Quark and Gluon Angular Momenta Contributions to Nucleon Spin (χQCD Collaboration)

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Introduction

- Understanding the nucleon spin structure has been a long standing issue both experimentally and theoretically.
- According to naive Quark Model, the nucleon spin is carried entirely by the valence quarks.
- The polarized Deep Inelastic Scattering experiment has revealed that it carries only a small fraction, $\sim 20\%$.
[EMC, J. Ashman *et al.*,1988].
- Subsequent experimental and Lattice QCD studies confirmed the EMC results, $20 - 25\%$.

Experiments	Polarised beam	Polarised target	Energy (GeV)
SLAC (completed)	e	p, n, d	≤ 50
EMC (completed)	μ	p	$100 - 200$
SMC (completed)	μ	p, d	$100, 190$
HERMES (analysing)	e	p, n, d	~ 30
COMPASS (running)	μ	p, d	160
JLAB (running)	e	p, n, d	≤ 6
BNL (running)	p	p	$\leq 250 + 250$

Quark Spin	COMPASS, 2010 (3 GeV^2)
Δu	$0.68(3)(3)$
Δd	$-0.29(6)(3)$
Δs	$-0.01(10)(10)$
$\Delta\Sigma/2$	$0.20(1)(2)$

- What are the other candidates for the missing proton spin? "Proton Spin Crisis"!!
- However QCD allows other candidates, namely the quark and gluon orbital angular momenta and gluon spin.
- How the nucleon spin of 1/2 is distributed among each quark flavor and gluons?

Spin Decomposition

$$J^i_{q,g} = \frac{1}{2}\,\epsilon^{ijk} \int d^3x\,(\mathcal{T}^{\{0k\}}_{q,g}\,x^j - \mathcal{T}^{\{0j\}}_{q,g}\,x^k) \tag{1}$$

$$\mathcal{T}^{\{0i\}}_q = \overline{\psi}_f \gamma^{\{0}(i\overleftrightarrow{\mathcal{D}})^{i\}}\psi_f, \quad \mathcal{T}^{\{0i\}}_g = -F^{0\alpha}F^i_\alpha \tag{2}$$

[Jaffe and Manohar, 1990; X. Ji, 1997]

J_q can decomposed into two gauge-invariant components:

$$\vec{J}_q = \int d^3x \frac{1}{2}\left[\overline{\psi}\vec{\gamma}\gamma^5\psi + \psi^\dagger\{\vec{x}\times(i\vec{D})\}\psi\right] = \frac{1}{2}\vec{\Sigma}_q + \vec{L}_q \tag{3}$$

J_g can not be decomposed:

$$\vec{J}_g = \int d^3x\left[\vec{x}\times(\vec{E}\times\vec{B})\right] \stackrel{?}{=} \frac{1}{2}\vec{\Sigma}_g + \vec{L}_g \tag{4}$$

The total angular momentum:

$$\vec{J} = \vec{J}_q + \vec{J}_g = \frac{1}{2}\vec{\Sigma}_q + \vec{L}_q + \vec{J}_g \tag{5}$$

Spin sum rule:

$$\frac{1}{2} = J_q + J_g = \frac{1}{2}\Sigma_q + L_q + J_g \tag{6}$$

$\frac{1}{2}\Sigma_q$: Total spin contributions from both valence and sea quarks.

L_q : Total orbital angular momentum from both valence and sea quarks.

J_g : Total angular momentum from gluons

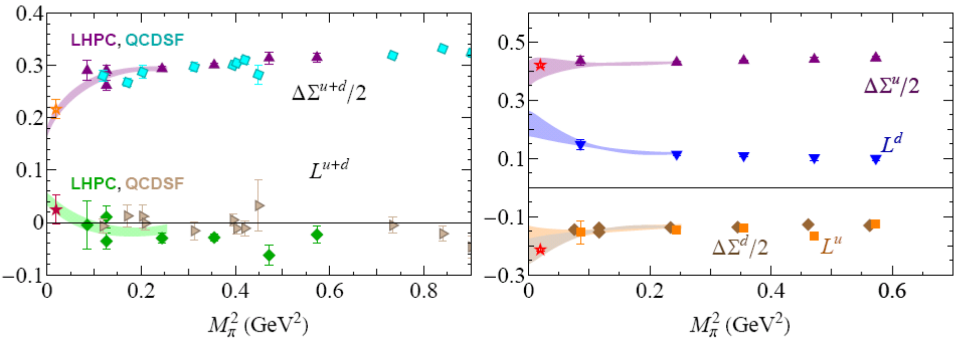
- One needs appropriate operators with well defined matrix elements between the initial and final hadronic states.

