QCD-N'12
October 22-26, 2012
Bilbao, Spain

TMD Phenomenology

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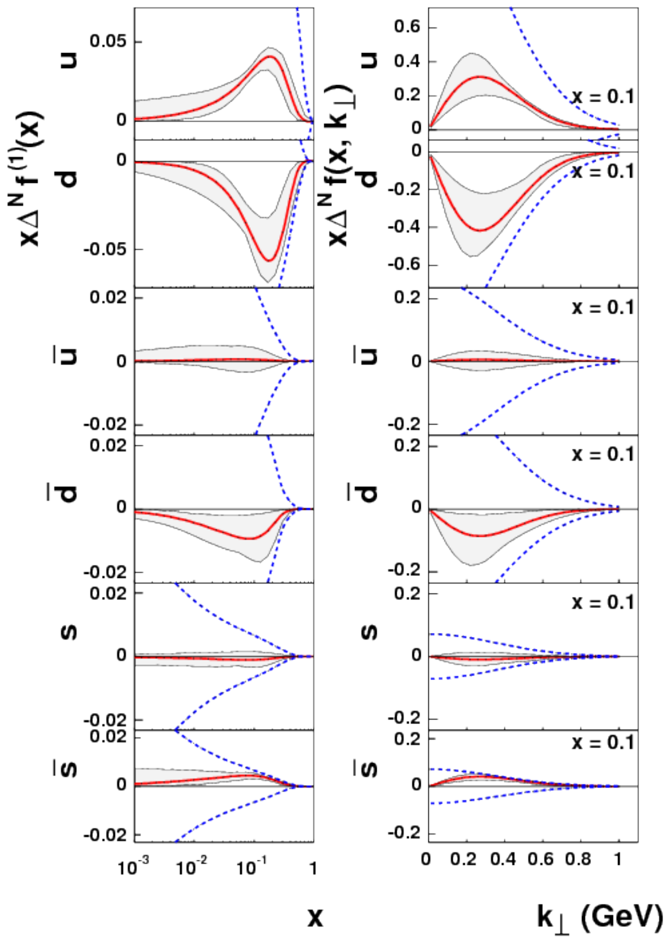
Outline

- Sivers function, fit and evolution
- Transversity and Collins functions
- Boer-Mulders & Cahn effect in SIDIS

The Sivers function from SIDIS data

Sivers function in SIDIS

- In 2009 we performed a fit of HERMES (2002-5) and COMPASS (Deuteron 2003-4) data on π and K production



✔Valence quark

- $\Delta^N f_{u/p^\uparrow} > 0 \quad \Rightarrow f_{1T}^{\perp u} < 0$
- $\Delta^N f_{d/p^\uparrow} < 0 \quad \Rightarrow f_{1T}^{\perp d} > 0$

✔Sea quarks

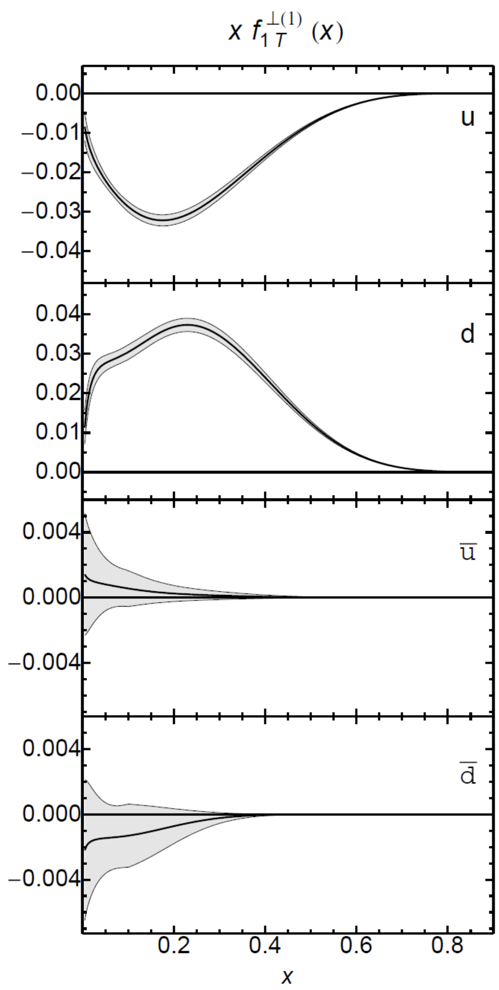
- $\Delta^N f_{\bar{s}/p^\uparrow} > 0 \quad \Rightarrow f_{1T}^{\perp \bar{s}} < 0$

→ $\Delta^N f_q^{(1)}(x) \equiv \int d^2 \boldsymbol{k}_\perp \frac{k_\perp}{4m_p} \Delta^N f_{q/p^\uparrow}(x, k_\perp) = -f_{1T}^{\perp(1)q}(x)$

Anselmino et al. , Eur. Phys. J. A39, 89-100 (2009)

Sivers function in SIDIS

- The sivers function can shed light on the partonic angular momentum



$$f_{1T}^{\perp(0)a}(x; Q_L^2) = -L(x)\, E^a(x,0,0;Q_L^2)$$

(Model dependent)

$$J^a(Q^2) = \frac{1}{2}\int_0^1 dx\, x\left(H^a(x,0,0;Q^2) + E^a(x,0,0;Q^2)\right)$$

Bacchetta and Radici, PRL 107 (2011) 212001

Sivers function in SIDIS

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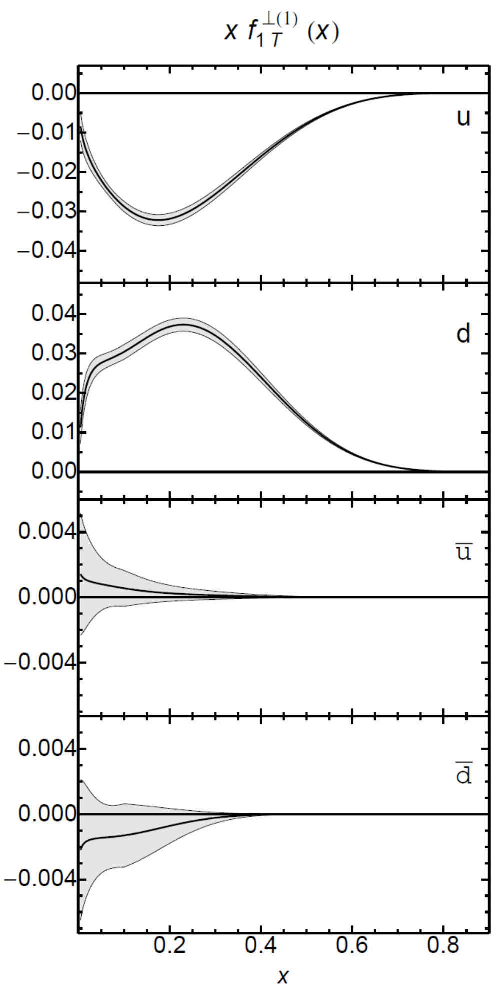
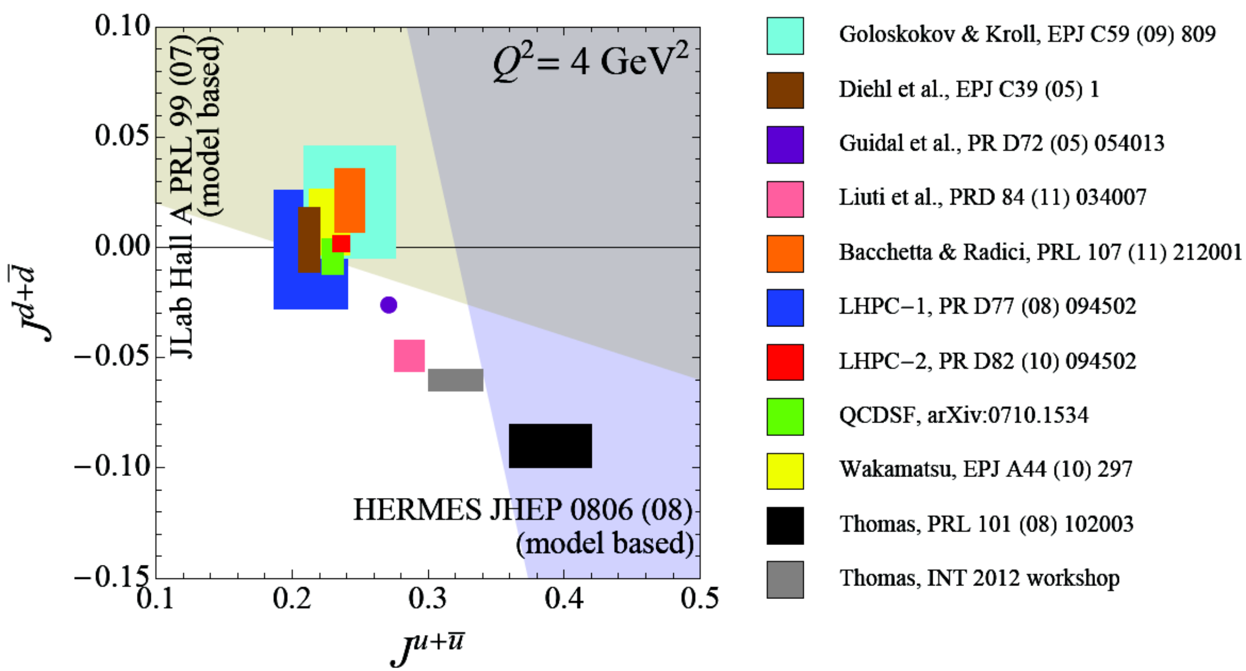




See M. Radici's talk!

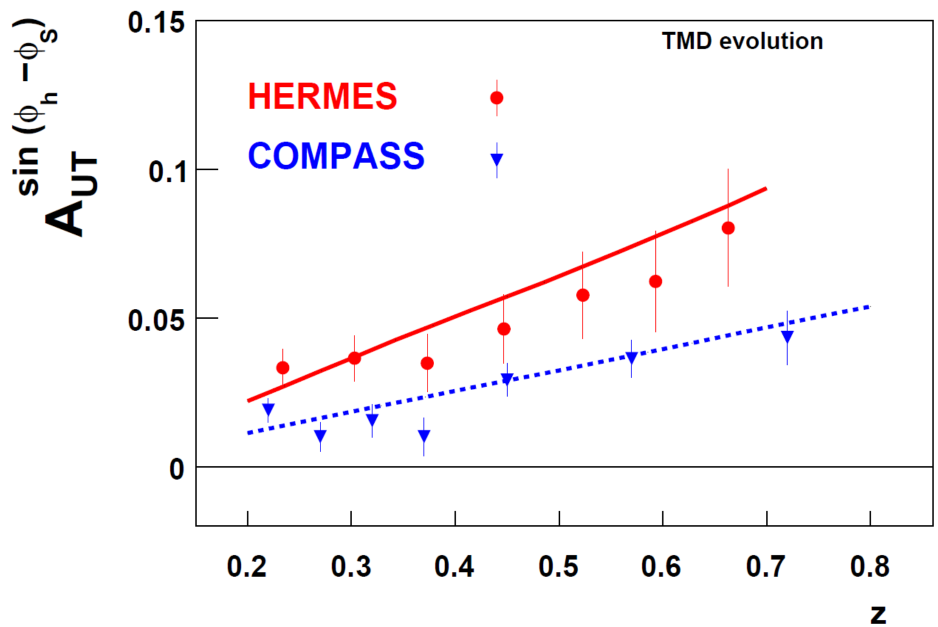
Bacchetta and Radici, PRL 107 (2011) 212001

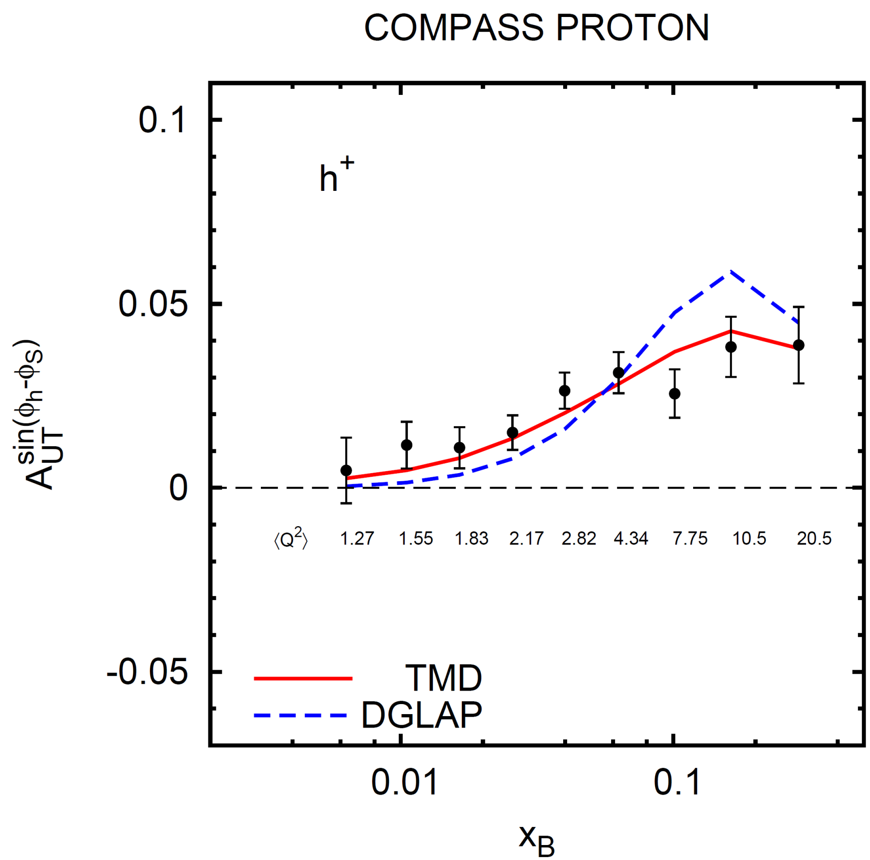
Sivers function in SIDIS

- New data from HERMES (2009) and from COMPASS (proton target, 2010-11)
- New theoretical tool: (Collins et al. & Scimemi et al.) TMD evolution equation

→ Can we see TMD evolution effects in the present SIDIS data??

