Nucleon Structure from lattice QCD

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3rd Workshop on the QCD Structure of the Nucleon
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Frank Gehry's breakthrough on the "structure of buildings"

Introduction

QCD-Gauge theory of the strong interaction
Lagrangian: formulated in terms of quarks and gluons

$$\mathcal{L}_{QCD} = -\frac{1}{4}F^a_{\mu\nu}F^{a\,\mu\nu} + \sum_{f=u,d,s,c,b,t}\bar{\psi}_f\left(i\gamma^\mu D_\mu - m_f\right)\psi_f$$

$$D_\mu = \partial_\mu - ig\frac{\lambda^a}{2}A^a_\mu$$

This "simple" Lagrangian produces the amazingly rich structure of strongly interacting matter in our universe.

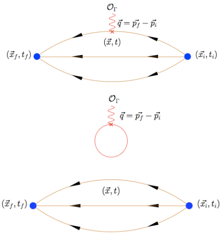
Numerical simulation of QCD provides essential input for a wide class of complex strong interaction phenomena
e.g. Hadron structure studied in experimental programs at JLab, Mainz, DESY

Focus of talk:

- Nucleon axial charge g_A and EM form factors
- Moments of parton distributions of quarks in the nucleon
- Nucleon TMDs (Hagler, Musch, Engelhardt, Negele, Schafer), scalar and tensor charge (Negele and collaboratotors)
- Nucleon σ-terms
- Hyperons and charmed baryons

Lattice QCD evaluation

Evaluation of two-point and three-point functions



$$G(\vec{q}, t) = \sum_{\vec{x}_f} e^{-i\vec{x}_f \cdot \vec{q}} \Gamma^4_{\beta\alpha} \langle J_\alpha(\vec{x}_f, t) \bar{J}_\beta(0) \rangle = \sum_{n=0,\cdots,\infty} A_n e^{-E_n(\vec{q})t}$$

$$\xrightarrow{t \to \infty} A_0 e^{-E_0(\vec{q})t}$$

$$G^{\mu\nu}(\Gamma, \vec{q}, t) = \sum_{\vec{x}_f, \vec{x}} e^{i\vec{x} \cdot \vec{q}} \Gamma_{\beta\alpha} \langle J_\alpha(\vec{x}_f, t_f) \mathcal{O}^{\mu\nu}(\vec{x}, t) \bar{J}_\beta(0) \rangle$$

- Noise to signal increases with time $\Longrightarrow$ Techniques to improve ground state dominance in correlators
- Large Euclidean time evolution gives ground state for given quantum numbers $\Longrightarrow$ enables determination of low-lying hadron properties
  Special techniques to extract excited states
- Connect lattice results to measurements: $\mathcal{O}_{\overline{\rm MS}}(\mu) = Z(\mu, a)\mathcal{O}_{\rm latt}(a)$
  Most collaborations evaluate $Z(\mu, a)$ non-perturbatively

$$aE_{\rm eff}(\vec{q}, t) = \ln\left[G(\vec{q}, t)/G(\vec{q}, t+a)\right]$$

$$= aE_0(\vec{q}) + \ln\left[\frac{1 + \sum_n B_n e^{-\Delta E_n t}}{1 + \sum_n B_n e^{-\Delta E_n (t+1)}}\right]$$

$$\to aE_0(\vec{q}) \overset{\vec{q}=0}{\to} am$$

$$R^{\mu\nu}(\Gamma, \vec{q}, t) = \frac{G^{\mu\nu}(\Gamma, \vec{q}, t)}{G(\vec{0}, t_f)} \sqrt{\frac{G(\vec{p}_i, t_f - t) G(\vec{0}, t) G(\vec{0}, t_f)}{G(\vec{0}, t_f - t) G(\vec{p}_i, t) G(\vec{p}_i, t_f)}}$$

$$\to \Pi^{\mu\nu}(\vec{q}, \Gamma)$$

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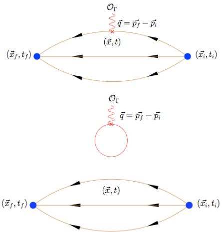

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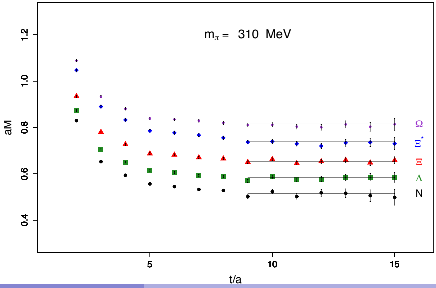

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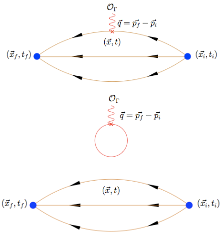

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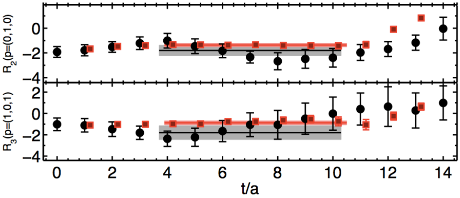

GPDs and generalized form factors

One-particle expectation values of light-cone correlation functions (GPDs⟶Talk by V. Guzey), X. Ji, J. Phys. G24 (1998) 1181

$$F_\Gamma(x,\xi,q^2)=\frac{1}{2}\int\frac{d\lambda}{2\pi}e^{ix\lambda}\langle p'|\bar\psi(-\lambda n/2)\Gamma\mathcal{P}e^{ig\int_{-\lambda/2}^{\lambda/2}d\alpha\, n\cdot A(n\alpha)}\psi(\lambda n/2)|p\rangle$$

