Fully differential MC generator and Bessel weighting

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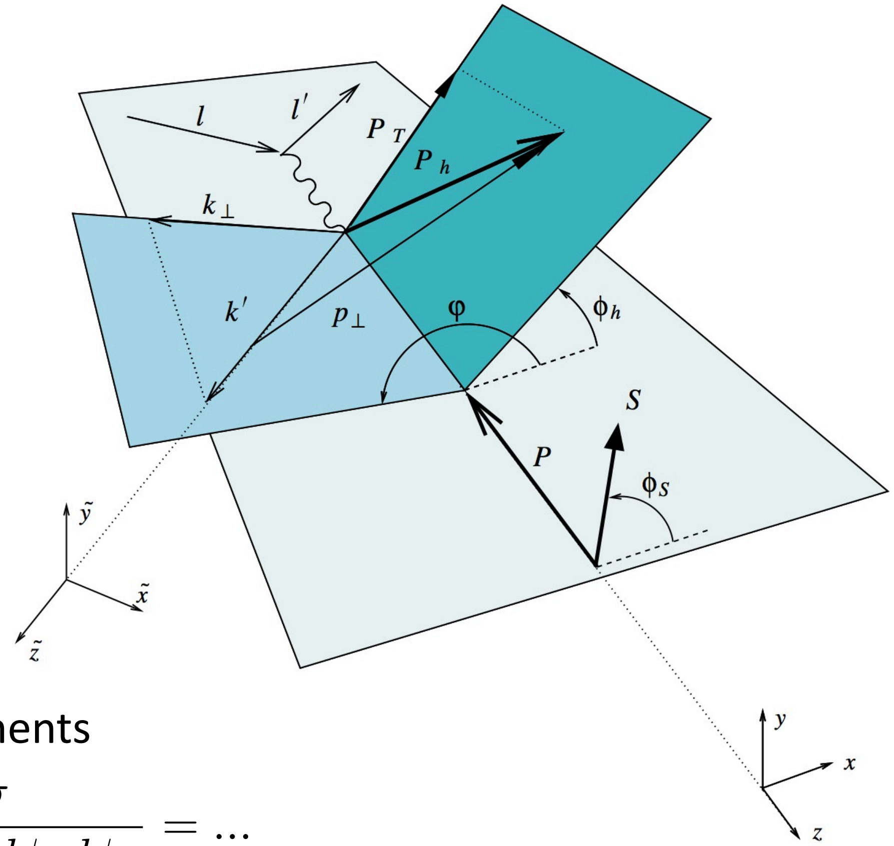
Outline

- SIDIS cross section
- Fully differential MC with
  - Factorized DF and FF
  - Non factorized DF and FF
- Bessel weighting
- Summary

SIDIS cross-section

Figure from **PRD 71, 074006 (2005).**



$$\frac{d\sigma}{dx\,dy\,dz\,dp_\perp^2\,dk_\perp^2\,d\phi_s\,d\phi_h\,d\phi_k} = \ldots$$

Assuming single photon exchange, after integration, the lepton-hadron cross section can be expressed in a model-independent way:

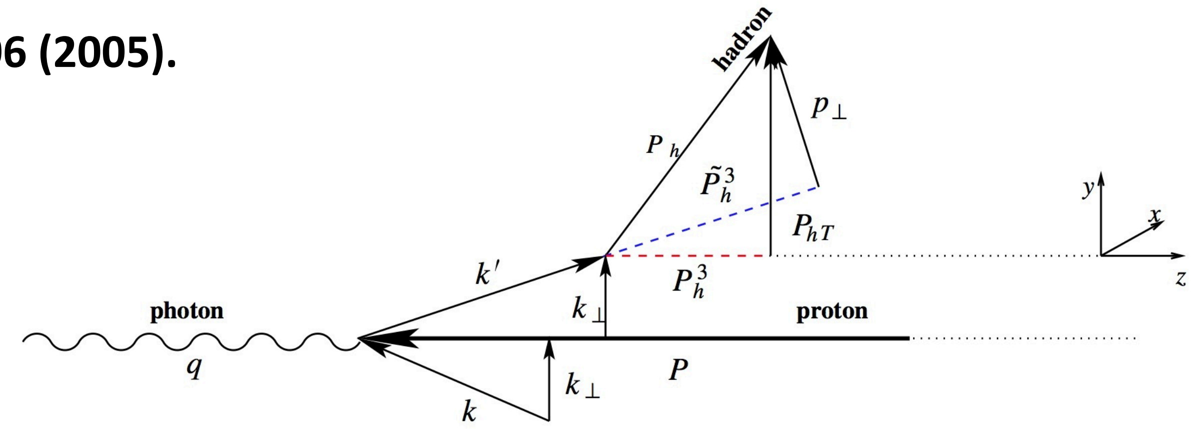
$$\frac{d\sigma}{dx\,dy\,dz\,dP_{h,T}^2\,d\phi_s\,d\phi_h} = \ldots$$

While usually we measure in the experiments

$$\frac{d\sigma}{dx\,dP_{h,T}^2\,d\phi_s\,d\phi_h} = \ldots$$ and/or $$\frac{d\sigma}{dz\,dP_{h,T}^2\,d\phi_s\,d\phi_h} = \ldots$$

Model for fully differential MC

PRD 71, 074006 (2005).



Quark intrinsic motion with Torino model: $M_p = 0$ $x_{LC} = k^- / P^-$

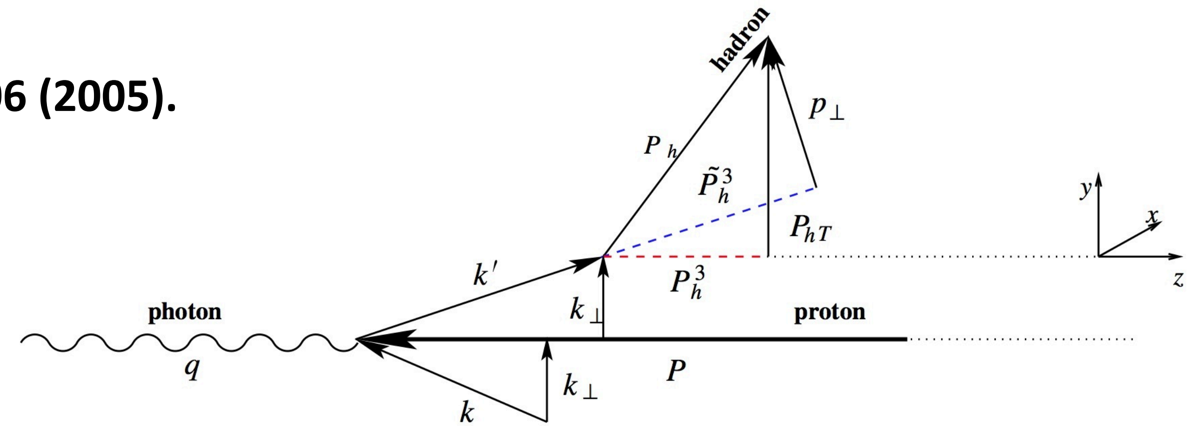
Quark inside the proton have the momentum:

$$k = \left(x_{LC} P_0 + \frac{k_\perp^2}{4 x_{LC} P_0}, \mathbf{k}_\perp, -x_{LC} P_0 + \frac{k_\perp^2}{4 x_{LC} P_0} \right)$$

Where $x_{LC} = \frac{1}{2} x \left(1 + \sqrt{1 + \frac{4k_\perp^2}{Q^2}} \right)$, and P_0 is the proton energy.

Fragmentation

PRD 71, 074006 (2005).