Sivers function in SIDIS





Aybat, Prokudin, Rogers, PRL 108 (2012) 242003

Anselmino, Boglione, Melis, PRD 86 (2012) 014028

TMD evolution formalism*

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- *J.C. Collins, Foundation of Perturbative QCD, Cambridge Monographs on Particle Physics, Nuclear Physics and Cosmology, No. 32, Cambridge University Press, 2011.*
- *S. M. Aybat and T. C. Rogers, Phys. Rev. D83, 114042 (2011), arXiv:1101.5057 [hep-ph]*
- *S. M. Aybat, J. C. Collins, J.-W. Qiu and T.C. Rogers, arXiv:1110.6428 [hep-ph]*

TMD evolution formalism

- Let us denote with $\widetilde{F}$ either a PDF (or a FF) or the first derivative of the Sivers function in the impact parameter space:

$$\widetilde{F}(x, \boldsymbol{b}_T; Q) \rightarrow \widetilde{f}_{q/p}(x, \boldsymbol{b}_T; Q)$$ Unpolarized PDF

$$\widetilde{F}(x, \boldsymbol{b}_T; Q) \rightarrow \widetilde{D}_{h/q}(z, \boldsymbol{b}_T; Q)$$ Unpolarized FF

$$\widetilde{F}(x, \boldsymbol{b}_T; Q) \rightarrow \widetilde{f}_{1T}^{\prime\perp f}(x, \boldsymbol{b}_T; Q)$$ First derivative of the Sivers function

TMD evolution formalism

- At LO the evolution equation can be summarized by the following expression:

$$\widetilde{F}(x, \boldsymbol{b}_T; Q) = \widetilde{F}(x, \boldsymbol{b}_T; Q_0)\, \widetilde{R}(Q, Q_0, b_T)\ \exp\left\{-g_K(b_T) \ln \frac{Q}{Q_0}\right\}$$

Corresponding to Eq. 44 of Ref [*] with $\tilde{K}$=0 and : $\mu^2 = \zeta_F = \zeta_D = Q^2$

- [*]S. M. Aybat, J. C. Collins, J.-W. Qiu and T.C. Rogers, arXiv:1110.6428 [hep-ph]

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Output function at the scale **Q**
in the impact parameter space

Input function at the scale $\mathbf{Q_0}$
in the impact parameter space

Evolution kernel

TMD evolution formalism

- At LO the evolution equation can be summarized by the following expression:

$$\widetilde{F}(x, \boldsymbol{b}_T; Q) = \widetilde{F}(x, \boldsymbol{b}_T; Q_0)\, \widetilde{R}(Q, Q_0, b_T)\, \exp\left\{-g_K(b_T) \ln \frac{Q}{Q_0}\right\}$$

- **Perturbative** part of the evolution kernel

TMD evolution formalism

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- **Perturbative** part of the evolution kernel

$$\widetilde{R}(Q,Q_0,b_T) \equiv \exp\left\{\ln\frac{Q}{Q_0}\int_{Q_0}^{\mu_b}\frac{\mathrm{d}\mu'}{\mu'}\gamma_K(\mu') + \int_{Q_0}^{Q}\frac{\mathrm{d}\mu}{\mu}\gamma_F\left(\mu,\frac{Q^2}{\mu^2}\right)\right\}$$

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$$\gamma_K(\mu) = \alpha_s(\mu) \frac{2\, C_F}{\pi}$$

$$\gamma_F(\mu; \frac{Q^2}{\mu^2}) = \alpha_s(\mu) \frac{C_F}{\pi} \left(\frac{3}{2} - \ln \frac{Q^2}{\mu^2}\right)$$

TMD evolution formalism

➤ At LO the evolution equation can be summarized by the following expression:

$$\widetilde{F}(x,\boldsymbol{b}_T;Q)=\widetilde{F}(x,\boldsymbol{b}_T;Q_0)\,\widetilde{R}(Q,Q_0,b_T)\,\exp\left\{-g_K(b_T)\ln\frac{Q}{Q_0}\right\}$$

➤ **Perturbative** part of the evolution kernel

$$\widetilde{R}(Q,Q_0,b_T)\equiv\exp\left\{\ln\frac{Q}{Q_0}\int_{Q_0}^{\mu_b}\frac{d\mu'}{\mu'}\gamma_K(\mu')+\int_{Q_0}^{Q}\frac{d\mu}{\mu}\gamma_F\left(\mu,\frac{Q^2}{\mu^2}\right)\right\}$$

Scale that separates the perturbative region from the non perturbative one

TMD evolution formalism

➤ At LO the evolution equation can be summarized by the following expression:

$$\widetilde{F}(x, \boldsymbol{b}_T; Q) = \widetilde{F}(x, \boldsymbol{b}_T; Q_0)\, \widetilde{R}(Q, Q_0, b_T)\, \exp\left\{-g_K(b_T) \ln \frac{Q}{Q_0}\right\}$$

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$$\widetilde{R}(Q, Q_0, b_T) \equiv \exp\left\{\ln \frac{Q}{Q_0} \int_{Q_0}^{\mu_b} \frac{d\mu'}{\mu'} \gamma_K(\mu') + \int_{Q_0}^{Q} \frac{d\mu}{\mu} \gamma_F\left(\mu, \frac{Q^2}{\mu^2}\right)\right\}$$

$$\mu_b = \frac{C_1}{b_*(b_T)} \qquad b_*(b_T) \equiv \frac{b_T}{\sqrt{1 + b_T^2/b_{\max}^2}} \qquad C_1 = 2e^{-\gamma_E}$$

One of the possible prescription to separate the perturbative region from the non perturbative one

TMD evolution formalism

- At LO the evolution equation can be summarized by the following expression:

$$\widetilde{F}(x, \boldsymbol{b}_T; Q) = \widetilde{F}(x, \boldsymbol{b}_T; Q_0)\, \widetilde{R}(Q, Q_0, b_T)\, \exp\left\{ -g_K(b_T) \ln \frac{Q}{Q_0} \right\}$$

- **Non Perturbative** (scale independent) part of the evolution kernel that needs to be empirically modeled

$$g_K(b_T) = \frac{1}{2}\, g_2\, b_T^2$$

Common choice used in the unpolarized DY data analyses in the CSS formalism

Parametrization of the input functions

$$\widetilde{F}(x,\boldsymbol{b}_T;Q) = \widetilde{F}(x,\boldsymbol{b}_T;Q_0)\,\widetilde{R}(Q,Q_0,b_T)\;\exp\left\{-g_K(b_T)\ln\frac{Q}{Q_0}\right\}$$

- Model/parametrization

Parametrization of the input functions

$$\widetilde{F}(x,\boldsymbol{b}_T;Q) = \widetilde{F}(x,\boldsymbol{b}_T;Q_0)\,\widetilde{R}(Q,Q_0,b_T)\ \exp\left\{-g_K(b_T)\ln\frac{Q}{Q_0}\right\}$$

Example: unpolarized pdf

$$\tilde{F}(x,b_T;Q_0) = \tilde{f}_{q/p}(x,b_T;Q_0) \xrightarrow{\text{Fourier transf.}} \widehat{f}_{q/p}(x,k_\perp;Q_0)$$

$$\widehat{f}_{q/p}(x,k_\perp;Q_0) = f_{q/p}(x,Q_0)\,\frac{1}{\pi\langle k_\perp^2\rangle}\,e^{-k_\perp^2/\langle k_\perp^2\rangle}$$

Parametrization of the input functions

$$\widetilde{F}(x, \boldsymbol{b}_T; Q) = \widetilde{F}(x, \boldsymbol{b}_T; Q_0)\, \widetilde{R}(Q, Q_0, b_T)\ \exp\left\{ -g_K(b_T) \ln \frac{Q}{Q_0} \right\}$$

$$\widetilde{f}_{q/p}(x, b_T; Q_0) = f_{q/p}(x, Q_0) \exp\left\{ -\alpha^2\, b_T^2 \right\}$$

$$\widehat{f}_{q/p}(x, k_\perp; Q_0) = f_{q/p}(x, Q_0)\, \frac{1}{\pi \langle k_\perp^2 \rangle}\, e^{-k_\perp^2/\langle k_\perp^2 \rangle}$$

$$\alpha^2 = \langle k_\perp^2 \rangle / 4$$

Parametrization ot the input functions

$$\widetilde{F}(x, \boldsymbol{b}_T; Q) = \widetilde{F}(x, \boldsymbol{b}_T; Q_0)\, \widetilde{R}(Q, Q_0, b_T) \; \exp\left\{-g_K(b_T) \ln \frac{Q}{Q_0}\right\}$$

$$\widetilde{D}_{h/q}(z, b_T; Q_0) = \frac{1}{z^2} D_{h/q}(z, Q_0) \; \exp\left\{-\beta^2\, b_T^2\right\}$$

$$\widehat{D}_{h/q}(z, p_\perp; Q_0) = D_{h/q}(z, Q_0)\, \frac{1}{\pi \langle p_\perp^2 \rangle}\, e^{-p_\perp^2/\langle p_\perp^2 \rangle}$$

$$\beta^2 = \langle p_\perp^2 \rangle / 4z^2$$

Parametrization ot the input functions

$$\widetilde{F}(x,\boldsymbol{b}_T;Q) = \widetilde{F}(x,\boldsymbol{b}_T;Q_0)\,\widetilde{R}(Q,Q_0,b_T)\ \exp\left\{-g_K(b_T)\ln\frac{Q}{Q_0}\right\}$$

$$\widetilde{f}'^{\perp}_{1T}(x,b_T;Q_0) = -2\,\gamma^2\,f^{\perp}_{1T}(x;Q_0)\,b_T\,e^{-\gamma^2\,b_T^2}$$

$$\widehat{f}^{\perp}_{1T}(x,k_\perp;Q_0) = f^{\perp}_{1T}(x;Q_0)\,\frac{1}{4\,\pi\,\gamma^2}\,e^{-k_\perp^2/4\gamma^2}$$

$$4\,\gamma^2 \equiv \langle k_\perp^2\rangle_S = \frac{M_1^2\,\langle k_\perp^2\rangle}{M_1^2+\langle k_\perp^2\rangle}$$

Parametrization of the input functions

- Then the evolution equations for unpolarized TMDs become simply:

$$\widetilde{f}_{q/p}(x, b_T; Q) = f_{q/p}(x, Q_0)\, \widetilde{R}(Q, Q_0, b_T)\; \exp\left\{-b_T^2 \left(\alpha^2 + \frac{g_2}{2} \ln \frac{Q}{Q_0}\right)\right\}$$

$$\widetilde{D}_{h/q}(z, b_T; Q) = \frac{1}{z^2} D_{h/q}(z, Q_0)\, \widetilde{R}(Q, Q_0, b_T)\; \exp\left\{-b_T^2 \left(\beta^2 + \frac{g_2}{2} \ln \frac{Q}{Q_0}\right)\right\}$$

- While for the Sivers function we have:

$$\widetilde{f}'^{\perp}_{1T}(x, b_T; Q) = -2\,\gamma^2\, f^{\perp}_{1T}(x; Q_0)\, \widetilde{R}(Q, Q_0, b_T)\, b_T\; \exp\left\{-b_T^2 \left(\gamma^2 + \frac{g_2}{2} \ln \frac{Q}{Q_0}\right)\right\}$$

TMD evolution formalism

➢One can get the TMD in the momentum space by Fourier trasforming:

$$\widehat{f}_{q/p}(x,k_\perp;Q) = \frac{1}{2\pi}\int_0^\infty \mathrm{d}b_T\, b_T\, J_0(k_\perp b_T)\, \widetilde{f}_{q/p}(x,b_T;Q)$$

$$\widehat{D}_{h/q}(z,p_\perp;Q) = \frac{1}{2\pi}\int_0^\infty \mathrm{d}b_T\, b_T\, J_0(\mathrm{k}_T b_T)\, \widetilde{D}_{h/q}(z,b_T;Q)$$

$$\widehat{f}_{1T}^{\perp f}(x,k_\perp;Q) = \frac{-1}{2\pi k_\perp}\int_0^\infty \mathrm{d}b_T\, b_T\, J_1(k_\perp b_T)\, \widetilde{f}_{1T}^{\prime\,\perp q}(x,b_T;Q)$$

$$\begin{aligned} f_{q/p^\uparrow}(x,\boldsymbol{k}_\perp,\boldsymbol{S};Q) &= f_{q/p}(x,k_\perp;Q) - f_{1T}^{\perp q}(x,k_\perp;Q)\frac{\epsilon_{ij}\,k_\perp^i\,S^j}{M_p} \\ &= f_{q/p}(x,k_\perp;Q) + \frac{1}{2}\Delta^N f_{q/p^\uparrow}(x,k_\perp;Q)\frac{\epsilon_{ij}\,k_\perp^i\,S^j}{k_\perp} \end{aligned}$$

Fit of HERMES and COMPASS SIDIS data

$$A_{UT}^{\sin(\phi_h-\phi_S)} = \frac{\sum_q \int d\phi_S\, d\phi_h\, d^2\boldsymbol{k}_\perp\, \Delta^N f_{q/p^\uparrow}(x,k_\perp,Q)\sin(\varphi-\phi_S)\,\frac{d\hat\sigma^{\ell q\to\ell q}}{dQ^2}\, D_q^h(z,p_\perp,Q)\sin(\phi_h-\phi_S)}{\sum_q \int d\phi_S\, d\phi_h\, d^2\boldsymbol{k}_\perp\, f_{q/p}(x,k_\perp,Q)\,\frac{d\hat\sigma^{\ell q\to\ell q}}{dQ^2}\, D_q^h(z,p_\perp,Q)}$$

$$\Delta^N \widehat{f}_{q/p^\uparrow}(x,k_\perp;Q_0) = 2\mathcal{N}_q(x)h(k_\perp)\widehat{f}_{q/p}(x,k_\perp;Q_0)$$

$$\mathcal{N}_q(x) = N_q\, x^{\alpha_q}(1-x)^{\beta_q}\,\frac{(\alpha_q+\beta_q)^{(\alpha_q+\beta_q)}}{\alpha_q^{\alpha_q}\beta_q^{\beta_q}}$$

$$h(k_\perp) = \sqrt{2e}\,\frac{k_\perp}{M_1}\,e^{-k_\perp^2/M_1^2}$$

$$\widehat{f}_{q/p}(x,k_\perp;Q_0) = f_{q/p}(x,Q_0)\,\frac{1}{\pi\langle k_\perp^2\rangle}\,e^{-k_\perp^2/\langle k_\perp^2\rangle}$$

$$\widehat{D}_{h/q}(z,p_\perp;Q_0) = D_{h/q}(z,Q_0)\,\frac{1}{\pi\langle p_\perp^2\rangle}\,e^{-p_\perp^2/\langle p_\perp^2\rangle}$$

11 free parameters

N_{u_v}	N_{d_v}	N_s
$N_{\bar u}$	$N_{\bar d}$	$N_{\bar s}$
α_{u_v}	α_{d_v}	α_{sea}
β	M_1 (GeV/c) .	