- No known operator for L_q.

- One can calculate the total quark angular momentum J_q as well as J_g.

  Corresponding operators : Energy-momentum tensor, $\mathcal{T}_{q,g}^{\{0i\}}$.

- Subtract the *independently* computed spin contributions to extract L_q.



- Lattice studies of L^{u+d} for connected insertions ~ 0.
- $\Delta G/G \sim 0$. [COMPASS, 2011; STAR, 2010]

⇒ "Dark Spin" Scenario ??

Experiments on angular momenta:

- JLab (6/12 GeV)
- COMPASS
- EIC/eRHIC
- RHIC
- GSI FAIR
- LHC

Matrix Elements

The matrix element of the energy-momentum tensor, $\mathcal{T}_{4i}$ is given by, [X. Ji, 1997]

$$
\begin{aligned}
(p,s|\mathcal{T}_{\{4i\}q,g}|p',s') &= \left(\frac{1}{2}\right)\bar{u}(p,s)\left[T_1(q^2)(\gamma_4\bar{p}_i+\gamma_i\bar{p}_4)\right.\\
&- \frac{1}{2m}T_2(q^2)(\bar{p}_4\sigma_{i\alpha}q_\alpha+\bar{p}_i\sigma_{4\alpha}q_\alpha)\\
&- \left.\frac{i}{m}T_3(q^2)q_4q_i\right]_{q,g}u(p',s')
\end{aligned} \tag{7}
$$

where, $q=p-p'$, $\bar{p}=(p+p')/2$.

Using Eq. (9) and taking $q^2\to 0$ limit, one can show that,

$$
J_{q,g} = \frac{1}{2}\left[T_1(0)+T_2(0)\right]_{q,g} \tag{8}
$$

Special Case

- Linear Momentum Operator : $P_i^{q,g} = \int d^3x \, \mathcal{T}^{\{0i\}q,g}$

  $\langle x \rangle_{q,g} = T_{1;q,g}(0)$: the first moment of the momentum fraction carried by the quarks or gluons.

- Momentum Sum Rule:

$$\langle x \rangle_q + \langle x \rangle_g = 1 \quad (9)$$

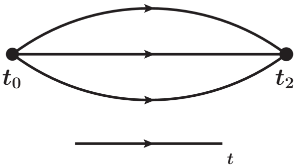
$p = p', q = 0 \Rightarrow$

$$(p, s|\mathcal{T}_{\{4i\}q,g}|p, s) = \left(\frac{1}{2}\right) \bar{u}(p, s) T_{1;q,g}(0)(\gamma_4 \bar{p}_i + \gamma_i \bar{p}_4) u(p, s) \quad (10)$$

- Can be computed independently [M. Deka *et al.*, 2008].

Two-point Correlation Functions

$$G_{NN}(t_2,\vec{p}) \quad = \quad \sum_{\vec{x}_2} e^{-i\vec{p}.(\vec{x}_2-\vec{x}_0)}\langle 0|T\left[\chi(x_2)\,\bar{\chi}(x_0)\right]|0\rangle \qquad (11)$$

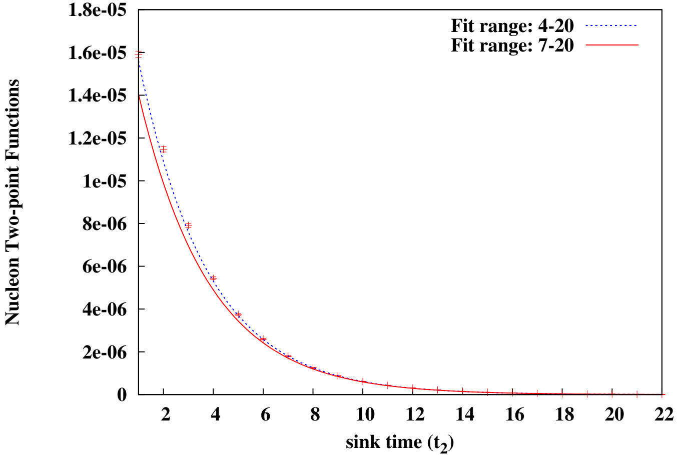


Inserting Energy eigenstates:

