frac{\epsilon_T^{p_T\{\mu}\,p_T^{\nu\}}}{2M^2}\,h_{1s}^{\perp g[U]}(x,p_T) - \frac{\epsilon_T^{p_T\{\mu}\,S_T^{\nu\}} + \epsilon_T^{S_T\{\mu}\,p_T^{\nu\}}}{4M}\,h_{1T}^{g[U]}(x,p_T^2).$$

- Note [U] dependence

Fermionic structure of TMDs



unpolarized quark distribution

with p_T

T-odd

helicity or chirality distribution

with p_T

$f_1^q(x) = q(x)$

$g_1^q(x) = \Delta q(x)$

$h_1^q(x) = \delta q(x)$

T-odd

with p_T

transverse spin distr. or transversity

with p_T

with p_T

$$f_1(x,p_T^2) =$$

$$\frac{\boldsymbol{p}_T \times \boldsymbol{S}_T}{M} f_{1T}^{\perp}(x,p_T^2) =$$

$$S_L\, g_{1L}(x,p_T^2) =$$

$$\frac{\boldsymbol{p}_T \cdot \boldsymbol{S}_T}{M} g_{1T}(x,p_T^2) =$$

$$S_T^{\alpha}\, h_{1T}(x,p_T^2) =$$

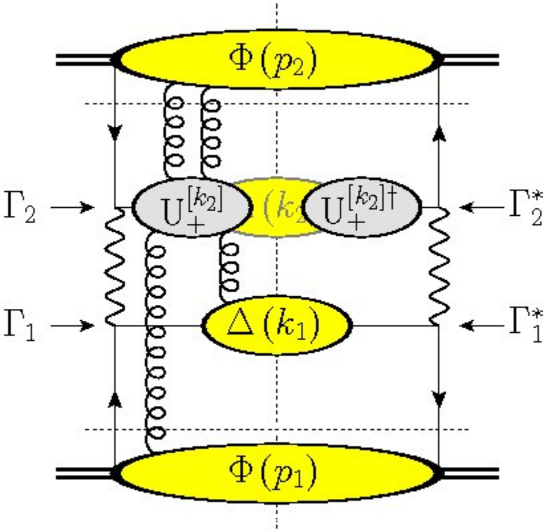
$$i\,\frac{p_T^{\alpha}}{M} h_1^{\perp}(x,p_T^2) =$$

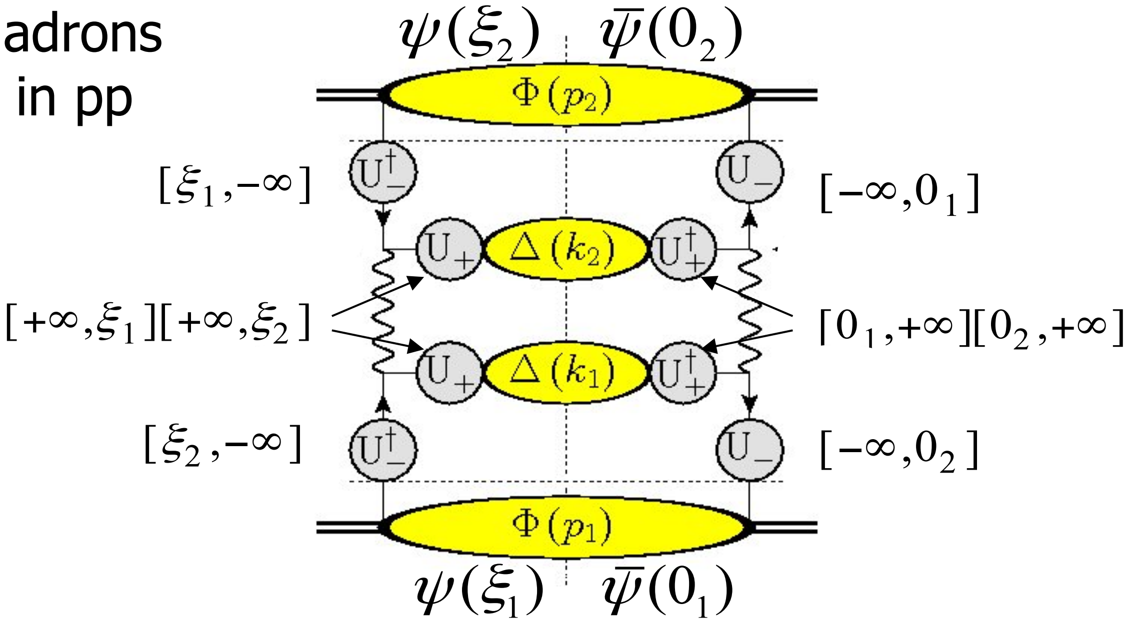
$$S_L \frac{p_T^{\alpha}}{M} h_{1L}^{\perp}(x,p_T^2) =$$

$$\frac{\boldsymbol{p}_T \cdot \boldsymbol{S}_T}{M}\frac{p_T^{\alpha}}{M} h_{1T}^{\perp}(x,p_T^2) =$$

Which gauge links?