Fixed parameters

$$\langle k_\perp^2\rangle = 0.25\ \mathrm{GeV}^2$$
$$\langle p_\perp^2\rangle = 0.20\ \mathrm{GeV}^2$$
$$g_2 = 0.68\ \mathrm{GeV}^2$$
$$b_{max} = 0.5\ \mathrm{GeV}^{-1}$$

Fit of HERMES and COMPASS SIDIS data

χ^2 tables

11 free parameters, 261 points

TMD evolution (exact)
$\chi^2_{\rm tot} = 255.8$
$\chi^2_{\rm d.o.f} = 1.02$

DGLAP evolution
$\chi^2_{\rm tot} = 315.6$
$\chi^2_{\rm d.o.f} = 1.26$

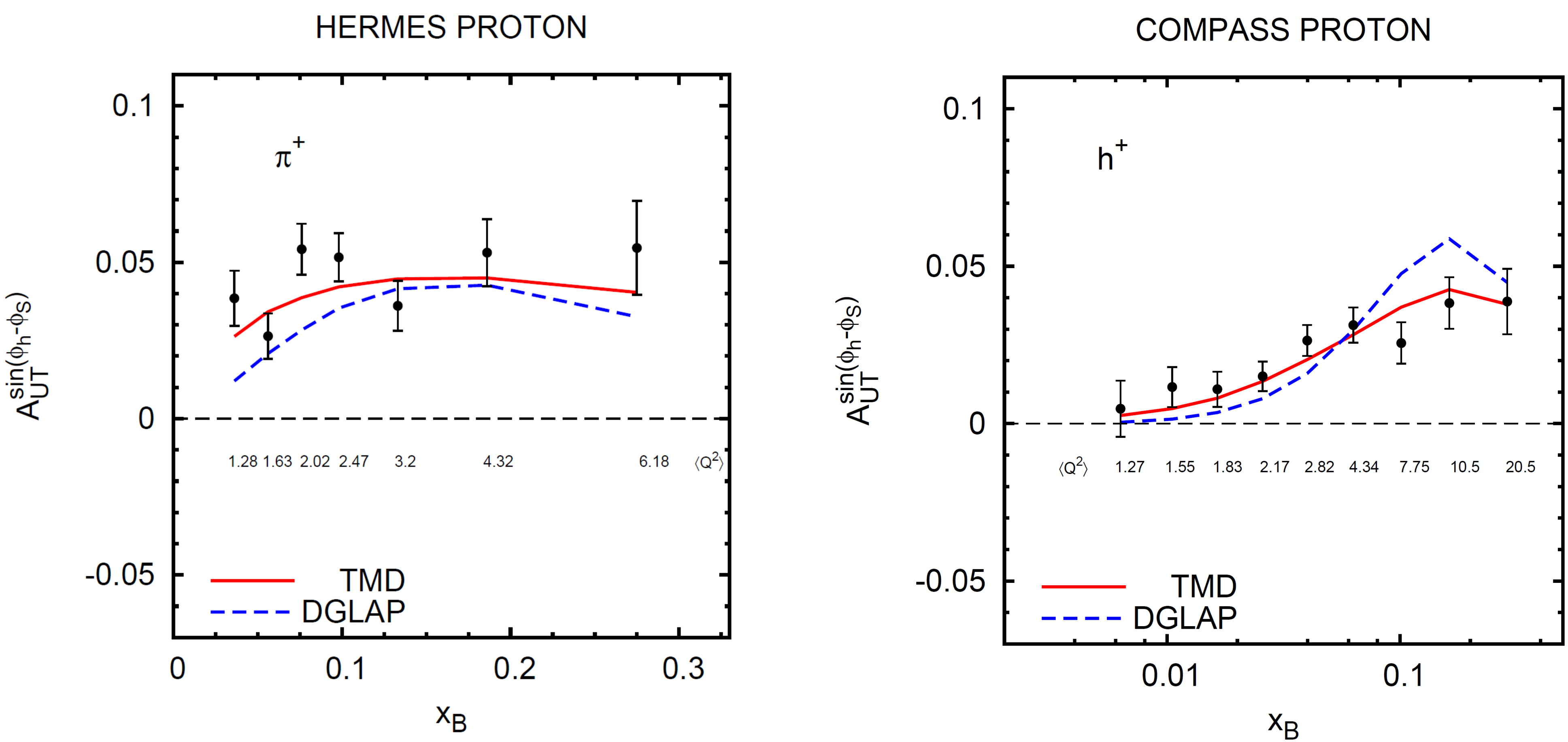
Fit of HERMES and COMPASS SIDIS data

χ^2 tables

11 free parameters, 261 points

	TMD Evolution (Exact)		DGLAP Evolution
	$\chi^2_{tot} = 255.8$		$\chi^2_{tot} = 315.6$
	$\chi^2_{d.o.f} = 1.02$		$\chi^2_{d.o.f} = 1.26$
HERMES π^+	$\chi^2_x = 10.7$	7 points	$\chi^2_x = 27.5$
	$\chi^2_z = 4.3$		$\chi^2_z = 8.6$
	$\chi^2_{P_T} = 9.1$		$\chi^2_{P_T} = 22.5$
COMPASS h^+	$\chi^2_x = 6.7$	9 points	$\chi^2_x = 29.2$
	$\chi^2_z = 17.8$		$\chi^2_z = 16.6$
	$\chi^2_{P_T} = 12.4$		$\chi^2_{P_T} = 11.8$

Fit of HERMES and COMPASS SIDIS data



Consequences on DY data and warnings

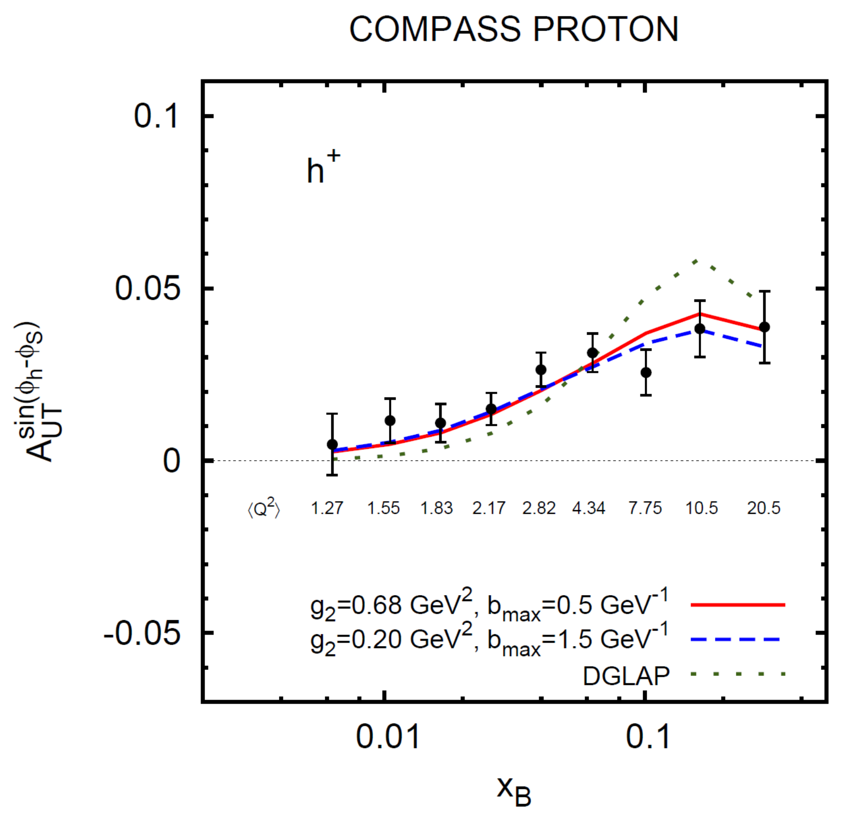
- A rigorous fit need a 'fresh restart' i.e. the analysis of the SIDIS and DY unpolarized data

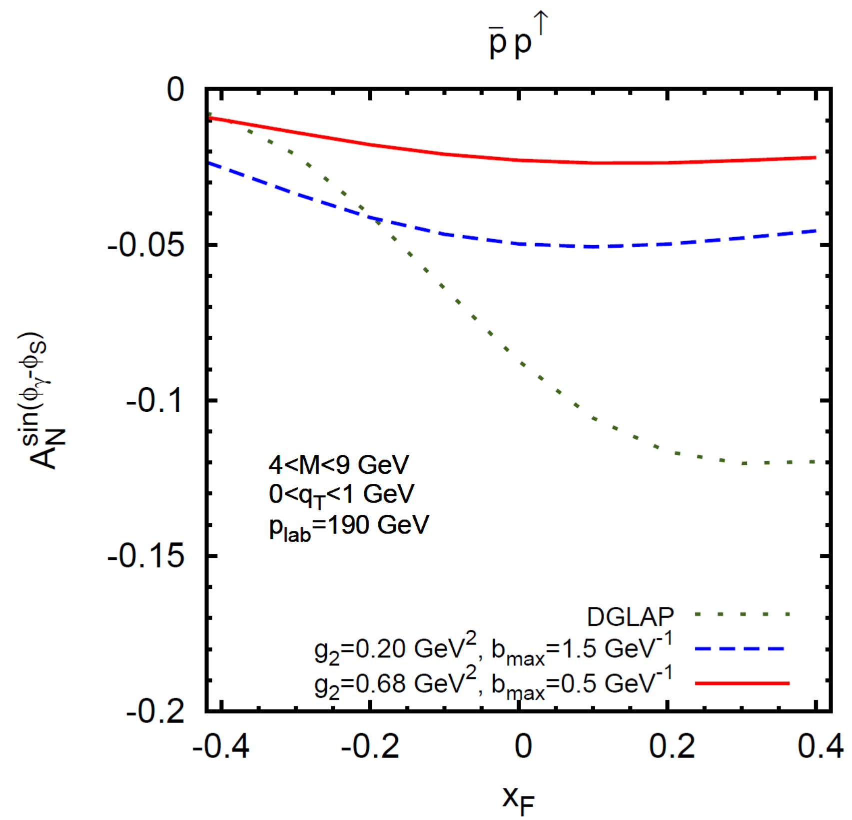
Fixed parameters in the fit

$$\langle k_\perp^2 \rangle = 0.25 \text{ GeV}^2$$
$$\langle p_\perp^2 \rangle = 0.20 \text{ GeV}^2$$
$$g_2 = 0.68 \text{ GeV}^2$$
$$b_{max} = 0.5 \text{ GeV}^{-1}$$

- In SIDIS, the Sivers asymmetry is not so strongly sensitive to these values.

- … however in DY they are crucial, in particular g_2


COMPASS PROTON
h+
$A_{UT}^{\sin(\phi_h-\phi_S)}$
x_B
⟨Q²⟩ 1.27 1.55 1.83 2.17 2.82 4.34 7.75 10.5 20.5
g_2=0.68 GeV², b_{max}=0.5 GeV⁻¹
g_2=0.20 GeV², b_{max}=1.5 GeV⁻¹
DGLAP


$\bar{p}\,p^{\uparrow}$
$A_N^{\sin(\phi_\gamma-\phi_S)}$
x_F
4<M<9 GeV
0<q_T<1 GeV
p_{lab}=190 GeV
DGLAP
g_2=0.20 GeV², b_{max}=1.5 GeV⁻¹
g_2=0.68 GeV², b_{max}=0.5 GeV⁻¹

Conclusions I

- Sivers functions are definitively different from zero!
- We can extract information on the partonic angular momentum from the Sivers function
- There are indications supporting TMD evolution in SIDIS
- Asymmetry in DY are more sensitive to TMD evolution

Polarized SIDIS& e+e- data:

Extraction of Transversity

Extraction of the transversity & Collins functions (The TMD way in Pavia slang...)

