More on the relation between the two physically inequivalent decompositions of the nucleon spin

Masashi Wakamatsu , Osaka University

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1. To start with

Nucleon Spin Decomposition Problem in QCD

"Is gauge-invariant complete decomposition of nucleon spin possible ? "

This is a fundamentally important question of **QCD as a gauge theory**.

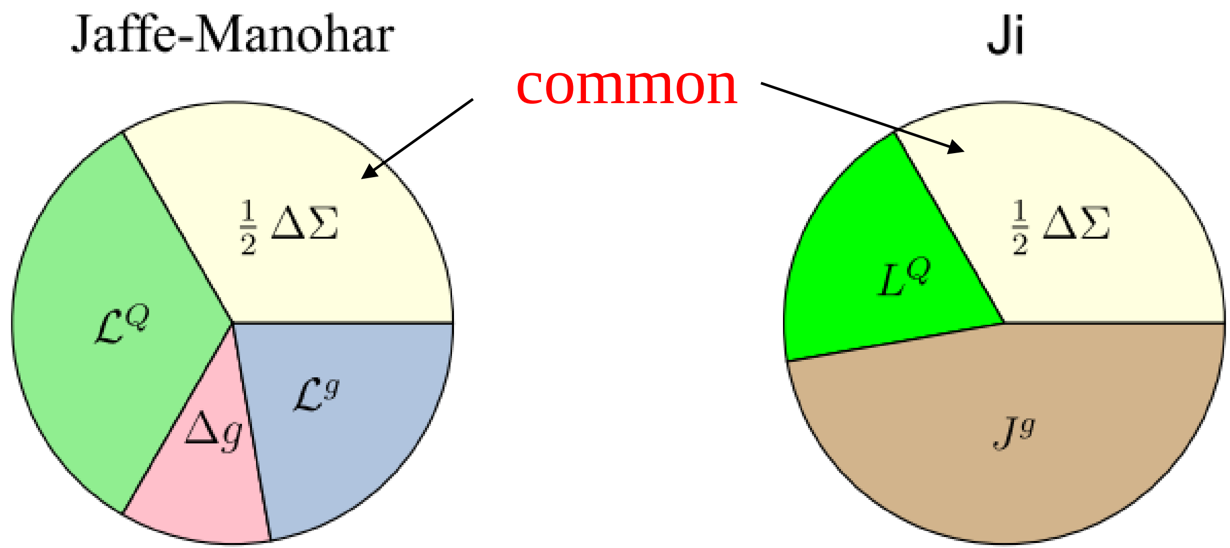
because the **gauge-invariance** is a **necessary condition** of **observability** !

Unfortunately, this is a very delicate problem, which is still under debate.

I feel that the recent INT workshop on "Orbital Angular Momentum in QCD"
increased controversy rather than settled it !

2. Controversies of nucleon spin decomposition problem ?

two popular decompositions of the nucleon spin



$$
\begin{aligned}
\boldsymbol{J}_{QCD} &= \int \psi^\dagger \frac{1}{2} \boldsymbol{\Sigma}\, \psi \, d^3 x \\
&+ \int \psi^\dagger \, \boldsymbol{x} \times \frac{1}{i} \nabla \, \psi \, d^3 x \\
&+ \int \boldsymbol{E}^a \times \boldsymbol{A}^a \, d^3 x \\
&+ \int E^{ai} \, \boldsymbol{x} \times \nabla \, A^{ai} \, d^3 x
\end{aligned}
$$

Each term is not separately gauge-invariant !

$$
\begin{aligned}
\boldsymbol{J}_{QCD} &= \int \psi^\dagger \frac{1}{2} \boldsymbol{\Sigma}\, \psi \, d^3 x \\
&+ \int \psi^\dagger \, \boldsymbol{x} \times \frac{1}{i} \boldsymbol{D} \, \psi \, d^3 x \\
&+ \int \boldsymbol{x} \times (\boldsymbol{E}^a \times \boldsymbol{B}^a) \, d^3 x
\end{aligned}
$$

J_g

No further GI decomposition !

Quite a lot of people believe that textbooks of QED say that the total angular momentum of the photon cannot be gauge-invariantly split into a intrinsic spin and an orbital part. (See, for example, Leader's talk at Spin2012.)

This is a delusion, however. If you suspect, please consult with the following very clearly written textbook of QED :

"Photons & Atoms" , C. Cohen Tannoudji et. al. (Wiley-VCH, 1989)

Somewhat unexpected for you might be the fact that the total photon angular momentum in a **coupled system of photons and charged-particles** is actually split into 3 gauge-invariant pieces, not 2 !

The decomposition is based on the standard transverse-longitudinal decomposition of the photon field given as

$$\boldsymbol{A} = \boldsymbol{A}_{\perp} + \boldsymbol{A}_{\parallel}, \quad \boldsymbol{E}_{\perp} = -\frac{\partial \boldsymbol{A}_{\perp}}{\partial t}, \quad \boldsymbol{E}_{\parallel} = -\nabla A^0 - \frac{\partial \boldsymbol{A}_{\parallel}}{\partial t}$$

with an important property that $\boldsymbol{A}_{\perp}$ is gauge-invariant !

The decomposition is given in the following way :

$$\begin{aligned} \boldsymbol{J}^{\gamma} &= \int d^3r\ \boldsymbol{r} \times (\boldsymbol{E} \times \boldsymbol{B}) \\ &= \int d^3r\ \boldsymbol{r} \times (\boldsymbol{E}_{\perp} \times \boldsymbol{B}) \ + \ \int d^3r\ \boldsymbol{r} \times (\boldsymbol{E}_{\parallel} \times \boldsymbol{B}) \\ &\equiv \quad \boldsymbol{J}_{trans} \quad + \quad \boldsymbol{J}_{long} \end{aligned}$$

Using the Gauss law $\nabla \cdot \boldsymbol{E}_{\parallel} = \rho$, the 2nd part can also be written in the form :

$$\boldsymbol{J}_{long} \ = \ \sum_i q_i\ \boldsymbol{r}_i \times \boldsymbol{A}_{\perp}(\boldsymbol{r}_i)$$

I called it the "**potential angular momentum**" term. It is solely gauge-invariant. It is also important to recognize that **this term vanishes for free photon**, i.e. if there is no charged particle sources for photon.