Scattered quark 4 momentum calculated: $k' = k + q$

Final hadron generated with the momentum:

$$P_{\tilde{x},h} = p_\perp \cos(\tilde{\phi}) \qquad P_{\tilde{y},h} = p_\perp \sin(\tilde{\phi}) \qquad P_{\tilde{z},h} = z_{LC} E_{k'} - \frac{p_\perp^2 + M_h^2}{4 z_{LC} E_{k'}}$$

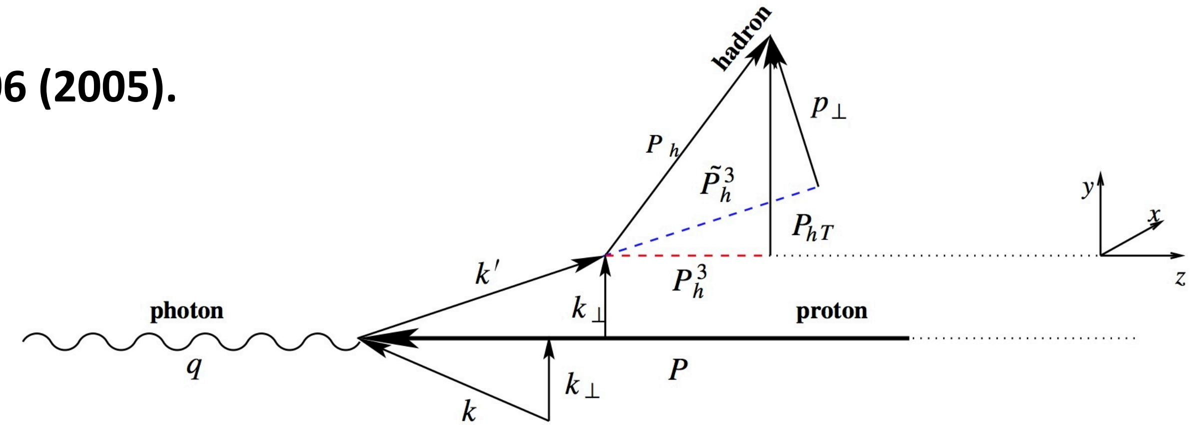
Strictly $z = z_h \neq z_{LC}$

Although, we assume $z = z_h \simeq z_{LC}$

To account and understand all the assumptions, integrations, correlations and more fully differential SIDIS cross-section should be studied.

Kinematics restrictions

PRD 71, 074006 (2005).

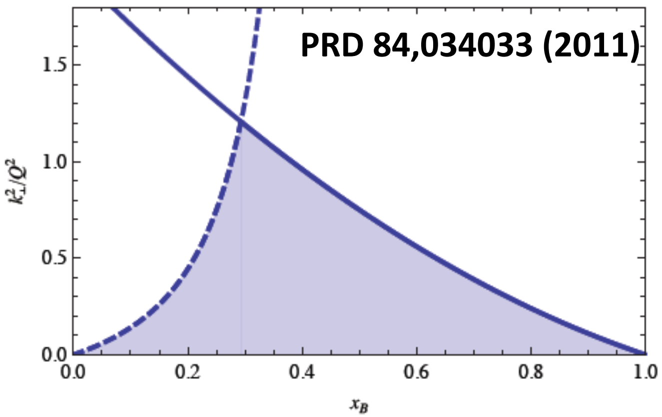


$$k_z \leq 0$$

$$M_p = M_q = 0$$

$$\frac{k_\perp^2}{Q^2} \leq \frac{x_B(1-x_B)}{(1-2x_B)^2}$$

$$x_B < 0.5$$



$$k_0 \leq P_0$$

$$M_p = M_q = 0$$

$$\frac{k_\perp^2}{Q^2} \leq (2-x_B)(1-x_B)$$

If one accounts for the non zero target mass, then one will obtain wider phase space, since bigger is target energy for given Q^2, bigger is quark momentum inside nucleon.

Fully differential MC with Gaussian FF and DF

$$\frac{d\sigma}{dxdydzdp_\perp^2 dk_\perp^2 d\phi_{l'} d\phi_h d\phi_k} = K(x,y)\left[J(x,Q^2,k_\perp)\left\{f_1(x,k_\perp)D_1(z,p_\perp)+\lambda g_1(x,k_\perp)D_1(z,p_\perp)\right\}\right]$$

Simple Gaussian DF and FF.

PRD 71, 074006 (2005).

$$f_1(x,k_\perp) = f_1(x)\frac{e^{-\frac{k_\perp^2}{<k_\perp^2>_{f_1}}}}{<k_\perp^2>_{f_1}}$$

$$f_1(x) = (1-x^3)x^{-1.313}$$

$$g_1(x,k_\perp) = g_1(x)\frac{e^{-\frac{k_\perp^2}{<k_\perp^2>_{g_1}}}}{<k_\perp^2>_{g_1}}$$

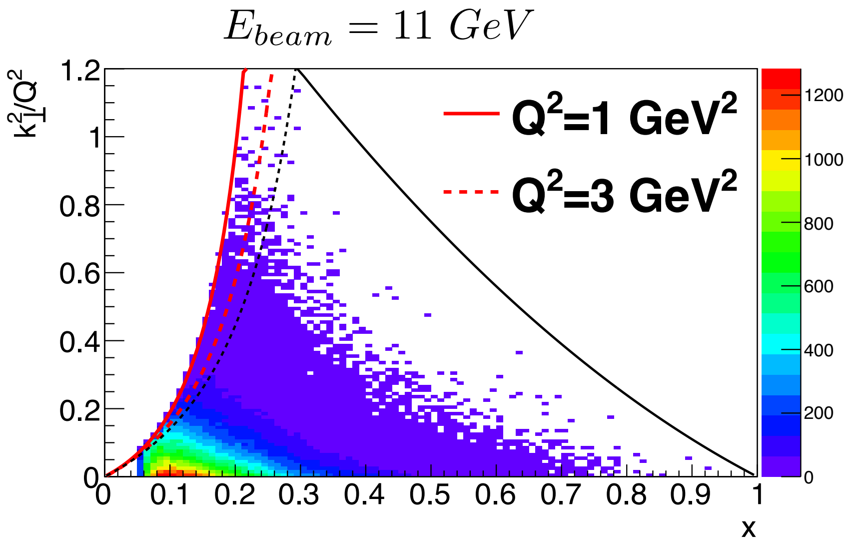
$$g_1(x) = f_1(x)\times x^{0.7}$$

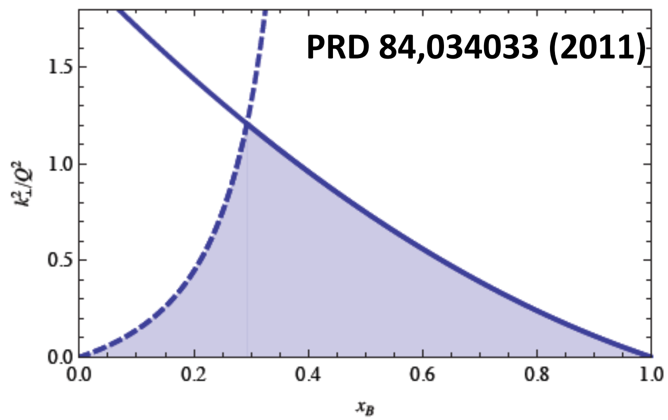
$$D_1(z,p_\perp) = D_1(z)\frac{e^{\frac{-p_\perp^2}{<p_\perp^2>}}}{<p_\perp^2>}$$

$$D_1(z) = 0.8*(1-z)^2$$

where: $J(x,Q^2,k_\perp) = \frac{x}{x_{LC}}\left(1+\frac{x^2}{x_{LC}^2}\frac{k_\perp^2}{Q^2}\right)$ $\quad K(x,y) = \frac{\alpha^2}{xyQ^2}\frac{y^2}{2(1-\varepsilon)}\left(1+\frac{\gamma^2}{2x}\right)$