- Azimuthal asymmetry in polarized SIDIS

$$\mathrm{d}\sigma^{\uparrow} - \mathrm{d}\sigma^{\downarrow} = \sum_q h_{1q}(x, \boldsymbol{k}_\perp) \otimes \mathrm{d}\Delta\hat{\sigma}(y, \boldsymbol{k}_\perp) \otimes \Delta^N D_{h/q^\uparrow}(z, \boldsymbol{p}_\perp)$$

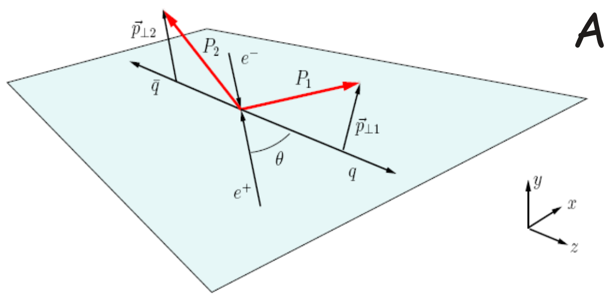
Transversity

Collins function

$$A_{UT}^{\sin(\phi+\phi_S)} \equiv 2\, \frac{\int \mathrm{d}\phi\, \mathrm{d}\phi_S\, [\mathrm{d}\sigma^{\uparrow} - \mathrm{d}\sigma^{\downarrow}]\, \sin(\phi+\phi_S)}{\int \mathrm{d}\phi\, \mathrm{d}\phi_S\, [\mathrm{d}\sigma^{\uparrow} + \mathrm{d}\sigma^{\downarrow}]}$$

Extraction of transversity & Collins functions

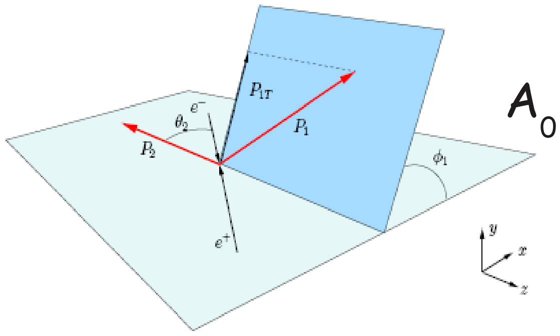
- $e^+e^- \to h_1\, h_2\, X$ BELLE Data



A_{12} asymmetry → Thrust axis method

$$A(z_1, z_2, \theta, \varphi_1 + \varphi_2) \equiv \frac{1}{\langle d\sigma \rangle} \frac{d\sigma^{e^+e^- \to h_1 h_2 X}}{dz_1\, dz_2\, d\cos\theta\, d(\varphi_1 + \varphi_2)}$$

$$= 1 + \frac{1}{8} \frac{\sin^2\theta}{1 + \cos^2\theta} \cos(\varphi_1 + \varphi_2) \frac{\sum_q e_q^2\, \Delta^N D_{h_1/q^\uparrow}(z_1)\, \Delta^N D_{h_2/\bar{q}^\uparrow}(z_2)}{\sum_q e_q^2 D_{h_1/q}(z_1)\, D_{h_2/\bar{q}}(z_2)}$$



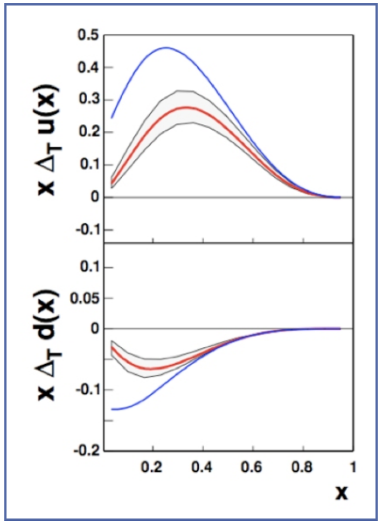
A_0 asymmetry → Hadronic plane method

$$A(z_1, z_2, \theta_2, \phi_1) = 1 + \frac{1}{\pi} \frac{z_1\, z_2}{z_1^2 + z_2^2} \frac{\sin^2\theta_2}{1 + \cos^2\theta_2} \cos(2\,\phi_1) \frac{\sum_q e_q^2\, \Delta^N D_{h_1/q^\uparrow}(z_1)\, \Delta^N D_{h_2/\bar{q}^\uparrow}(z_2)}{\sum_q e_q^2 D_{h_1/q}(z_1)\, D_{h_2/\bar{q}}(z_2)}$$

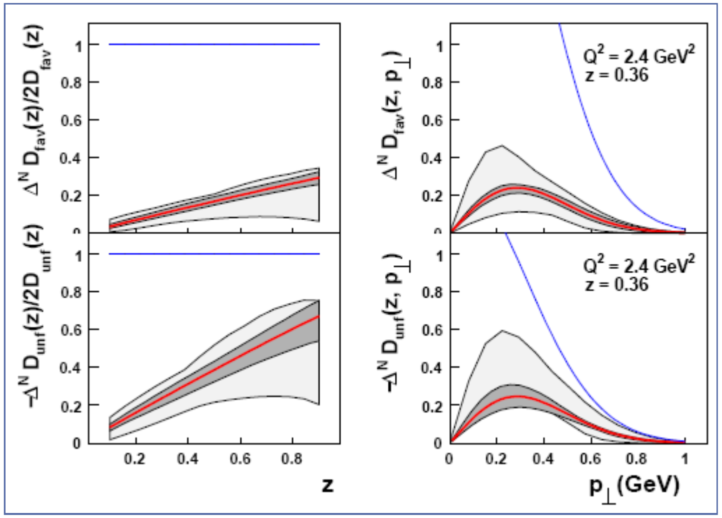
Extraction of transversity & Collins functions

- Simultaneous fit of HERMES, COMPASS and BELLE data

χ^2_{dof}=1.3



Transversity



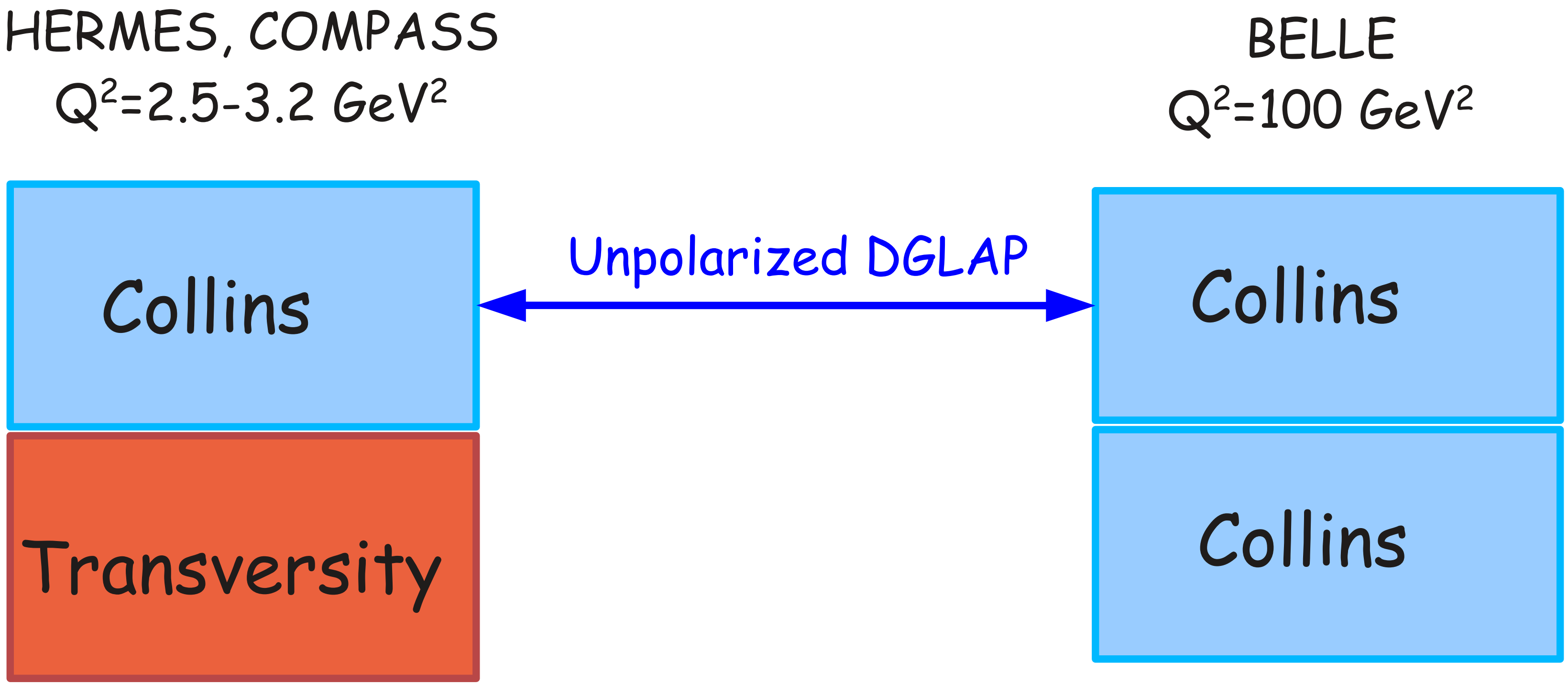
Collins functions

$N_u^T = 0.64 \pm 0.34$	$N_d^T = -1.00 \pm 0.02$
$\alpha = 0.73 \pm 0.51$	$\beta = 0.84 \pm 2.30$
$N_{fav}^C = 0.44 \pm 0.07$	$N_{unf}^C = -1.00 \pm 0.06$
$\gamma = 0.96 \pm 0.08$	$\delta = 0.01 \pm 0.05$
$M_h^2 = 0.91 \pm 0.52$ GeV^2	

- Anselmino et. al arXiv: 0812.4366v1

TMD evolution

- TMD evolution for the Collins function is still unknown.
- TMD evolution can suppress the Collins function at large Q^2 (Boer, 2001)



TMD evolution

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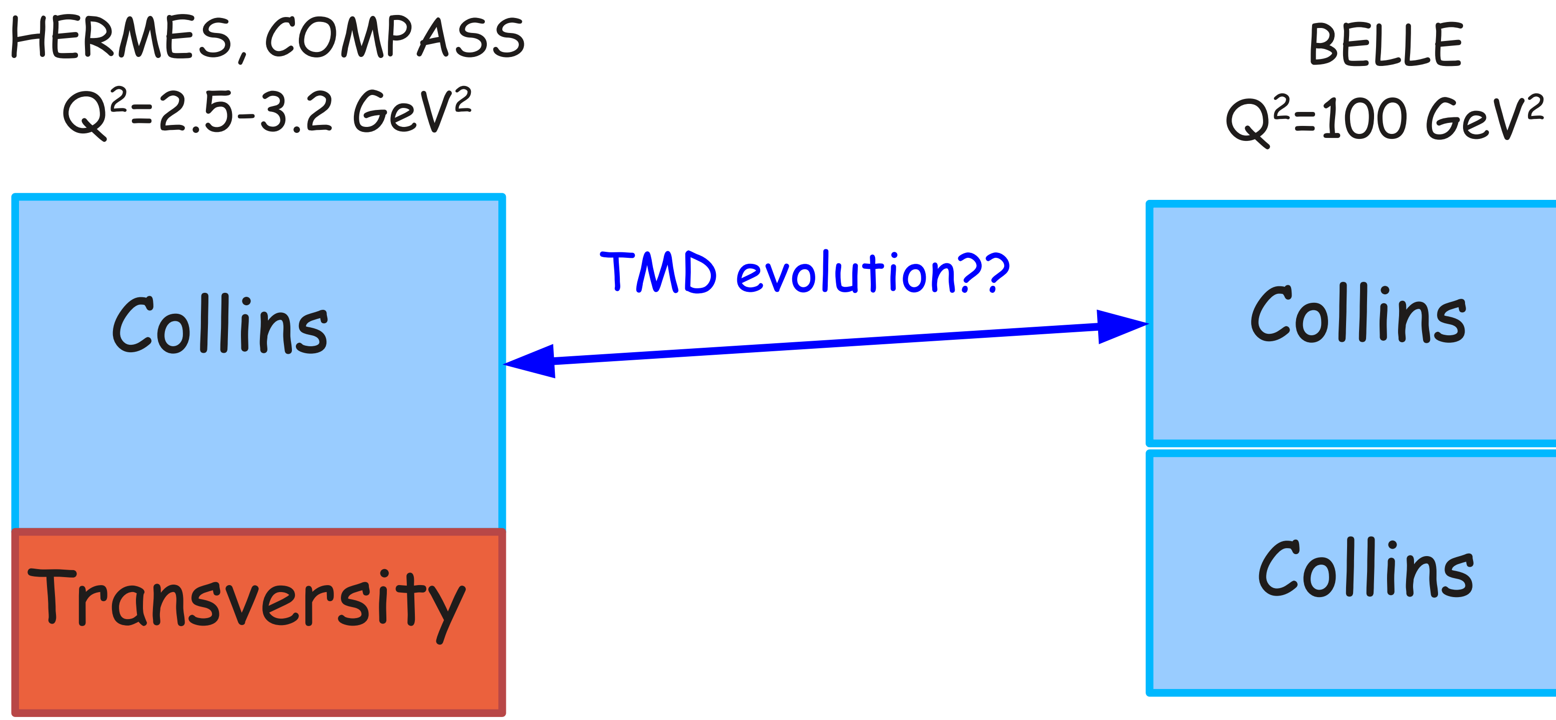
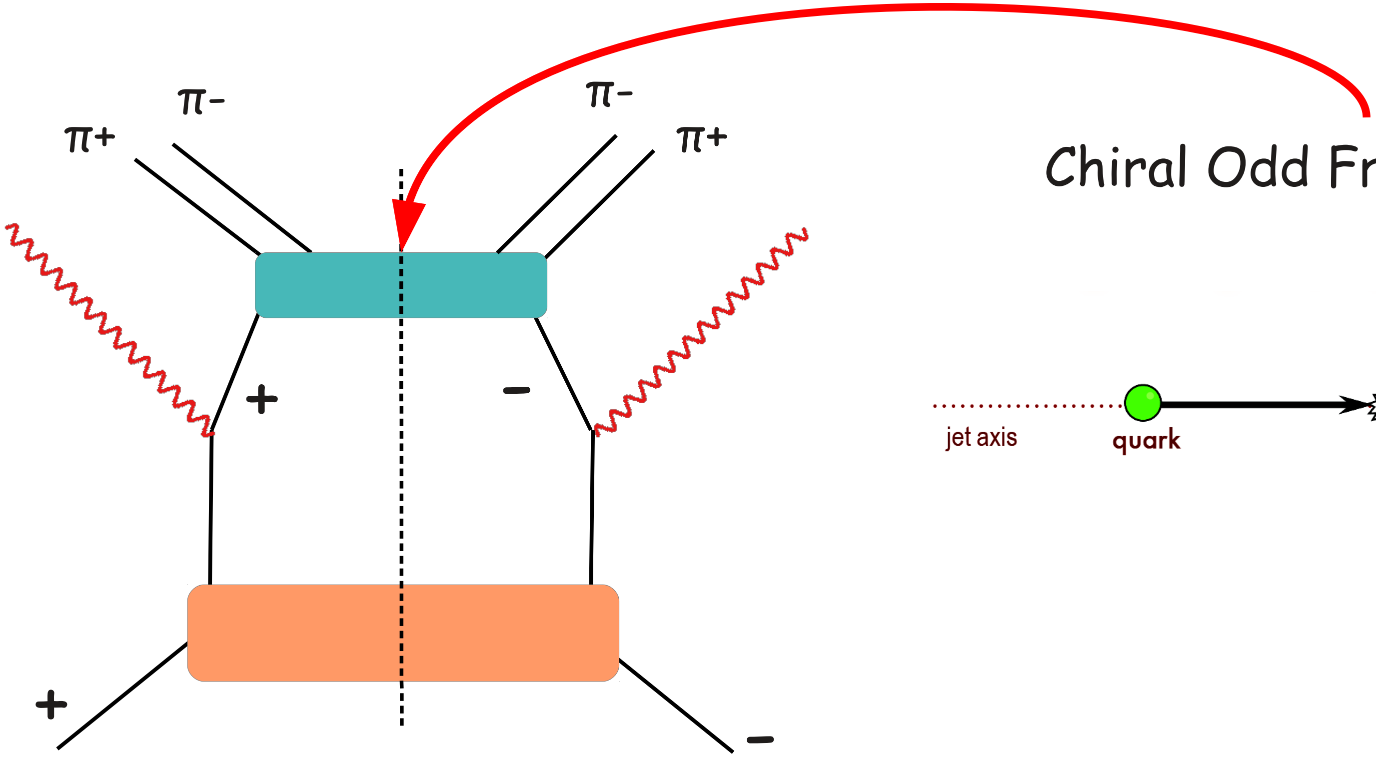
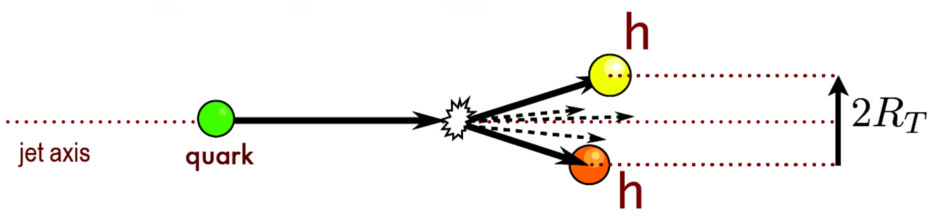