The 1st part can further be split into 2 pieces in a gauge-invariant way :

$$\boldsymbol{J}_{trans} \ = \ \int d^3r\ E^l_{\perp}\ (\boldsymbol{r} \times \nabla)\ A^l_{\perp} \ + \ \int d^3r\ \boldsymbol{E}_{\perp} \times \boldsymbol{A}_{\perp}$$

photon canonical OAM **intrinsic photon spin**

definitely gauge-invariant !

The presence of the potential angular momentum term, which is solely gauge-invariant, introduces an arbitrariness in the spin decomposition. (We shall come back later to this issue.)

Leaving it aside, an important lesson from the above demonstration is as follows :

Standard textbooks of QED never says that the angular momentum of the photon cannot be split in a gauge-invariant way into a spin part and an orbital part.

It is in fact possible to isolate **intrinsic spin of the photon gauge-invariantly** !

It is quite natural to expect that the same holds for the gluon spin in the nucleon.

$$\Delta G$$

Once the gauge-invariance of the gluon spin were established, the gluon OAM satisfying the relation

$$L_G = J_G - \Delta G$$

must be definitely gauge-invariant, so that the vital question we must answer in the nucleon spin decomposition problem **reduces** to the following :

Is ΔG really gauge-invariant or not ?

3. Gauge-invariant decomposition of covariant angular-momentum tensor

QCD energy momentum tensor : from the paper of Jaffe and Manohar

$$T^{\mu\nu} = T_q^{\mu\nu} + T_G^{\mu\nu} \quad \text{with} \quad T^{\mu\nu} = T^{\nu\mu}, \quad \partial_\mu T^{\mu\nu} = 0$$

where

$$T_q^{\mu\nu} = \frac{1}{2} \bar{\psi} (\gamma^\mu i D^\nu + \gamma^\nu i D^\mu) \psi,$$

$$T_G^{\mu\nu} = 2 \, \mathrm{Tr} (F^{\mu\alpha} F_\alpha^\nu - \frac{1}{4} g^{\mu\nu} F^2)$$

symmetric Belinfante tensor

QCD angular-momentum tensor

$$M^{\mu\nu\lambda} \equiv x^\nu T^{\mu\lambda} - x^\lambda T^{\mu\nu}$$

$M^{\mu\nu\lambda}$ is conserved if $T^{\mu\nu}$ is symmetric and conserved !

$$\partial_\mu M^{\mu\nu\lambda} = 0$$

It is known that, by using the identity

$$\bar{\psi}\,(\,x^\nu\,\gamma^\lambda \;-\; x^\lambda\,\gamma^\nu\,)\,i\,D^\mu\,\psi \;-\; \bar{\psi}\,\gamma^\mu\,(\,x^\nu\,i\,D^\lambda \;-\; x^\lambda\,i\,D^\nu\,)\,\psi$$
$$= \;\epsilon^{\mu\nu\lambda\beta}\,\bar{\psi}\,\gamma_\beta\,\gamma_5\,\psi \;-\; \frac{1}{2}\,\partial_\alpha\,[\,(x^\nu\,\epsilon^{\mu\lambda\alpha\beta} \;-\; x^\lambda\,\epsilon^{\mu\nu\alpha\beta}\,)\,\bar{\psi}\,\gamma_\beta\,\gamma_5\,\psi\,]$$

The quark part of $M^{\mu\nu\lambda}$ can gauge-invariantly be decomposed as

$$M_q^{\mu\nu\lambda} \;=\; \frac{1}{2}\,\epsilon^{\mu\nu\lambda\beta}\,\bar{\psi}\,\gamma_\beta\,\gamma_5\,\psi \;+\; \bar{\psi}\,\gamma^\mu\,(\,x^\nu\,i\,D^\lambda \;-\; x^\lambda\,i\,D^\nu\,)\,\psi$$

up to a surface term.

On the contrary, it is widely believed that the gluon part of $M^{\mu\nu\lambda}$

$$M_G^{\mu\nu\lambda} \;=\; 2\,\mathrm{Tr}\,[\,x^\nu\,F^{\mu\alpha}\,F_\alpha^\lambda - x^\lambda\,F^{\mu\alpha}\,F_\alpha^\nu\,] \;+\; \frac{1}{2}\,\mathrm{Tr}\,F^2\,[\,x^\nu\,g^{\mu\nu} - x^\lambda\,g^{\mu\lambda}\,]$$

cannot be gauge-invariantly decomposed into the intrinsic spin and OAM parts.

However, by following a similar idea as proposed by Chen et al., we can make it !
The key point is the decomposition of gluon field

$$A^\mu \;=\; A^\mu_{phys} \;+\; A^\mu_{pure}$$

Here, we impose only the following very general conditions :

$$F^{\mu\nu}_{pure} \;\equiv\; \partial^\mu A^\nu_{pure} - \partial^\nu A^\mu_{pure} - i\, g\, [A^\mu_{pure}, A^\nu_{pure}] \;=\; 0$$

and

$$A^\mu_{phys}(x) \;\rightarrow\; U(x)\, A^\mu_{phys}(x)\, U^{-1}(x)$$

$$A^\mu_{pure}(x) \;\rightarrow\; U(x) \left(A^\mu_{pure}(x) \;+\; \frac{i}{g}\, \partial^\mu \right) U^{-1}(x)$$

- As a matter of course, these conditions are not enough to uniquely fix the form of A^μ_{phys} and A^μ_{pure} , which is not unrelated to the process of gauge-fixing !

- However, the point of our treatment is that we can postpone a complete gauge-fixing until later stage, while accomplishing a gauge-invariant decomposition of $M^{\mu\nu\lambda}$ based on the above conditions only.