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Expansion of the light cone operator leads to a tower of local twist-2 operators $\mathcal{O}^{\mu_1\cdots\mu_n}$ $\Longrightarrow$ expectation value of $\mathcal{O}^{\mu_1\cdots\mu_n}$ can be calculated

Decomposition of nucleon matrix elements into generalized form factors (GFFs):

$$\langle N(p',s')|\mathcal{O}_q^{\mu\mu_1\cdots\mu_n}|N(p,s)\rangle=\bar u_N(p',s')\Big[\sum_{i=0,2,\ldots}^{n}\Big(A_{n+1,i}(q^2)\gamma^{\{\mu}+B_{n+1,i}(q^2)\frac{i\sigma^{\{\mu\alpha}q_\alpha}{2m}\Big)q^{\mu_1}\ldots q^{\mu_i}\overline{P}^{\mu_{i+1}}\ldots\overline{P}^{\mu_n\}}$$
$$+\mathrm{mod}(n,2)C_{n+1,0}(q^2)\frac{1}{m}q^{\{\mu}q^{\mu_1}\ldots q^{\mu_n\}}\Big]u_N(p,s)$$

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Similarly for $\mathcal{O}_{\Delta q}^{\mu\mu_1\cdots\mu_n}$ (in terms of $\tilde A_{ni}(q^2)$, $\tilde B_{ni}(q^2)$) and $\mathcal{O}_{\delta q}^{\mu\mu_1\cdots\mu_n}$ (in terms of $A^T_{ni},\ B^T_{ni},\ C^T_{ni},\ D^T_{ni}$).

Special cases:

- Ordinary nucleon form factors

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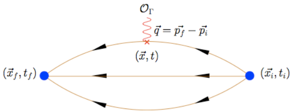
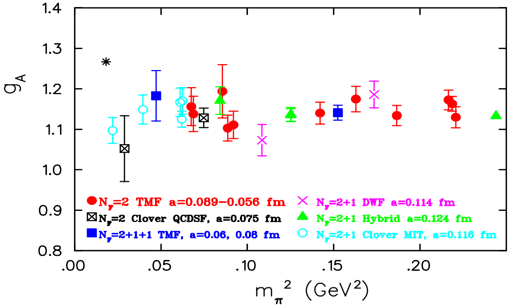
Nucleon Structure: axial charge

- Many lattice studies down to lowest pion mass of $m_\pi \sim 300$ MeV
  $\Longrightarrow$ Lattice data in general agreement
- Axial-vector FFs: $A^a_\mu = \bar\psi\gamma_\mu\gamma_5\frac{\tau^a}{2}\psi(x)$
  $\Longrightarrow \frac{1}{2}\left[\gamma_\mu\gamma_5 G_A(q^2) + \frac{q^\mu\gamma_5}{2m}G_p(q^2)\right]$



Axial charge is well known experimentally, straight forward to compute in lattice QCD



- Agreement among recent lattice results - all use $\mathcal{O}(a)$-improvement, non-perturbative Z_A
- Results using various lattice spacings and volumes - no visible cut-off nor volume effects, within current statistical errors
- Weak light quark mass dependence

- $N_F = 2+1$ Clover: J. R. Green *et al.*, Lattice2012
- $N_F = 2$ and $N_F = 2+1+1$ TMF: C. A. *et al.* (ETMC), PRD 83 (2011) 045010, and in preparation
- DWF: T. Yamazaki *et al.*, (RBC-UKQCD), PRD 79 (2009) 14505; S. Ohta, arXiv:1011.1388
- Hybrid: J. D. Bratt *et al.* (LHPC),PRD 82 (2010) 094502
- $N_F = 2$ Clover:D. Pleiter *et al.* (QCDSF), arXiv:1101.2326

Results on the Nucleon $\langle x \rangle_{u-d}$ and $\langle x \rangle_{\Delta u - \Delta d}$

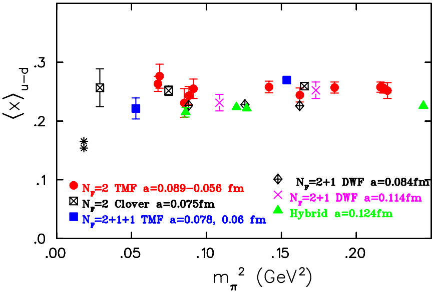
Moments of parton distributions:

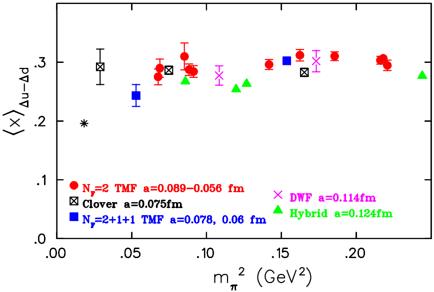
$$\langle x \rangle_q = \int_0^1 dxx\,[q(x) + \bar{q}(x)]\ , \qquad \langle x \rangle_{\Delta q} = \int_0^1 dxx\,[\Delta q(x) - \Delta\bar{q}(x)]$$

$q = q_\downarrow + q_\uparrow,\ \Delta q = q_\downarrow - q_\uparrow$

Extracted from nucleon matrix elements of $\mathcal{O}_q^{\mu_1\mu_2} = \bar\psi\gamma^{\{\mu_1} i \overleftrightarrow{D}^{\mu_2\}}\psi$ and $\mathcal{O}_{\Delta q}^{\mu_1\mu_2} = \bar\psi\gamma^{\{\mu_1}\gamma_5 i \overleftrightarrow{D}^{\mu_2\}}\psi$

Summary of $N_F = 2$, $N_F = 2+1$ and $N_F = 2+1+1$ results in the $\overline{MS}$ scheme at $\mu = 2$ GeV.