The dihadron way



Chiral Odd Fragmentation!



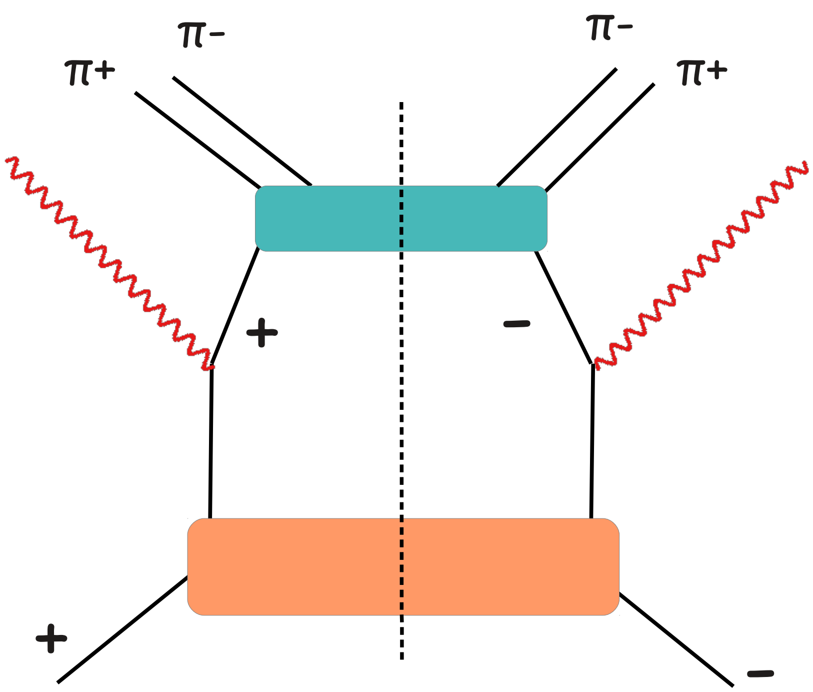
$$D_1^{q\rightarrow h_1 h_2}(z, M_h^2)$$

➢Unpolarized DiFF

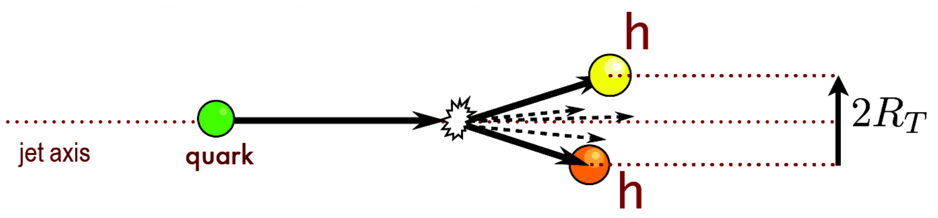
$$H_1^{\sphericalangle q}(z, M_h^2)$$

➢Chiral-Odd DiFF

The dihadron way



Collinear evolution!



$$D_1^{q\to h_1 h_2}(z, M_h^2)$$

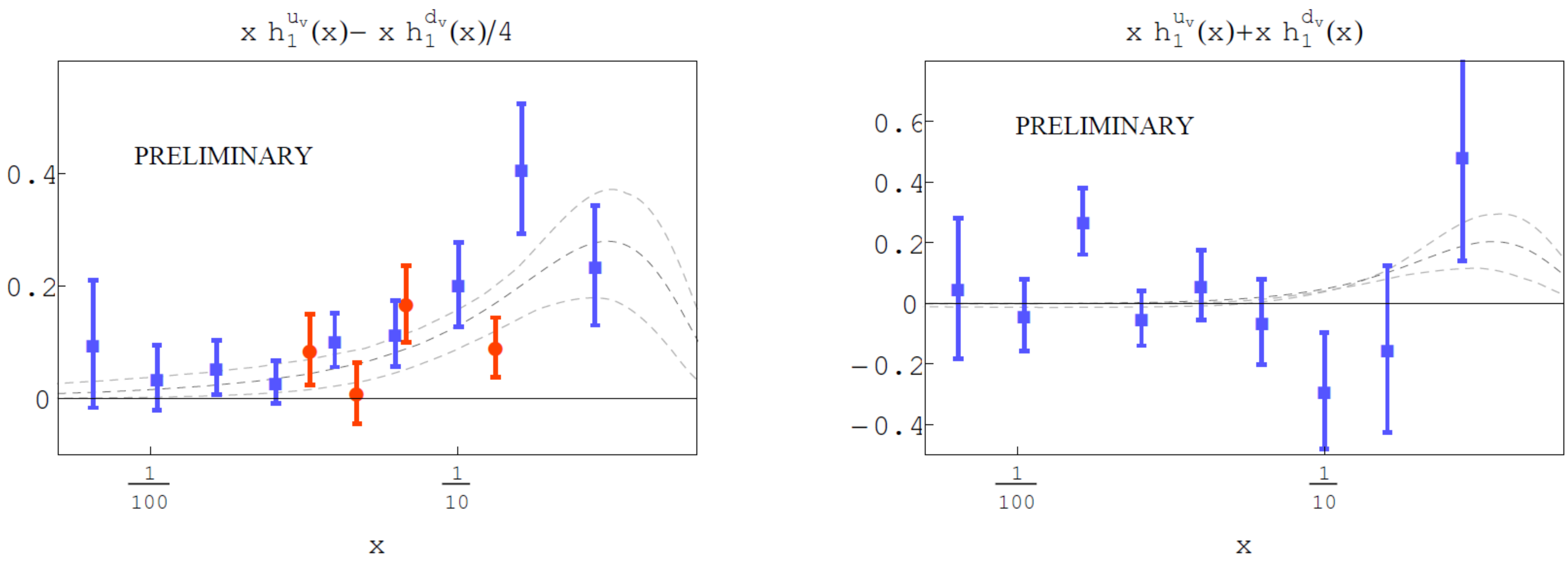
- Unpolarized DiFF

$$H_1^{\sphericalangle q}(z, M_h^2)$$

- Chiral-Odd DiFF

The dihadron way: Pavia group extraction

- Comparison Pavia-Torino



A. Courtoy's talk

Bacchetta, Courtoy, Radici.,arXiv:1206.1836

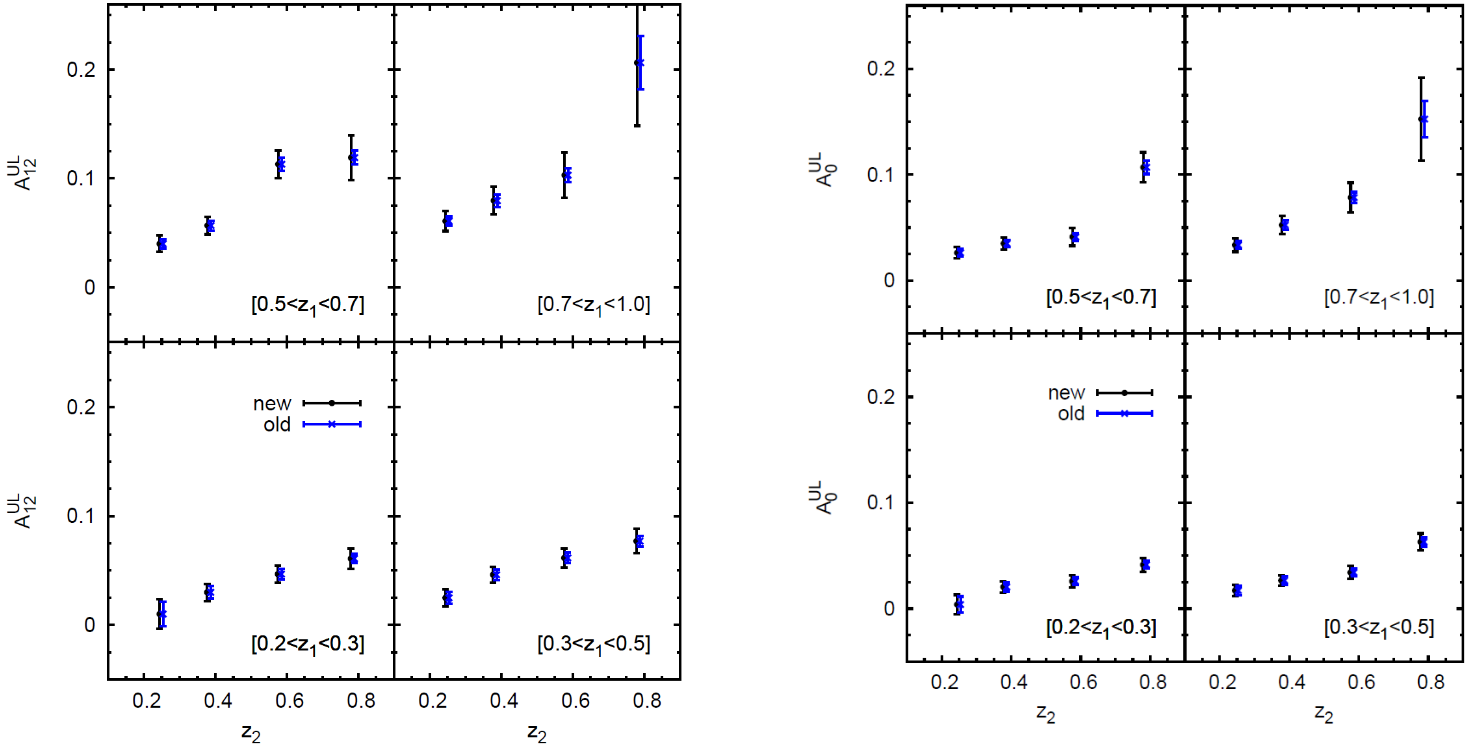
News on the Collins function

- New data from COMPASS (proton target, 2010-11)
- BELLE Erratum: R. Seidl, PRD 86 (2012) 039905