- With more (initial state) hadrons color gets entangled, e.g. in pp





- Outgoing color contributes future pointing gauge link to $\Phi(p_2)$ and future pointing part of a loop in the gauge link for $\Phi(p_1)$

- Can be color-detangled if only p_T of one correlator is relevant (using polarization, …) but include Wilson loops in final U

T.C. Rogers, PJM, PR D81 (2010) 094006

MGAB, PJM, JHEP 07 (2011) 065

Operator structure in collinear case (reminder)

- Collinear functions and x-moments

$$\Phi^q(x) = \int \frac{d(\xi.P)}{(2\pi)} e^{i\,p.\xi} \left\langle P \middle| \overline{\psi}(0) U^{[n]}_{[0,\xi]} \psi(\xi) \middle| P \right\rangle_{\xi.n=\xi_T=0}$$

$$x^{N-1}\,\Phi^q(x) = \int \frac{d(\xi.P)}{(2\pi)} e^{i\,p.\xi} \left\langle P \middle| \overline{\psi}(0) (\partial^n)^{N-1} U^{[n]}_{[0,\xi]} \psi(\xi) \middle| P \right\rangle_{\xi.n=\xi_T=0}$$

$$= \int \frac{d(\xi.P)}{(2\pi)} e^{i\,p.\xi} \left\langle P \middle| \overline{\psi}(0) U^{[n]}_{[0,\xi]} (D^n)^{N-1} \psi(\xi) \middle| P \right\rangle_{\xi.n=\xi_T=0}$$

- Moments correspond to local matrix elements with calculable anomalous dimensions, that can be Mellin transformed to splitting functions

$$\Phi^{(N)} = \left\langle P \middle| \overline{\psi}(0) (D^n)^{N-1} \psi(0) \middle| P \right\rangle$$

- All operators have same twist since dim(D^n) = 0

Operator structure in TMD case

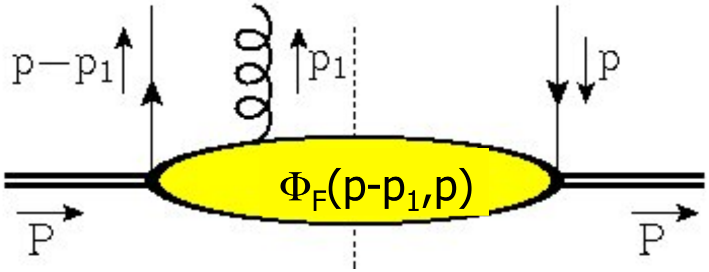
- For TMD functions one can consider transverse moments

$$\Phi(x,p_T;n)=\int\frac{d(\xi.P)d^2\xi_T}{(2\pi)^3}e^{i\,p.\xi}\left\langle P\middle|\bar{\psi}(0)\;U^{[\pm]}\;\psi(\xi)\middle|P\right\rangle_{\xi.n=0}$$

$$p_T^\alpha\Phi^{[\pm]}(x,p_T;n)=\int\frac{d(\xi.P)d^2\xi_T}{(2\pi)^3}e^{i\,p.\xi}\left\langle P\middle|\bar{\psi}(0)UD_T^\alpha(\pm\infty)U\psi(\xi)\middle|P\right\rangle_{\xi.n=0}$$

- Transverse moments involve collinear twist-3 multi-parton correlators Φ_D and Φ_F built from non-local combination of three parton fields

$$\Phi_F^\alpha(x-x_1,x_1\,|\,x)=\int\frac{d\xi.P\,d\eta.P}{(2\pi)^2}e^{i(p-p_1).\xi+ip_1.\eta}\left\langle P\middle|\bar{\psi}(0)F^{n\alpha}(\eta)\,\psi(\xi)\middle|P\right\rangle_{\xi.n=\xi_T=0}$$



$$\Phi_D^\alpha(x)=\int dx_1\,\Phi_D^\alpha(x-x_1,x_1\,|\,x)$$

$$\Phi_A^\alpha(x)\underset{\uparrow}{=}PV\int dx_1\,\frac{1}{x_1}\Phi_F^{n\alpha}(x-x_1,x_1\,|\,x)$$