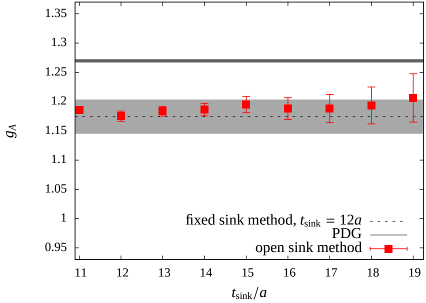
Physical point:

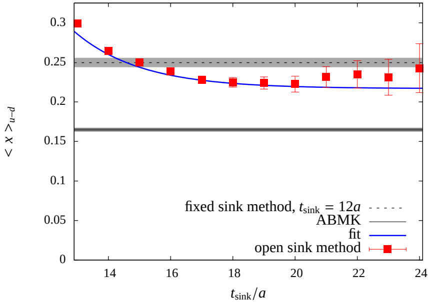
- $\langle x \rangle_{u-d}$ from S. Alekhin, J. Blümlein, S. Moch, arXiv:1202.2281; R. D. Ball arXiv:1107.2652
- $\langle x \rangle_{\Delta u - \Delta d}$ from J. Blümlein *et al.* arXiv:1005.3113

Study of excited state contributions

$N_F = 2 + 1 + 1$ with $m_\pi \sim 390$ MeV and $a = 0.08$ fm

Vary source- sink separation:





Statistics: $\sim$ 7,000

Physical value from a global analysis, S. Alekhin, J. Blumlein, S. Klein, and S. Moch. Phys.Rev., D81, 014032 (2010)

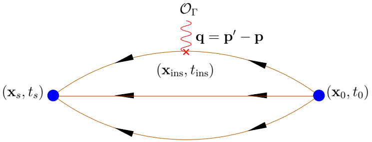
Statistics: 23,000

g_A unaffected, $\langle x \rangle_{u-d}$ 10% lower

$\Longrightarrow$ Excited contributions are operator dependent

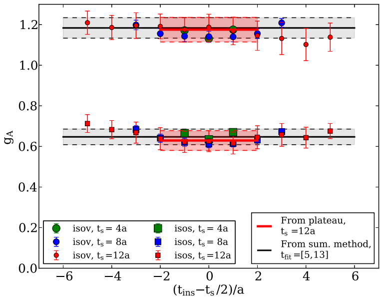
S. Dinter, C.A., M. Constantinou, V. Drach, K. Jansen and D. Renner, arXiv: 1108.1076

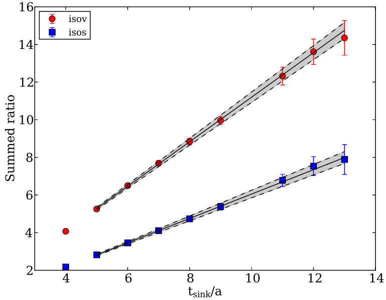
g_A with the summation method



Sum over time of the current insertion:
$\sum_{t_{\rm ins}} \longrightarrow a + g_A t_{\rm sink}$

Contamination due to excited states $\sim e^{-\Delta E t_s}$ instead of $\sim e^{-\Delta E t_{ins}}$. However need to extract the slope.
One twisted mass ensemble, a = 0.08 fm, m_π = 390 MeV,
iso-scalar (only connected) and iso-vector g_A



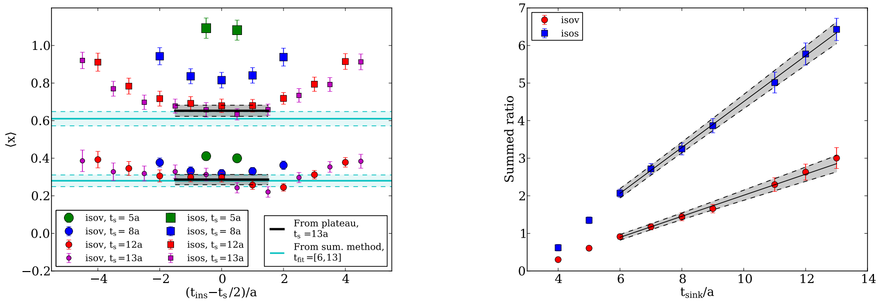


No detectable excited states contamination, agrees with high precision study S. Dinter *et al.*, arXiv:1108.1076 and C. Alexandrou *et al.*, arXiv:1112.2931

- Same plateau for multiple t_{sink}s
- No curvature in summed ratio, consistent results for various fit-ranges

$\langle \mathbf{x} \rangle_{u-d}$ with the summation method

One twisted mass ensemble, $a = 0.08$ fm, $m_\pi = 390$ MeV,
iso-scalar (only connected) and iso-vector $\langle x \rangle$



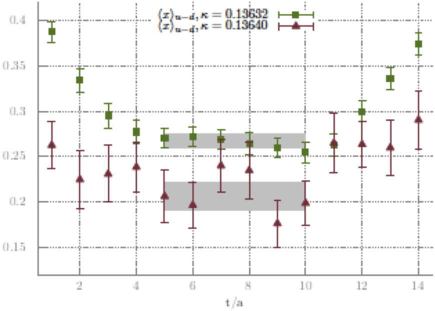
- Noticeable contamination, especially for the iso-scalar
- Summation method uses 7 sink-source time separations
- For the plateau method one needs to show convergence by varying the sink-source time separation $\rightarrow$ also requires a number of sequential inversions

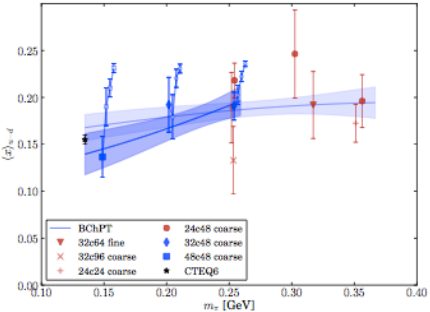
$\Longrightarrow$ Plateau and summation method give consistent results.