Skipping intermediate steps, let us write down the final answer :

$$M^{\mu\nu\lambda} = M^{\mu\nu\lambda}_{q-spin} + M^{\mu\nu\lambda}_{q-OAM} + M^{\mu\nu\lambda}_{G-spin} + M^{\mu\nu\lambda}_{G-OAM} + M^{\mu\nu\lambda}_{boost} + \text{total divergence}$$

where

$$M^{\mu\nu\lambda}_{q-spin} = \frac{1}{2}\,\epsilon^{\mu\nu\lambda\sigma}\,\bar{\psi}\,\gamma_\sigma\,\gamma_5\,\psi,$$

$$M^{\mu\nu\lambda}_{q-OAM} = \bar{\psi}\,\gamma^\mu\,(\,x^\nu\,i\,D^\lambda \;-\; x^\lambda\,i\,D^\nu\,)\,\psi$$

$$M^{\mu\nu\lambda}_{G-spin} = 2\,\mathrm{Tr}\,[\,F^{\mu\lambda}\,A^\nu_{phys} \;-\; F^{\mu\nu}\,A^\lambda_{phys}\,],$$

$$M^{\mu\nu\lambda}_{G-OAM} = -\,2\,\mathrm{Tr}\,[\,F^{\mu\alpha}\,(\,x^\nu\,D^\lambda_{pure} \;-\; x^\lambda\,D^\nu_{pure}\,)\,A^{phys}_\alpha\,],$$
$$+\,2\,\mathrm{Tr}\,[\,(\,D_\alpha\,F^{\alpha\mu}\,)\,(\,x^\nu\,A^\lambda_{phys} \;-\; x^\lambda\,A^\nu_{phys}\,)\,],$$

decomposition (I)

Check of gauge-invariance

$$M^{\mu\nu\lambda}_{q-spin}, \ M^{\mu\nu\lambda}_{q-OAM} \quad : \quad \texttt{trivial}$$

$$M^{\mu\nu\lambda}_{G-spin} \quad : \quad \texttt{almost trivial}$$

$$M^{\mu\nu\lambda}_{G-OAM} \quad : \quad \texttt{less trivial}$$

However, since

$$\begin{aligned}
D^{\lambda}_{pure} A^{phys}_{\alpha} &\equiv \partial^{\lambda} A^{phys}_{\alpha} - i g [A^{\lambda}_{pure}, A^{phys}_{\alpha}] \\
&\rightarrow \partial^{\lambda} (U A^{phys}_{\alpha} U^{-1}) - i g [U (A^{\lambda}_{pure} - \frac{i}{g} \partial^{\lambda}) U^{-1}, U A^{phys}_{\alpha} U^{-1}] \\
&= U (\partial^{\lambda} A^{phys}_{\alpha} - i g [A^{\lambda}_{pure}, A^{phys}_{\alpha}]) U^{-1} \\
&= U D^{\lambda}_{pure} A^{phys}_{\alpha} U^{-1} \quad : \quad \texttt{covariantly transform}
\end{aligned}$$

one finds

$$\begin{aligned}
M^{\mu\nu\lambda}_{g-OAM} &= -2 \, \mathrm{Tr} [F^{\mu\alpha} (x^{\nu} D^{\lambda}_{pure} - x^{\lambda} D^{\nu}_{pure}) A^{phys}_{\alpha}], \\
&\quad + 2 \, \mathrm{Tr} [(D_{\alpha} F^{\alpha\mu}) (x^{\nu} A^{\lambda}_{phys} - x^{\lambda} A^{\nu}_{phys})] \\
&\rightarrow \quad \texttt{invariant}
\end{aligned}$$

Incidentally, the generalized potential angular momentum term

$$2\,\mathrm{Tr}\,\{\,(D_\alpha\,F^{\alpha\mu})\,(x^\nu\,A^\lambda_{phys} - x^\lambda\,A^\nu_{phys})\,\} \;=\; -\,g\,\bar{\psi}\,\gamma^\mu\,(\,x^\nu\,A^\lambda_{phys} - x^\lambda\,A^\nu_{phys}\,)\,\psi$$

is solely gauge-invariant, so that one may combine it with the quark OAM part.

Then, one gets another gauge-invariant decomposition, which is nothing but a covariant generalization of the Chen decomposition.

$$\begin{aligned} M'^{\mu\nu\lambda} \;&=\; \frac{1}{2}\,\epsilon^{\mu\nu\lambda\sigma}\,\bar{\psi}\,\gamma_\sigma\,\gamma_5\,\psi \\ &+\; \bar{\psi}\,\gamma^\mu\,(\,x^\nu\,i\,D^\lambda_{pure} \;-\; x^\lambda\,i\,D^\nu_{pure}\,)\,\psi \\ &+\; 2\,\mathrm{Tr}\,\{\,F^{\mu\lambda}\,A^\nu_{phys} \;-\; F^{\mu\nu}\,A^\lambda_{phys}\,\} \\ &-\; 2\,\mathrm{Tr}\,\{\,F^{\mu\alpha}\,(\,x^\nu\,D^\lambda_{pure} \;-\; x^\lambda\,D^\nu_{pure}\,)\,A^{phys}_\alpha\,\} \\ &+\; M^{\mu\nu\lambda}_{boost} \;+\; \text{total divergence} \end{aligned}$$