T-invariant definition

Operator structure in TMD case

- For TMD functions one can consider transverse moments

$$\Phi(x,p_T;n) = \int \frac{d(\xi.P)\,d^2\xi_T}{(2\pi)^3}\, e^{i\,p.\xi} \left\langle P\middle|\bar{\psi}(0)\; U^{[\pm]}\; \psi(\xi)\middle|P\right\rangle_{\xi.n=0}$$

$$p_T^\alpha \Phi^{[\pm]}(x,p_T;n) = \int \frac{d(\xi.P)\,d^2\xi_T}{(2\pi)^3}\, e^{i\,p.\xi} \left\langle P\middle|\bar{\psi}(0)\,U D_T^\alpha(\pm\infty) U\,\psi(\xi)\middle|P\right\rangle_{\xi.n=0}$$

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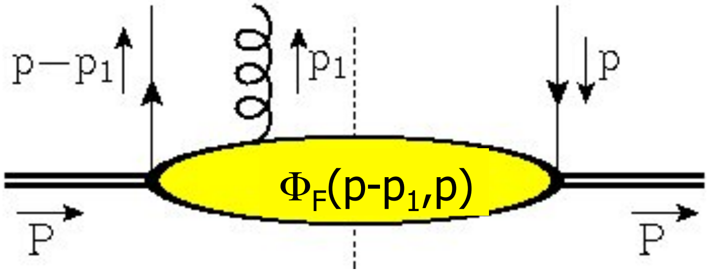


$$\Phi_D^\alpha(x) = \int dx_1\, \Phi_D^\alpha(x-x_1,x_1\,|\,x)$$

$$\Phi_A^\alpha(x) \underset{\uparrow}{=} PV\int dx_1\, \frac{1}{x_1}\, \Phi_F^{n\alpha}(x-x_1,x_1\,|\,x)$$

T-invariant definition

Operator structure in TMD case

- Transverse moments can be expressed in these particular collinear multi-parton twist-3 correlators
- $\Phi_{\partial}^{\alpha[U]}(x) = \int d^2p_T\; p_T^{\alpha}\Phi^{[U]}(x,p_T;n) = \tilde{\Phi}_{\partial}^{\alpha}(x) + C_G^{[U]}\pi\Phi_G^{\alpha}(x)$

T-even

T-odd (gluonic pole or ETQS m.e.)

$$\boxed{\tilde{\Phi}_{\partial}^{\alpha}(x) = \Phi_{D}^{\alpha}(x) - \Phi_{A}^{\alpha}(x)} \qquad \boxed{\Phi_{G}^{\alpha}(x) = \Phi_{F}^{n\alpha}(x,0\,|\,x)}$$

- This gives rise to process dependence in PDFs, for unpolarized case

$$\frac{1}{M}\Phi_{\partial}^{\alpha[U]}(x) = \;\ldots\, h_1^{\perp(1)[U]}(x) = \;\ldots\, C_G^{[U]}\, h_1^{\perp(1)}(x)$$

  - Weightings defined as

$$h_1^{\perp(n)}(x) = \int d^2p_T\left(-\frac{p_T^2}{2M^2}\right)^n h_1^{\perp}(x,p_T^2)$$

Operator structure in TMD case

- Transverse moments can be expressed in these particular collinear multi-parton twist-3 correlators
- $$\Phi_\partial^{\alpha[U]}(x) = \int d^2p_T\; p_T^\alpha \Phi^{[U]}(x,p_T;n) = \tilde{\Phi}_\partial^\alpha(x) + C_G^{[U]}\pi\Phi_G^\alpha(x)$$

T-even

T-odd (gluonic pole or ETQS m.e.)

$$\boxed{\tilde{\Phi}_\partial^\alpha(x) = \Phi_D^\alpha(x) - \Phi_A^\alpha(x)}$$

$$\boxed{\Phi_G^\alpha(x) = \Phi_F^{n\alpha}(x,0\,|\,x)}$$

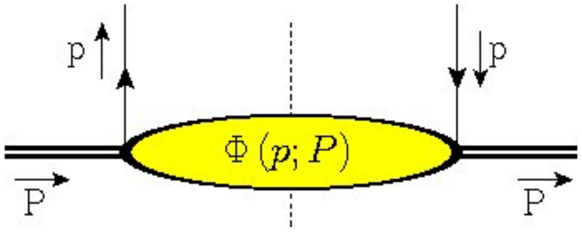
- For a polarized nucleon:

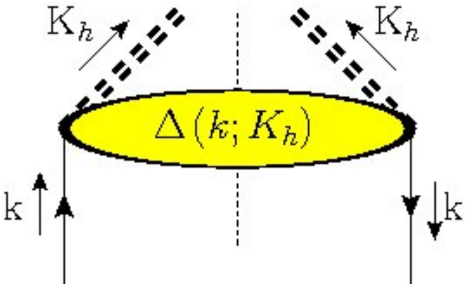
$$\frac{1}{M}\Phi_\partial^{\alpha[U]}(x) = \Big(\ldots g_{1T}^{\perp(1)}(x) + \ldots h_{1L}^{\perp(1)}(x)\Big) + \ldots C_G^{[U]}\, f_{1T}^{\perp(1)}(x)$$