Phase space in MC: simple Gaussian DF and FF



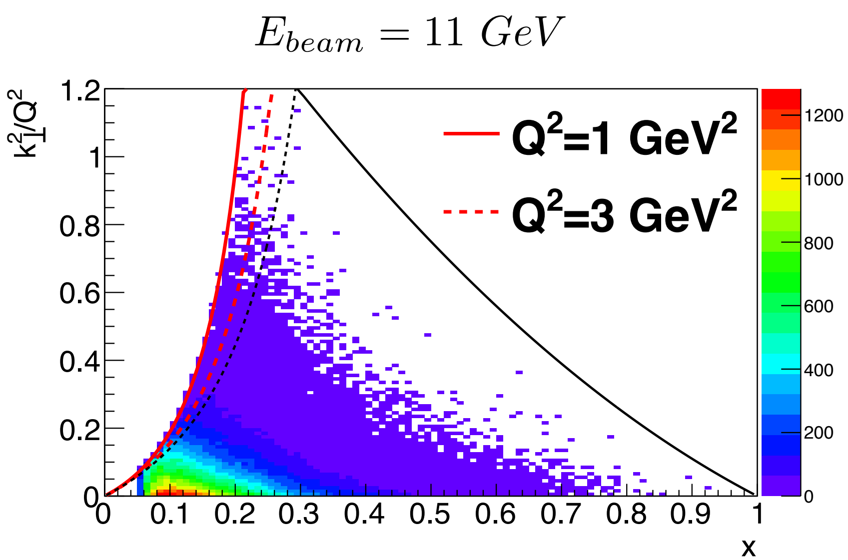


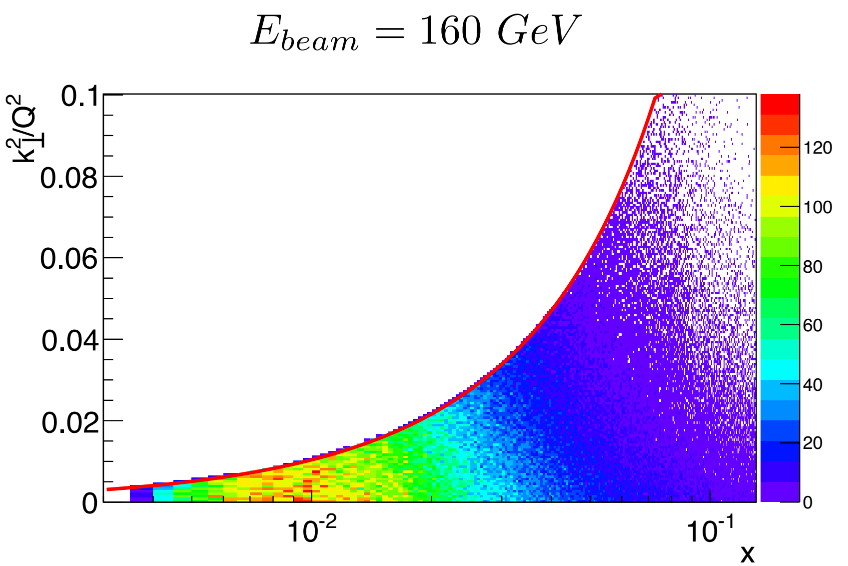
Input parameters:

$$< k_\perp^2 >_{f_1} = 0.2 GeV^2, < k_\perp^2 >_{g_1} = 0.16 GeV^2, < p_\perp^2 > = 0.14 GeV^2$$

x and $k_\perp$ are not factorized even in Gaussian approach.

Phase space in MC: simple Gaussian DF and FF





$$< k_\perp^2 >_{f_1} = 0.2 GeV^2, < k_\perp^2 >_{g_1} = 0.16 GeV^2, < p_\perp^2 >= 0.14 GeV^2$$

Kinematic cut-off is sharper at higher beam energy and smaller x

More input functions

- How MC will behave with different functions?
- How strong result will depend on input functions and parameters?

➢ Since I can not apply exact kinematic restriction of the model to the experimental data, how big is the effect with and without kinematic restrictions within the model?

For simple Gaussian input DF and FF MC describes data only with kinematic restrictions! (+ Torino fits)

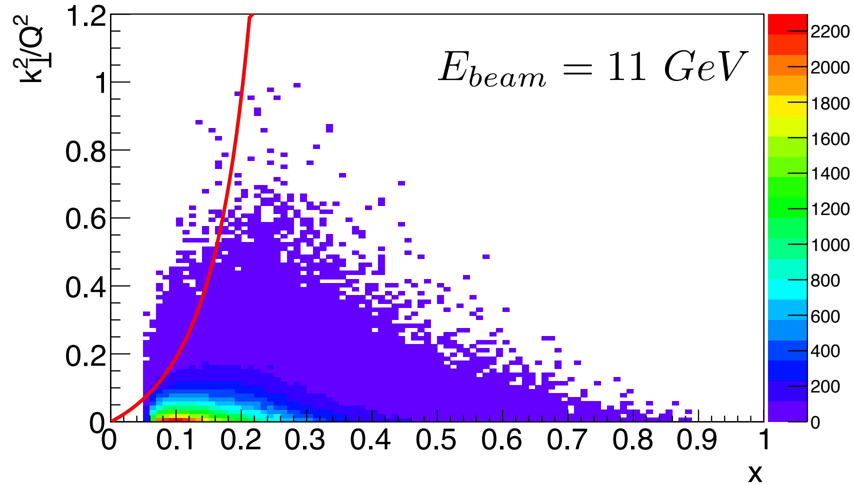
Modified Gaussian FF and DF

Stan Brodsky, "Novel Features of Hadron Dynamics and Light-Front Holography"
Warsaw - July 3 - 6, 2012

$$f_1(x,k_\perp) = f_1(x)\frac{e^{-\frac{k_\perp^2}{2<k_\perp^2>_{f_1}x(1-x)}}}{<k_\perp^2>_{f_1}\sqrt{x(1-x)}}$$

$$g_1(x,k_\perp) = g_1(x)\frac{e^{-\frac{k_\perp^2}{2<k_\perp^2>_{g_1}x(1-x)}}}{<k_\perp^2>_{g_1}\sqrt{x(1-x)}}$$

$$D_1(z,p_\perp) = D_1(z)\frac{e^{-\frac{p_\perp^2}{2<p_\perp^2>z(1-z)}}}{<p_\perp^2>\sqrt{z(1-z)}}$$

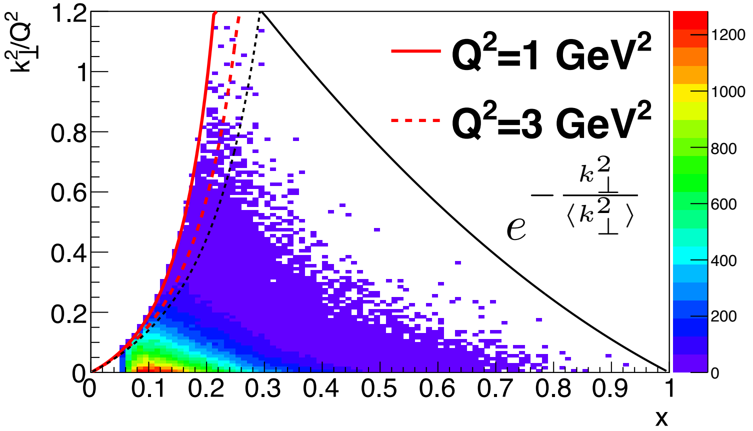


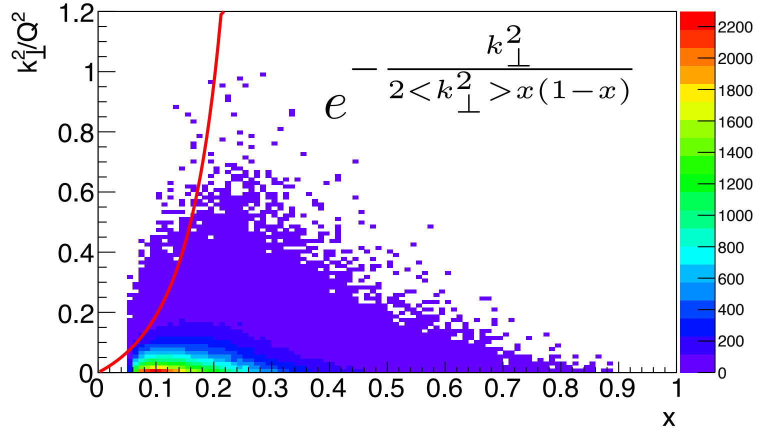
No cuts with red lines.
Only energy and momentum conservation.

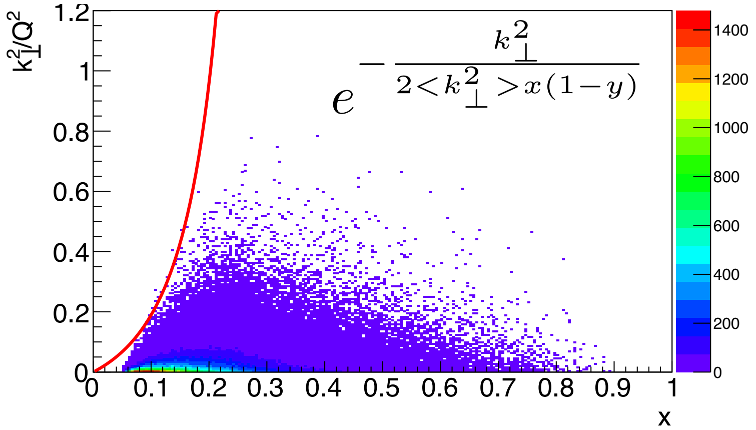
$$<k_\perp^2>_{f_1} = 0.375GeV^2, <k_\perp^2>_{g_1} = 0.3GeV^2, <p_\perp^2> = 0.14GeV^2$$

$k_z \leq 0$ requirement is satisfied automatically for 95-99% of events.

