Gluon TMDs in pp-collisions

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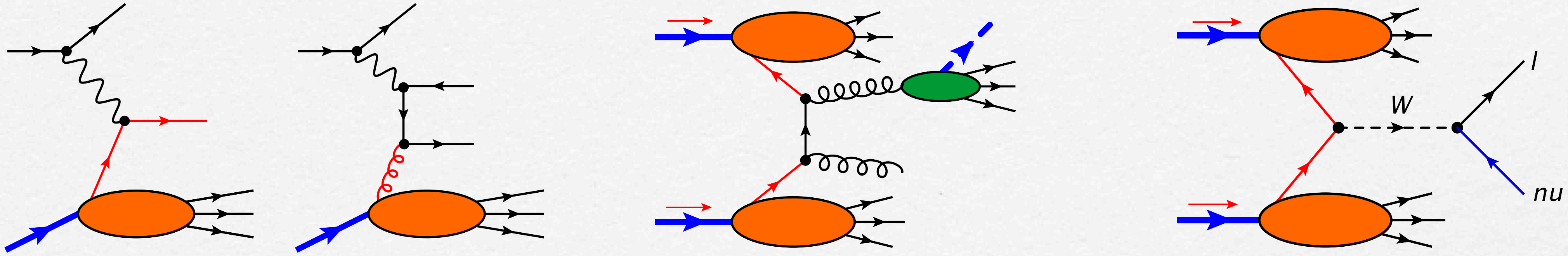
TMD vs. collinear factorization

Collinear factorization in pQCD

- applicable to one-scale processes, e.g. 1-particle inclusive processes



$$\boxed{\frac{d\sigma}{dx\, dQ^2}}$$

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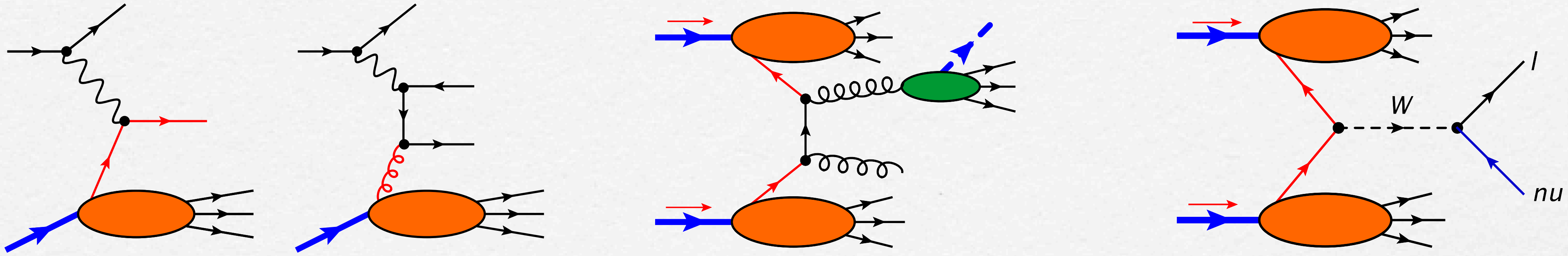

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☐ Cross sections at high energies → (hard part) × (soft parts)

☐ hard part → pQCD (NLO, NNLO,...) ; soft parts → universal, 1-dim

collinear parton distributions

$$\boxed{q(x,\mu),\ \Delta q(x,\mu),\ \delta q(x,\mu)} \qquad \boxed{G(x,\mu),\ \Delta G(x,\mu)}$$

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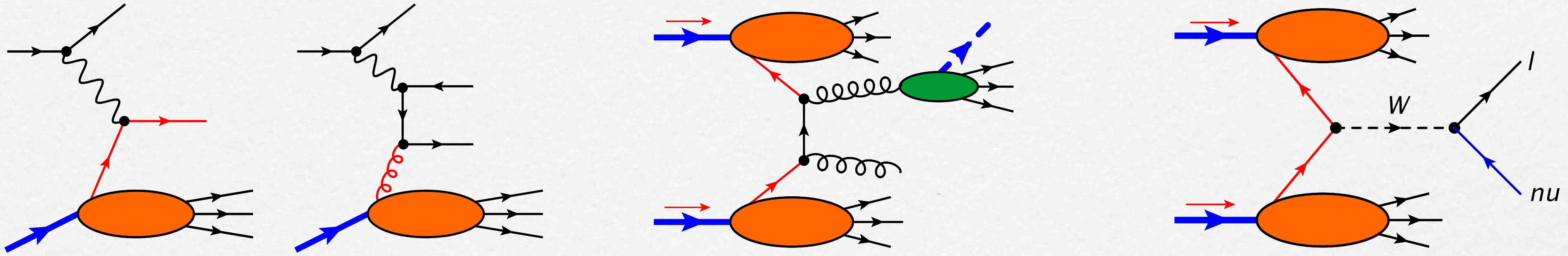

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☐ also for higher twist observables → Single-Spin Asymmetries (ETQS)

☐ Collinear factorization: 2 (or more...)-particle inclusive processes

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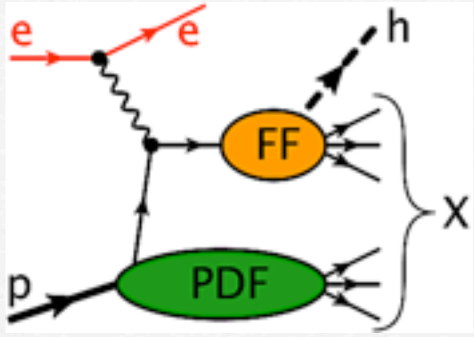

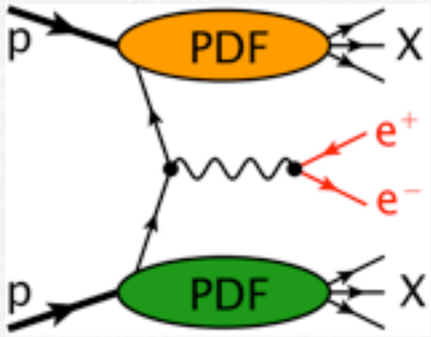

Drell-Yan

two scales: hard scale Q + final state transverse momentum q_T

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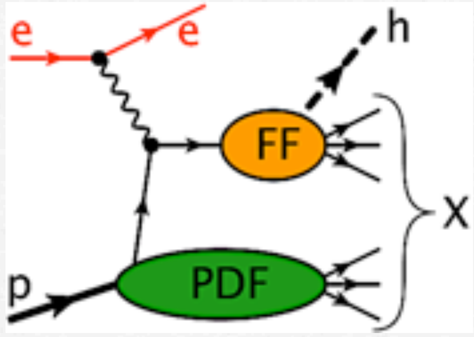

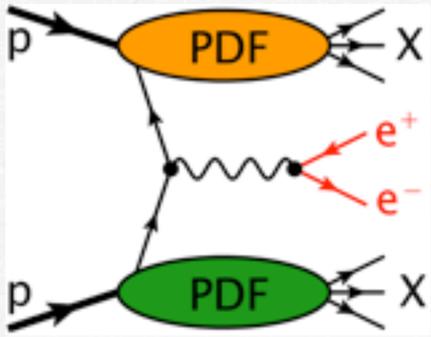

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→ integrated observables $$\int d^2 q_T \, w(q_T) \, \frac{d\sigma}{dx \, dQ^2 \, dq_T} \equiv \langle w(q_T) \rangle$$

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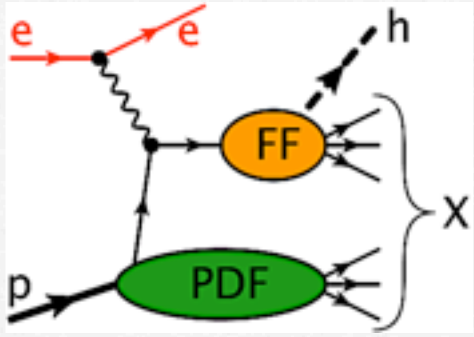

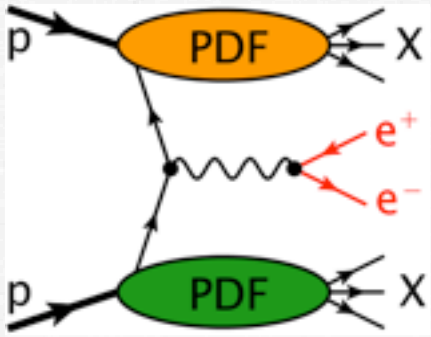

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$$\boxed{\frac{d\sigma}{dq_T}(q_T \sim Q)}$$

one scale → collinear factorization ok,
transverse momentum generated perturbatively in hard part