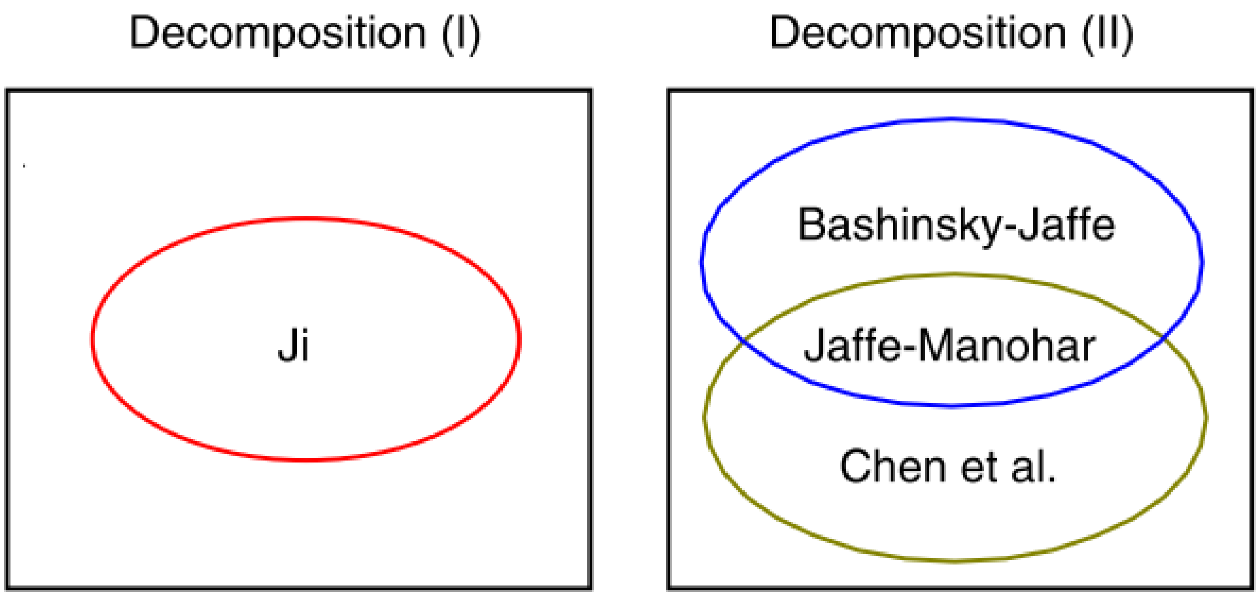
each term of which is also separately gauge invariant.

decomposition (II)

We thus confirmed the existence of 2 physically inequivalent gauge-invariant decompositions of the nucleon spin.

The decomposition (I) is essentially the Ji decomposition, but we are claiming that the gluon total angular momentum can also be split into spin and orbital parts in a gauge invariant way.

The decomposition (II) reduces to either of the **Chen decomposition** or the **Bashinsky-Jaffe decomposition** after (partial) gauge-fixing in an appropriate Lorentz frame. (Remember that the latter two reduces to the **Jaffe-Manohar decomposition** after (full) gauge fixing in an appropriate frame.)



After the INT workshop, a conflicting view has rapidly spread. See, for example,

- X. Ji, Y. Xu, and Y. Zhao, arXiv : JHEP 1208 (2012) 082.

According to them, the **Chen decomposition** is a gauge-invariant extension (GIE) of the **Jaffe-Manohar decomposition** based on the Coulomb gauge, while the **Bashinsky-Jaffe decomposition** is a GIE of the Jaffe-Manohar decomposition based on the light-cone gauge.

They claim that, since the way of GIE is not unique, there is no need that the 2 decompositions give the same physical predictions.

This makes Ji rehash his longstanding claim that the gluon spin ΔG has a meaning only in the light-cone gauge, and it is not gauge-invariant quantity in a true or traditional sense, **although** it is measurable in DIS scatterings.

⇩

One should recognize a self-contradiction inherent in this claim.

In fact, first remember a **fundamental proposition of physics** :

"Observables must be gauge-invariant ! "

The **contraposition** of this proposition (it is always correct) is

"gauge-variant quantities cannot be observables ! "

This dictates that, if ΔG is observable, it must be gauge-invariant !

Another comment on GIE

- C. Lorcé, arXiv : 1205.6483 [hep-ph]

Lorcé claimed that the Chen decomposition is a **GIE** based on Stückelberg trick.

This is misleading, since the Stückelberg is a trick, with which one can make a non-gauge theory into a gauge theory. The key is to introduce extra degrees of freedom by hand, which are called the **compensator** or **compensating field**.

We emphasize that the Chen decomposition is not a Stückelberg, since the color SU(3) gauge degrees of freedom is present from the very beginning in QCD.

4. The Chen decomposition is not a GIE a la Stückelberg

We clarify the following two facts in easier QED case given by the Hamiltonian :

$$H \;=\; \sum_i \frac{1}{2} m_i \dot{\boldsymbol{r}}_i^2 \;+\; \frac{1}{2} \int d^3 r \, [\, \boldsymbol{E}^2 + \boldsymbol{B}^2 \,]$$

(1) The Chen decomposition is not a GIE a la Stückelberg.
(2) There can be 2 independent GI decompositions of total angular momentum.

We start with the expression for the total angular momentum of this system.

$$\boldsymbol{J} \;=\; \sum_i \boldsymbol{r}_i \times m_i \dot{\boldsymbol{r}}_i + \int d^3 r \; \boldsymbol{r} \times [\, \boldsymbol{E} \times \boldsymbol{B} \,]$$

There is no doubts that 2 terms of the r.h.s are both gauge-invariant.

As is well-known, the vector potential $\boldsymbol{A}$ of the photon field can be decomposed into longitudinal and transverse components as

$$\boldsymbol{A} \;=\; \boldsymbol{A}_{\parallel} \;+\; \boldsymbol{A}_{\perp}$$

with the property

$$\nabla \times \boldsymbol{A}_{\parallel} \;=\; 0, \qquad \nabla \cdot \boldsymbol{A}_{\perp} \;=\; 0$$

This longitudinal-transverse decomposition is unique, once the Lorentz frame of reference is fixed. Under a general gauge-transformation given by

$$A^0 \rightarrow A'^0 = A^0 - \frac{\partial}{\partial t}\Lambda(x), \quad \boldsymbol{A} \rightarrow \boldsymbol{A}' = \boldsymbol{A} + \nabla\Lambda(x)$$

the longitudinal and transverse components transform as

$$\boldsymbol{A}_{||} \rightarrow \boldsymbol{A}'_{||} + \nabla\Lambda(x), \quad \boldsymbol{A}_{\perp} \rightarrow \boldsymbol{A}'_{\perp} = \boldsymbol{A}_{\perp}$$

indicating that $\boldsymbol{A}_{||}$ carries **unphysical gauge degrees of freedom** !

To avoid misunderstanding, we emphasize that the above longitudinal-transverse decomposition should be clearly distinguished from the Coulomb gauge fixing.