More modifications





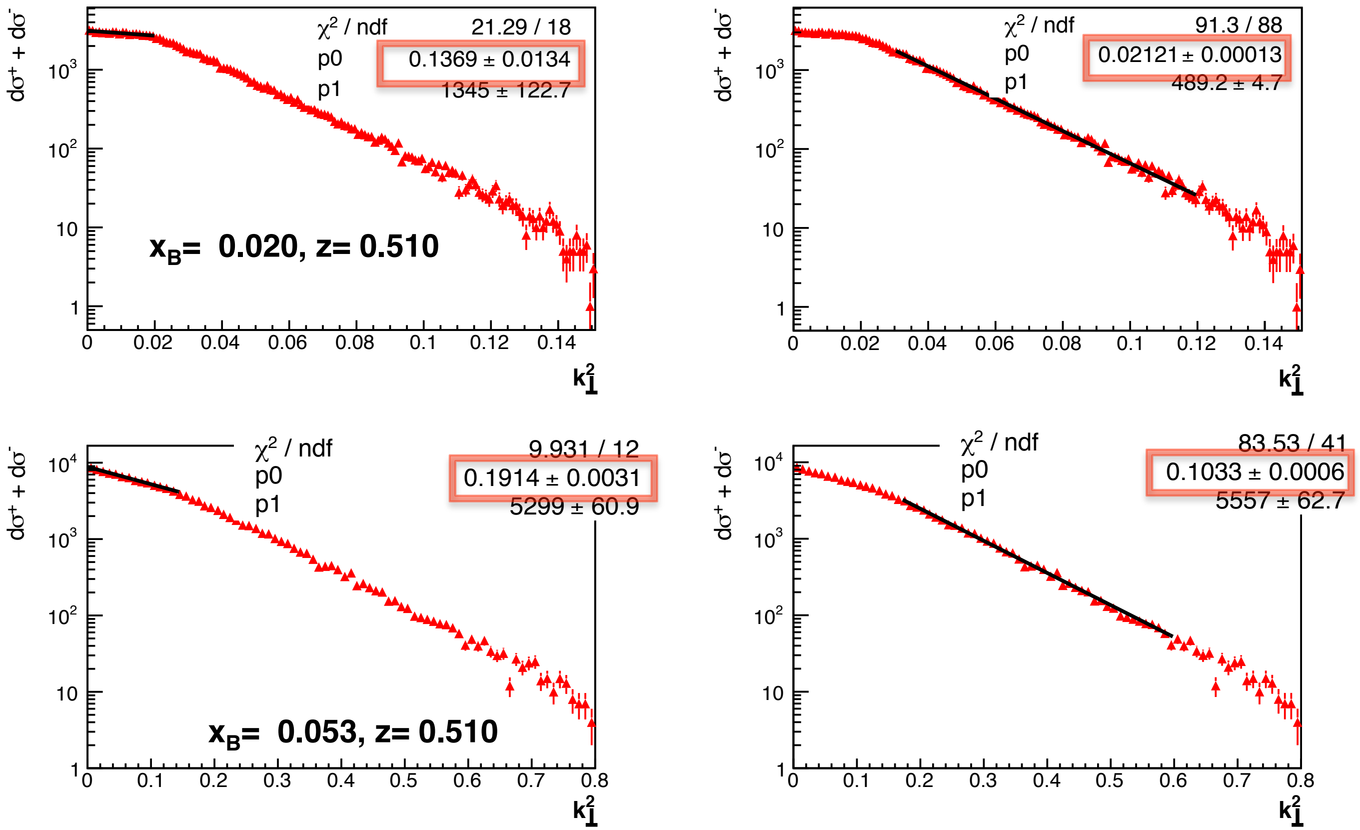


If we account not only *x* but also *y*, requirement $k_z \leq 0$ fulfilled automatically! Like positivity bounds from PRL 85,712(2000) one can automatically fulfill with proper choice of the functions.

$k_\perp^2$ dependence for different x bins simple Gaussian DF and FF

$$< k_\perp^2 >_{f_1} = 0.2 GeV^2, < k_\perp^2 >_{g_1} = 0.16 GeV^2, < p_\perp^2 > = 0.14 GeV^2$$

$E_{beam} = 160\ GeV$

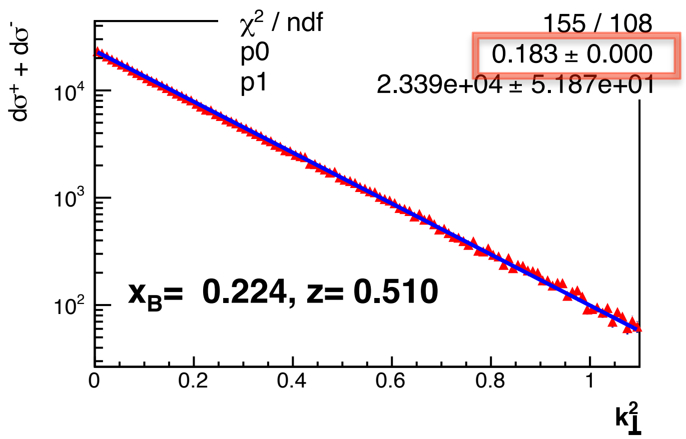


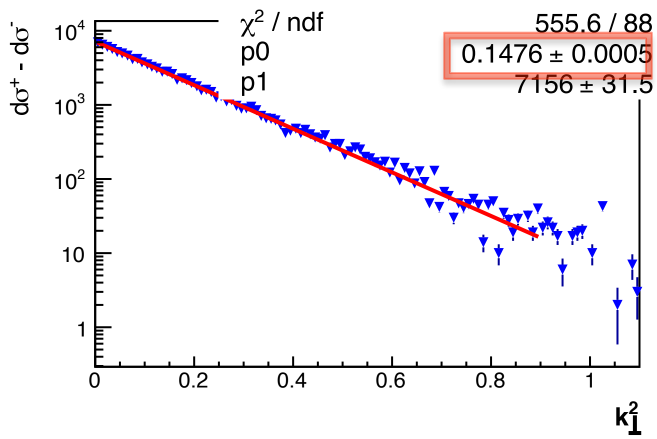
At low $k_\perp^2$ and higher x the outcome is close to implemented value for small $k_\perp^2$

$k_\perp^2$ dependence for different x bins simple Gaussian DF and FF

$$< k_\perp^2 >_{f_1} = 0.2 GeV^2, < k_\perp^2 >_{g_1} = 0.16 GeV^2, < p_\perp^2 >= 0.14 GeV^2$$

$$E_{Beam} = 6\ GeV$$





Output of MC is close to the implemented values for wider range!
Is it connected to the cut $k_z \leq 0$?

$k_\perp^2$ dependence for different x bins

$< k_\perp^2 >_{f_1} = 0.2GeV^2, < k_\perp^2 >_{g_1} = 0.16GeV^2, < p_\perp^2 > = 0.14GeV^2$

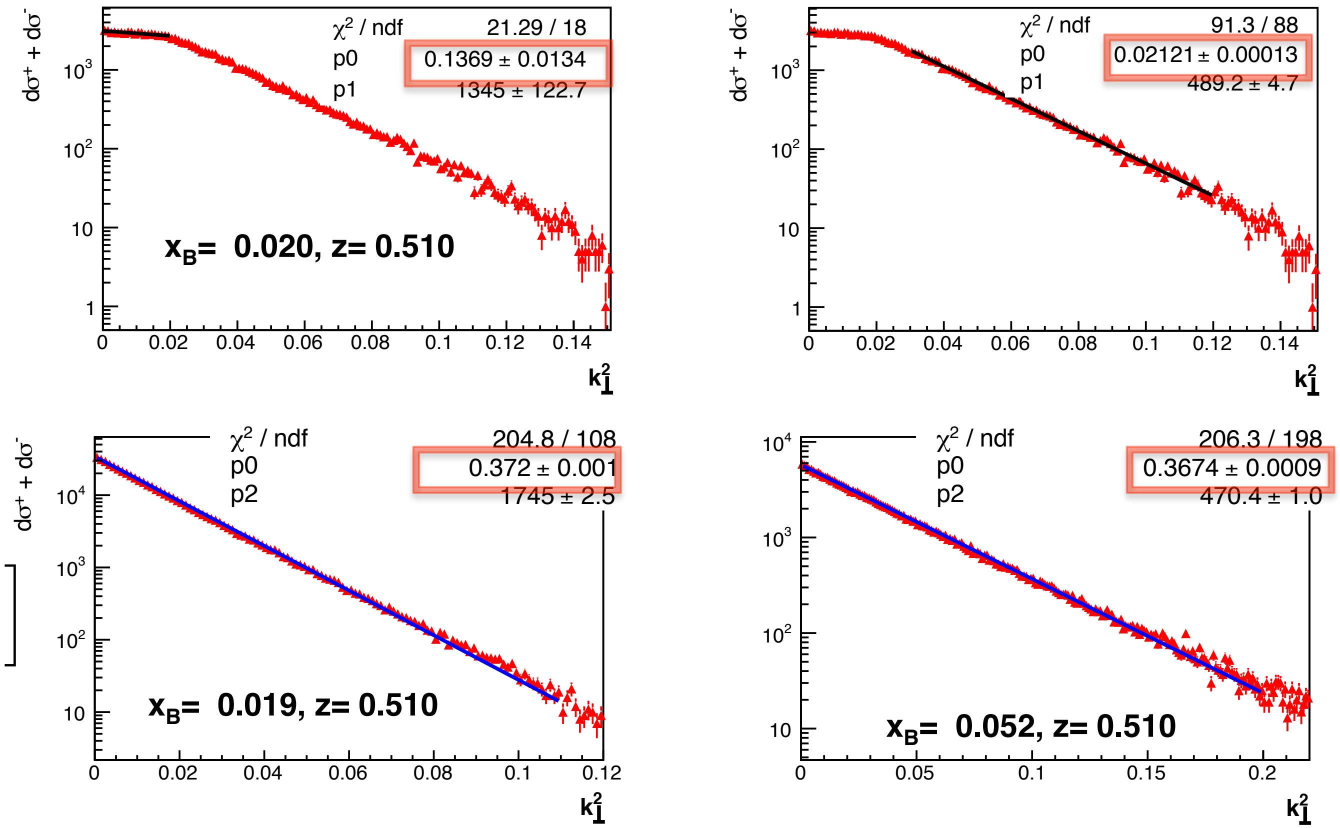
$$\exp\left[-\frac{k_\perp^2}{\langle k_\perp^2 \rangle}\right]$$