Results at almost physical pion mass

Very recent results claim correct value of $\langle x \rangle_{u-d}$:

- $N_F = 2$ Clover at $m_\pi = 157(6)$ MeV, $a = 0.07$ fm and $m_\pi L = 2.74$ using time separation ~ 1.07 fm, G. Bali *et al.* arXiv:1207.1110
- $N_F = 2+1$ BMW configurations at $m_\pi = 149$ MeV, $a = 0.116$ fm and $m_\pi L = 4.2$ using 3 time separations up to 1.4 fm in combination with summation method, J. R. Green *et al.* arXiv:1209.1687





But not g_A, J. R. Green *et al.* arXiv:1209.1687

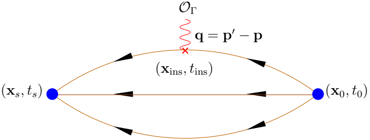
Nucleon spin

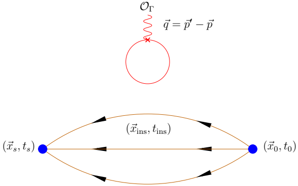
Contributions to the spin of the nucleon
Spin sum: $\frac{1}{2} = \frac{1}{2}\Delta\Sigma + L_q + J_G$

Non-relativistic quark model:

If $\Delta\Sigma_{u,d} = \Delta u + \Delta d = 1 \Rightarrow L_q = 0$ and $J_G = 0$, as well as $\Delta s = 0$, where Δq contains both the spin of q and $\bar{q}$.
Lattice QCD: Need both connected and disconnected contributions to evaluate contributions to spin

$\Delta u + \Delta d + \Delta s$ = 0.45 (4)(9) with $\Delta s = -0.020(10)(4)$ at $\mu = \sqrt{7.4}$ GeV, Bali *et al.* (QCDSF), Phys.Rev.Lett. 108 (2012) 222001
Also from mixed action approach: $\Delta s = -0.031(17)$ at $\mu = 2$ GeV , M. Engelhardt, arXiv:1210.0025





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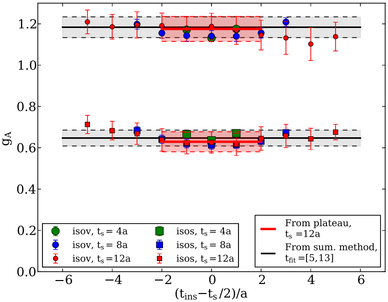
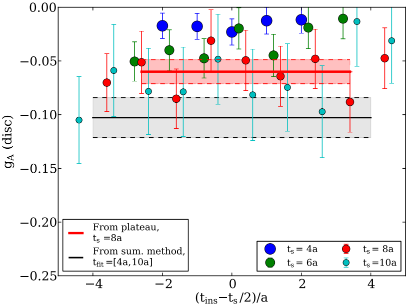


Connected



Disconnected

Lattice results on the nucleon spin

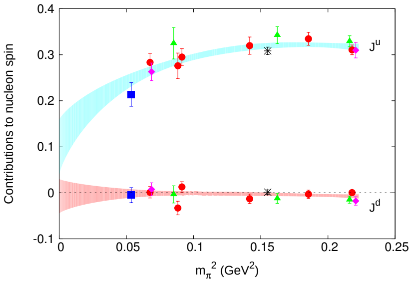
$J_q = \frac{1}{2}[A_{20}(0) + B_{20}(0)] = \frac{1}{2}\Delta\Sigma + L_q$
$\Delta\Sigma = \tilde{A}_{10}$

Only connected contribution

Results using $N_F = 2$ TMF for 270 MeV $< m_\pi < 500$ MeV, C. Alexandrou *et al.* (ETMC), arXiv:1104.1600 and $N_F = 2+1+1$ at $m_\pi \sim 230$ MeV and 390 MeV

In agreement with A. Sternbeck *et al.* (QCDSF) arXiv:1203.6579

In qualitative agreement with J. D. Bratt *et al.* (LHPC), PRD82 (2010) 094502



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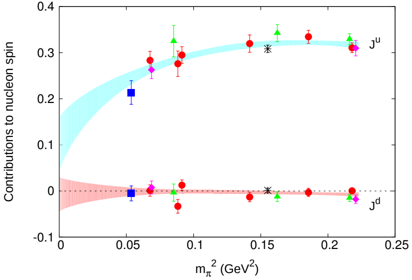

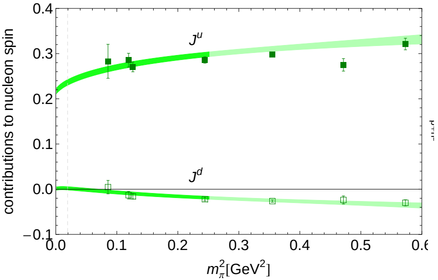

$\Longrightarrow$ Total spin for u-quarks $J^u \sim 0.25$ and for d-quark $J^d \sim 0$