The Coulomb gauge fixing is to require $\nabla \cdot \boldsymbol{A} = 0$.

Because $\nabla \cdot \boldsymbol{A}_{\perp} = 0$ by definition, this is equivalent to requiring that

$$\nabla \cdot \boldsymbol{A}_{||} = 0 \quad : \quad \text{Coulomg gauge condition}$$

Now that $\boldsymbol{A}_{||}$ is divergence-free as well as irrotational by definition, one can set

$$\boldsymbol{A}_{||} = 0 \quad \text{in Coulomb gauge}$$

Another important remarks

Naturally, the longitudinal-transverse decomposition of the 3-vector potential is Lorentz-frame dependent. (Anyhow, the whole treatment above is non-covariant !)

It is true that the Coulomb gauge condition $\nabla \cdot \boldsymbol{A} = 0$ is not preserved, once we move to different Lorentz frame. Here, we need another gauge-transformation to get vector potential satisfying the Coulomb gauge condition.

An equivalent way of doing the above procedure is to assume somewhat unusual Lorentz transformation property of the 4-vector potential (or gauge field)

$$U(\epsilon)\, A^\mu(x)\, U^{-1}(\epsilon) \;=\; A^\mu(x') \;-\; \epsilon^{\mu\nu}\, A_\nu(x') \;+\; \frac{\partial \Lambda(x', \epsilon)}{\partial x'_\mu}$$

as described in the **textbooks** of **Bjorken-Drell** and **Weinberg**.

In any case, the Lorentz-frame dependence of the longitudinal-transverse decomposition does not make any trouble, because one can start this decomposition in an arbitrarily chosen Lorentz frame.

After all, the gauge- and frame-independence of observables is the core of the celebrated Maxwell's electrodynamics as a Lorentz-invariant gauge theory !

Now we come back to our original task. We have already pointed out that the total angular momentum of the photon can be split into 3 pieces as

$$\boldsymbol{J}^{\gamma} = \boldsymbol{J}_{long} + \boldsymbol{J}_{trans}$$

$$\boldsymbol{J}_{long} = \sum_i q_i \boldsymbol{r}_i \times \boldsymbol{A}_{\perp}(\boldsymbol{r}_i) \Rightarrow \texttt{potential angular momentum}$$

$$\boldsymbol{J}_{trans} = \int d^3r \, E^l_{\perp} (\boldsymbol{r} \times \nabla) A^l_{\perp} + \int d^3r \, \boldsymbol{E}_{\perp} \times \boldsymbol{A}_{\perp}$$

What happens if we combine the potential angular momentum term with the "mechanical angular momentum" of charged particles ? We get

$$\sum_i \boldsymbol{r}_i \times m_i \dot{\boldsymbol{r}}_i + \sum_i \boldsymbol{r}_i \times q_i \boldsymbol{A}_{\perp}(\boldsymbol{r}_i) = \sum_i \boldsymbol{r}_i \times \left(\boldsymbol{p}_i - q_i \boldsymbol{A}_{||}(\boldsymbol{r}_i) \right)$$

Here, we have used the usual definition of the canonical momentum $\boldsymbol{p}_i$

$$\begin{aligned} \boldsymbol{p}_i &\equiv m_i \dot{\boldsymbol{r}}_i - q_i \boldsymbol{A}(\boldsymbol{r}_i) \\ &= m_i \dot{\boldsymbol{r}}_i - q_i \left(\boldsymbol{A}_{||}(\boldsymbol{r}_i) + \boldsymbol{A}_{\perp}(\boldsymbol{r}_i) \right) \end{aligned}$$

This leads to a gauge-invariant decomposition corresponding to Chen's.

$$\boldsymbol{J} \;=\; \boldsymbol{L}'_p \,+\, \boldsymbol{S}'_\gamma \,+\, \boldsymbol{L}'_\gamma$$

where

$$\begin{aligned}
\boldsymbol{L}'_p &= \sum_i \boldsymbol{r}_i \times (\boldsymbol{p}_i - q_i\, \boldsymbol{A}_{||}(\boldsymbol{r}_i)) \;\Rightarrow\; \sum_i \boldsymbol{r}_i \times \frac{1}{i} \boldsymbol{D}_{i,pure} \\
\boldsymbol{S}'_\gamma &= \int d^3 r\, \boldsymbol{E}_\perp \times \boldsymbol{A}_\perp \\
\boldsymbol{L}'_\gamma &= \int d^3 r\, E^k_\perp\, (\boldsymbol{r} \times \nabla)\, A^k_\perp
\end{aligned}$$

The gauge-invariance of the 1st term can easily be convinced from the gauge transformation property of the longitudinal component

$$\boldsymbol{A}_{||}(\boldsymbol{r}_i) \;\rightarrow\; \boldsymbol{A}_{||}(\boldsymbol{r}_i) \,+\, \nabla \Lambda(\boldsymbol{r}_i)$$

and the gauge transformation property of quantum mechanical w.f. of charged particle system :

$$\Psi(\boldsymbol{r}_1, \cdots, \boldsymbol{r}_N) \;\rightarrow\; \left(\prod_i^N e^{i\, q_i\, \Lambda(\boldsymbol{r}_i)} \right) \Psi(\boldsymbol{r}_1, \cdots \boldsymbol{r}_N)$$

Let me emphasize again that the pure gauge derivative in the Chen formalism appears automatically or quite naturally.

The gauge degrees of freedom, carried by the longitudinal component

$$\boldsymbol{A}_{pure} \equiv \boldsymbol{A}_{||}$$

is not introduced by hand. **It exists from the beginning in the original theory !**

The Chen decomposition is not a GIE by the Stückelberg trick !

basically a **transverse-longitudinal decomposition**.

Note however that the Chen decomposition is not only one GI decomposition !