$k_z \leq 0$

$$\exp\left[-\frac{k_\perp^2}{2\langle k_\perp^2 \rangle x(1-x)}\right]$$

~~$k_z \leq 0$~~

$E_{beam} = 160\ GeV$



$< k_\perp^2 >_{f_1} = 0.375GeV^2, < k_\perp^2 >_{g_1} = 0.3GeV^2, < p_\perp^2 > = 0.14GeV^2$

In modified Gaussian MC we recover implemented value even at small *x*!

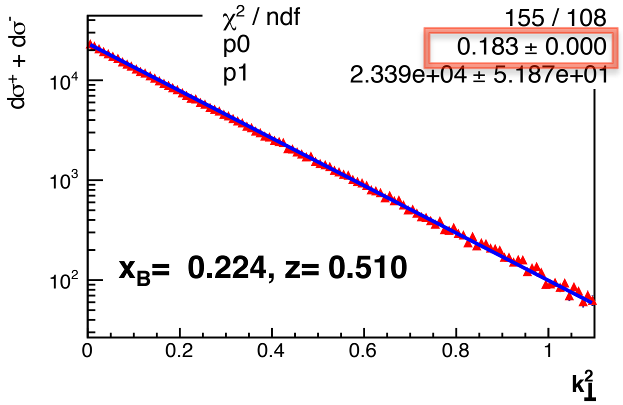
$k_\perp^2$ dependence for different x bins

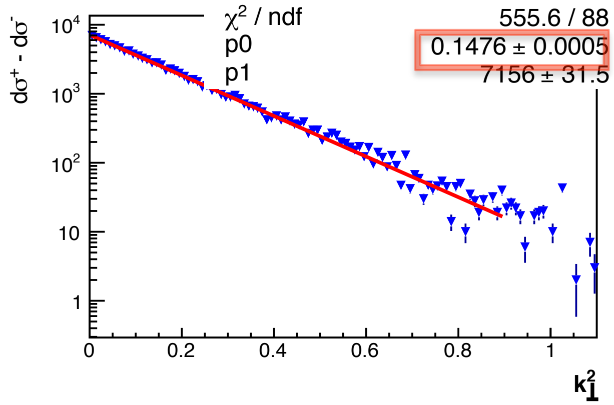
$E_{Beam} = 6\ GeV$

$< k_\perp^2 >_{f_1} = 0.2 GeV^2, < k_\perp^2 >_{g_1} = 0.16 GeV^2, < p_\perp^2 >= 0.14 GeV^2$

$$\exp\left[-\frac{k_\perp^2}{\langle k_\perp^2 \rangle}\right]$$

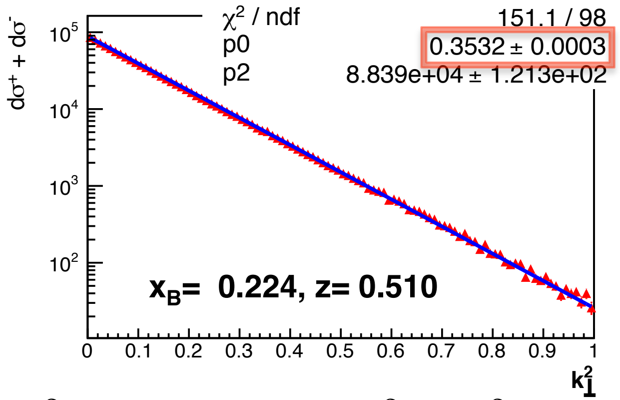
$k_z \leq 0$

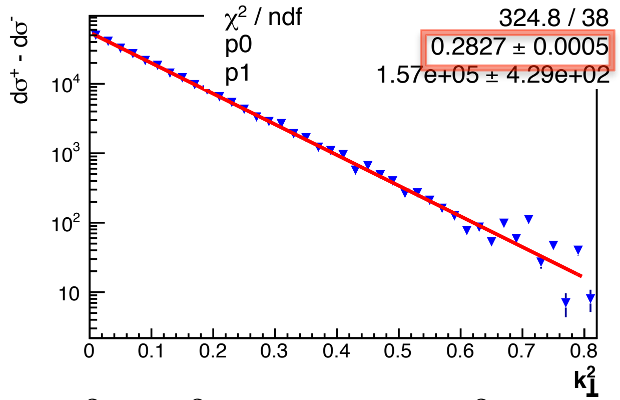




$$\exp\left[-\frac{k_\perp^2}{2\langle k_\perp^2 \rangle x(1-x)}\right]$$

~~$k_z \leq 0$~~





$< k_\perp^2 >_{f_1} = 0.375 GeV^2, < k_\perp^2 >_{g_1} = 0.3 GeV^2, < p_\perp^2 >= 0.14 GeV^2$

Variation of the implemented and observed widths is due to the energy and momentum conservation!

$\langle k_\perp^2 \rangle$ dependence vs x

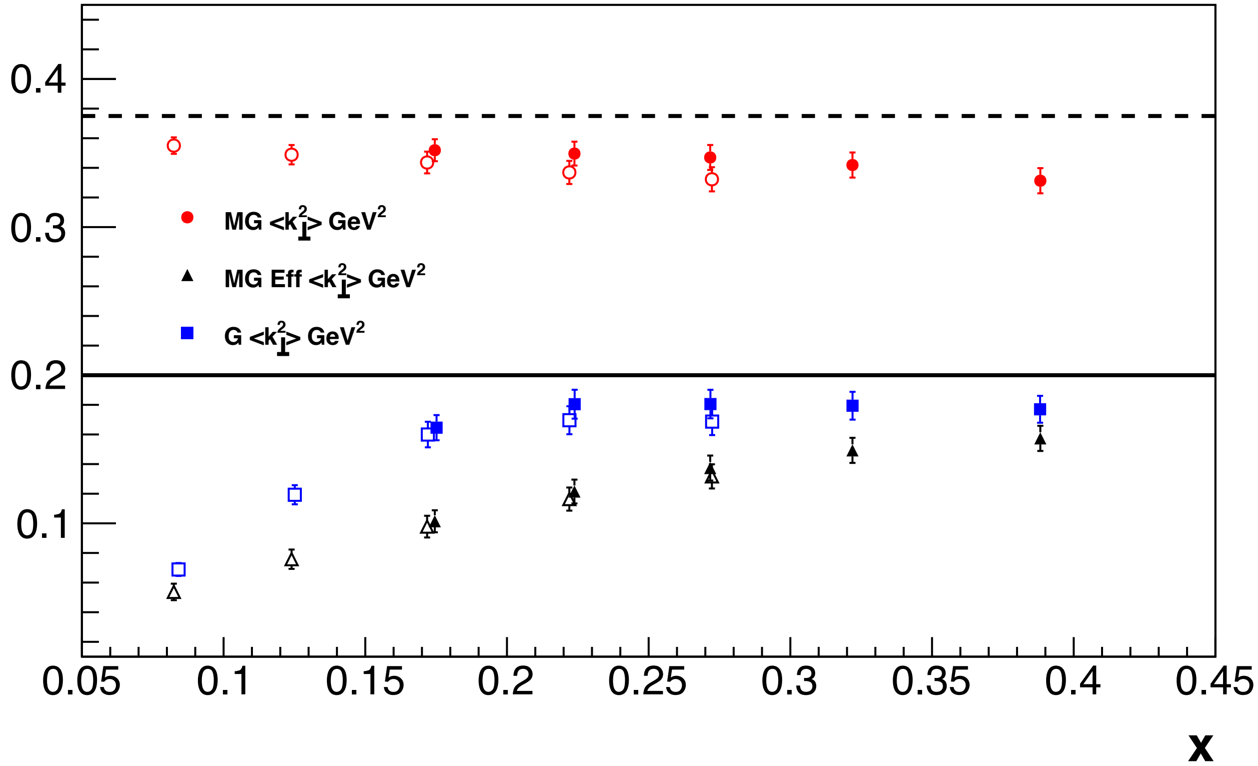
Red: parameter for the MC, Black: actual

Blue: simple Gaussian DF with restrictions

$$\exp\left[-\frac{k_\perp^2}{2\langle k_\perp^2\rangle x(1-x)}\right]$$

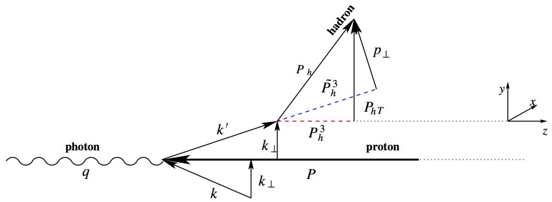
$$\exp\left[-\frac{k_\perp^2}{\langle k_\perp^2\rangle}\right]$$

$$k_z \leq 0$$



Modified Gaussian DF and FF allows to use one fixed parameter for different *x*!