Lattice results on the nucleon spin

$J_q = \frac{1}{2}[A_{20}(0) + B_{20}(0)] = \frac{1}{2}\Delta\Sigma + L_q$

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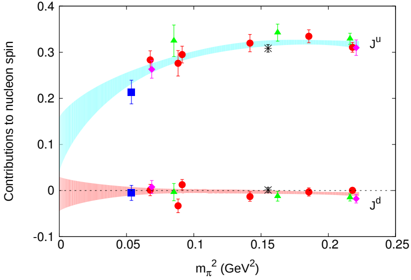
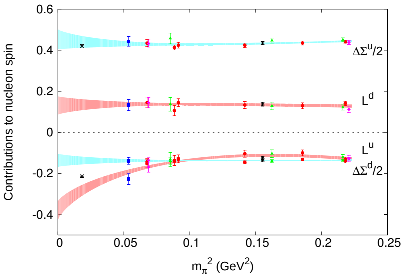
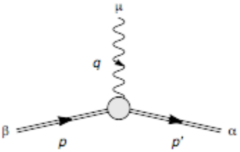




- Lattice results for $\Delta\Sigma^{u-d}$ and L^{u-d} in good agreement
- $L^{u+d} \sim 0$ at physical point.
- How about the disconnected contributions to L_q and contributions from J_g? K.-F. Liu *et al.* (χQCD), arXiv:1203.6388 claim they are large $\Longrightarrow$ Need to be confirmed using dynamical quarks, larger volumes and lighter quark masses $\Longrightarrow$ Please wait!!!!

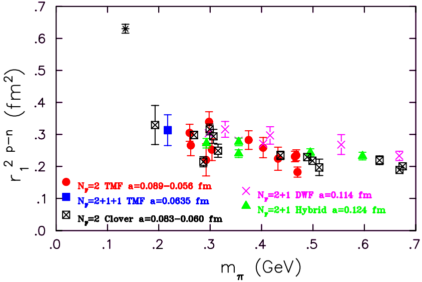
Nucleon Dirac and Pauli isovector radii

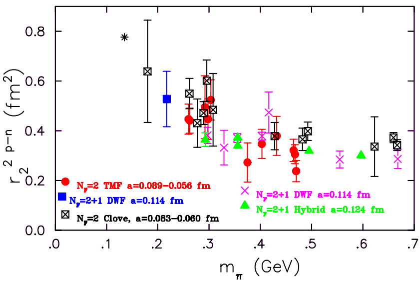


$$\langle N(p',s')|j^\mu(0)|N(p,s)\rangle = \bar{u}_N(p',s')\left[\gamma^\mu F_1(q^2) + \frac{i\sigma^{\mu\nu}q_\nu}{2m}F_2(q^2)\right]u_N(p,s)$$

Dirac and Pauli radii: $r^2_{1,2} = -\frac{6}{F_{1,2}(0)}\frac{dF_{1,2}}{dq^2}|_{q^2=0}$

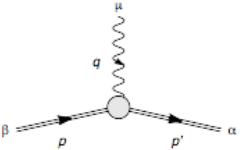
Use a dipole Ansatz to fit the q^2-dependence of F_1 and F_2.





- TMF: C. A. *et al.* (ETMC), PRD83 (2011) 094502
- Clover: S. Collins *et al.* (QCDSF), Phys.Rev. D84 (2011) 074507
- DWF: S. N. Syritsyn *et al.* (LHPC), PRD 81, 034507 (2010); T. Yamazaki *et al.* (RBC-UKQCD), PRD 79, 114505 (2009)
- Hybrid:J. D. Bratt *et al.* (LHPC), Phys. Rev. D82, 094502 (2010)

Nucleon Dirac and Pauli isovector radii



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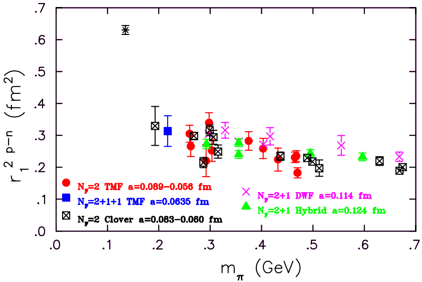

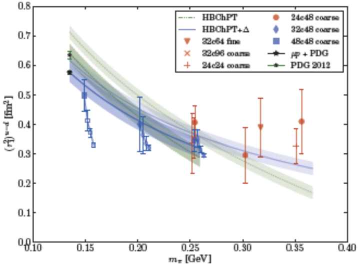

J. R. Green *et al.* 1209.1687

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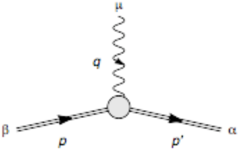

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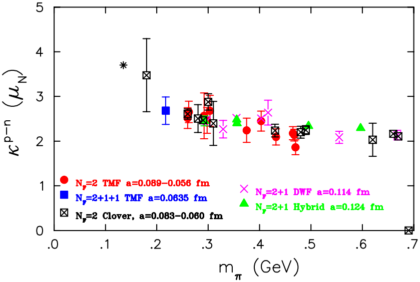
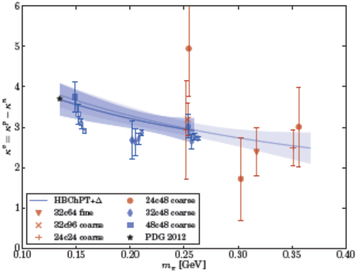
Anomalous magnetic moment: $F_2(0)\,\frac{m_N^{\rm phys}}{m_N^{\rm lat}}$





J. R. Green *et al.* 1209.1687

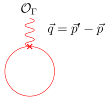
Nucleon σ-terms

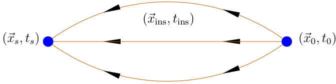
- $\sigma_l \equiv m_l \langle N|\bar{u}u + \bar{d}d|N\rangle$: measures the explicit breaking of chiral symmetry
  Extracted from analysis of low-energy pion-proton scattering data
- In lattice QCD it can be obtained via the Feynman-Hellman theorem: $\sigma_l = m_l \frac{\partial m_N}{\partial m_l}$
- Similarly $\sigma_s \equiv m_s \langle N|\bar{s}s|N\rangle >= m_s \frac{\partial m_N}{\partial m_s}$
- The strange quark content of the nucleon: $y_N = \frac{2\langle N|\bar{s}s|N\rangle}{\langle N|\bar{u}u+\bar{d}d|N\rangle}$
- A number of groups have use the spectral method to extract the σ-terms.
- Can also be calculated directly.