Because the potential angular momentum

$$\boldsymbol{J}_{long} = \sum_i q_i \, \boldsymbol{r}_i \times \boldsymbol{A}_{\perp}(\boldsymbol{r}_i) = \int d^3r \, \boldsymbol{r} \times (\boldsymbol{E}_{||} \times \boldsymbol{B}_{\perp})$$

is solely gauge-invariant, we can leave it in the photon part, which leads to another GI decomposition.

decomposition (I) according to our classification decomposition (I)

$$\boldsymbol{J} \;=\; \boldsymbol{L}_p + \boldsymbol{S}_\gamma + \boldsymbol{L}_\gamma$$

where

$$\boldsymbol{L}_p = \sum_i m_i \boldsymbol{r}_i \times \dot{\boldsymbol{r}}_i = \sum_i m_i \boldsymbol{r}_i \times (\boldsymbol{p}_i - q_i \boldsymbol{A}(\boldsymbol{r}_i)) \Rightarrow \sum_i \boldsymbol{r}_i \times \frac{1}{i} \boldsymbol{D}_i$$

$$\boldsymbol{S}_\gamma = \boldsymbol{S}'_\gamma = \int d^3 r \, \boldsymbol{E}_\perp \times \boldsymbol{A}_\perp$$

$$\boldsymbol{L}_\gamma = \int d^3 r \, E^k_\perp (\boldsymbol{r} \times \nabla) A^k_\perp + \int d^3 r \, \boldsymbol{r} \times (\boldsymbol{E}_{||} \times \boldsymbol{B}_\perp)$$

canonical OAM term **potential OAM term**

characteristic features

- The difference with the decomposition (II) exist only in orbital parts.
- The intrinsic photon spin part is just common in the two decompositions.
- This intrinsic photon spin part is definitely gauge-invariant !

another important remark

interaction term ?

It is a wide-spread belief that, among the following two quantities :

$$\boldsymbol{L}_{can} = \boldsymbol{r} \times \boldsymbol{p} \quad \Longleftrightarrow \quad \boldsymbol{L}_{mech} = \boldsymbol{r} \times (\boldsymbol{p} - e\,\boldsymbol{A}_\perp)$$

what is closer to physical image of orbital motion is the former, because the latter appears to contain an extra **interaction term** with the gauge field !

The fact is just opposite !

$$\begin{aligned} \boldsymbol{L}_{\text{“}can\text{”}} &= \boldsymbol{L}_{mech} + \sum_i \boldsymbol{r}_i \times q_i \boldsymbol{A}_\perp(\boldsymbol{r}_i) \\ &= \sum_i m_i \boldsymbol{r}_i \times \dot{\boldsymbol{r}}_i + \int d^3r\, \boldsymbol{r} \times (\boldsymbol{E}_{||} \times \boldsymbol{B}_\perp) \end{aligned}$$

orbital motion !

- It is the “mechanical” angular momentum $\boldsymbol{L}_{mech}$ not the “canonical” angular momentum $\boldsymbol{L}_{\text{“}can\text{”}}$ that has a natural physical interpretation as orbital motion of particles !
- It may sound **paradoxical**, but what contains an extra interaction term is rather the “canonical” angular momentum than the “mechanical” angular momentum !

5. What is needed to settle the controversies

We have shown that each term of our nucleon spin decomposition (I) and (II) is separately gauge invariant, as long as the two parts of the decomposition

$$A^{\mu} = A^{\mu}_{phys} + A^{\mu}_{pure}$$

satisfy the following conditions under general color SU(3) gauge transformation :

$$A^{\mu}_{phys}(x) \rightarrow U(x)\, A^{\mu}_{phys}(x)\, U^{\dagger}(x)$$
$$A^{\mu}_{pure}(x) \rightarrow U(x) \left(A^{\mu}_{pure}(x) + \frac{i}{g}\, \partial^{\mu} \right) U^{\dagger}(x)$$

However, the fact that we did not give explicit formula for $A^{\mu}_{phys}(x)$ and $A^{\mu}_{pure}(x)$ was sometimes criticized, and has been a cause of misunderstanding.

To answer this criticism, we first recall the fact that the above decomposition is essentially (or physically) the **transverse-longitudinal decomposition**.

From the **physical viewpoint**, the massless gauge field has only 2 transverse degrees of freedom, and the other components are unphysical gauge D. O. F.

As was pointed out before, however, the transverse-longitudinal decomposition can be made, only after specifying a particular Lorentz frame.

Fortunately, there exists a convenient method, with which we can make this decomposition in a form, which is convenient for **perturbative calculations**.

The key is a introduction of a constant 4-vector n^μ.

[Example] **Coulomb gauge-type projector in QED case**

- M. Lavelle and D. McMullan, Phys. Lett. B312 (1993) 211.

$$A_\mu^{phys}(x) = P_{\mu\nu} A^\nu(x)$$

where

$$P_{\mu\nu} = g_{\mu\nu} - \frac{\partial_\mu \partial_\nu + \Box n_\mu n_\nu - (n_\mu \partial_\nu + n_\nu \partial_\nu) n \cdot \partial}{\Box - (n \cdot \partial)^2}$$

with n being the temporal vector $n^\mu = (1,0,0,0)$.

One can easily check that this projector satisfies the **transversality condition** :

$$\partial^\mu P_{\mu\nu} = P_{\mu\nu} \partial^\nu = 0$$

general axial-gauge type projector in QCD

$$A^{phys}_{\mu\nu}(x) = P_{\mu\nu}\, A^{\nu}(x)$$

$$P_{\mu\nu} = g_{\mu\nu} - \frac{n_\mu\, \partial_\nu + n_\nu\, \partial_\mu}{n \cdot \partial} + \frac{\Box\, n_\mu\, n_\nu}{(n \cdot \partial)^2}$$

with n^μ being an arbitrary constant 4-vector $(n^2 > 0, \text{ or } n^2 = 0, \text{ or } n^2 < 0)$