$\langle p_\perp^2 \rangle$ VS Z



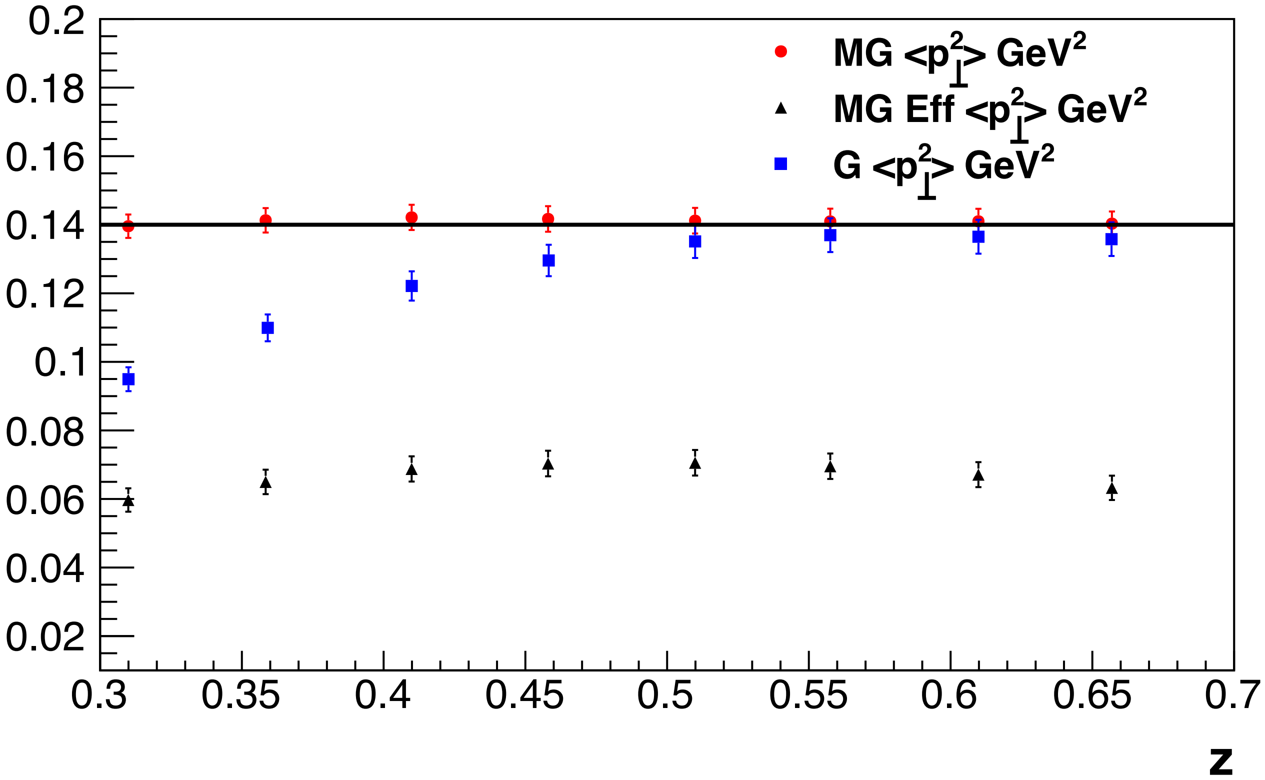
Red: parameter for the MC, Black: actual

Blue: simple Gaussian DF with restrictions

$$\exp\left[-\frac{p_\perp^2}{2\langle p_\perp^2\rangle z(1-z)}\right]$$

$$\exp\left[-\frac{p_\perp^2}{\langle p_\perp^2\rangle}\right]$$

$$P_{h,\tilde{z}} \geq 0$$



Modified Gaussian DF and FF allows to use one fixed parameter for different *z*!

Bessel-weighted extraction of the double spin asymmetry A_{LL}

JHEP10(2011)021

$$A_{LL}^{J_0(b_T P_{h,T})}(b_T) = \frac{\tilde{\sigma}^+(b_T) - \tilde{\sigma}^-(b_T)}{\tilde{\sigma}^+(b_T) + \tilde{\sigma}^-(b_T)} = \frac{\tilde{\sigma}_{LL}(b_T)}{\tilde{\sigma}_{UU}(b_T)} = \sqrt{1-\varepsilon^2}\,\frac{\sum_q \tilde{g}_1^q(x, z^2 b_T^2)\tilde{D}_1^q(z, b_T^2)}{\sum_q \tilde{f}_1^q(x, z^2 b_T^2)\tilde{D}_1^q(z, b_T^2)}$$

where

$$\tilde{\sigma}^{\pm}(b_T) = \int \frac{d\sigma^{\pm}}{dP_{h,T}} J_0(b_T P_{h,T}) P_{h,T} dP_{h,T}$$

Or for MC events

$$\tilde{\sigma}^{\pm}(b_T) \simeq S^{\pm} = \sum_{i=1}^{N^{\pm}} J_0(b_T P_{h,T,i})$$

In Fourier space convolution of transverse momentum dependent parton DF and FF become simple products!

In simple words / equations

Let assume we can present:

$$\frac{d\sigma_{XY}}{dP_{h,T}} = C_{XY} e^{-\frac{P_{h,T}^2}{<P_{h,T}^2>_{XY}}}$$

$$\tilde{\sigma}_{XY}(b_T) = C_{XY} \int_0^\infty e^{-\frac{P_{h,T}^2}{<P_{h,T}^2>_{XY}}} J_0(b_T P_{h,T}) P_{h,T} dP_{h,T} = C_{XY} \frac{1}{2} e^{-\frac{<P_{h,T}^2>_{XY} b_T^2}{4}}$$

Assuming: $\langle P_{h,T}^2 \rangle = \langle k_\perp^2 \rangle z^2 + \langle p_\perp^2 \rangle$ $\qquad \tilde{\sigma}_{LL}(b_T) = C \frac{1}{2} \tilde{g}_1(x, z b_T) \times \tilde{D}_1(z, b_T)$

Where:

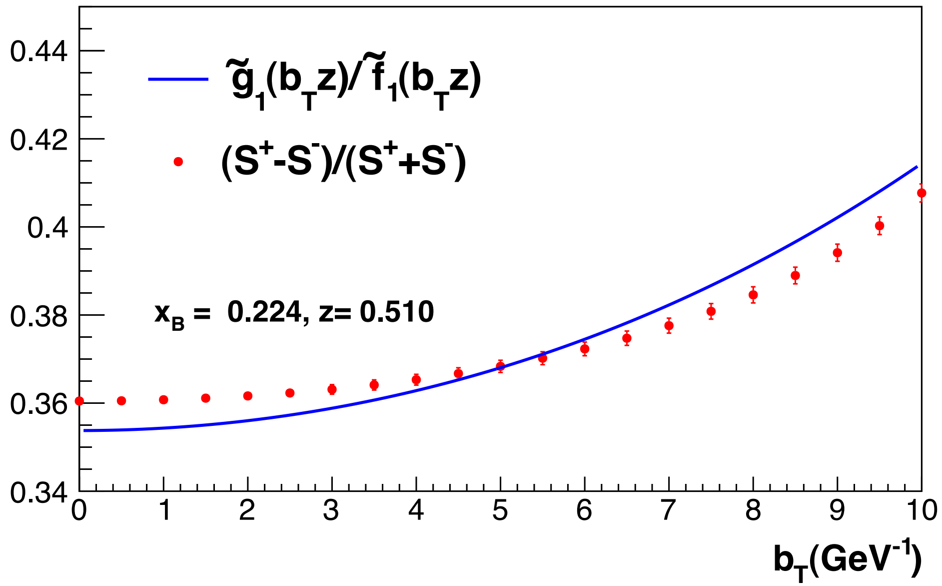
$$\tilde{g}_1(b_T) = 2 \int_0^\infty J_0(b k_\perp) e^{-\frac{k_\perp^2}{<k_\perp^2>_{g_1}}} k_\perp dk_\perp = e^{\frac{-<k_\perp^2>_{g_1} b_T^2}{4}}$$

$$\tilde{D}_1(z, b_T) = \int_0^\infty J_0(b p_\perp) e^{-\frac{p_\perp^2}{<p_\perp^2>}} p_\perp dp_\perp = e^{\frac{-<p_\perp^2> b_T^2}{4}}$$