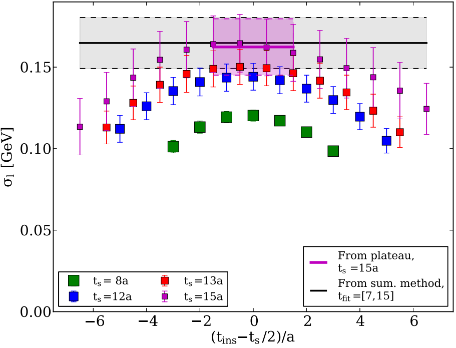
Nucleon σ-terms

Advantages for twisted mass fermions:

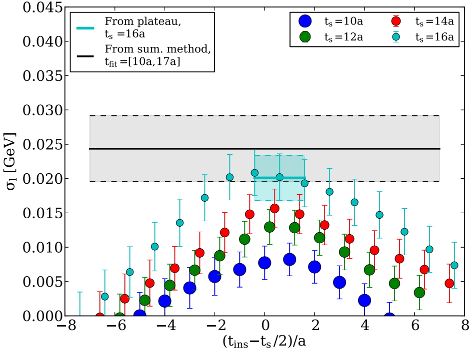
- In the twisted basis the scalar $\bar{u}u + \bar{d}d$ becomes: $i(\bar{u}\gamma_5 u - \bar{d}\gamma_5 d)$
  From the TM action: $D_u - D_d = 2i\mu\gamma_5$
  $\rightarrow D_u^{-1} - D_d^{-1} = -2i\mu D_d^{-1}\gamma_5 D_u^{-1}$ with noise suppression due to small value of μ
- The light quark loops can be computed by calculating stochastically $D_u^{-1}\,(2i\mu\gamma_5)\,D_d^{-1}$ using the **one-end-trick** to further improve the signal, S. Dinter *et al.* 1202.1480
- Renormalization straight forward







Connected



Disconnected

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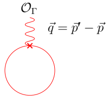
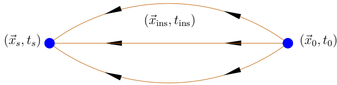
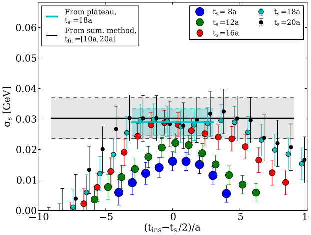
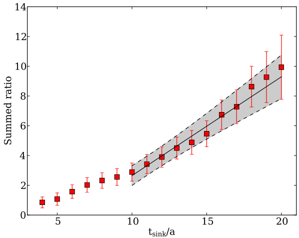
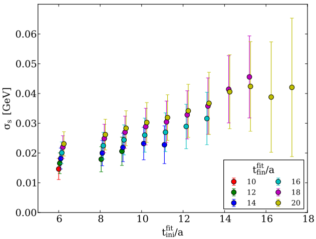








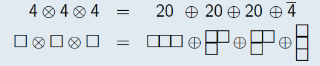


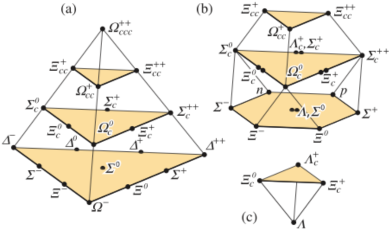
We use Osterwalder-Seiler s-quarks
and compute $\frac{1}{2}\,(\bar{s}_+ s_+ + \bar{s}_- s_-)$
At $m_\pi = 390$ MeV a consistent result is obtained with the two approaches
Investigate $y_N = \frac{2\langle N|\bar{s}s|N\rangle}{\langle N|\bar{u}u+\bar{d}d|N\rangle}$ as a function of m_π.

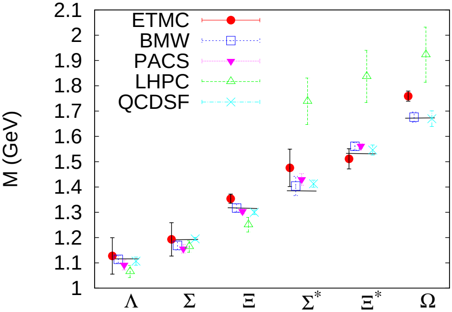
Hyperons and Charmed baryons

SU(4) representations:

$$4 \otimes 4 \otimes 4 = 20 \oplus 20 \oplus 20 \oplus \bar{4}$$

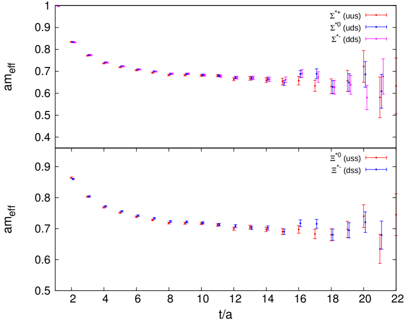






$N_F = 2+1$ clover, hybrid and $N_F = 2$ twisted mass fermions.