R. Seidl's talk

News on the Collins function

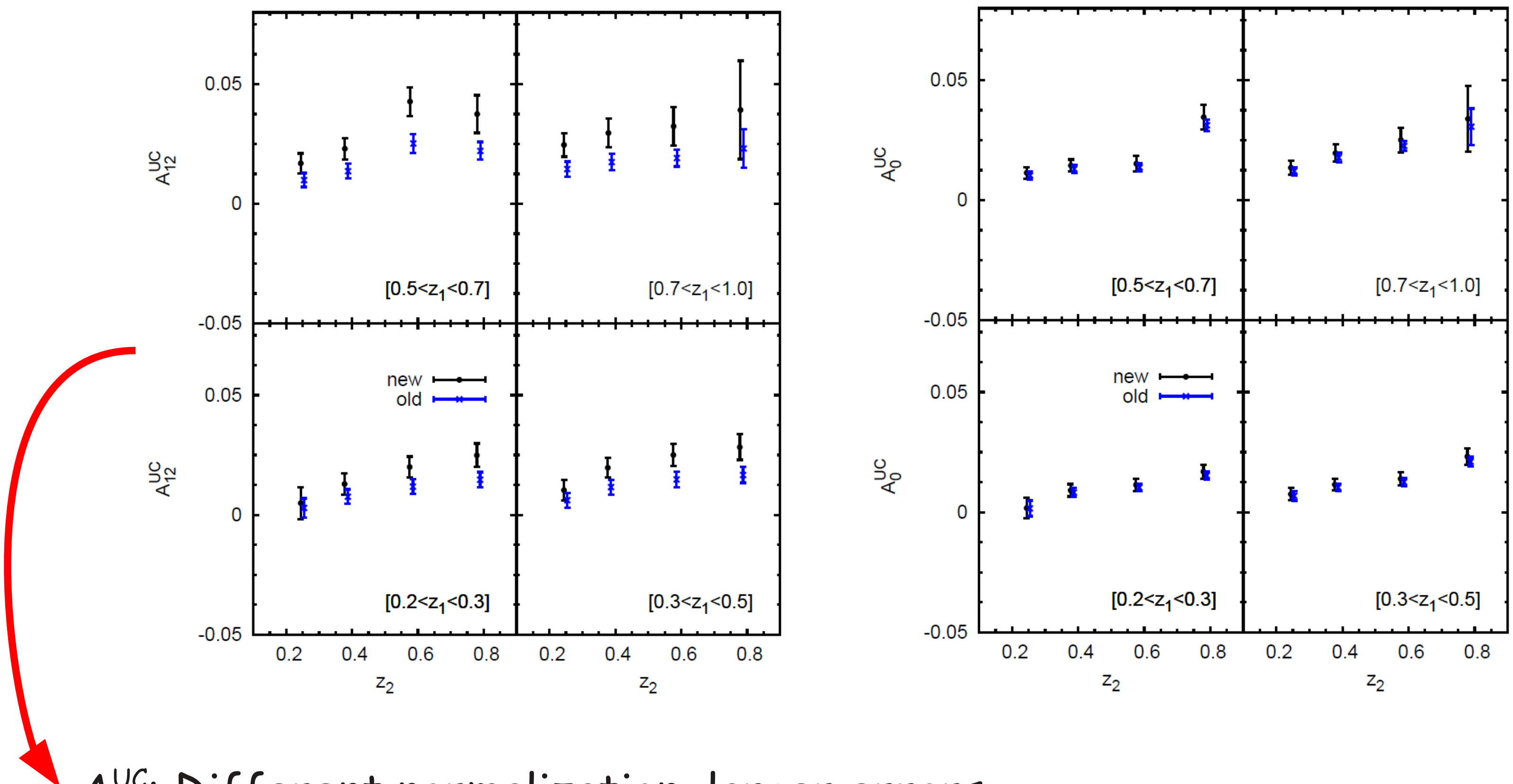
- New data from COMPASS (proton target, 2010-11)
- BELLE Erratum: R. Seidl, PRD 86 (2012) 039905



A^{UL}: Same central values, larger errors

News on the Collins function

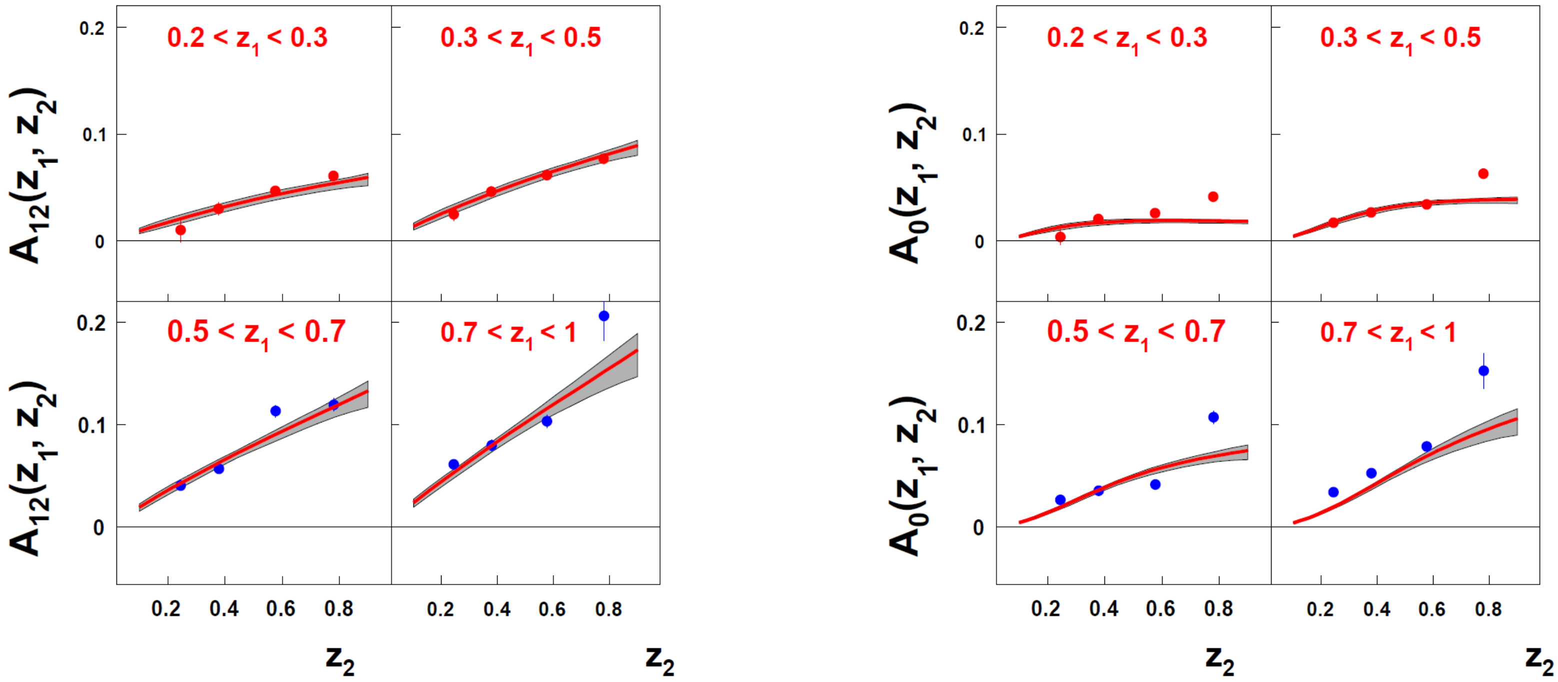
- New data from COMPASS (proton target, 2010-11)
- BELLE Erratum: R. Seidl, PRD 86 (2012) 039905



A^{UC}: Different normalization, larger errors

News on the Collins function

- New data from COMPASS (proton target, 2010-11)
- BELLE Erratum: R. Seidl, PRD 86 (2012) 039905



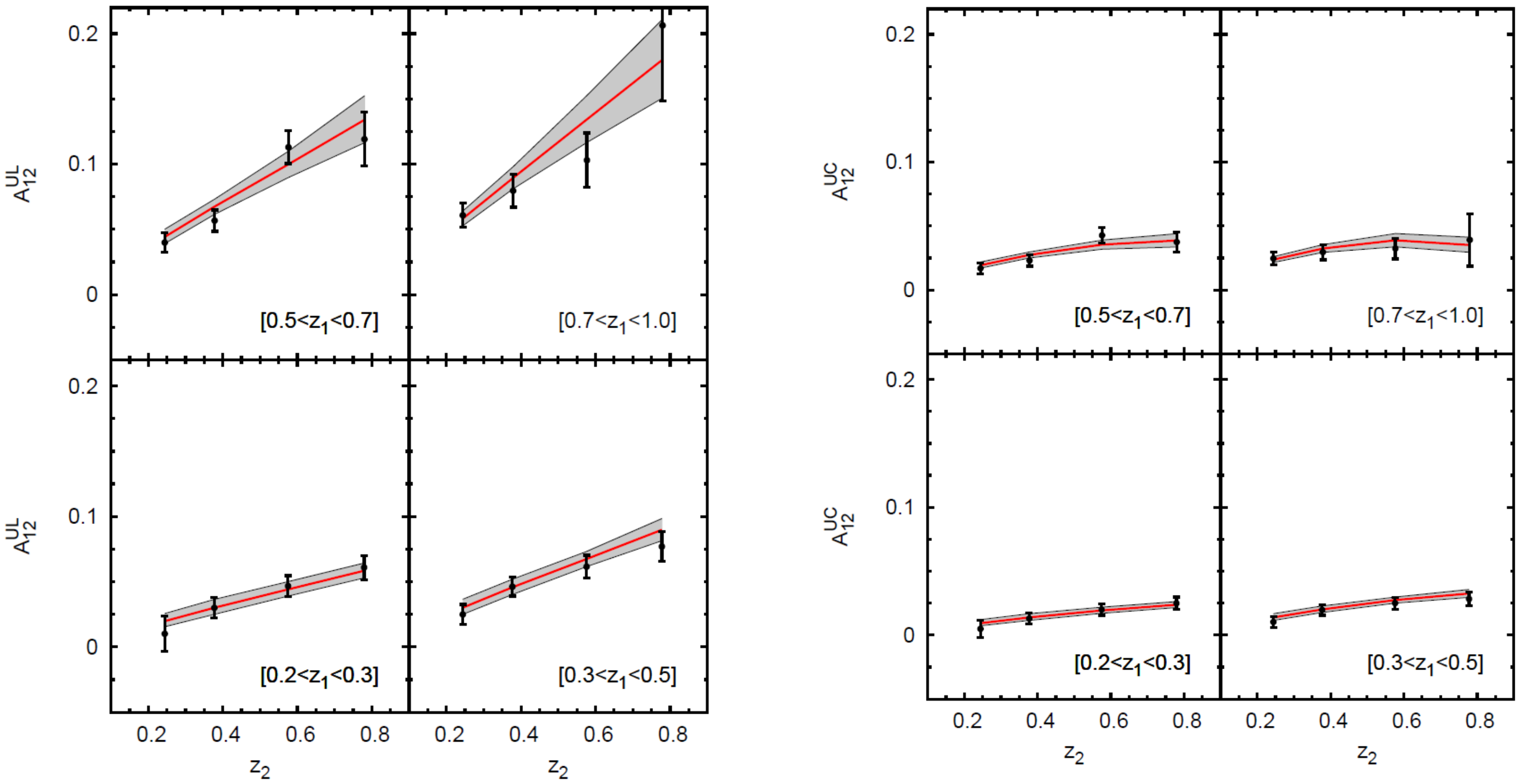
◇ R. Seidl et al., Phys. Rev. D78

Good news! Previously partial incompatibility between the sets

News on the Collins function

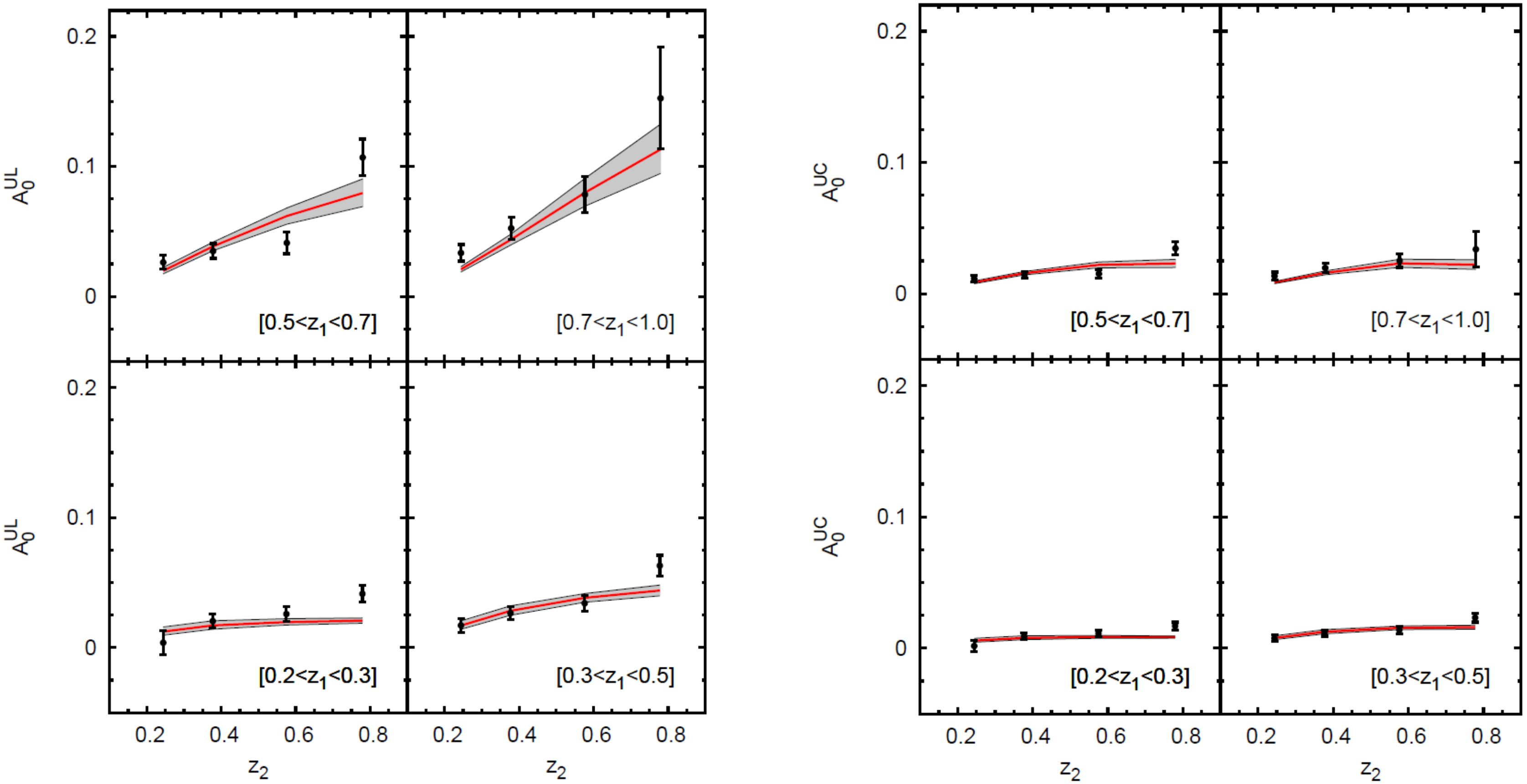
- New analysis:
  - HERMES (2009) $\pi+$ $\pi-$
  - COMPASS Deuteron (2004) $\pi+$ $\pi-$
  - COMPASS Proton (2011) h+ h-
  - BELLE all sets