$$\mathrm{Tr}[\Gamma\, G_{NN}(t_2,\vec{p})] \quad = \quad A\, e^{-E_p(t_2-t_0)} + \text{Higher states} \qquad (12)$$

Wick Contraction:

$$\mathrm{Tr}[\Gamma\, G_{NN}(t_2,\vec{p})] \quad = \quad \sum_{\vec{x}_2} e^{-i\vec{p}.(\vec{x}_2-\vec{x}_0)}\,\frac{1}{Z}\int \mathcal{D}U\,[\det M(U)]\,e^{-S_g}$$
$$\mathrm{Tr}\left\{f\left[M^{-1}(x_2,x_0)\,M^{-1}(x_2,x_0)\,M^{-1}(x_2,x_0);U\right]\right\} \qquad (13)$$



$$\sum_{\vec{x}_2} e^{-i\vec{p}.(\vec{x}_2-\vec{x}_0)} \sum_{\{U\}} \mathrm{Tr}\{f\left[M^{-1}(x_2,x_0)\,M^{-1}(x_2,x_0)\,M^{-1}(x_2,x_0);U\right]\}$$

$$\xrightarrow{(t_2-t_0)\gg 1} A\,e^{-E_p(t_2-t_0)} \tag{14}$$

Three-point Correlation Functions

The three point-function for an operator, $\mathcal{T}$,

$$\begin{aligned} G_{N\mathcal{T}N}(\vec{p},t_2;\vec{q},t_1;\vec{p}',t_0) &= \sum_{\vec{x}_2,\vec{x}_1} e^{-i\vec{p}.\vec{x}_2} e^{i\vec{q}.\vec{x}_1} e^{i\vec{p}'.\vec{x}_0} \langle 0|\mathrm{T}(\chi(x_2)\mathcal{T}(x_1)\bar{\chi}(x_0))|0\rangle \\ &= \sum_{\vec{x}_2} e^{-i\vec{p}.(\vec{x}_2-\vec{x}_0)} e^{+i\vec{q}.(\vec{x}_1-\vec{x}_0)} \\ &\quad \sum_{\{U\}} f\left[M^{-1}M^{-1}M^{-1}M^{-1};U\right] \end{aligned} \tag{15}$$

$$\mathrm{Tr}\left[\Gamma G_{N\mathcal{T}_{4i}N}(\vec{p},t_2;\vec{q},t_1;\vec{p},t_0)\right] \xrightarrow{t_1 \gg t_0,\; t_2 \gg t_1}$$

$$B\, e^{-E_p(t_2-t_1)} e^{-E_{p'}(t_1-t_0)} [a_1\, T_1(q^2) + a_2\, T_2(q^2) + a_3\, T_3(q^2)] \tag{16}$$

Ratios

$$\frac{\mathrm{Tr}\left[\Gamma_{\mathrm{pol,unpol}} G_{N\mathcal{T}_{4i}N}(\vec{p},t_2;\vec{q},t_1;\vec{p}\,',t_0)\right]}{\mathrm{Tr}\left[\Gamma_{\mathrm{unpol}} G_{NN}(\vec{p},t_2)\right]}$$

$$\times \sqrt{\frac{\mathrm{Tr}\left[\Gamma_{\mathrm{unpol}} G_{NN}(\vec{p}\,',t_2-t_1+t_0)\right]}{\mathrm{Tr}\left[\Gamma_{\mathrm{unpol}} G_{NN}(\vec{p},t_2-t_1+t_0)\right]}}$$

$$\times \sqrt{\frac{\mathrm{Tr}\left[\Gamma_{\mathrm{unpol}} G_{NN}(\vec{p},t_1)\right]}{\mathrm{Tr}\left[\Gamma_{\mathrm{unpol}} G_{NN}(\vec{p}\,',t_1)\right]} \cdot \frac{\mathrm{Tr}\left[\Gamma_{\mathrm{unpol}} G_{NN}(\vec{p},t_2)\right]}{\mathrm{Tr}\left[\Gamma_{\mathrm{unpol}} G_{NN}(\vec{p}\,',t_2)\right]}}$$