C. A., J. Carbonell, D. Christaras, V. Drach, M. Gravina, M. Papinutto, arXiv:1205.6856

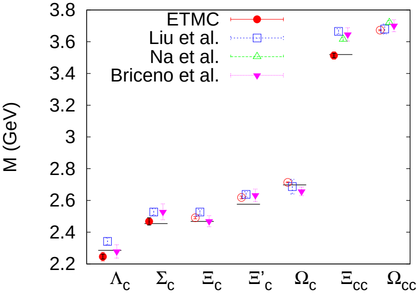


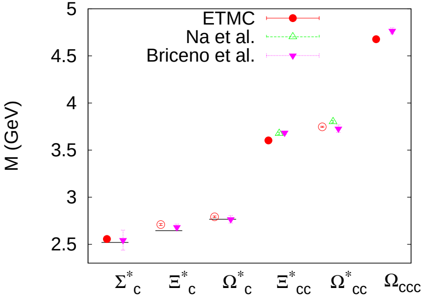
New results using $N_F = 2+1+1$ TM configurations at $m_\pi \sim 390$ MeV and $a = 0.078$ fm

Mass of charmed baryons

All use a mixed action approach:

- ETMC: TM $N_F = 2$ fermions gauge configurations
- Other collaborations use staggered $N_F = 2 + 1$ quarks, and a relativistic heavy quark or clover action for the charm quark





C. A., J. Carbonell, D. Christaras, V. Drach, M. Gravina, M. Papinutto, arXiv:1205.6856

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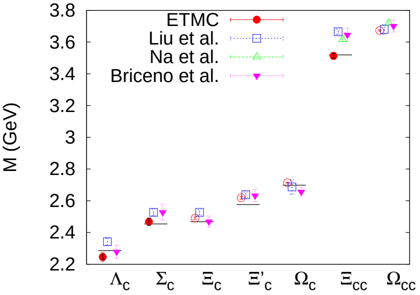
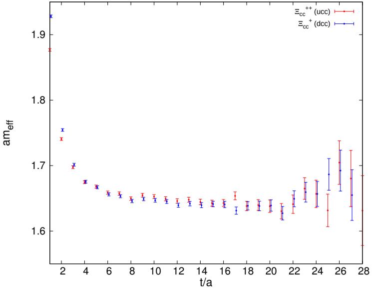
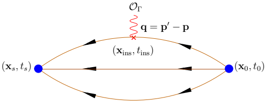




$N_F = 2 + 1 + 1$ at $m_\pi \sim 390$ MeV and $a = 0.078$ fm

Axial charge for hyperons

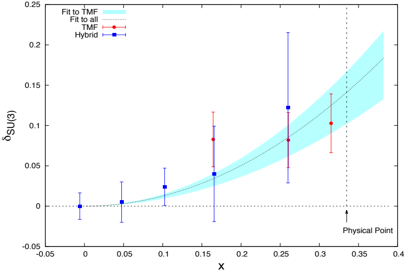
- Given by the hadron matrix element at zero momentum transfer: $\langle h|\bar{\psi}\gamma_\mu\gamma_5\psi|h\rangle|_{q^2=0}$
- Efficient to calculate (connected contribution with fixed current method) - computational cost for all hadrons about twice that required for one hadron (cost of additional contractions)



If exact SU(3) flavor symmetry:

- $g_A^N = F + D$, $g_A^\Sigma = 2F$, $g_A^\Xi = -D + F \Longrightarrow g_A^N - g_A^\Sigma + g_A^\Xi = 0$

Probe deviation: $\delta_{\rm SU(3)} = g_A^N - g_A^\Sigma + g_A^\Xi$ versus $x = (m_K^2 - m_\pi^2)/4\pi^2 f_\pi^2$, H.- W. Lin and K. Orginos, PRD 79, 034507 (2009)

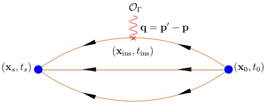


Breaking $\sim x^2$ leads to about 15% at the physical point

$x_{\rm phy} = 0.33$

Axial charge for hyperons

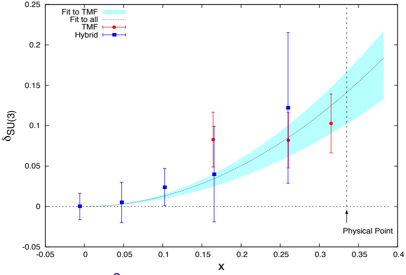
- Given by the hadron matrix element at zero momentum transfer:$\langle h|\bar{\psi}\gamma_\mu\gamma_5\psi|h\rangle|_{q^2=0}$
- Efficient to calculate (connected contribution with fixed current method) - computational cost for all hadrons about twice that required for one hadron (cost of additional contractions)

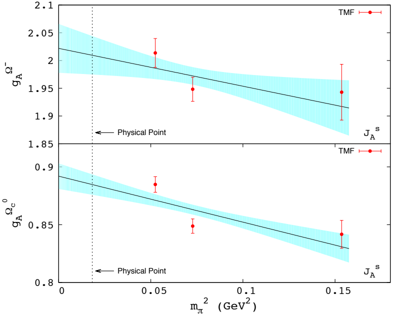


If exact SU(3) flavor symmetry:

- $g_A^N = F + D,\ g_A^\Sigma = 2F,\ g_A^\Xi = -D + F \Longrightarrow g_A^N - g_A^\Sigma + g_A^\Xi = 0$