T-even

T-odd

Distributions versus fragmentation





- Operators:

$$\Phi^{[\pm]}(p\,|\,p) \sim \left\langle P\,|\,\overline{\psi}(0)U_{\pm}\psi(\xi)\,|\,P\right\rangle$$

$$\Phi_{\partial}^{\alpha}(x) = \tilde{\Phi}_{\partial}^{\alpha}(x) \pm \pi\,\Phi_{G}^{\alpha}(x)$$

T-even

T-odd (gluonic pole)

$$\boxed{\Phi_{G}^{\alpha}(x) = \Phi_{F}^{n\alpha}(x,0\,|\,x) \neq 0}$$

- Operators:

out state

$$\Delta(k\,|\,k)$$

$$\sim \sum_{X}\left\langle 0\,|\,\psi(\xi)\,|\,K_h X\right\rangle\left\langle K_h X\,|\,\overline{\psi}(0)\,|\,0\right\rangle$$

$$\boxed{\Delta_{G}^{\alpha}(x) = \Delta_{F}^{n\alpha}(\tfrac{1}{Z},0\,|\,\tfrac{1}{Z}) = 0}$$

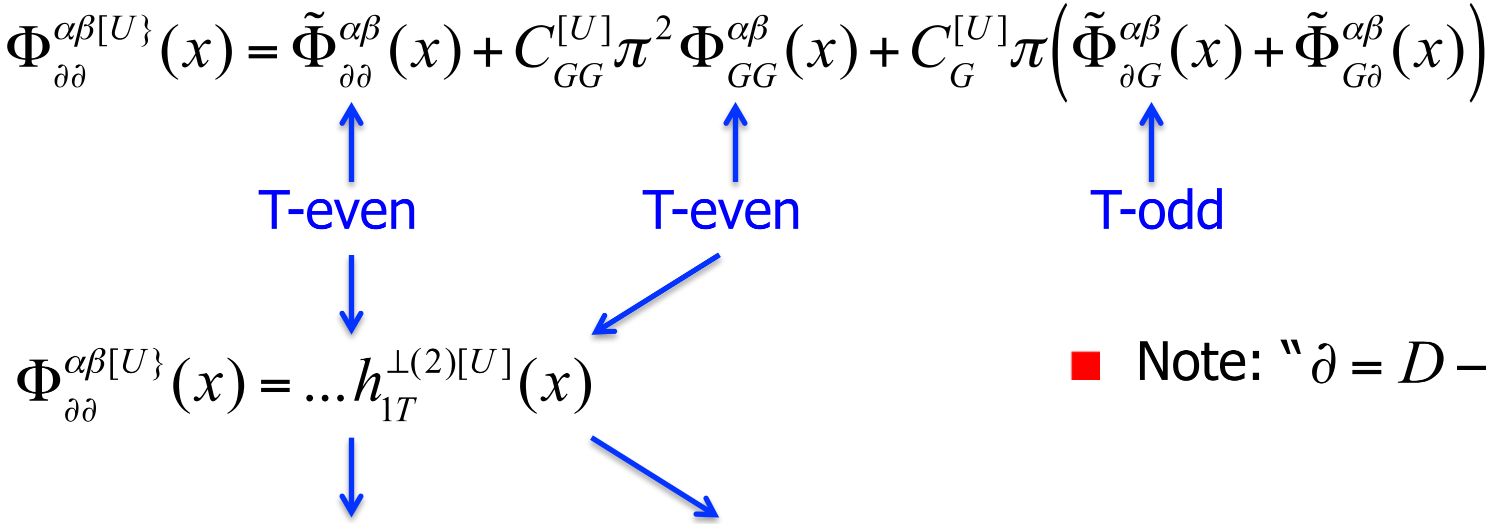
$$\Delta_{\partial}^{\alpha[U]}(x) = \tilde{\Delta}_{\partial}^{\alpha}(x)$$

T-even operator combination, but no T-constraints!

Collins, Metz; Meissner, Metz; Gamberg, M, Mukherjee, PR D 83 (2011) 071503

Double transverse weighting

- The double transverse weighted distribution function contains multiple 4-parton matrix elements



$$\Phi_{\partial\partial}^{\alpha\beta[U\}}(x) = \tilde{\Phi}_{\partial\partial}^{\alpha\beta}(x) + C_{GG}^{[U]}\pi^2\,\Phi_{GG}^{\alpha\beta}(x) + C_{G}^{[U]}\pi\left(\tilde{\Phi}_{\partial G}^{\alpha\beta}(x) + \tilde{\Phi}_{G\partial}^{\alpha\beta}(x)\right)$$