News on the Collins function



Full compatibility between UL e UC

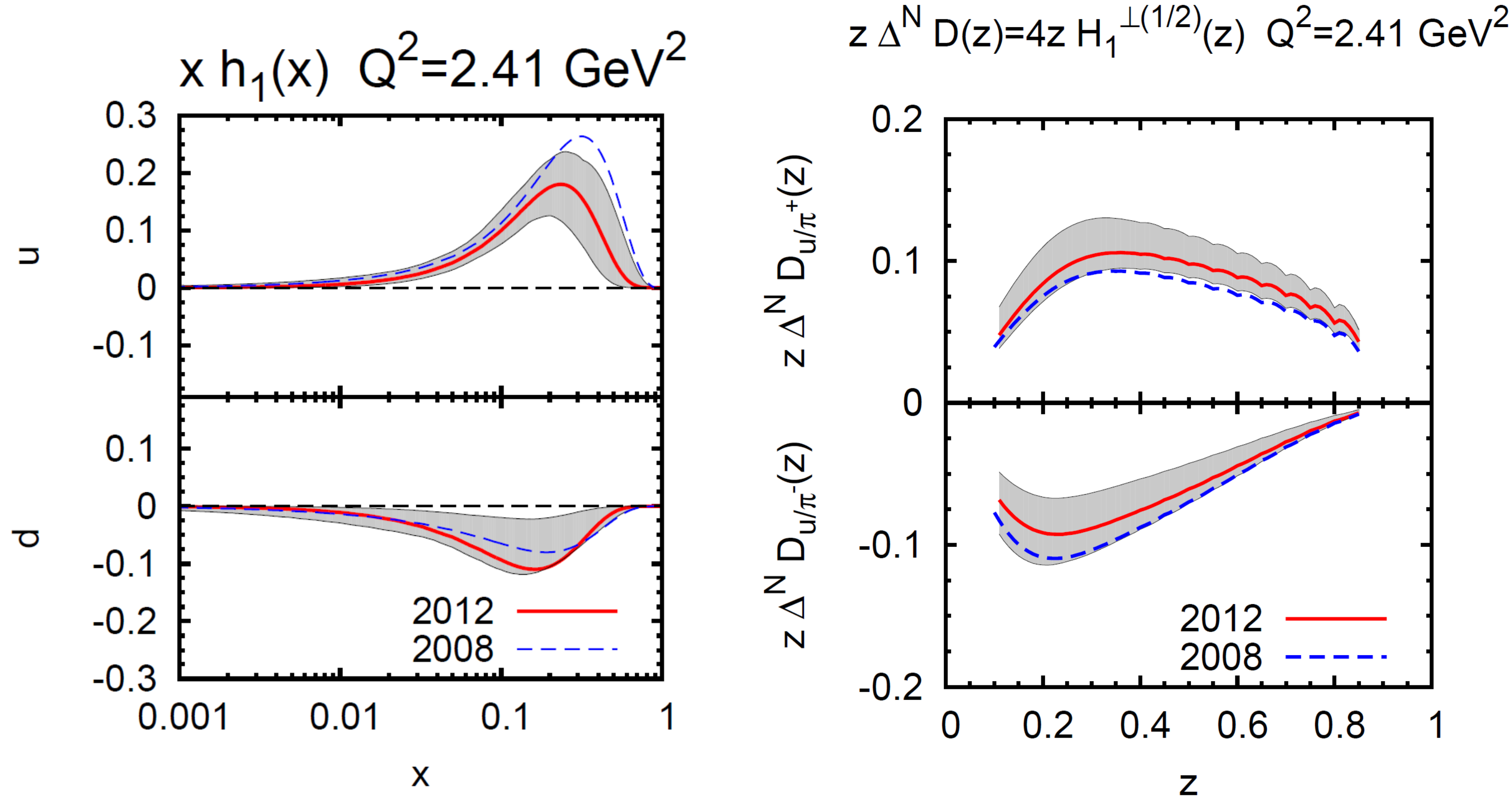
News on the Collins function



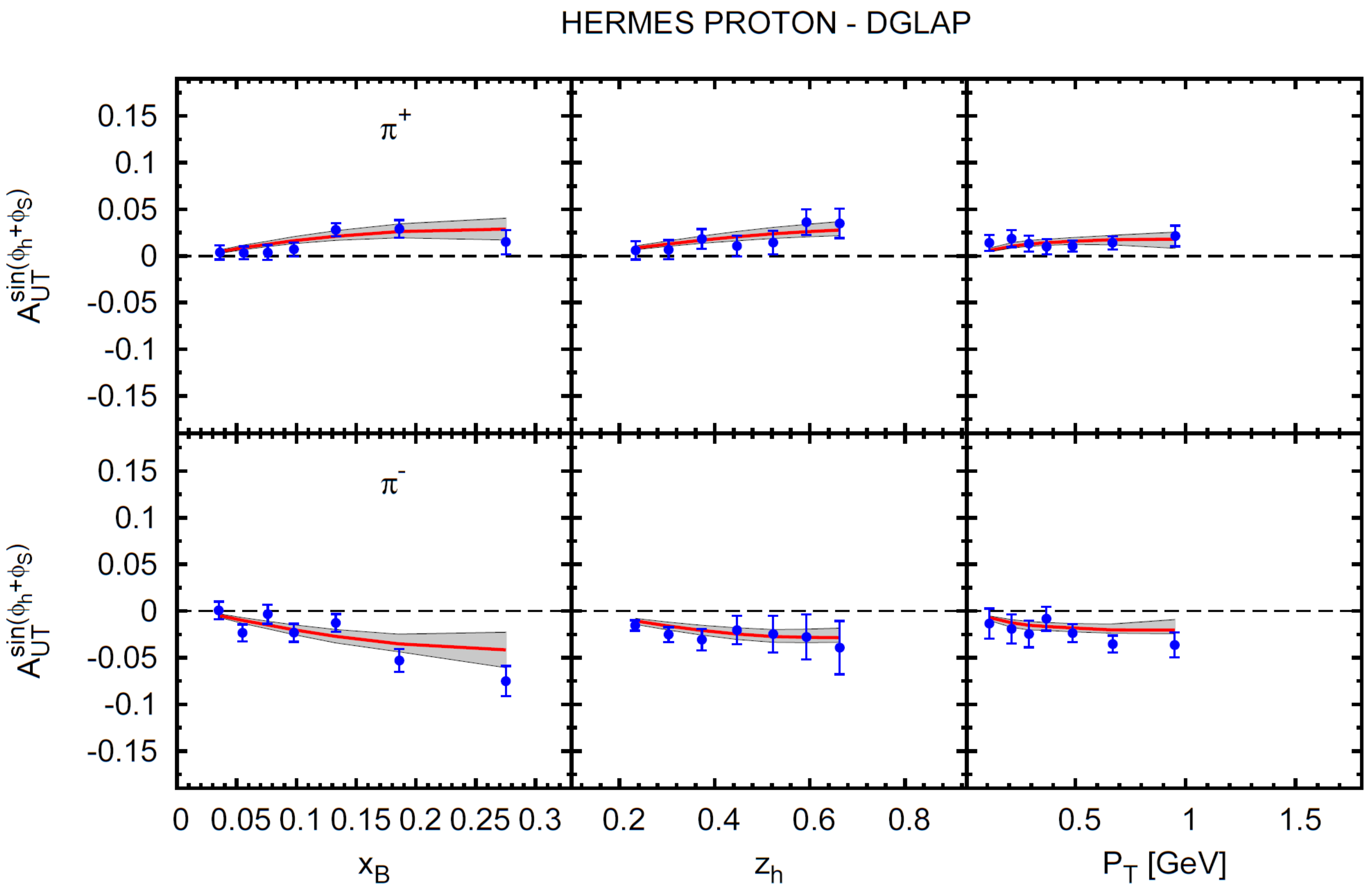
Still tension between the method 12 and 0. A_{12} is described better

News on the Collins function

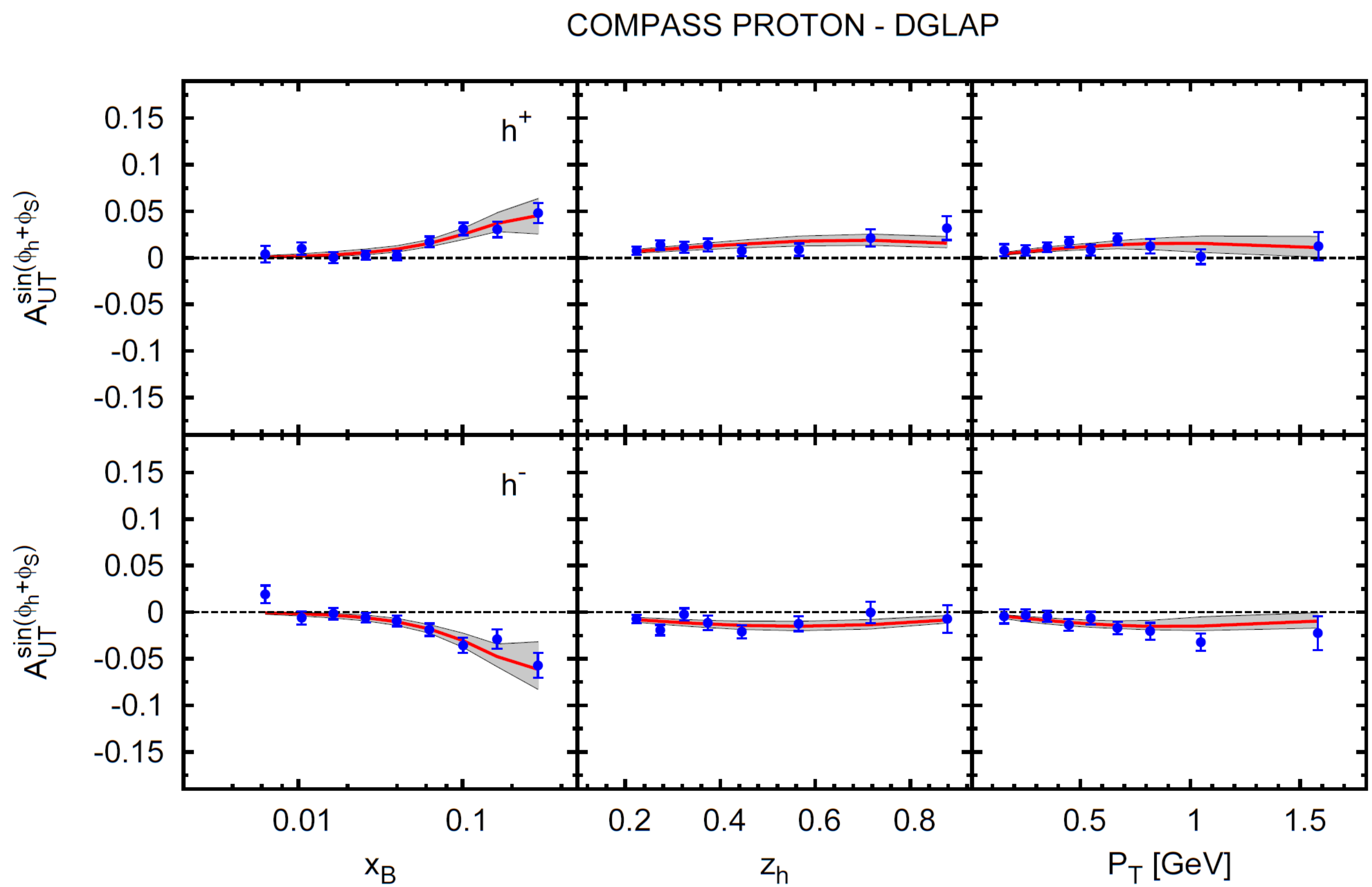
$$\chi^2_{d.o.f} = 1.07 \qquad \chi^2_{tot} = 217 \qquad \#\text{points} = 146\ (SIDIS) + 64\ (e^+e^-)$$