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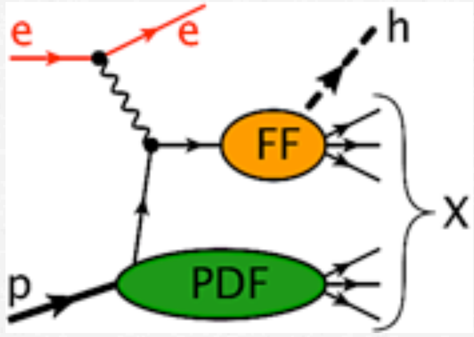

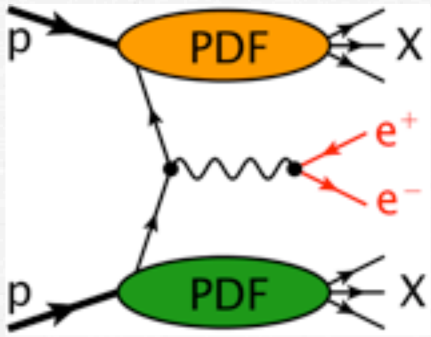

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large logs in the hard part (gluon radiation) $\log^n(q_T/Q)$

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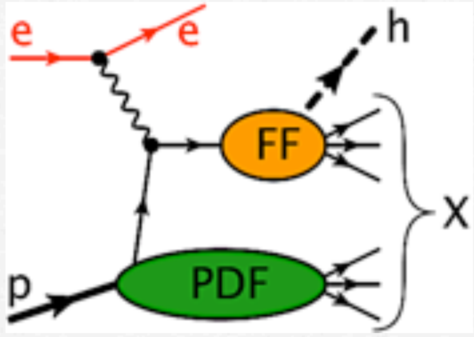

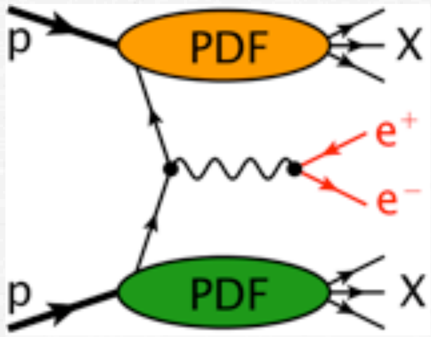

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→ Transverse momentum dependent (TMD) factorization!

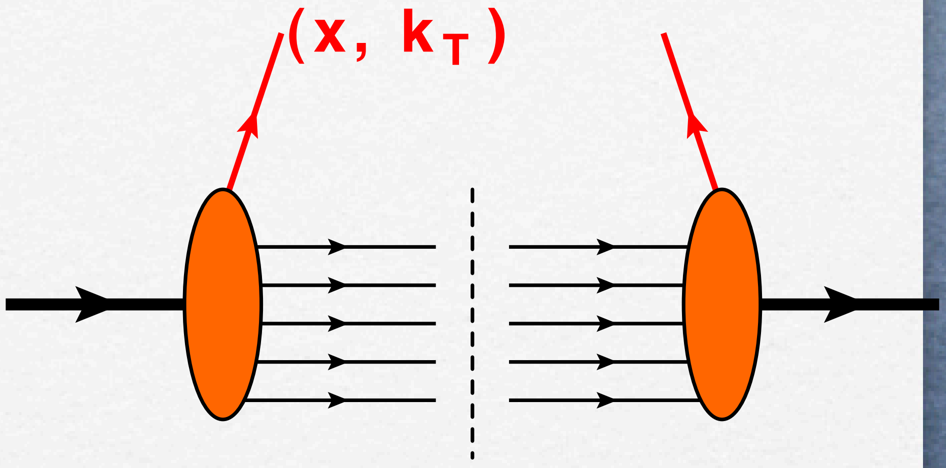
Idea of TMDs:

transverse momentum q_T from "intrinsic" transverse parton momentum k_T

→ different kind of factorization

→ additional degree of freedom of partonic motion

→ study different aspects of hadron spin structure (e.g. 3-d momentum structure, spin-orbit correlations, etc.)



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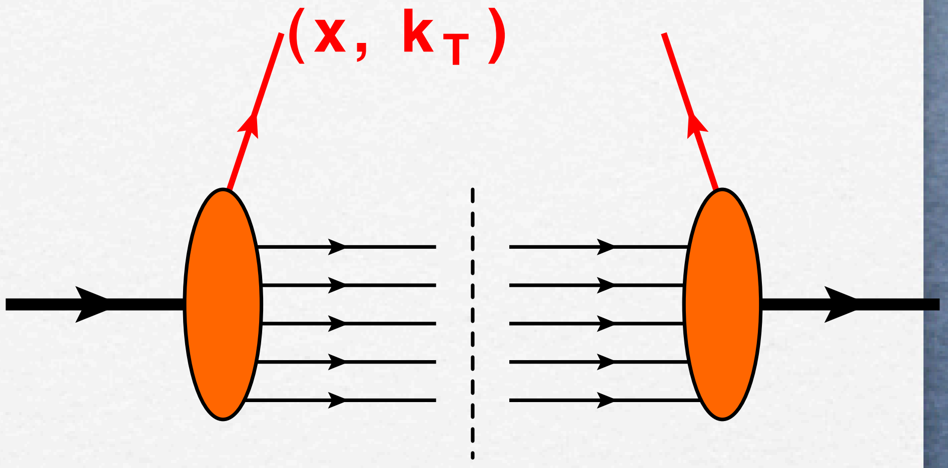
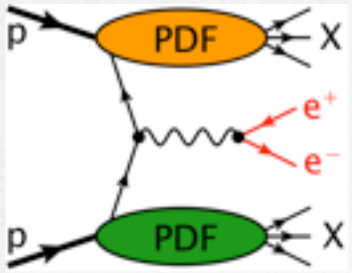


☐ All-order factorization theorem for, e.g., Drell-Yan



$$\boxed{W^{\mu\nu} \sim \int d^2k_{aT}d^2k_{bT}\,\delta^{(2)}(\vec{k}_{aT}+\vec{k}_{bT}-\vec{q}_T)\,\mathrm{Tr}[\hat{M}^{\mu}\Phi(x_a,\vec{k}_{aT})(\hat{M}^{\nu})^{\dagger}\bar{\Phi}(x_b,\vec{k}_{bT})] + Y^{\mu\nu}}$$

$q_T \ll Q$

$q_T \simeq Q$

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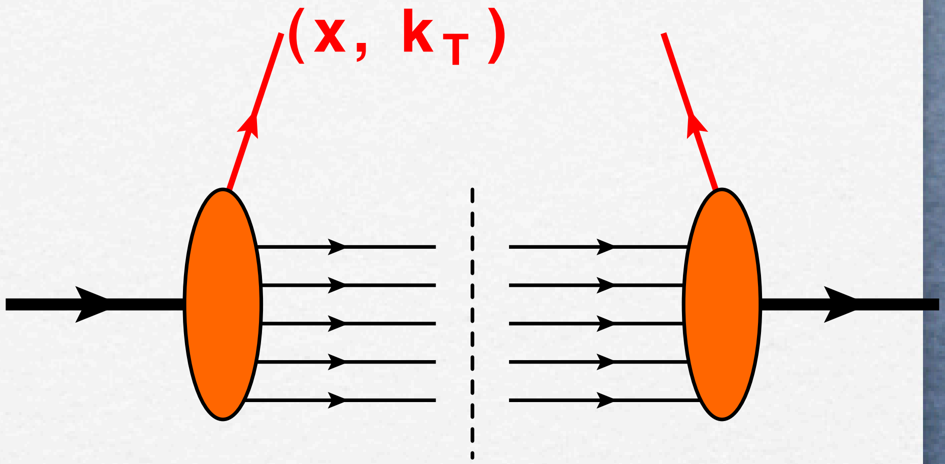

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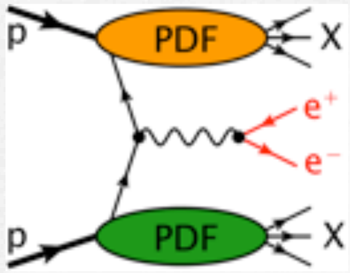

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$q_T \ll Q$ $\qquad$ $q_T \simeq Q$

☐ proven for SIDIS + pp - collisions with color singlet final states

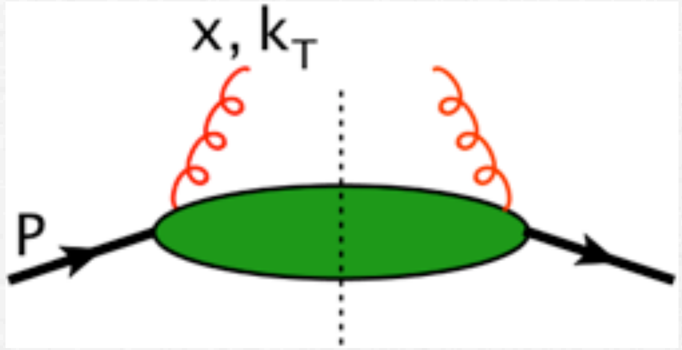
[Collins; Ji, Ma, Yuan; Qiu; Rogers, Mulders; ...]