The above projector also satisfies the **transversality condition** :

$$\partial^\mu\, P_{\mu\nu} = P_{\mu\nu}\, \partial^\nu = 0$$

gauge transformation property of $A^{phys}_\mu(x)$:

$$\begin{aligned} A^{phys}_\mu(x) &\equiv P_{\mu\nu}\, A^\nu(x) \\ &\rightarrow P_{\mu\nu}\, U(x) \left(A^\nu(x) + \frac{i}{g}\, \partial^\nu \right) U^\dagger(x) \\ &= U(x) \left(P_{\mu\nu}\, A^\nu(x) + \frac{i}{g}\, P_{\mu\nu}\, \partial^\nu \right) U^\dagger(x) \\ &= U(x)\, A^{phys}_\mu(x)\, U^\dagger(x) \end{aligned}$$

desirable covariant or homogeneous gauge transformation law seemingly **covariant- and gauge-invariant gluon spin operator**

$$M^{\mu\nu\lambda}_{G-spin} \; = \; 2\,\mathrm{Tr}\left[F^{\mu\lambda} A^{\nu}_{phys} \; + \; F^{\nu\mu} A^{\lambda}_{phys} \right]$$

where

$$A^{phys}_{\mu\nu}(x) \; = \; P_{\mu\nu} A^{\nu}(x)$$

with

$$P_{\mu\nu} \; = \; g_{\mu\nu} \; - \; \frac{n_\mu \partial_\nu \; + \; n_\nu \partial_\mu}{n \cdot \partial} \; + \; \frac{n_\mu n_\nu \Box}{(n \cdot \partial)^2}$$

We have calculated the 1-loop anomalous dimension of the above gluon spin operator, and found that it reproduces the standardly-known answer :

$$\begin{pmatrix} \Delta\gamma_{qq} & \Delta\gamma_{qG} \\ \Delta\gamma_{Gq} & \Delta\gamma_{GG} \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ \frac{\alpha_S}{2\pi} \cdot \frac{3}{2} C_F & \frac{\alpha_S}{2\pi} \left(\frac{11}{6} C_A \; - \; \frac{1}{3} n_f \right) \end{pmatrix}$$

irrespectively of the choice of n^μ !

⇩

gauge-independence of ΔG

6. What is a pitfall of GIE approach ?

Lorcé and Pasquini gave a useful relation between OAM and Wigner distribution :

$$L_{\mathcal{U}} = \int dx\, d^2 \boldsymbol{b}_\perp\, d^2 \boldsymbol{k}_\perp\, (\boldsymbol{b}_\perp \times \boldsymbol{k}_\perp)_z\, W^{\mathcal{U}}(x, \boldsymbol{b}_\perp, \boldsymbol{k}_\perp)$$

GI definition of Wigner distribution depends on the path $\mathcal{U}$ of gauge link.

$$W^{\mathcal{U}}(x, \boldsymbol{b}_\perp, \boldsymbol{k}_\perp) = \int \frac{d^2 \boldsymbol{q}_\perp}{(2\pi)^2} \int \frac{d^2 \boldsymbol{\xi}\perp\, d\xi^-}{(2\pi)^3} \times e^{i(x P^+ \xi^- - \boldsymbol{k}_\perp \cdot \boldsymbol{\xi}_\perp)} \langle P' S' \,|\, \bar{q}(0)\, \gamma^+ \mathcal{L}_{\mathcal{U}}[0, \xi]\, q(\xi) \,|\, P S \rangle$$

Hatta showed that the LC-like path choice gives

$$L_{\text{“can”}} = \int dx\, d^2 \boldsymbol{b}_\perp\, d^2 \boldsymbol{k}_\perp\, (\boldsymbol{b}_\perp \times \boldsymbol{k}_\perp)_z\, W^{LC}(x, \boldsymbol{b}_\perp, \boldsymbol{k}_\perp)$$

On the other hand, Ji, Xiong, and Yuan claim that the straight path connecting $(0^-, \boldsymbol{0}_\perp)$ and $(\xi^-, \boldsymbol{\xi}_\perp)$ gives

$$L_{\text{“dyn”}} = \int dx\, d^2 \boldsymbol{b}_\perp\, d^2 \boldsymbol{k}_\perp\, (\boldsymbol{b}_\perp \times \boldsymbol{k}_\perp)_z\, W^{straight}(x, \boldsymbol{b}_\perp, \boldsymbol{k}_\perp)$$

could be wrong ? $\Longrightarrow$ C. Lorcé, arXiv : 1210.2581 [hep-ph].

Burkardt showed that the difference between the two OAMs is the change in OAM as the quark moves through the color field created by the spectator :

$$L^q_{\text{“}can\text{''}} - L^q_{\text{“}dyn\text{''}} = \frac{\int d^3r \, \langle PS \,|\, \bar{q}(\boldsymbol{r}) \, \gamma^+ \int_{r^-}^{\infty} dy^- \, T^z(y^-, \boldsymbol{r}_\perp) \, q(\boldsymbol{r}) \,|\, Ps \rangle}{\langle PS \,|\, Ps \rangle}$$

with

$$T^z(\boldsymbol{r}) \equiv g \left(x \, F^{+y}(\boldsymbol{r}) - y \, F^{+x}(\boldsymbol{r}) \right)$$

According to him, $L^q_{\text{“}dyn\text{''}}$ represents a local and manifestly gauge-invariant OAM of the quark **before** it has been struck by the virtual photon, while $L^q_{\text{“}can\text{''}}$ does a gauge-invariant OAM **after** it has left the nucleon and move to the infinity.

He, however, confesses that no practical experiment has been identified yet to measure the OAM of quarks **after** they have been ejected in DIS.

At any rate, Burkardt's paper vividly illustrates model-dependent nature of the GIE approach with introduction of the gauge link.

In my opinion, the GIE approach is equivalent to the standard treatment of gauge theory, only when its extension by means of **gauge link** is **path-independent**, or equivalently **process-independent** !

By the **standard treatment of the gauge theory**, I mean the following :

- ♣ Start with a gauge-invariant quantity or expression.
- ♣ Fix gauge according to the necessity of practical calculation.
- ♣ Answer should be independent of gauge choice.

Our QED example shows that, except for the choice of Lorentz frame, there is no arbitrariness in the decomposition $A^\mu = A^\mu_{phys} + A^\mu_{pure}$, as related to the Stückelberg-like transformation of Lorcé.

Although A^μ_{pure} changes arbitrarily under the gauge-transformation, A^μ_{phys} is essentially a unique object, constrained by the transversality condition.