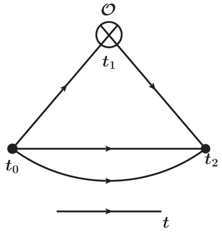
$$\xrightarrow{t_1 \gg t_0,\; t_2 \gg t_1} \frac{\left[a_1 T_1(q^2) + a_2 T_2(q^2) + a_3 T_3(q^2)\right]}{4\sqrt{E_p(E_p+m)E_{p'}(E_{p'}+m)}} \tag{17}$$

$$\frac{\mathrm{Tr}\left[\Gamma_{\mathrm{unpol}} G_{N\mathcal{T}_{4i}N}(\vec{p},t_2;\vec{0},t_1;\vec{p},t_0)\right]}{\mathrm{Tr}\left[\Gamma_{\mathrm{unpol}} G_{NN}(\vec{p},t_2)\right]} \xrightarrow{t_1 \gg t_0,\; t_2 \gg t_1} T_1(0) = \langle x \rangle \tag{18}$$

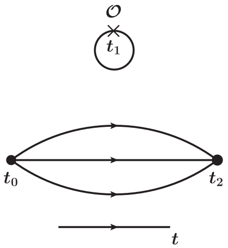
Connected and Disconnected Insertions

$$G_{N\mathcal{T}N}(\vec{p},t_2;\vec{q},t_1;\vec{p}',t_0) \quad = \quad \sum_{\vec{x}_2,\vec{x}_1} e^{-i\vec{p}.\vec{x}_2} e^{i\vec{q}.\vec{x}_1} e^{i\vec{p}'.\vec{x}_0} \langle 0|\mathrm{T}(\chi(x_2)\mathcal{T}(x_1)\bar{\chi}(x_0))|0\rangle$$

The three-point functions for quarks have two different contributions: connected (CI) and disconnected insertions (DI).



- CI three-point functions are relatively easier to compute.



- DI three-point functions are computationally challenging: Involves propagators from *all-to-all* points.

$$\mathrm{Tr}\,[\text{Three-point Func.}]_{\mathrm{DI}} \sim \mathrm{Tr}\,[\text{two-point Func.}] \times \mathrm{Tr}\,[\text{loop}]$$

- *strange* quark contributions come only from DI. *up* and *down* quarks have contributions both from CI and DI.

Computation of DI

- Loops are stochastically estimated by using complex Z_2 noise [S. J. Dong and K. F. Liu, 1992].

$$\langle \eta_i \rangle = \lim_{L\to\infty} \frac{1}{L} \sum_{n=1}^{L} \eta_i^n = 0, \qquad \langle \eta_i^\dagger \eta_j \rangle = \lim_{L\to\infty} \frac{1}{L} \sum_{n=1}^{L} \eta_i^{\dagger n} \eta_j^n = \delta_{ij} \quad (19)$$

- Solve for X:

$$MX = \eta \quad \Rightarrow \quad \langle \eta_j^\dagger X_i \rangle = \sum_k M_{ik}^{-1} \langle \eta_k^\dagger \eta_j \rangle \simeq M_{ij}^{-1} \quad (20)$$

Improvement:

- Unbiased Subtraction.
- Discrete Symmetries and Transformations.
- Summation over insertion time.
- Multiple nucleon sources.

Glue Operator

- Need suitable operator.
- We construct from the overlap operator, D_{OV}, [K. F. Liu *et al.*, 2008; T. Doi *et al.*, 2008]

$$F_{\mu\nu}(x) = \text{const.} \times \mathrm{Tr}^s \left[\sigma_{\mu\nu} D_{\mathrm{OV}}(x,x)\right] \tag{21}$$

- High mode fluctuations are expected to be suppressed.
- $D_{\mathrm{OV}}(x,x)$ is estimated stochastically using complex Z_2 noise.
- Color and spin are computed exactly.
- Space-time dilution is perfoemed by a separation of two sites on top of odd/even dilution. "taxi driver distance" $= 4$.