Gluon TMDs

$$\Gamma^{ij}(x,\vec{k}_T) = \frac{1}{xP^+}\int\frac{dz^-\,d^2z_T}{(2\pi)^3}\,\mathrm{e}^{ik\cdot z}\langle P,S|\,F^{+i}(0)\,\mathcal{W}[0\,;\,z]\,F^{+j}(z)\,|P,S\rangle\Big|_{z^+=0}$$

$\Gamma^{[T-even]}(x,\vec{k}_T)$			$\Gamma^{[T-odd]}(x,\vec{k}_T)$	
		flip		flip
U	f_1^g	$h_1^{\perp g}$		
L	$g_{1L}^{\perp g}$			$h_{1L}^{\perp g}$
T	$g_{1T}^{\perp g}$		$f_{1T}^{\perp g}$	h_1^g $h_{1T}^{\perp g}$

[Mulders, Rodriues, PRD 63,094021]



- ✳ gluonic correspondence to "Boer-Mulders": T-even
- ✳ unpolarized gluons in transversely pol. proton: gluon Sivers function
- ✳ gluonic transversity / pretzelosity / wormgears: T-odd
- ✳ no chirality
- ✳ two collinear PDFs

Processes sensitive to gluon TMDs

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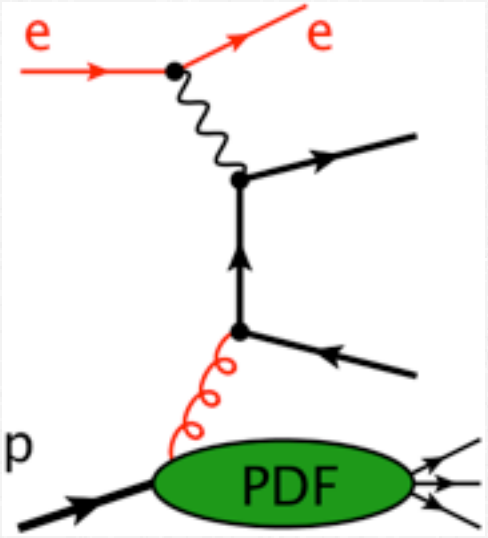

Heavy Quark production in e + p - collisions

[Boer, Brodsky, Mulders, Pisano, PRL 106, 132001]

$$\boxed{e\,+\,p^{(\uparrow)}\,\rightarrow e'\,+\,\mathrm{jet}(c,b)\,+\,\mathrm{jet}(\bar{c},\bar{b})\,+\,X}$$

TMD factorization ok!

Spin dependent (+independent...) gluon TMDs: EIC would be ideal!

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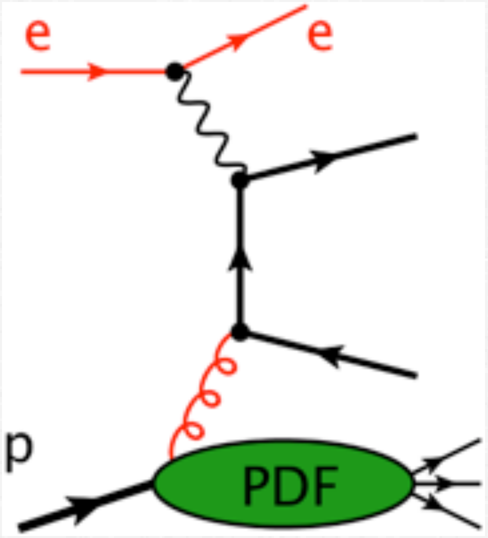

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unpolarized cross section:

$$\boxed{\frac{d\sigma_{UU}}{dq_T} \propto \left(F_1 + F_2 \cos(2\phi)\right)}$$

azimuthally independent term: $F_1 \propto f_1^g(x, q_T)$ → unpol. gluon distribution

azimuthally dependent term: $F_2 \propto h_1^{\perp g}(x, q_T)$ → linearly pol. gluons

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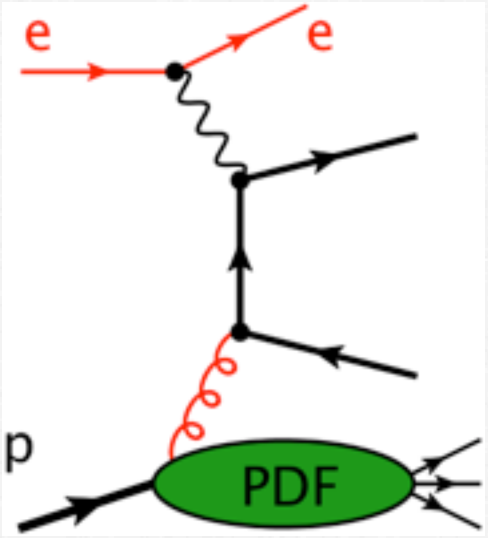

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Heavy "back-to-back" dijet production in pp - collisions:

TMD factorization problematic!

pp - collisions

Gluon TMDs measurable in pp-collisions with color singlet final states

$$p + p \to \Big(\cancel{(l\bar{l})}, (\gamma\gamma), (\gamma\, l\bar{l}), (l'\bar{l}')(l\bar{l}), ... \Big) + X \qquad \text{(RHIC / LHC)}$$

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Photon Pair production

[Qiu, M.S., Vogelsang, PRL 107, 062001 (2011)]

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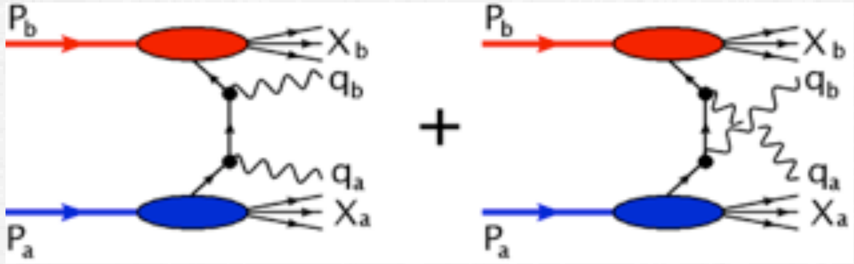
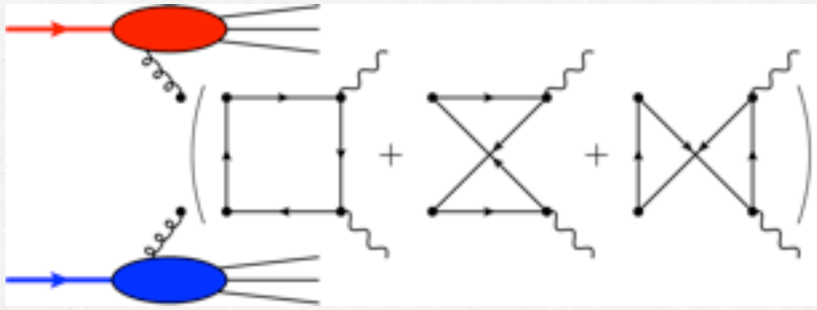


gluon TMDs at $O(\alpha_s^2)$



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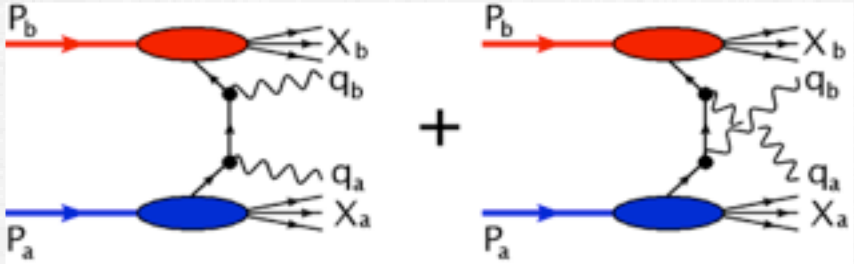