T-even

T-even

T-odd

$$\Phi_{\partial\partial}^{\alpha\beta[U\}}(x) = \ldots h_{1T}^{\perp(2)[U]}(x)$$

$$\boxed{h_{1T}^{\perp(2)[U]}(x) = h_{1T}^{\perp(2)(A)}(x) + C_{GG}^{[U]}h_{1T}^{\perp(2)(B1)}(x)}$$

- Note: "$\partial = D - A$"

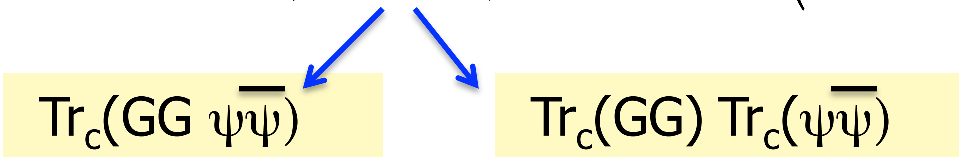
- Separation in T-even and T-odd parts is no longer enough to isolate process dependent parts ⟶ also Pretzelocity function is non-universal
- …. although $C_{GG}^{[+]} = C_{GG}^{[-]} = 1$ (so not different in DY and SIDIS)

MGA Buffing, A Mukherjee, PJM, PRD2012 , Arxiv: 1207.3221 [hep-ph]

Double transverse weighting

- Pretzelocity type of correlations come actually in three matrix elements and have to be parametrized using three functions

$$\Phi_{\partial\partial}^{\alpha\beta[U]}(x) = \tilde{\Phi}_{\partial\partial}^{\alpha\beta}(x) + C_{GG,c}^{[U]}\pi^2\Phi_{GG,c}^{\alpha\beta}(x) + C_G^{[U]}\pi\left(\tilde{\Phi}_{\partial G}^{\alpha\beta}(x) + \tilde{\Phi}_{G\partial}^{\alpha\beta}(x)\right)$$



$\mathrm{Tr}_c(GG\,\psi\overline{\psi})$

$\mathrm{Tr}_c(GG)\,\mathrm{Tr}_c(\psi\overline{\psi})$

$$\boxed{h_{1T}^{\perp(2)[U]}(x) = h_{1T}^{\perp(2)(A)}(x) + C_{GG,1}^{[U]}h_{1T}^{\perp(2)(B1)}(x) + C_{GG,2}^{[U]}h_{1T}^{\perp(2)(B2)}(x)}$$

U	$U^{[\pm]}$	$U^{[+]}\,U^{[\square]}$	$\frac{1}{N_c}\,\mathrm{Tr}_c(U^{[\square]})\,U^{[+]}$
$\Phi^{[U]}$	$\Phi^{[\pm]}$	$\Phi^{[+\square]}$	$\Phi^{[(\square)+]}$
$C_G^{[U]}$	±1	3	1
$C_{GG,1}^{[U]}$	1	9	1
$C_{GG,2}^{[U]}$	0	0	4

MGA Buffing, A Mukherjee, PJM, PRD2012 , Arxiv: 1207.3221 [hep-ph]

The next step: TMDs of definite rank

- Expansion into TMDs of definite rank

$$\begin{aligned}\Phi^{[U]}(x,p_T) = {} & \tilde{\Phi}(x,p_T^2) + C_G^{[U]}\pi p_{Ti}\tilde{\Phi}_G^i(x,p_T^2) + C_{GG,c}^{[U]}\pi^2 p_{Tij}\tilde{\Phi}_{GG,c}^{ij}(x,p_T^2) + \ldots \\ & + p_{Ti}\tilde{\Phi}_\partial^i(x,p_T^2) + C_G^{[U]}\pi p_{Tij}\tilde{\Phi}_{\{\partial G\}}^{ij}(x,p_T^2) + \ldots \\ & + p_{Tij}\tilde{\Phi}_{\partial\partial}^{ij}(x,p_T^2) + \ldots \\ & + \ldots\end{aligned}$$

- Depending on spin and type of operators, only a finite number needed
- Example 1: quarks in an unpolarized target

$$\tilde{\Phi}(x,p_T^2) = \Big(f_1(x,p_T^2)\Big)\frac{\not{P}}{2} \qquad \pi\tilde{\Phi}_G^\alpha(x,p_T^2) = \left(i h_1^\perp(x,p_T^2)\frac{\gamma_T^\alpha}{M}\right)\frac{\not{P}}{2}$$