Extraction of T_1 and T_2

$$\text{ratio} = \frac{a_1\, T_1(q^2) + a_2\, T_2(q^2) + a_3\, T_3(q^2)}{4\sqrt{E_p(E_p + m)E_{p'}(E_{p'} + m)}} \tag{22}$$

- Select appropriate kinematics for which $\vec{p} \neq \vec{p}' \neq 0$, and set up enough number of equations for each q^2.
- CI: Obtained from fitting a constant.

  DI: Summation over insertion time $\Rightarrow$ Obtained from the slope.
- Separate $T_1(q^2)$ and $T_2(q^2)$ at a few different values of q^2.
- Separately extrapolate them to $q^2 \to 0$.

Numerical Parameters

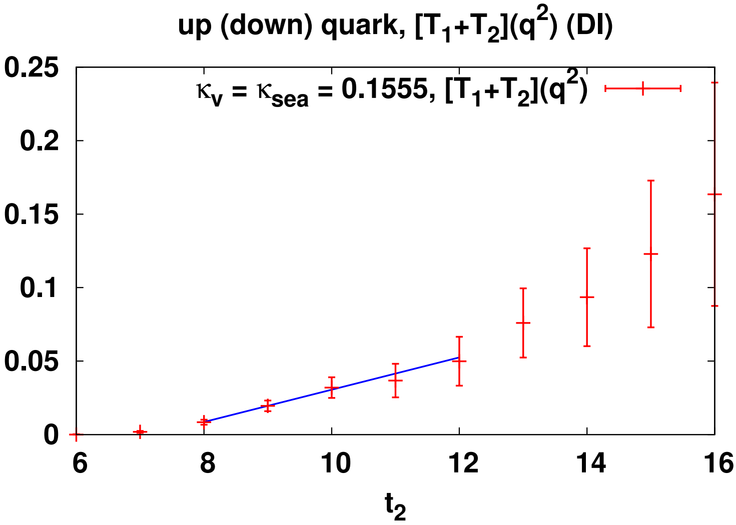
- Standard Wilson Action. The gauge fields are generated in the quenched approximation with Wilson gauge action.
- $16^3 \times 24$ lattice.
- The lattice spacing is $a \sim 0.11$ fm $\simeq$ 2 Gev.
- $\kappa = 0.154, 0.155, 0.1555$. The corresponding pion masses are $650\ (3),\ 538\ (4)$ and $478\ (4)$ MeV respectively.
- The number of configurations 500. For quarks, the number of Z_2 noise estimator is 500 on each gauge configuration.