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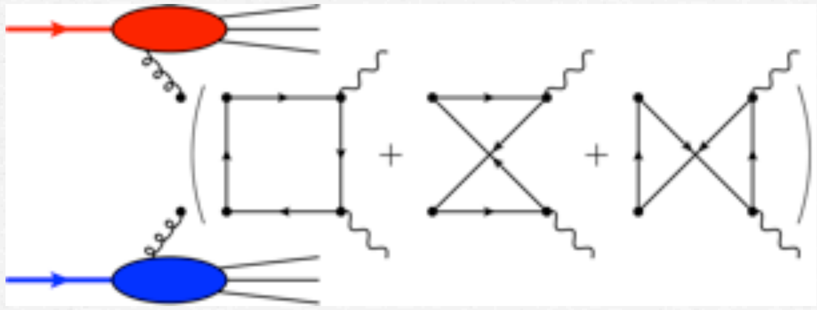
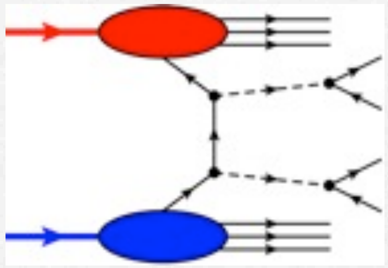
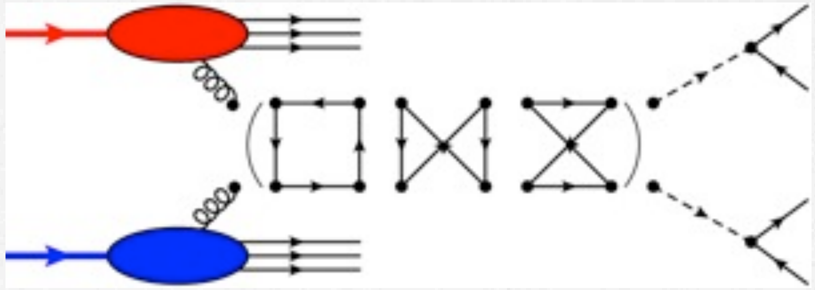


Other leptonic final states (2lγ, 4l)

[Boer, den Dunnen, Pisano, M.S., Vogelsang, in prep.]





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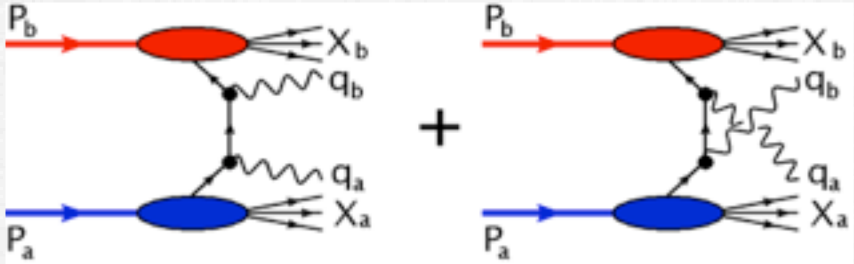

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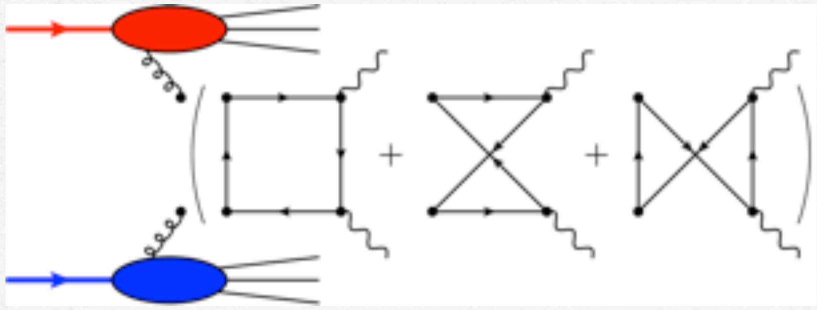

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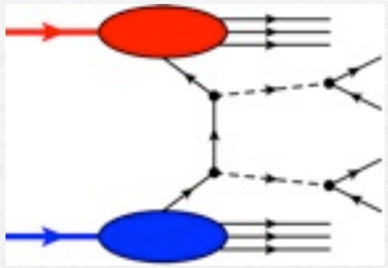

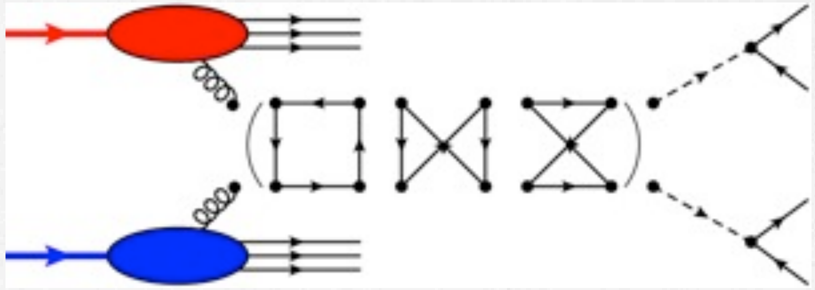

☐ no colored final state ⇒ TMD factorization ok

☐ Initial state interactions only, past-pointing Wilson lines

☐ gauge invariance ⇒ box finite ⇒ effectively tree-level

☐ potentially large gluon distributions

Unpolarized pp→$\gamma\gamma$X Cross-Section at $q_T << Q$

$$\frac{d\sigma_{UU}}{d^4q\,d\Omega} \sim \left(\frac{2}{\sin^2\theta}\right)\left((1+\cos^2\theta)[f_1^q \otimes f_1^{\bar{q}}] + \cos(2\phi)\sin(2\theta)[h_1^{\perp q} \otimes h_1^{\perp \bar{q}}]\right)$$

quark contributions → almost identical to DY

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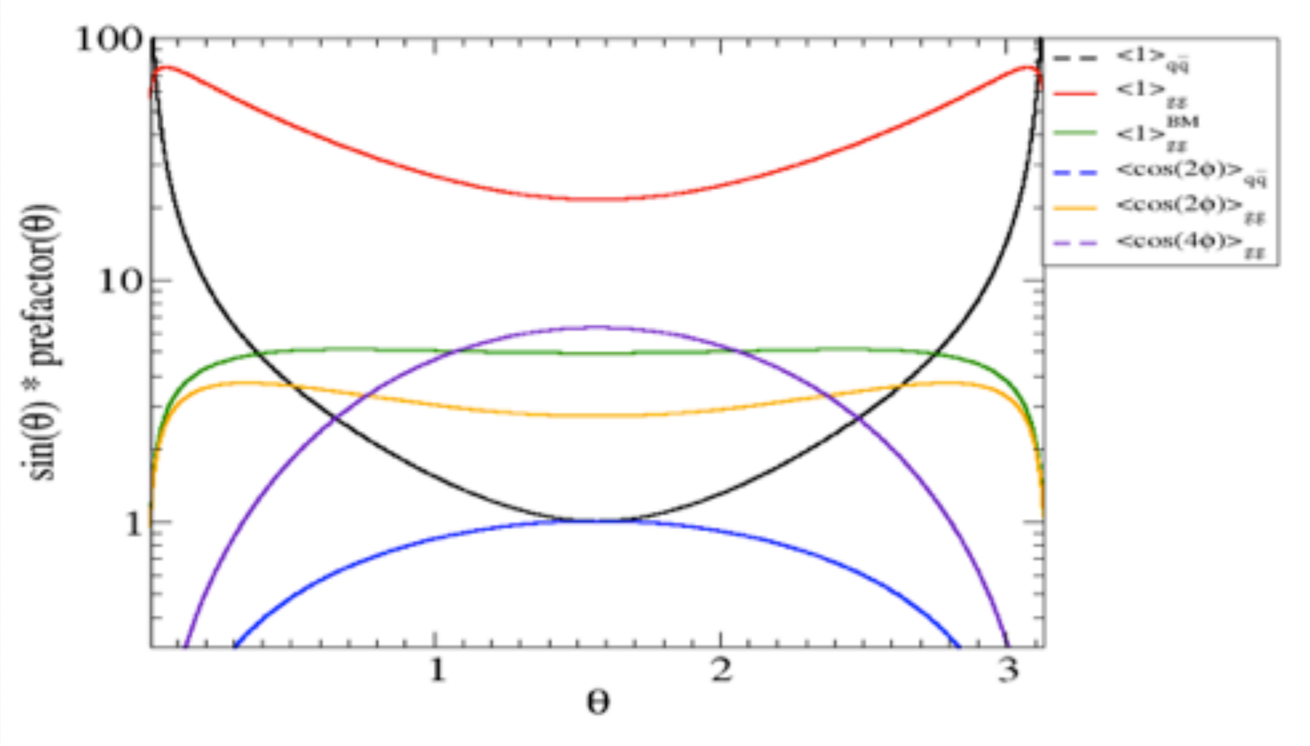
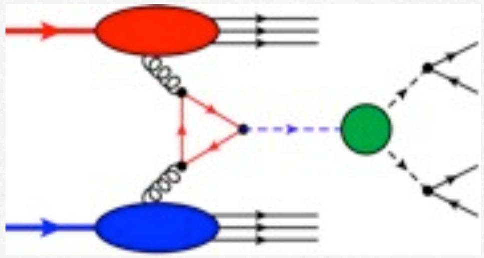


- cos(4φ) modulation a pure gluonic effect
- cos(2φ) → sign of gluon $h_1^\perp$
- requires p_T isolation cuts for the photons
- powerful in combination with DY
  → map out quark TMDs in DY
  → gluon TMDs in γγ

Can gluonic TMDs be useful for the LHC?