Examples

- Rank expansion of TMDs

$\tilde{\Phi}(x,p_T^2)$	$C_G^{[U]}\pi p_{Ti}\tilde{\Phi}_G^i(x,p_T^2)$	$C_{GG,c}^{[U]}\pi^2 p_{Tij}\tilde{\Phi}_{GG,c}^{ij}(x,p_T^2)$	$\cdots$
$p_{Ti}\tilde{\Phi}_\partial^i(x,p_T^2)$	$C_G^{[U]}\pi p_{Tij}\tilde{\Phi}_{\{\partial G\}}^{ij}(x,p_T^2)$	$\cdots$	
$p_{Tij}\tilde{\Phi}_{\partial\partial}^{ij}(x,p_T^2)$	$\cdots$		
$\cdots$			

- Example 1: quarks in an unpolarized target

$f_1(x,p_T^2)$	$h_1^\perp(x,p_T^2)$	$-$
$-$	$-$	$-$
$-$	$-$	$-$

Examples

- General identification for quarks in a nucleon (spin ½)

$$\Phi(x,p_T^2) = \Big\{ f_1(x,p_T^2) + S_L\, g_1(x,p_T^2)\gamma_5 + h_1(x,p_T^2)\gamma_5\, \not S_T \Big\} \frac{\not P}{2},$$

$$\frac{p_{Ti}}{M}\,\widetilde{\Phi}^i_\partial(x,p_T^2) = \Big\{ h_{1L}^\perp(x,p_T^2)\, S_L\, \frac{\gamma_5\, \not p_T}{M} - g_{1T}(x,p_T^2)\, \frac{p_T\cdot S_T}{M}\gamma_5 \Big\} \frac{\not P}{2},$$

$$\frac{p_{Ti}}{M}\,\Phi^i_G(x,p_T^2) = \Big\{ - f_{1T}^\perp(x,p_T^2)\, \frac{\epsilon_T^{\rho\sigma}\, p_{T\rho} S_{T\sigma}}{M} + i h_1^\perp(x,p_T^2)\, \frac{\not p_T}{M} \Big\} \frac{\not P}{2},$$

$$\frac{p_{Tij}}{M^2}\,\widetilde{\Phi}^{ij}_{\partial\partial}(x,p_T^2) = h_{1T}^{\perp(A)}(x,p_T^2)\, \frac{p_{Tij} S_T^i\, \gamma_5 \gamma_T^j}{M^2}\, \frac{\not P}{2},$$

$$\frac{p_{Tij}}{M^2}\,\Phi^{ij}_{GG,1}(x,p_T^2) = \frac{1}{\pi^2} h_{1T}^{\perp(B1)}(x,p_T^2)\, \frac{p_{Tij} S_T^i\, \gamma_5 \gamma_T^j}{M^2}\, \frac{\not P}{2},$$

$\mathrm{Tr}_c(GG\psi\overline{\psi})$

$$\frac{p_{Tij}}{M^2}\,\Phi^{ij}_{GG,2}(x,p_T^2) = \frac{1}{\pi^2} h_{1T}^{\perp(B2)}(x,p_T^2)\, \frac{p_{Tij} S_T^i\, \gamma_5 \gamma_T^j}{M^2}\, \frac{\not P}{2},$$

$\mathrm{Tr}_c(GG)\mathrm{Tr}_c(\psi\overline{\psi})$

$$\frac{p_{Tij}}{M^2}\,\widetilde{\Phi}^{ij}_{\{\partial G\}}(x,p_T^2) = 0.$$

Examples

- Rank expansion of TMDs

$\tilde{\Phi}(x,p_T^2)$	$C_G^{[U]}\pi p_{Ti}\tilde{\Phi}_G^i(x,p_T^2)$	$C_{GG,c}^{[U]}\pi^2 p_{Tij}\tilde{\Phi}_{GG,c}^{ij}(x,p_T^2)$	$\cdots$
$p_{Ti}\tilde{\Phi}_\partial^i(x,p_T^2)$	$C_G^{[U]}\pi p_{Tij}\tilde{\Phi}_{\{\partial G\}}^{ij}(x,p_T^2)$	$\cdots$	
$p_{Tij}\tilde{\Phi}_{\partial\partial}^{ij}(x,p_T^2)$	$\cdots$		
$\cdots$			

- Example 2: TMD PDFs for a longitudinally (L) polarized spin ½ target

g_1	–	–
$h_{1L}^\perp$	–	–
–	–	–

Examples

- Expansion

$\tilde{\Phi}(x, p_T^2)$	$C_G^{[U]}\pi p_{Ti}\tilde{\Phi}_G^i(x, p_T^2)$	$C_{GG,c}^{[U]}\pi^2 p_{Tij}\tilde{\Phi}_{GG,c}^{ij}(x, p_T^2)$	$\cdots$
$p_{Ti}\tilde{\Phi}_\partial^i(x, p_T^2)$	$C_G^{[U]}\pi p_{Tij}\tilde{\Phi}_{\{\partial G\}}^{ij}(x, p_T^2)$	$\cdots$	
$p_{Tij}\tilde{\Phi}_{\partial\partial}^{ij}(x, p_T^2)$	$\cdots$		
$\cdots$			