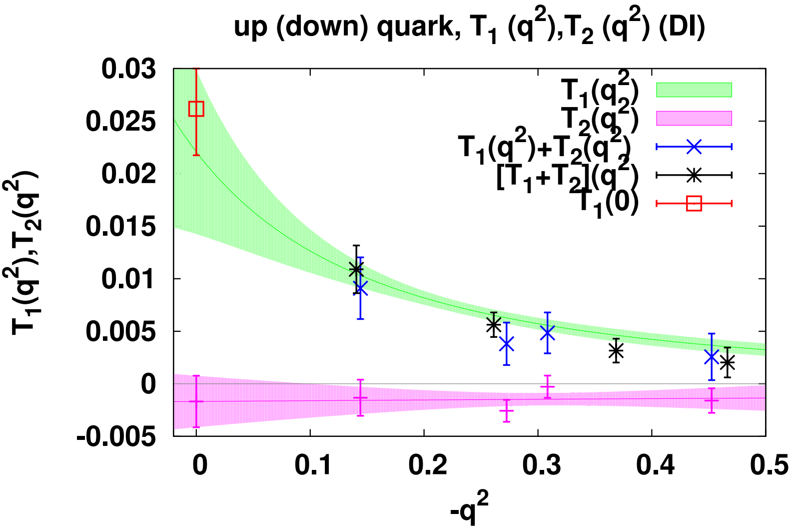
Analysis

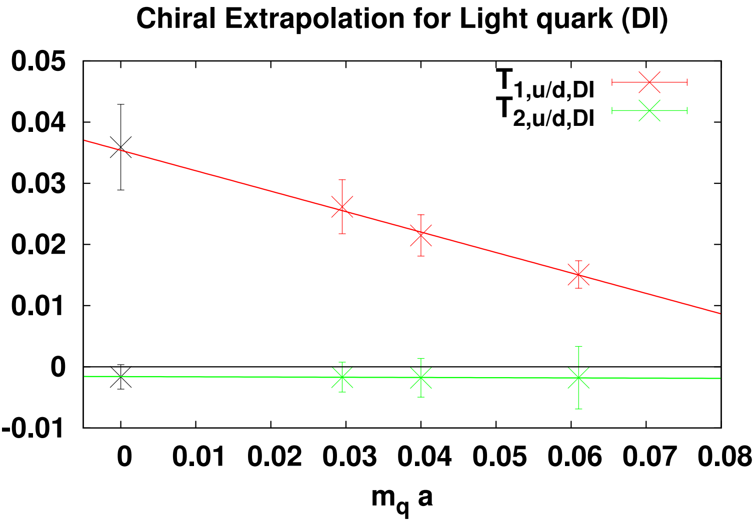
- Close to 30 TB of data has been produced from more than one million processor hours of computing. Has to be analyzed in various sequential steps.
- The error analysis is performed by using the jackknife procedure.
- The correlation among different quantities are taken into account by constructing the corresponding covariance matrices.
- 4 different q^2 values are considered.
- q^2 extrapolations are performed using dipole approximation,

$$\left[T_i(q^2)\right]\Big|_{i=1,2} = \frac{T_i(0)}{\left[1+\frac{q^2}{\Lambda_i^2}\right]^2}\Bigg|_{i=1,2} \tag{23}$$