[Boer, den Dunnen, Pisano, M.S., Vogelsang, PRL 108, 032002 (2012)]

Main Higgs production mechanism: gluon fusion



A new boson (Higgs!?) is found at LHC...

want to determine its spin (0 or 2), parity, etc.

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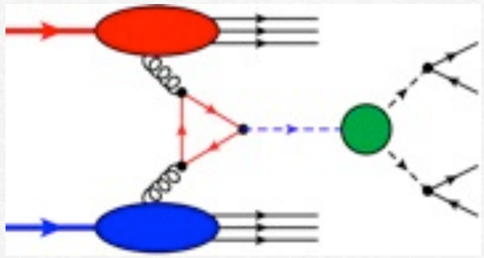
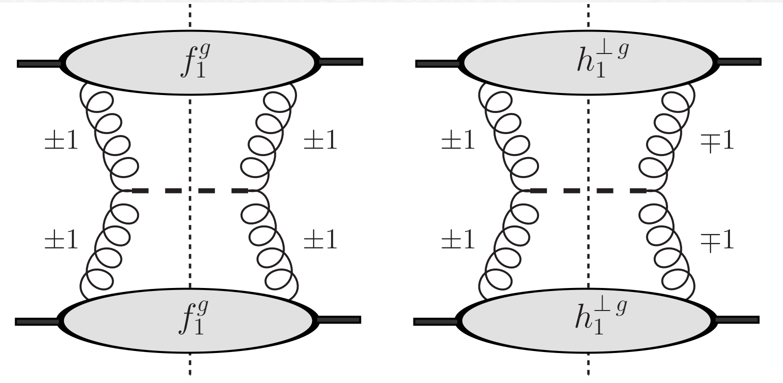


pure Higgs production



linearly polarized gluons sensitive to Higgs parity

$$\frac{d\sigma}{d^3q} \propto [f_1^g \otimes f_1^g] \pm [h_1^{\perp g} \otimes h_1^{\perp g}]$$

+: scalar Higgs   -: pseudoscalar Higgs

→ precise q_T measurement may offer a way to determine Higgs parity

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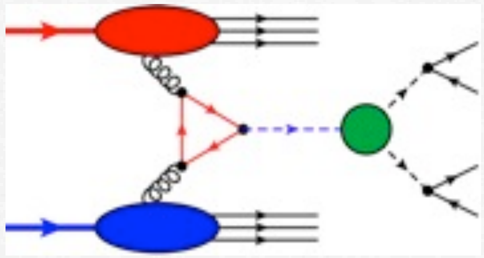

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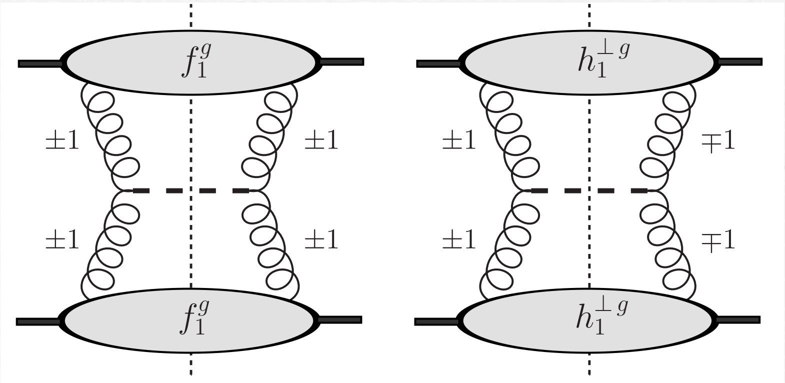

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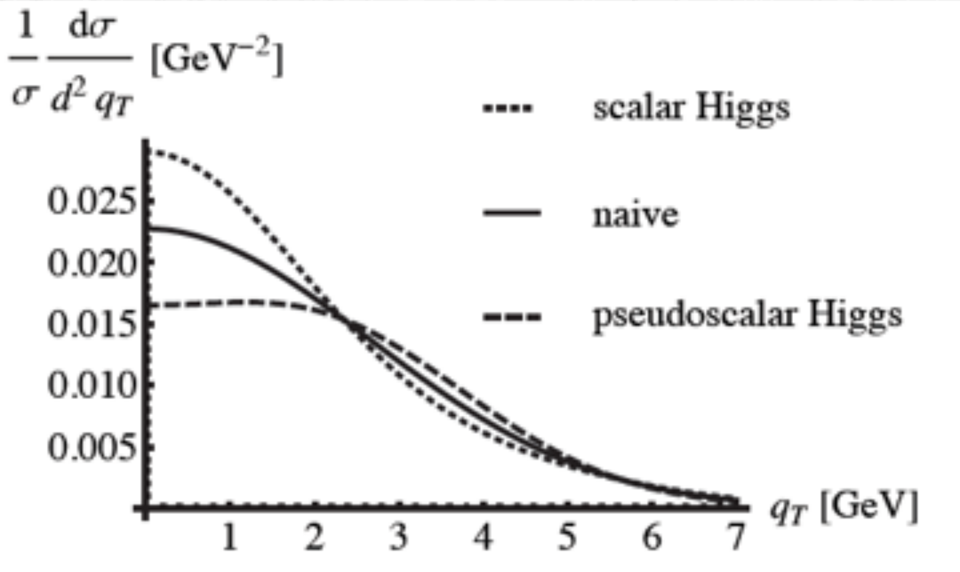


Numerical estimate:

Gaussian ansatz +

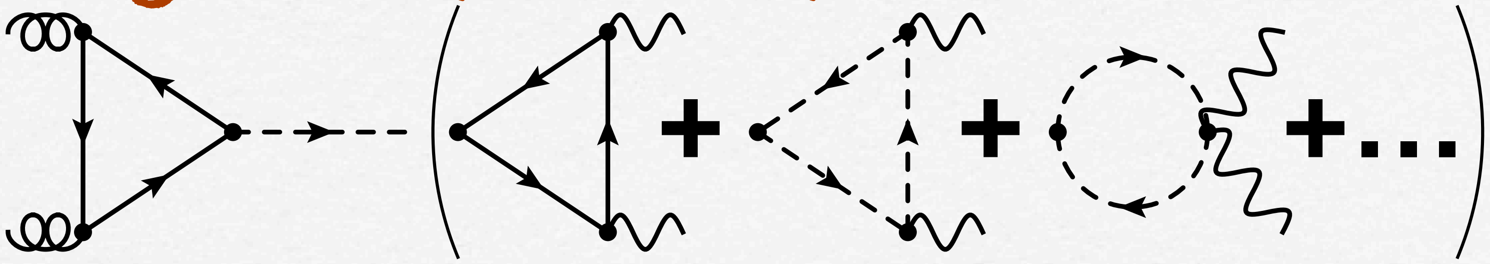
saturation of positivity bounds

$\langle p_T^2 \rangle = 7$ GeV^2

CS evolution of unpol./lin. pol. gluons studied in

[Sun, Xiao, Yuan, PRD 84, 094005]

Higgs decay into photon pairs: gg → H/A → γγ



φ - integrated cross section of Higgs + box:

$$\int d\phi \frac{d\sigma^{gg}}{d^4q\, d\Omega} \propto \bar{\mathcal{F}}_1 \left[f_1^g \otimes f_1^g\right] + \bar{\mathcal{F}}_2 \left[h_1^{\perp g} \otimes h_1^{\perp g}\right.$$

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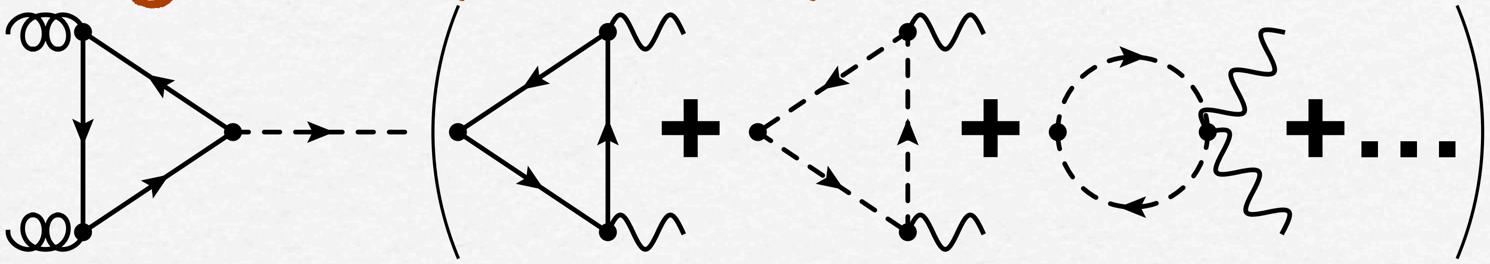