- Example 3: TMD PDFs for a transversely (T) polarized spin ½ target

h_1	$f_{1T}^{\perp}$	$h_{1T}^{\perp(B1)},\ h_{1T}^{\perp(B2)}$
g_{1T}	–	–
$h_{1T}^{\perp(A)}$	–	–

Examples

- Rank expansion

$\tilde{\Phi}(x, p_T^2)$	$C_G^{[U]}\pi p_{Ti}\tilde{\Phi}_G^i(x, p_T^2)$	$C_{GG,c}^{[U]}\pi^2 p_{Tij}\tilde{\Phi}_{GG,c}^{ij}(x, p_T^2)$	$\cdots$
$p_{Ti}\tilde{\Phi}_\partial^i(x, p_T^2)$	$C_G^{[U]}\pi p_{Tij}\tilde{\Phi}_{\{\partial G\}}^{ij}(x, p_T^2)$	$\cdots$	
$p_{Tij}\tilde{\Phi}_{\partial\partial}^{ij}(x, p_T^2)$	$\cdots$		
$\cdots$			

- Example 4: TMD PFFs for spin ½ fragment

D_1, G_1, H_1		
$H_1^\perp, H_{1L}^\perp, G_{1T}, D_{1T}^\perp$		
$H_{1T}^\perp$		

Summarizing quark TMDs up to spin 1 targets

GLUONIC POLE RANK			
0	1	2	3
$\Phi(x, p_T^2)$	$\pi C_G^{[U]}\,\Phi_G$	$\pi^2 C_{GG,c}^{[U]}\,\Phi_{GG,c}$	$\pi^3 C_{GGG,c}^{[U]}\,\Phi_{GGG,c}$
$\widetilde{\Phi}_\partial$	$\pi C_G^{[U]}\,\widetilde{\Phi}_{\{\partial G\}}$	$\pi^2 C_{GG,c}^{[U]}\,\widetilde{\Phi}_{\{\partial GG\},c}$	$\ldots$
$\widetilde{\Phi}_{\partial\partial}$	$\pi C_G^{[U]}\,\widetilde{\Phi}_{\{\partial\partial G\}}$	$\ldots$	$\ldots$
$\widetilde{\Phi}_{\partial\partial\partial}$	$\ldots$	$\ldots$	$\ldots$

PDFs FOR SPIN 0 HADRONS		
f_1	$h_1^\perp$	

PDFs FOR SPIN 1/2 HADRONS		
g_1, h_1	$f_{1T}^\perp$	$h_{1T}^{\perp(B1)}$, $h_{1T}^{\perp(B2)}$
g_{1T}, $h_{1L}^\perp$		
$h_{1T}^{\perp(A)}$		

PDFs FOR TENSOR POLARIZED SPIN 1 HADRONS			
f_{1LL}, ~~h_{1LT}~~	$h_{1LL}^\perp$, g_{1LT}, h_{1TT}	$f_{1TT}^{(Bc)}$	$h_{1TT}^{\perp(Bc)}$
f_{1LT}	$h_{1LT}^\perp$, g_{1TT}		
$f_{1TT}^{(A)}$	$h_{1TT}^{\perp(A)}$		

MGA Buffing, A Mukherjee, PJM, PRD2012 , Arxiv: 1207.3221 [hep-ph]

Time reversal constraints

- After all a ‘forbidden’ TMD because of time reversal symmetry: h_{1LT}

PDFs FOR TENSOR POLARIZED SPIN 1 HADRONS			
$f_{1LL}, \cancel{h_{1LT}}$	$h_{1LL}^{\perp}, g_{1LT}, h_{1TT}$	$f_{1TT}^{(Bc)}$	$h_{1TT}^{\perp(Bc)}$
f_{1LT}	$h_{1LT}^{\perp}, g_{1TT}$		
$f_{1TT}^{(A)}$	$h_{1TT}^{\perp(A)}$		

- H_{1LT} is allowed, D_{1TT} is unique, …

PFFs FOR TENSOR POLARIZED SPIN 1 HADRONS			
D_{1LL}, H_{1LT}			
$D_{1LT}, H_{1LL}^{\perp}, G_{1LT}, H_{1TT}$			
$D_{1TT}, H_{1LT}^{\perp}, G_{1TT}$			
$H_{1TT}^{\perp}$			

MGA Buffing, A Mukherjee, PJM, PRD2012 , Arxiv: 1207.3221 [hep-ph]