Probe deviation: $\delta_{\rm SU(3)} = g_A^N - g_A^\Sigma + g_A^\Xi$ versus $x = (m_K^2 - m_\pi^2)/4\pi^2 f_\pi^2$, H.- W. Lin and K. Orginos, PRD 79, 034507 (2009)



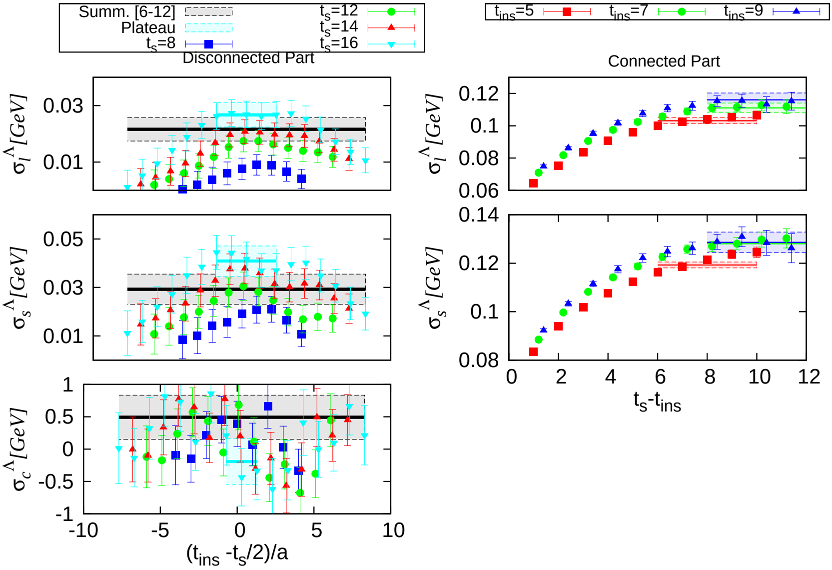


Breaking $\sim x^2$ leads to about 15% at the physical point
$x_{\rm phy} = 0.33$

σ-terms for hyperons and charmed baryons

Need both connected and disconnected pieces

First results using $N_F = 2+1+1$ TMF at $m_\pi = 390$ MeV



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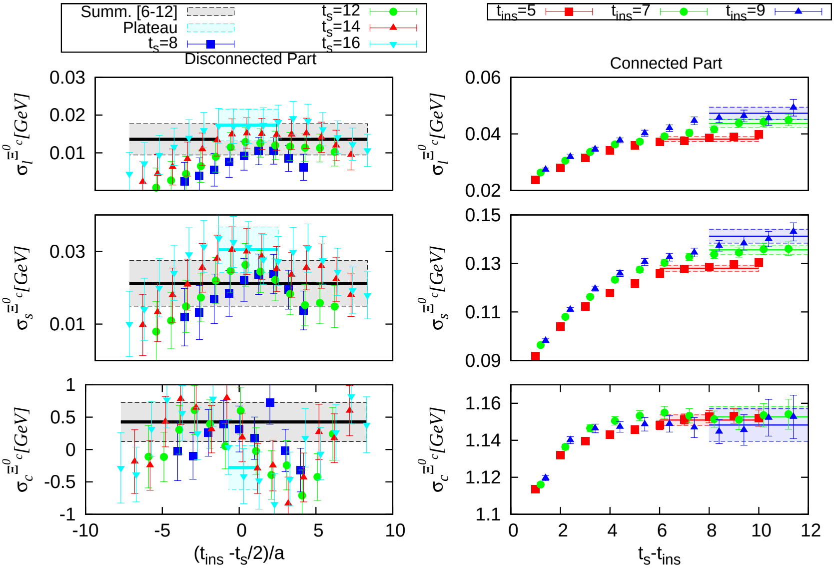

Conclusions

- Nucleon structure is a benchmark for the LQCD approach
  Some puzzles remain like g_A. Others need to be confrim by other groups like $\langle x \rangle_{u-d}$
  $\Longrightarrow$ simulations of the full theory at near physical parameters will eliminate umbiguities due chiral extrapolations
- Evaluation of quark loop diagrams has become feasible
- Predictions for other hadron observables are beginning to emerge e.g. axial charge of hyperons and charmed baryons
- Studying baryon resonances $\rightarrow$ provides insight into the structure of hadrons giving information that is difficult to extract experimentally.

As simulations at the physical pion mass and more computer time become available we expect many physical results on these key hadron observables

Acknowledgments

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Thank you for your attention

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our ideas, actions for development

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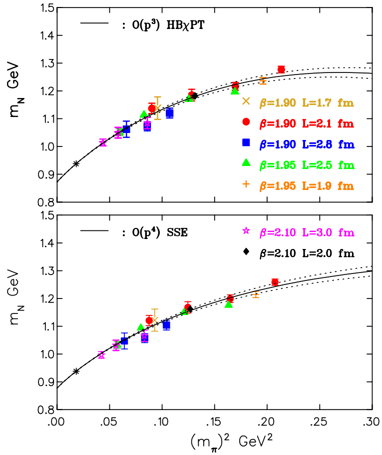
Lattice spacing determination

- Use nucleon mass at physical limit
- Extrapolate using LO expansion: $m_N = m_N^0 - 4c_1 m_\pi^2 - \frac{3g_A^2}{16\pi f_\pi^2} m_\pi^3$
- Systematic error from $\mathcal{O}(p^4)$ SSE HBχPT
- Simultaneous fits to $\beta = 1.9$, $\beta = 1.95$ and $\beta = 2.1$ results



$\beta = 1.90 \quad : a = 0.095(1)(1)$

$\beta = 1.95 \quad : a = 0.086(1)(2)$

$\beta = 2.10 \quad : a = 0.066(1)(1)$