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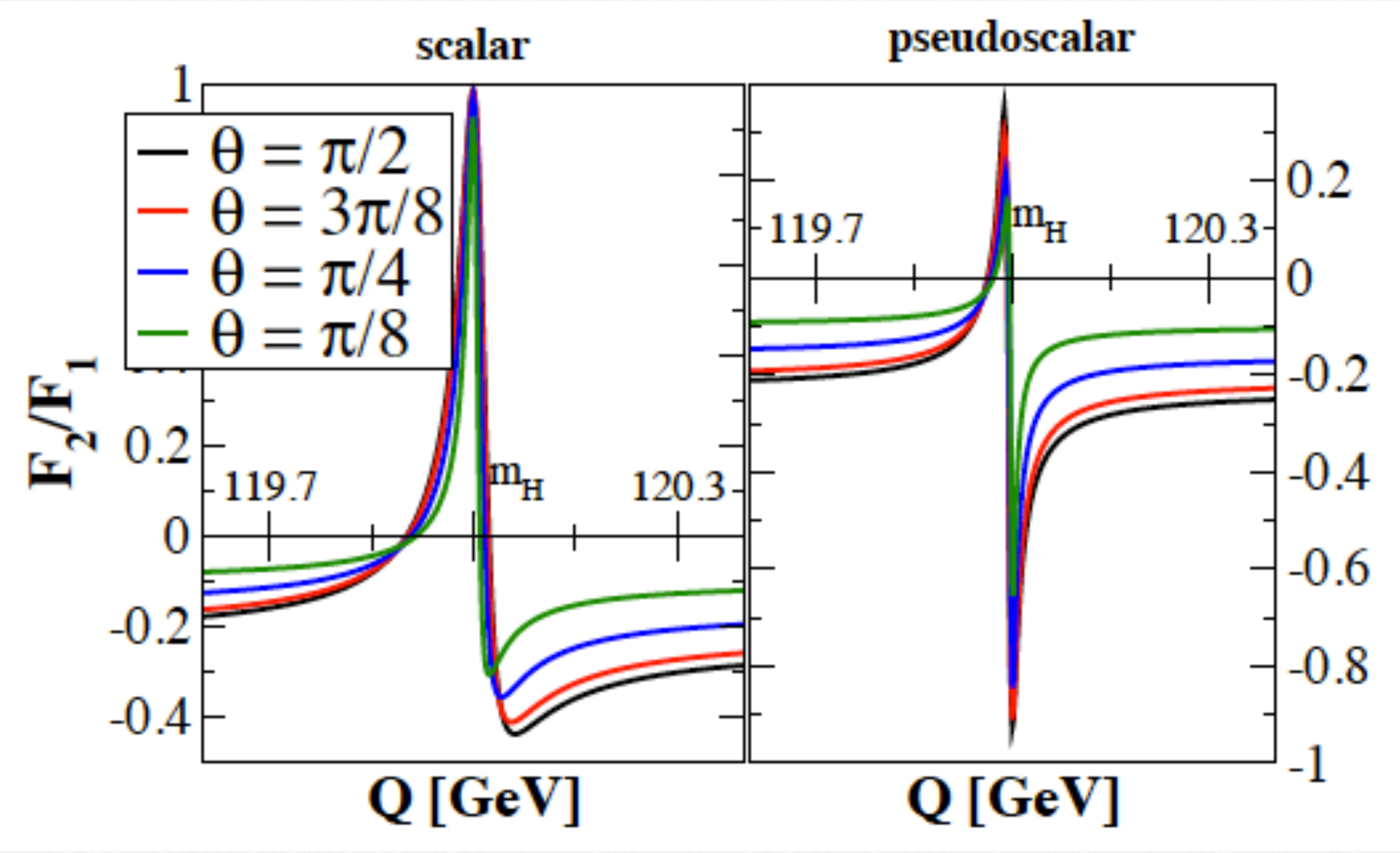


Q ≠ m_H: $\bar{\mathcal{F}}_1 \gg \bar{\mathcal{F}}_2$

box dominant

Q ~ m_H: $\bar{\mathcal{F}}_1 \simeq \bar{\mathcal{F}}_2$

Higgs dominant (pole of the propagator)

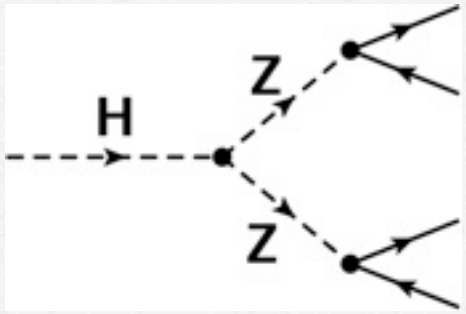
Sign signature preserved at the pole!
small total Higgs width → good Q resolution

Higgs decay into 4 leptons: gg → H/A → 4l

[Boer, den Dunnen, Pisano, MS, Vogelsang, in prep.]

Different decay channels for scalar and pseudoscalar Higgs:

SM Higgs: tree-level vertex



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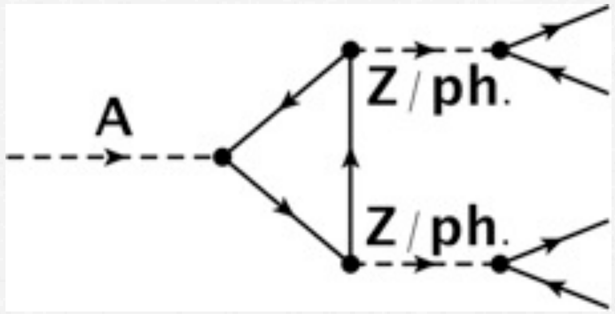

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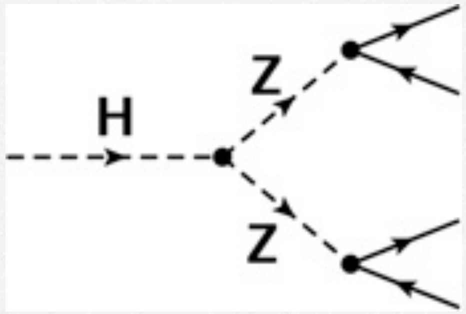

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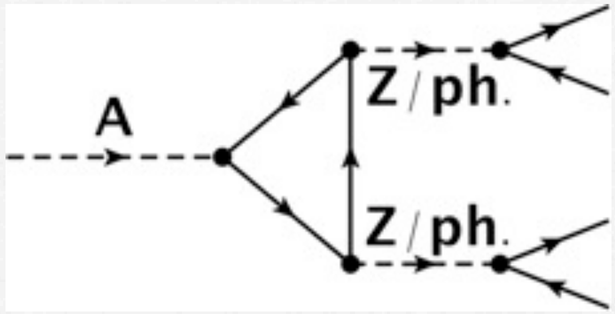
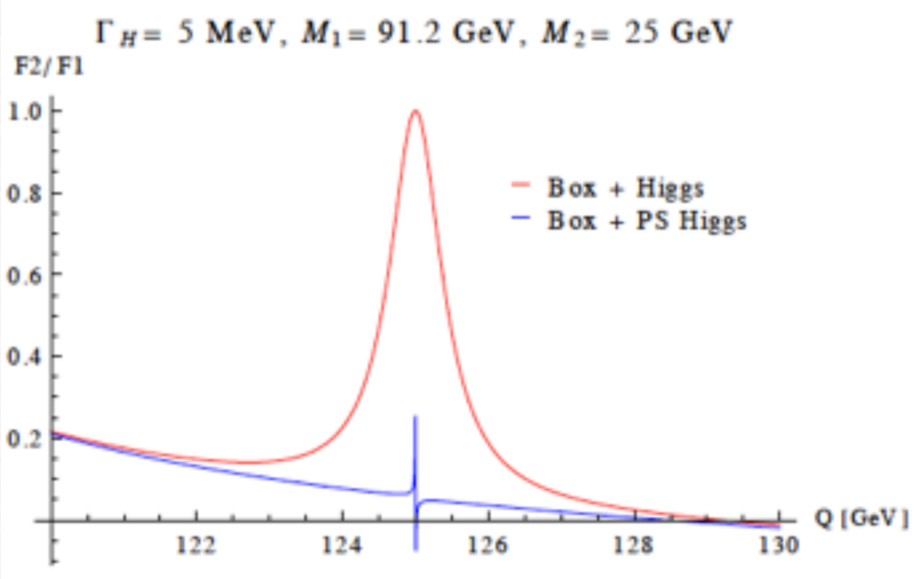