Gluon TMDs in a nucleon

- For gluons one can have rank 0 – 3

GLUONIC POLE RANK			
0	1	2	3
$\Gamma(x,p_T^2)$	$\Gamma_{G,c}^{[U]}$	$\Gamma_{GG,c}^{[U]}$	$\Gamma_{GGG,c}^{[U]}$
$\widetilde{\Gamma}_{\partial}$	$\widetilde{\Gamma}_{\{\partial G\},c}^{[U]}$	$\widetilde{\Gamma}_{\{\partial GG\},c}^{[U]}$	$\ldots$
$\widetilde{\Gamma}_{\partial\partial}$	$\widetilde{\Gamma}_{\{\partial\partial G\},c}^{[U]}$	$\ldots$	$\ldots$
$\widetilde{\Gamma}_{\partial\partial\partial}$	$\ldots$	$\ldots$	$\ldots$

- Color structure GG,c includes a.o. $\mathrm{Tr}_c([G,F][G,F])$ and $\mathrm{Tr}_c(\{G,F\}\{G,F\})$
- PDFs:

PDFs FOR GLUONS			
$f_1^g,\ g_{1L}^g$	$f_{1T}^{\perp g(Ac)},\ h_{1T}^{g(Ac)}$	$h_1^{\perp g(Bc)}$	$h_{1T}^{\perp g(Bc)}$
g_{1T}^g	$h_{1L}^{\perp g(Ac)}$		
$h_1^{\perp g(A)}$	$h_{1T}^{\perp g(Ac)}$		

MGA Buffing, A Mukherjee, PJM, PRD2012 , in preparation

Bessel transforms

- The universal TMDs of definite rank are natural objects that can be studied in impact parameter space

$$\frac{p_{Ti_1\ldots i_m}}{M^m}\,\widetilde{\Phi}^{i_1\ldots i_m}_{\ldots}(x,p_T^2) \qquad \text{or} \qquad \widetilde{\Phi}^{(m/2)}_{\ldots}(x,p_T^2)\,e^{\pm im\varphi_p}$$

- Bessel transforms for rank m involve (m/2)-moments

$$\widetilde{f}^{(m/2)}_{\ldots}(x,|p_T|) = \int_0^\infty db\,\sqrt{|p_T|b}\,J_m(|p_T|b)\,f^{(m/2)}_{\ldots}(x,b)$$

MGA Buffing, A Mukherjee, PJM, PRD2012 , Arxiv: 1207.3221 [hep-ph]

Process dependent complications (preliminary)

- There are remaining process-dependent complications in convolutions

$$
\begin{aligned}
\sigma_{DY}(x_1,x_2,q_T) = & \tfrac{1}{N_c} Conv\left\{\Phi(x_1,p_{1T}^2)\Phi(x_2,p_{2T}^2)\hat{\sigma}\right\} \\
& + \tfrac{1}{N_c} Conv\left\{p_{1T\alpha}\tilde{\Phi}_{\partial}^{\alpha}(x_1,p_{1T}^2)\Phi(x_2,p_{2T}^2)\hat{\sigma}\right\} \\
& + \tfrac{1}{N_c} Conv\left\{C_G^{[-]}p_{1T\alpha}\Phi_G^{\alpha}(x_1,p_{1T}^2)\Phi(x_2,p_{2T}^2)\hat{\sigma}\right\} \\
& + \tfrac{1}{N_c} Conv\left\{\Phi(x_1,p_{1T}^2)p_{2T\alpha}\tilde{\Phi}_{\partial}^{\alpha}(x_2,p_{2T}^2)\hat{\sigma}\right\} \\
& + \tfrac{1}{N_c} Conv\left\{\Phi(x_1,p_{1T}^2)C_G^{[-]}p_{2T\alpha}\Phi_G^{\alpha}(x_2,p_{2T}^2)\hat{\sigma}\right\} \\
& + \tfrac{1}{N_c} Conv\left\{C_{GG}^{[-]}p_{1T\alpha\beta}\Phi_{GG}^{\alpha\beta}(x_1,p_{1T}^2)\Phi(x_2,p_{2T}^2)\hat{\sigma}\right\} \\
& + \tfrac{1}{N_c(N_c^2-1)} Conv\left\{C_G^{[-]}p_{1T\alpha}\Phi(x_1,p_{1T}^2)C_G^{[-]}p_{2T\alpha}\Phi_G^{\alpha}(x_2,p_{2T}^2)\hat{\sigma}\right\} \\
& + \ldots.
\end{aligned}
$$

- … but these complications are not worse than collinear twist-3 squared

MGA Buffing, PJM, JHEP 07 (2011) 065

Conclusions

- (Generalized) universality using definite rank functions
- Rank m is coupled to cos(mϕ) and sin(mϕ) azimuthal asymmetries
- Multiple distribution functions showing up in azimuthal asymmetries (depending on color structure of operators), e.g. three pretzelocities.
- In principle distinguishable in different experiments (with different color flow in tree-level diagrams)
- Factorization is the next step