This is just very simple presentation based on chain of assumption…

<u>Bessel-weighting strategy does not depend on a Gaussian approach at all!</u>

Bessel-weighted A_{LL}

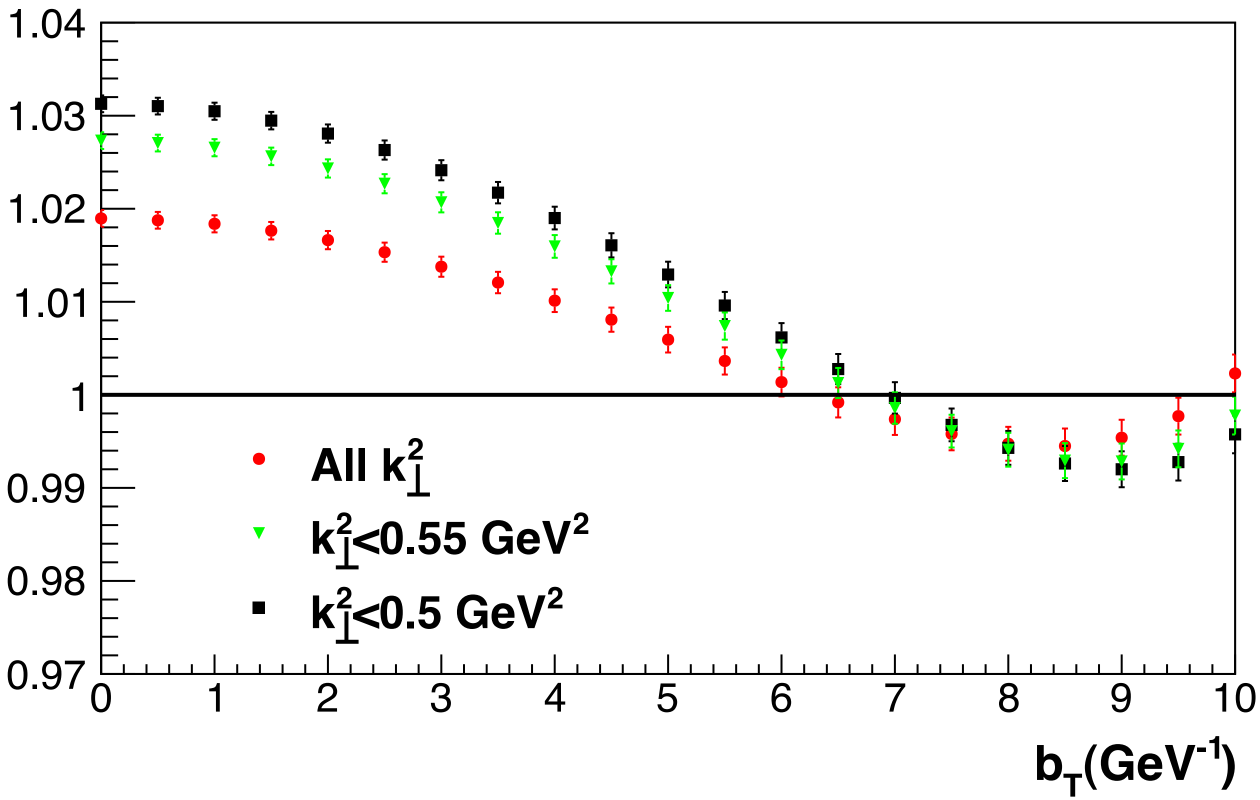


$$points = \frac{1}{\sqrt{1-\varepsilon^2}} \frac{S^+ - S^-}{S^+ + S^-}$$ where $$S^{\pm} = \sum_{i=1}^{N^{\pm}} J_0(P_{h,T} b_T)$$

Curve calculated: $\tilde{f}_1(x, b_T) = f_1(x) e^{\frac{-<k_\perp^2>_{f_1} b_T^2}{4}}$ $\tilde{g}_1(x, b_T) = g_1(x) e^{\frac{-<k_\perp^2>_{g_1} b_T^2}{4}}$

$\langle k_\perp^2 \rangle$ were obtained using fits on $k_\perp^2$ distributions for given bin of the MC sample!

Systematic discrepancy

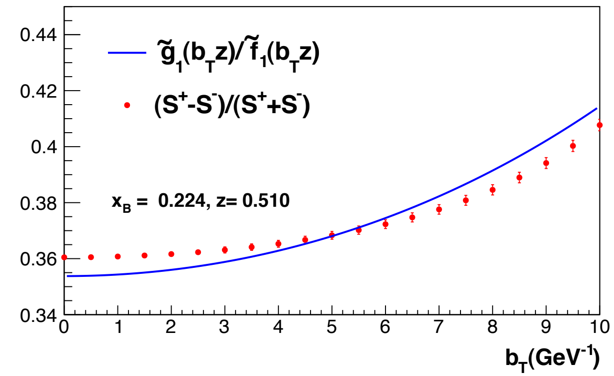


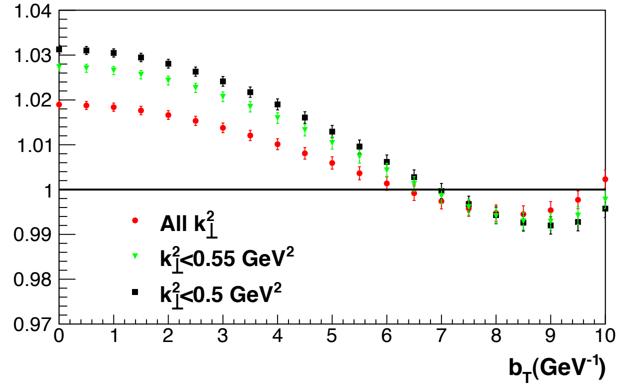
The ratio of the extraction to the curve has systematic shift, which is increasing with decrease of $k_\perp^2$ range (similar behavior is observed also cutting $P_{h,T}$).

The reason of the systematic few percent discrepancy

$$\tilde{T}(b_T) = 2\int_0^{\infty} J_0(b_T t)\frac{e^{-t^2/a^2}}{a^2} t dt = e^{-\frac{a^2 b_T^2}{4}} > 0$$

$$2\int_{t_{min}}^{t_{max}} J_0(b_T t)\frac{e^{-t^2/a^2}}{a^2} t dt \gtreqless 0$$





Red points are integrated for the range from 0 to maximum value!
While curve is integrated from 0 to infinity!

Correction

$$\int_0^{t_{max}} J_0(b_T t)\frac{e^{-t^2/a^2}}{a^2} t dt = \int_0^{\infty} J_0(b_T t)\frac{e^{-t^2/a^2}}{a^2} t dt - \int_{t_{max}}^{\infty} J_0(b_T t)\frac{e^{-t^2/a^2}}{a^2} t dt = \frac{1}{2} e^{-\frac{a^2 b_T^2}{4}} \times (1-\epsilon)$$

where

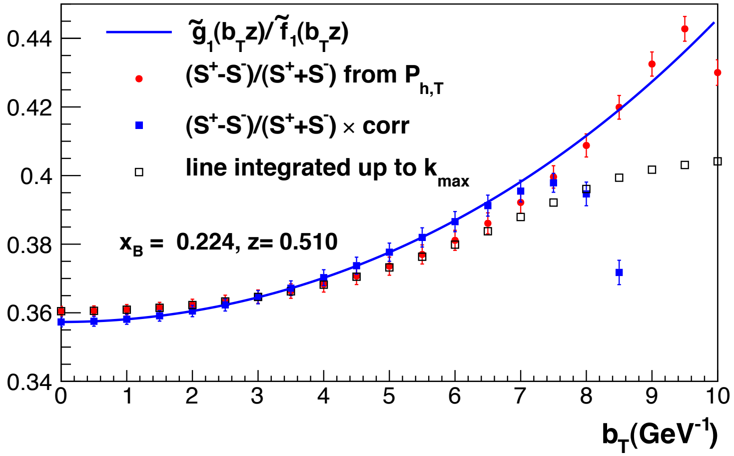
$$\epsilon = \frac{\int_{t_{max}}^{\infty} J_0(b_T t)\frac{e^{-t^2/a^2}}{a^2} t dt}{\int_0^{\infty} J_0(b_T t)\frac{e^{-t^2/a^2}}{a^2} t dt} = 2e^{\frac{a^2 b_T^2}{4}} \int_{t_{max}}^{\infty} J_0(b_T t)\frac{e^{-t^2/a^2}}{a^2} t dt$$

$$A_{LL,measured}^{J_0(b_T P_{h,T})}(b_T) = \sqrt{1-\varepsilon^2}\frac{\tilde{g}_1(x, z^2 b_T^2)}{\tilde{f}_1(x, z^2 b_T^2)} \times \frac{1-\epsilon_{g_1}}{1-\epsilon_{f_1}} = A_{LL}^{J_0(b_T P_{h,T})}(b_T) \times \frac{1-\epsilon_{g_1}}{1-\epsilon_{f_1}}$$