one on-shell Z: gg → ZZ*



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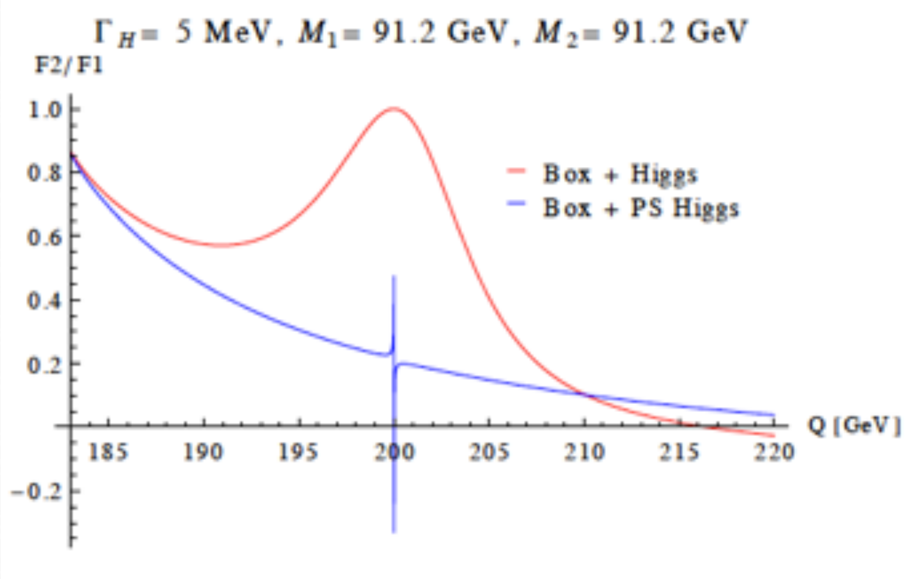

→ more difficult w.r.t. parity distinction, clean process experimentally

Warning: multi-parton scattering!

May use also azimuthal cos(2φ) modulation...

[Boer, den Dunnen, Pisano, MS, Vogelsang, in prep.]

$$\langle q_T^2 \cos(2\phi)\rangle = \int d^2 q_T\, d\phi\, q_T^2 \cos(2\phi) \frac{d\sigma}{d^4q\, d\Omega} \sim \bar{\mathcal{F}}_3(\theta, Q^2) \Big[f_1^g \otimes h_1^{\perp g} + h_1^{\perp g} \otimes f_1^g$$

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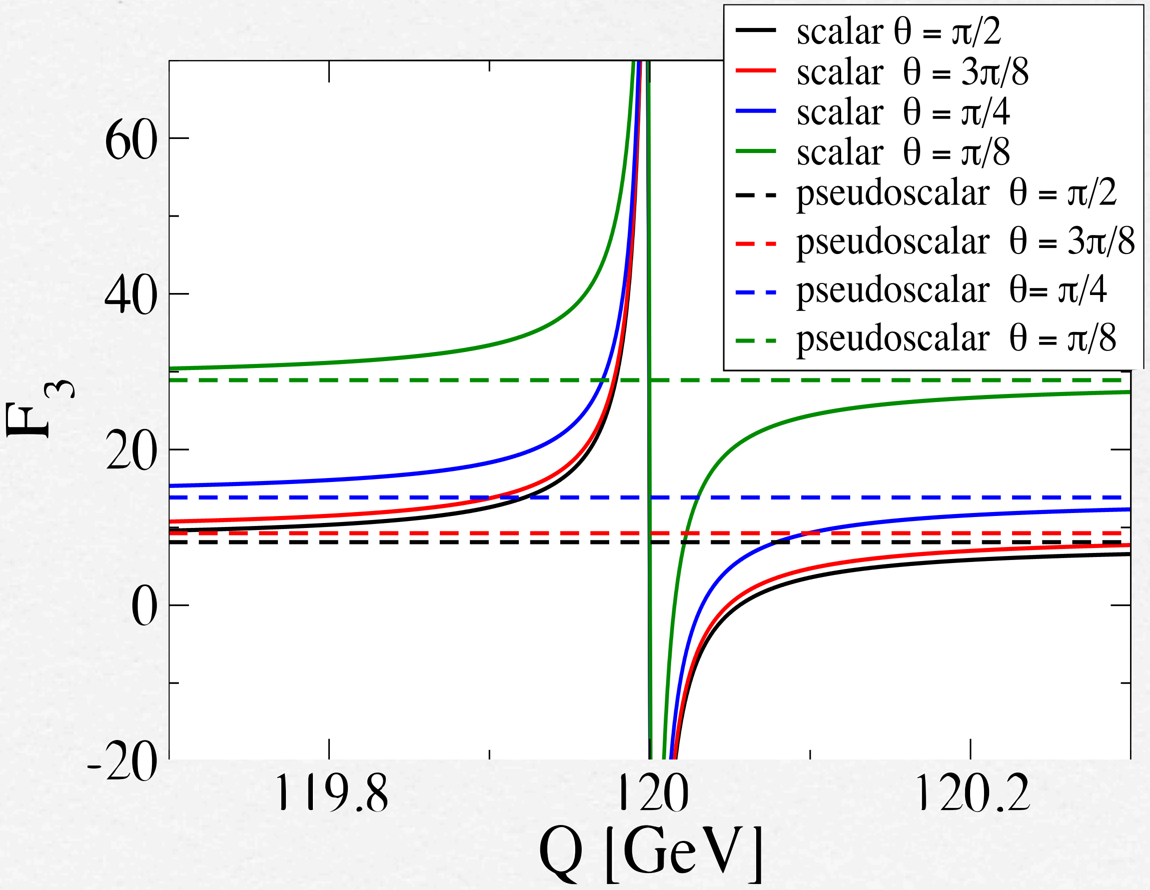
Plot for photon pair production



Summary

- Important progress on evolution of TMDs → can be studied at EIC
- Gluon TMDs can be studies at EIC
- Gluon TMDs from pp - collisions with leptonic final states at RHIC / LHC
- Gluon Boer-Mulders effect may offer a way to determine parity of the Higgs boson at LHC

(Naive) definition of the quark TMD correlator

$$\boxed{\Phi^{[\Gamma]}(x,k_T) = \int \frac{dz^- d^2 z_T}{2(2\pi)^3} \mathrm{e}^{ik\cdot z} \langle P,S|\,\bar{q}(0)\,\Gamma\,\mathcal{W}[0,z]\,q(z)\,|P,S\rangle\Big|_{z^+=0}}$$

- Wilson line: initial/final state interactions (sign change, color entanglement, etc.)

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All-order definition beyond tree-level

[Aybat, Rogers, PRD83, 114042; Collins' "Foundations of pQCD"]

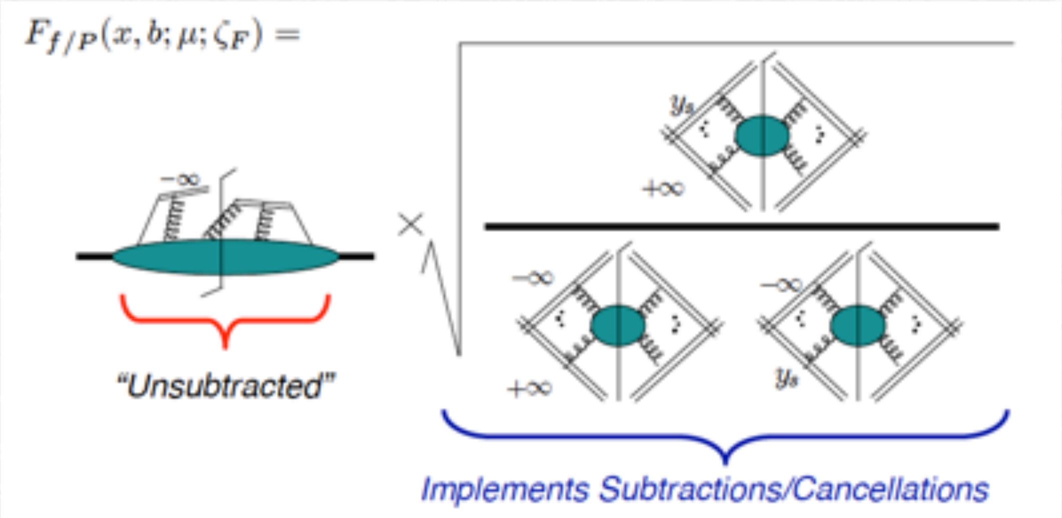
→ Wilson lines off the lightcone

→ regulator ξ → "unsubtracted"