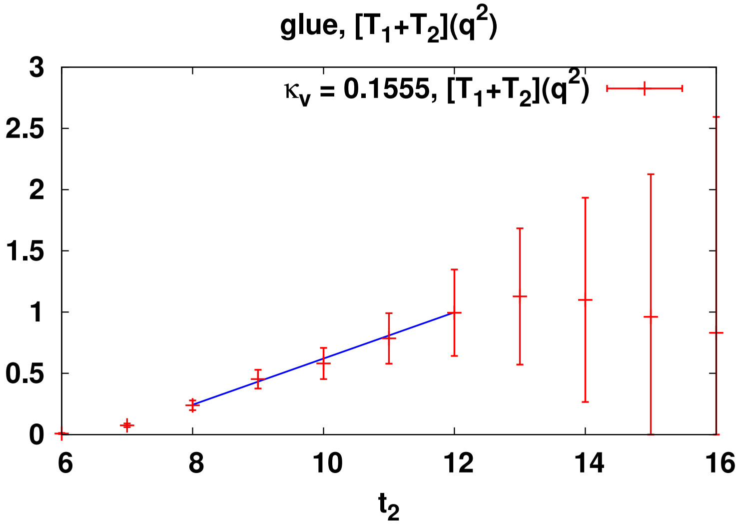
Disconnected Insertions

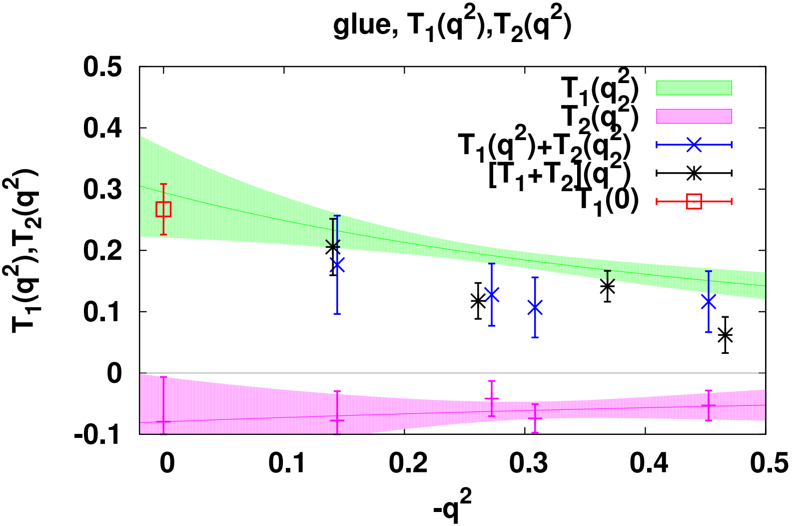


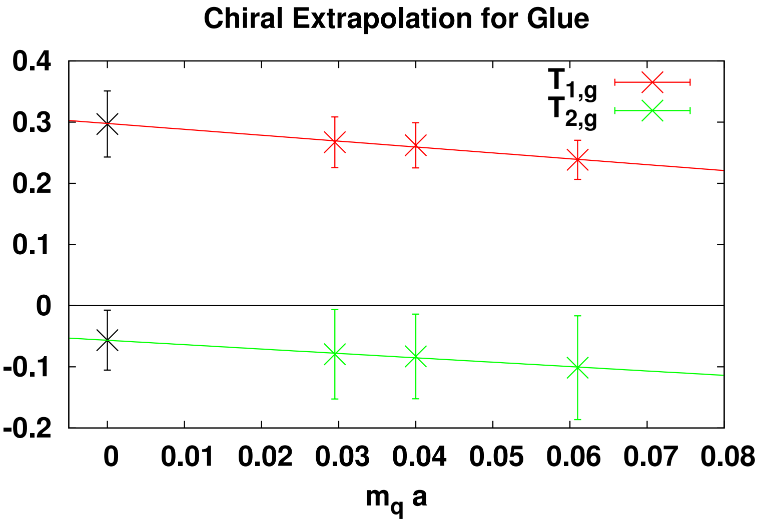

up (down) quark, T1 (q2),T2 (q2) (DI)
T1(q2)
T2(q2)
T1(q2)+T2(q2)
[T1+T2](q2)
T1(0)
T1(q2),T2(q2)
-q2
0.03
0.025
0.02
0.015
0.01
0.005
0
-0.005
0
0.1
0.2
0.3
0.4
0.5


Chiral Extrapolation for Light quark (DI)
T1,u/d,DI
T2,u/d,DI
0.05
0.04
0.03
0.02
0.01
0
-0.01
0
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08
mq a

Glue




glue, T1(q2),T2(q2)
T1(q2)
T2(q2)
T1(q2)+T2(q2)
[T1+T2](q2)
T1(0)
T1(q2),T2(q2)
0.5
0.4
0.3
0.2
0.1
0
-0.1
0
0.1
0.2
0.3
0.4
0.5
-q2


Chiral Extrapolation for Glue
0.4
0.3
0.2
0.1
0
-0.1
-0.2
0
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08
m_q a
$T_{1,g}$
$T_{2,g}$

Renormalization

- Renormalized operators

$$\mathcal{T}(\mu) \quad = \quad Z_{\mathcal{T}}(a\mu, g(a))\, \mathcal{T}(a)$$

- Lattice renormalization are performed using sum rule

$$Z_q(a)T_1(0)_q + Z_g(a)T_1(0)_g = 1$$

$$Z_q(a)[T_1(0)_q + T_2(0)_q] + Z_g(a)[T_1(0)_g + T_2(0)_g] = \frac{1}{2}$$

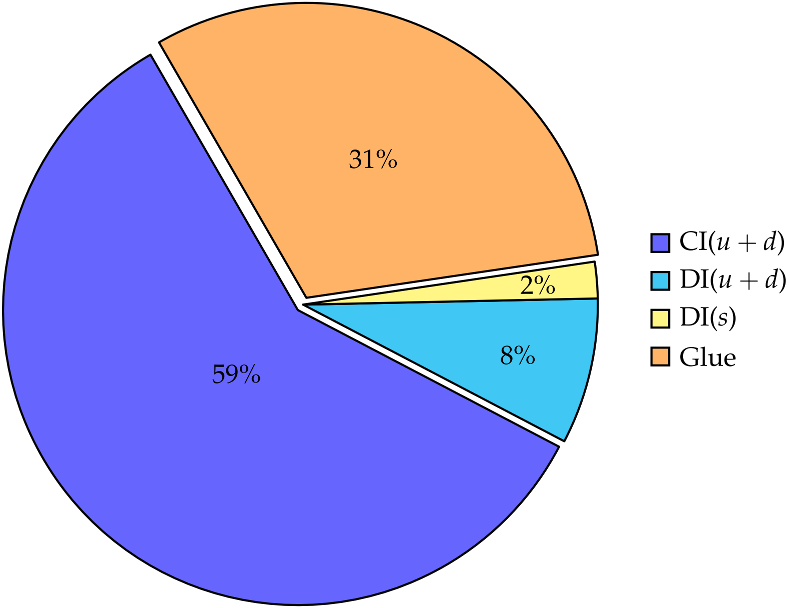
$$\Rightarrow \qquad Z_q = 1.05\ ,\ Z_g = 1.05$$