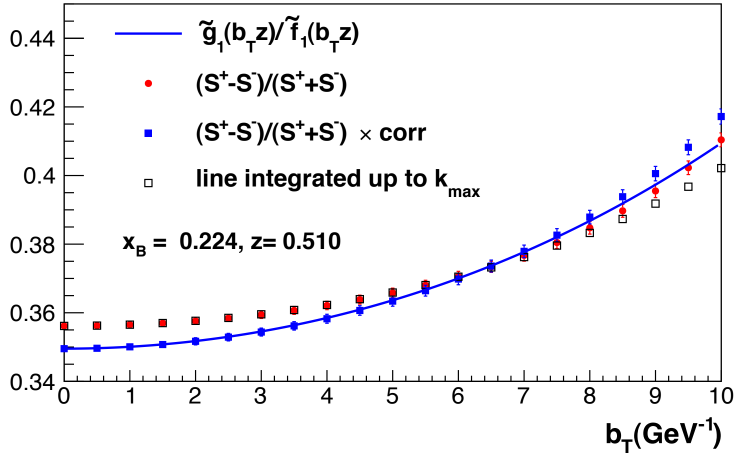
One can use for example Gaussian approach to correct extracted points!

Correction and calculation

Gaussian MC $\exp\left[-\frac{k_\perp^2}{\langle k_\perp^2\rangle}\right]$



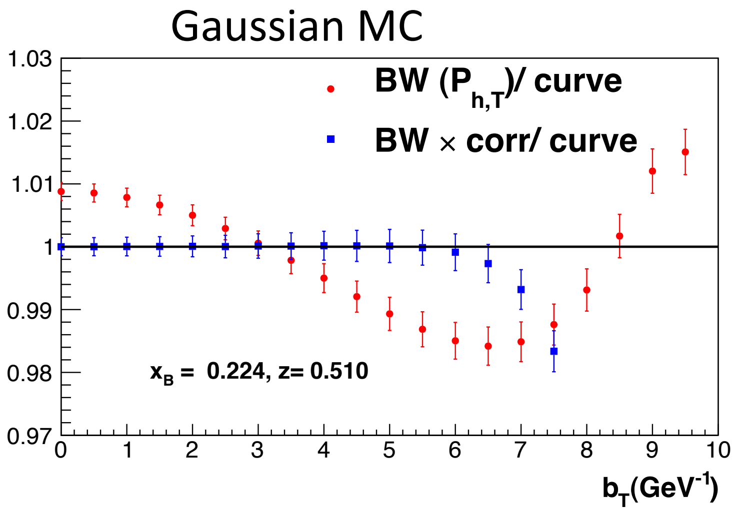
Modified Gaussian MC $\exp\left[-\frac{k_\perp^2}{2\langle k_\perp^2\rangle x(1-x)}\right]$

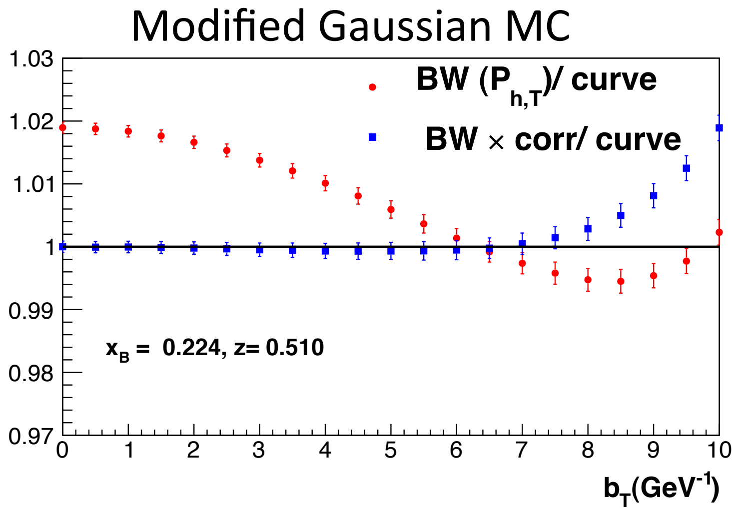


One can correct red points using series of assumptions, which is presented with blue points.

Or, more precise, one can do calculations for the exact bin using integration (from minimum) up to the maximum value, which is presented with black open squares !

Ratio or accuracy





If we consider b_T range up to 6-7 GeV^{-1}, $b_T < 1.2\ fm$
Bessel weighted extraction provides accuracy bellow 1%

Summary

- Multi-dimensional MC developed to study TMD extraction from SIDIS data.
- Target mass corrections has been studied and found to be important at intermediate x.
- Realistic DF with *x* and $k_\perp$ (also *y*) similar way *z* and $p_\perp$ in FF helps to automatically fulfill with tree level parton model kinematic restrictions.
- We present, model independent Bessel-weighting strategy on our MC sample.
- The procedure was checked with different input functions (simple Gaussian $k_\perp$ and x distributions factorized, modified not factorized Gaussian case).

Support

Generalization to non zero proton mass

1) Generating x and y I construct q.
2) Do the boost to the CM frame of virtual photon and target.

Assume that quark inside the proton have the momentum:

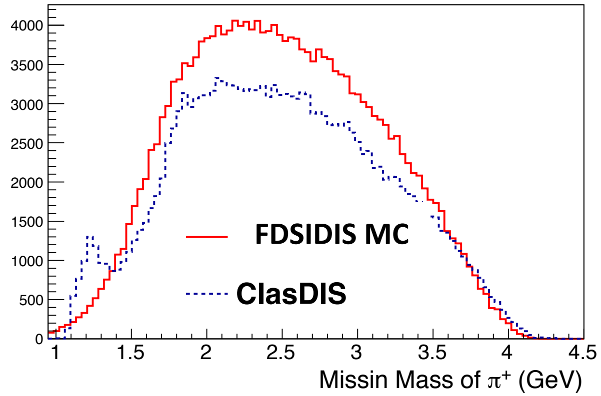
$$k = \left(x'_{LC} P_0 + \frac{k_\perp^2}{4x'_{LC} P_0}, \mathbf{k}_\perp, -x'_{LC} P_0 + \frac{k_\perp^2}{4x'_{LC} P_0} \right)$$

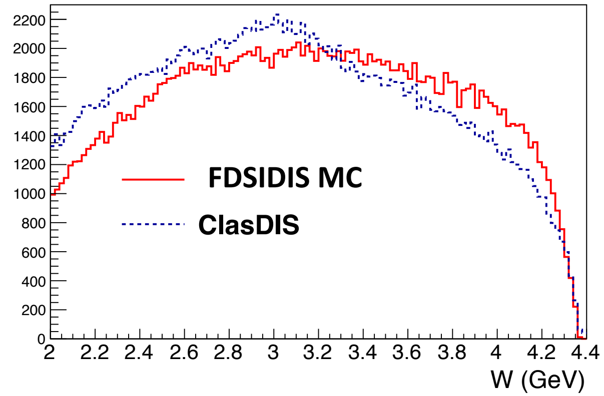
Where $x'_{LC} = \frac{1}{2} x \left(1 + \sqrt{1 + \frac{4k_\perp^2}{Q^2}} \right)$, and P_0 is the proton energy with <u>non zero proton mass</u>.

$$x_{LC} = k^- / P^- = x'_{LC} \times (1 + ...)$$

$$x = \frac{1}{(1 + \sqrt{1 + 4M^2 x_B^2 / Q^2})} \frac{x_B}{2} (1 + \sqrt{1 + 4k_\perp^2 / Q^2})$$

FDSIDIS MC vs CLAS-DIS





PRD 71, 074006 (2005).