This reconfirms our claim that there exist only 2 physically inequivalent GI decompositions of the nucleon spin :

decomposition (I) & decomposition (II)

7. Summary

- We have established the existence of **two** physically inequivalent GI decompositions of the nucleon spin, the **decompositions (I) and (II)**, with particular emphasis upon the existence of two types of OAM, i.e.

dynamical OAM & "canonical" OAM

- We confirmed that the dynamical OAMs of quarks and gluons appearing in the decomposition (I) can in principle be extracted model-independently from combined analysis of GPD and polarized DIS measurements.

$$
\begin{aligned}
L_q &= \langle p \uparrow | \, M^{012}_{q-OAM} \, | p \uparrow \rangle \\
&= \frac{1}{2} \int_{-1}^{1} x \, [\, H^q(x,0,0) + E^q(x,0,0) \,] \, dx - \frac{1}{2} \int_{-1}^{1} \Delta q(x) \, dx \\
L_G &= \langle p \uparrow | \, M^{012}_{G-OAM} \, | p \uparrow \rangle \\
&= \frac{1}{2} \int_{-1}^{1} x \, [\, H^g(x,0,0) + E^g(x,0,0) \,] \, dx - \int_{-1}^{1} \Delta g(x) \, dx
\end{aligned}
$$

- This means that we now have at least one satisfactory solution to the nucleon spin decomposition problem, which has observational basis.

- ♣ On the other hand, the observability of the OAM appearing in the decomposition (II), i.e. the generalized “canonical’’ OAM, is not clear yet !

- ♣ This is partly because the relation between this “canonical “ OAM and observables is given through the Wigner distributions, the path-independence or process-independence of which should be convinced more carefully !

- ♣ Moreover, once quantum loop effects is included, the very existence of TMDs as well as Wigner distributions satisfying gauge-invariance and factorization (or universality) simultaneously is being questioned !

$L_{\text{“}can\text{''}}$ ⇨ Is process-independent extraction possible or not ?

Still a challenging open question !

Stückelberg transformation (abelian case)

$$A_\mu^{pure}(x) \rightarrow A_\mu^{pure,g}(x) = A_\mu^{pure}(x) - \partial_\mu C(x)$$
$$A_\mu^{phys}(x) \rightarrow A_\mu^{phys,g}(x) = A_\mu^{phys}(x) + \partial_\mu C(x)$$
$$C(x) \;:\; \texttt{arbitrary funcion of space-time}$$

contradiction with standard longitudinal-transverse decomposition

$$\boldsymbol{A} = \boldsymbol{A}_{\parallel} + \boldsymbol{A}_{\perp} \quad \texttt{with} \quad \nabla \times \boldsymbol{A}_{\parallel} = 0, \quad \nabla \cdot \boldsymbol{A}_{\perp} = 0$$

In fact, under Stückelberg

$$\boldsymbol{A}_{\parallel} \rightarrow \boldsymbol{A}_{\parallel}^{g} = \boldsymbol{A}_{\parallel} - \nabla C(x)$$
$$\boldsymbol{A}_{\perp} \rightarrow \boldsymbol{A}_{\perp}^{g} = \boldsymbol{A}_{\perp} + \nabla C(x)$$

then

$$\nabla \times \boldsymbol{A}_{\parallel}^{g} = \nabla \times (\boldsymbol{A}_{\parallel} - \nabla C(x)) = \nabla \times \boldsymbol{A}_{\parallel} \quad (\texttt{O.K.})$$

but

$$\nabla \cdot \boldsymbol{A}_{\perp}^{g} = \nabla \cdot (\boldsymbol{A}_{\perp} + \nabla C(x)) = \nabla \cdot \boldsymbol{A}_{\perp} + \Delta C(x)$$
$$\neq \nabla \cdot \boldsymbol{A}_{\perp} \quad \texttt{unless} \quad \Delta C(x) = 0$$

transverse condition is not preserved by Stückelberg !

Nontrivial problems in the Coulomb gauge calculation of evolution matrix

- Lorentz-frame dependence ?
- Role of instantaneous Coulomb interaction ?
- Coulomb gauge Ward-identity generally requires ghost field !
- Ambiguous nature of loop-integral ?

[ex.] might need sophisticated regularization method like split dimensional regularization of Leibbrandt ?

$$d^4 q \;=\; dq_4\, d^3\boldsymbol{q} \;\Rightarrow\; d^{2\sigma}\, d^{2\,\omega}\, \boldsymbol{q}\Big|_{\omega \to (3/2)^+}^{\sigma \to (1/2)^+}$$

- Might need a certain limiting procedure ?

[ex.] taking a Coulomb-gauge limit of interpolating gauge between the Coulomb gauge and the Landau gauge etc. ?

transverse (or physical) propagator in Coulomb-gauge-like treatment

$$D^{phys}_{\mu\nu}(k) = \int \frac{d^4 k}{(2\pi)^4} e^{i\,k\cdot(x-y)} \left\langle T \left(A^{phys}_{\mu}(x)\, A^{phys}_{\nu}(y) \right) \right\rangle$$

$$\stackrel{L.O.}{=} \frac{-i}{k^2 + i\,\varepsilon} \left[g_{\mu\nu} - \frac{k_\mu k_\nu + k^2 n_\mu n_\nu - k\cdot n\,(k_\mu n_\nu + k_\nu n_\mu)}{k^2 - (k\cdot n)^2} \right]$$

This is slightly different from the standard Coulomb-gauge propagator given as

$$D^{Coulomb}_{\mu\nu}(k) = \int \frac{d^4 k}{(2\pi)^4} e^{i\,k\cdot(x-y)} \langle T \left(A_{\mu}(x)\, A_{\nu}(y) \right) \rangle$$

$$\stackrel{L.O.}{=} \frac{-i}{k^2 + i\,\varepsilon} \left[g_{\mu\nu} - \frac{k_\mu k_\nu - k\cdot n\,(k_\mu n_\nu + k_\nu n_\mu)}{k^2 - (k\cdot n)^2} \right]$$

The difference is that the latter contains instantaneous Coulomb interaction !