- Perturbative mixing and matching to the $\overline{MS}$ scheme has not been completed.

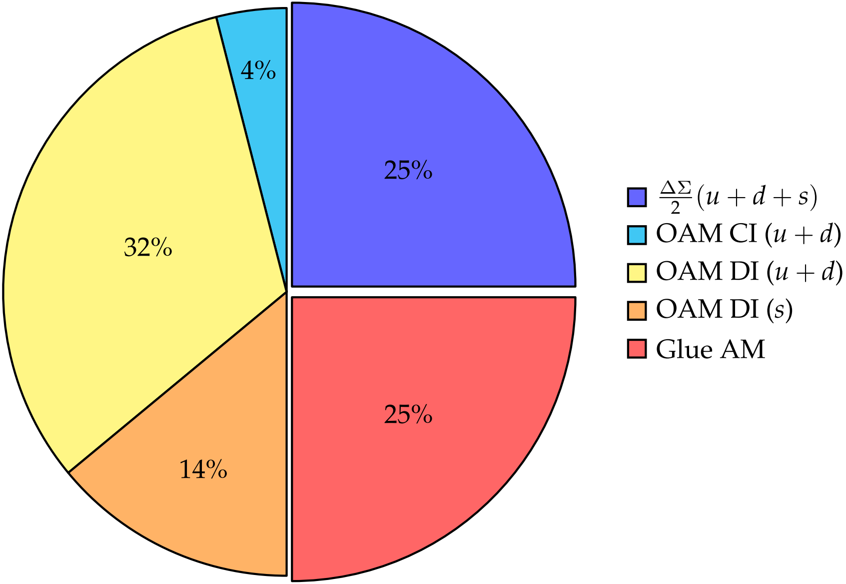
Table: Lattice renormalized value with renormalization constants, $Z_q = Z_g = 1.05$.

	CI(u)	CI(d)	CI(u+d)	DI(u,d)	DI(s)	Glue
$\langle x \rangle$	0.428(40)	0.156(20)	0.586(45)	0.038(7)	0.024(6)	0.313(56)
$T_2(0)$	0.297(112)	-0.228(80)	0.064(22)	-0.002(2)	-0.001(3)	-0.059(52)
2J	0.726(128)	-0.072(82)	0.651(51)	0.036(7)	0.023(7)	0.254(76)
2L	-0.18(18)	0.23(14)	0.04(10)	0.16(2)	0.14(2)	–

Decomposition of First Moment



Decomposition of Total Angular Momenta



Conclusion and Outlook

- A complete calculation of the quark and glue momenta and angular momenta in the nucleon has been carried out.
- Renormalization constants in $\overline{MS}$ scheme needed to be calculated. Work is in Progress.
- The glue momentum fraction of $0.313(56)$ is smaller than CTEQ4M results of 0.42 at $Q = 1.6$ GeV. [H.L. Lai *et al.*, 1997]
- $\langle x \rangle_s = 0.024(6)$ is in the range of CTEQ6M results: $0.018 < \langle x \rangle_s < 0.040$. [H. L. Lai *et al.*, 2007]
- In progress:
  - Dynamical domain-wall fermion gauge (RBC + UKQCD) configurations, lowest pion mass $\sim$ 140 MeV on a 5.5 fm box.
  - Quark loops with low mode averaging and improved nucleon propagator.

Thank you !!