"subtract" soft factor

$$\boxed{\Phi_{ij}(x,\vec{k}_T;S;\xi,\mu)}$$

Collins-Soper evolution equations for ξ, μ



Solution of evolution equations

[Aybat, Rogers, Collins, Qiu; Anselmino et al.,...]

$$\boxed{f_1^q(x, \vec{b}_T^2; \mu; \xi) = \sum_{q'} \left(\tilde{C}_{qq'} \otimes q(x) \right) \Big|_{\mu \propto 1/b_*} \mathrm{e}^{S_{\mathrm{pert}}(b_*)} \Big|_{\mu \propto 1/b_*} \mathrm{e}^{g_q(x,b_T) + \frac{1}{2} g_K(b_T) \ln \frac{\xi}{\xi_0}}}$$

TMD at large k_T perturbative Sudakov factor non-perturbative input

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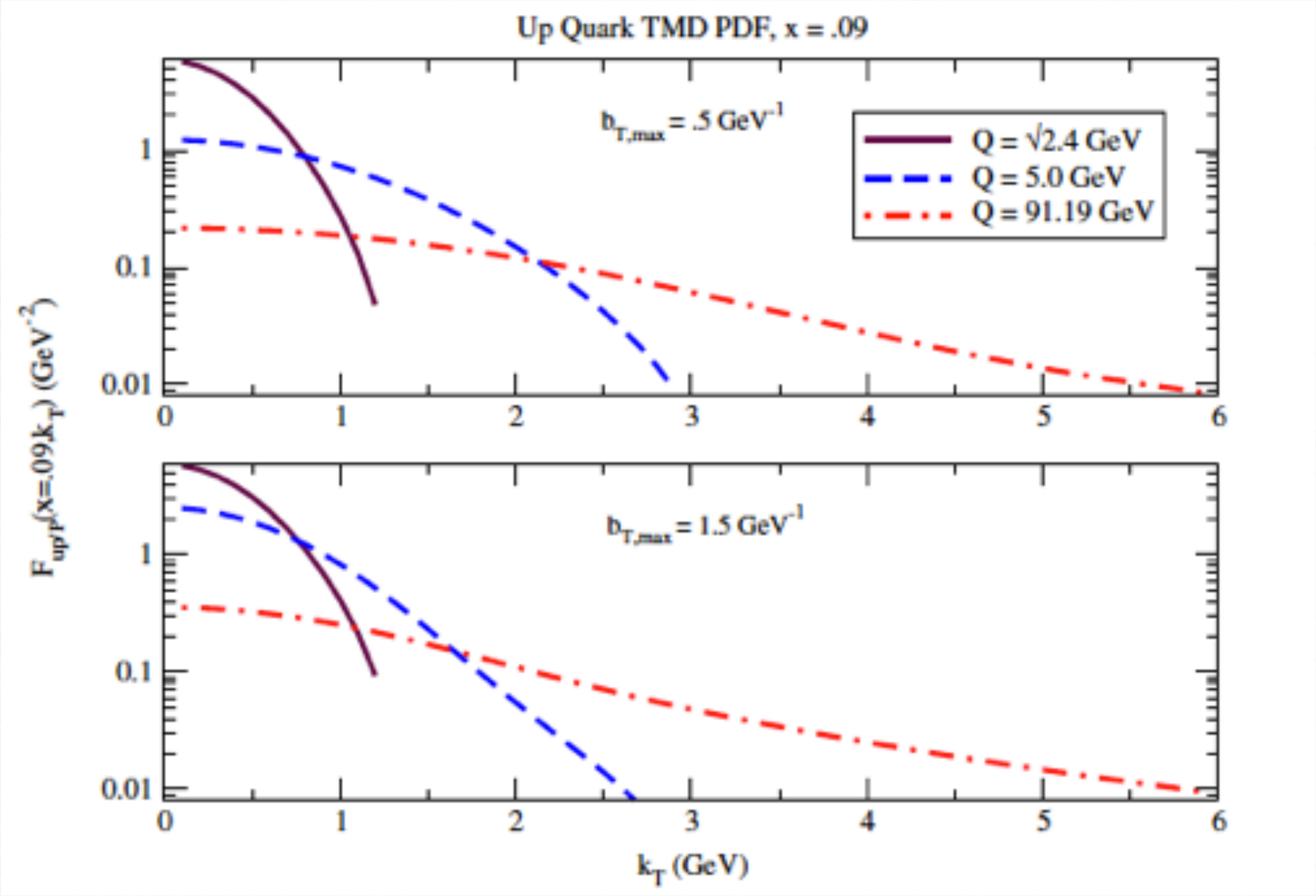
TMD at large k_T

perturbative Sudakov factor

non-perturbative input



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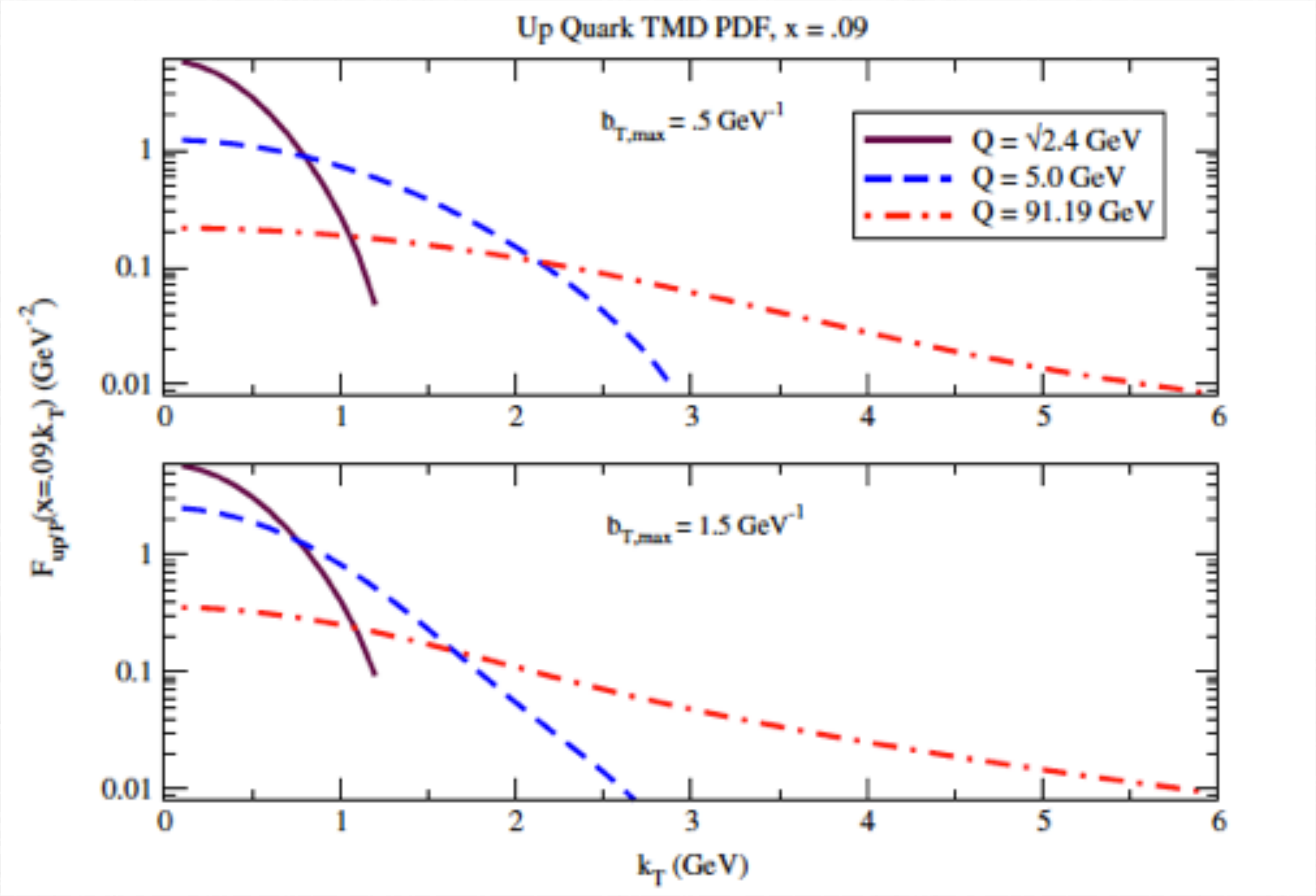
TMD at large k_T — perturbative Sudakov factor — non-perturbative input



Sivers function $f_{1T}^{\perp}$

☐ similar treatment available
[Aybat, Collins, Qiu, Rogers, PRD85, 034043]

☐ first attempt to implement evolution into data fits
[Anselmino et al., arXiv:1204.1239]

→ improved χ^2 compared to previous fits