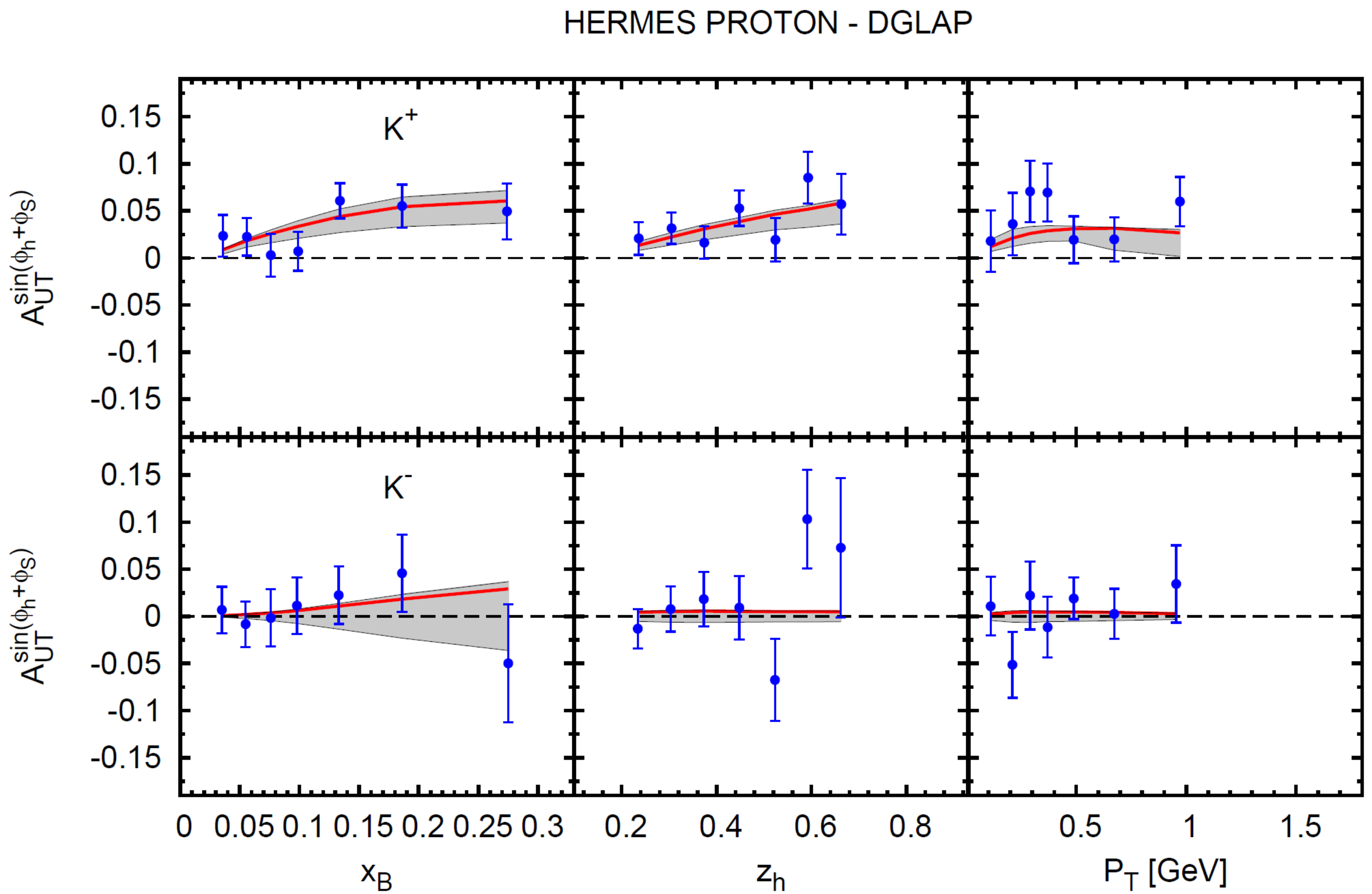
Extraction of transversity & Collins functions



Extraction of transversity & Collins functions



Kaons Collins functions



Conclusions II

- Transversity functions are definitively different from zero!
- Now we have two complementary way to extract transversity
- BELLE Erratum: Good News, better description of data

Boer-Mulders function and Cahn effect in unpolarized SIDIS

Boer-Mulders functions in unpolarized SIDIS

- The angular distribution in the unpolarized SIDIS can be written as

$$d\sigma = A + B\cos\phi + C\cos 2\phi$$

- $A =\propto f_1 \otimes D_1$ is the usual ϕ-independent contribution
- $B \propto \frac{1}{Q}(f_1 \otimes D_1 + h_1^\perp \otimes H_1^\perp)$ subleading Cahn+Boer-Mulders effect
- $C \propto h_1^\perp \otimes H_1^\perp + \frac{1}{Q^2} f_1 \otimes D_1$ BM effect+Twist-4 Cahn effect

Boer-Mulders functions in unpolarized SIDIS

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- $C \propto h_1^{\perp} \otimes H_1^{\perp} + \frac{1}{Q^2} f_1 \otimes D_1$ BM effect+Twist-4 Cahn effect+???

Extraction of the Boer-Mulders function

➢ The angular distribution in the unpolarized SIDIS can be written as

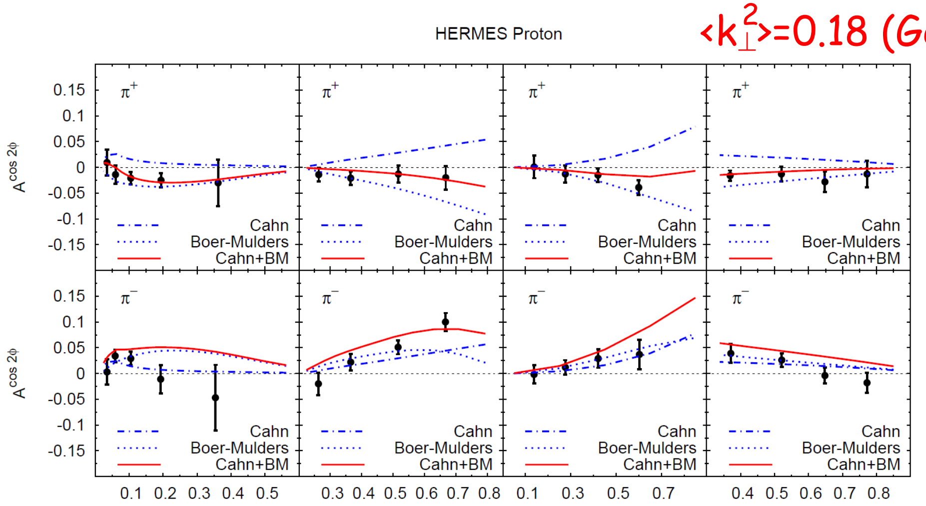
$$d\sigma = A + B\cos\phi + C\cos 2\phi$$

- $A =\propto f_1 \otimes D_1$ is the usual ϕ-independent contribution
- $C \propto h_1^\perp \otimes H_1^\perp + \frac{1}{Q^2} f_1 \otimes D_1$ BM effect+Twist-4 Cahn effect

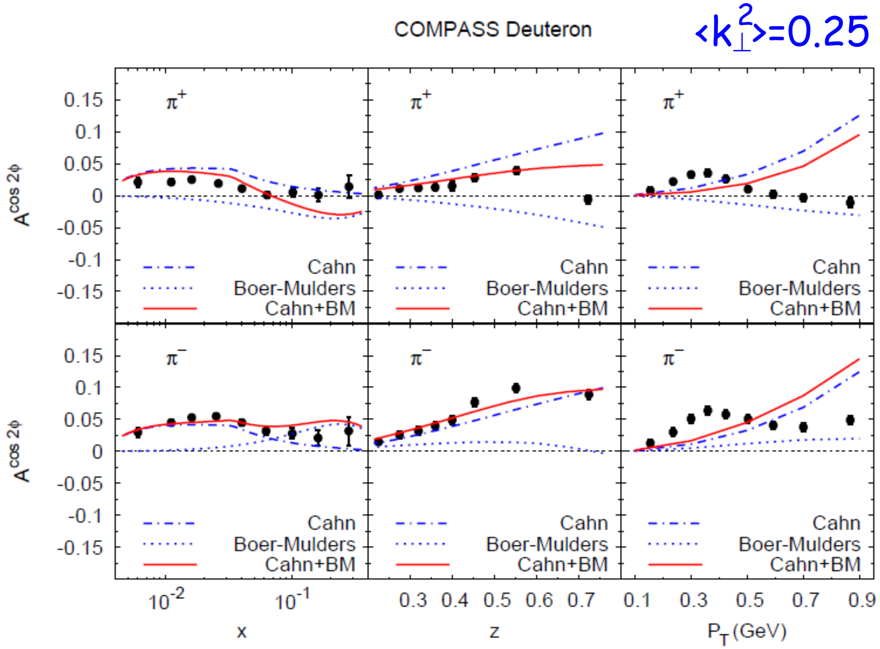
$$A^{\cos 2\phi} = 2\frac{\int d\sigma \cos 2\phi}{\int d\sigma} = \frac{C}{A}$$

Extraction of the Boer-Mulders function

$\langle k_\perp^2 \rangle = 0.18\ (GeV/c)^2$



$\langle k_\perp^2 \rangle = 0.25\ (GeV/c)^2$



$$h_1^{\perp q}(x, k_\perp) = \lambda_q\, f_{1T}^{\perp q}(x, k_\perp)$$

$\Rightarrow h_1^{\perp d}$ and $h_1^{\perp u}$ both negative

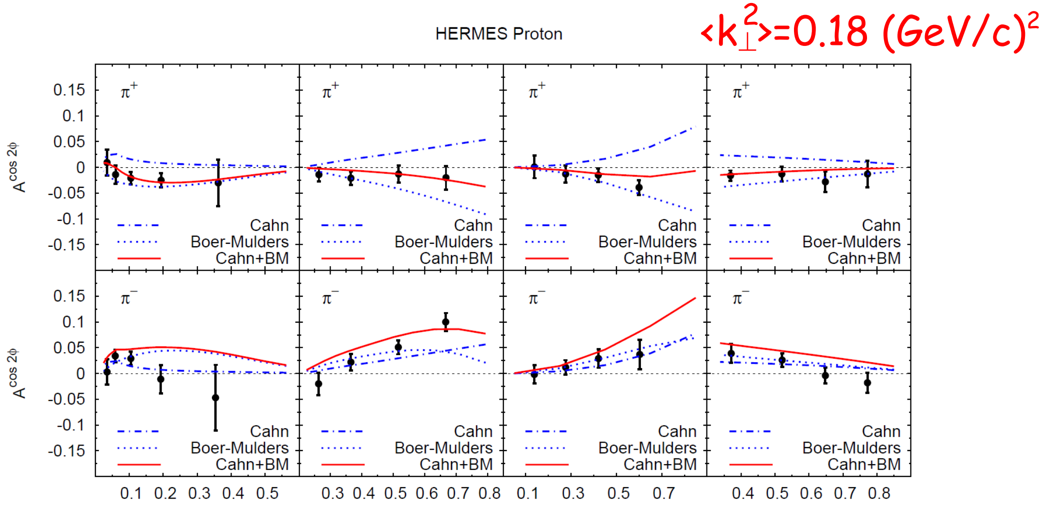
Compatible with models predictions

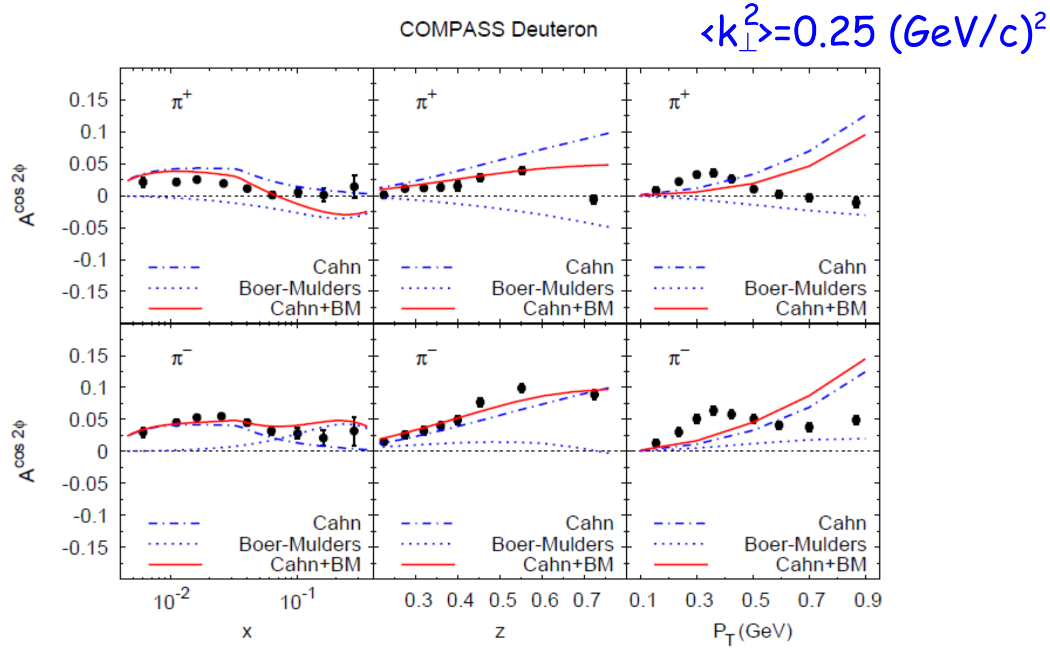
$\diamond \chi^2/d.o.f. = 2.41$

- $\lambda_u = 2.1 \pm 0.1$
- $\lambda_d = -1.11^{+0.00}_{-0.02}$

V. Barone, S. Melis and A. Prokudin Phys. Rev. D81, 114026 (2010)

Extraction of the Boer-Mulders function





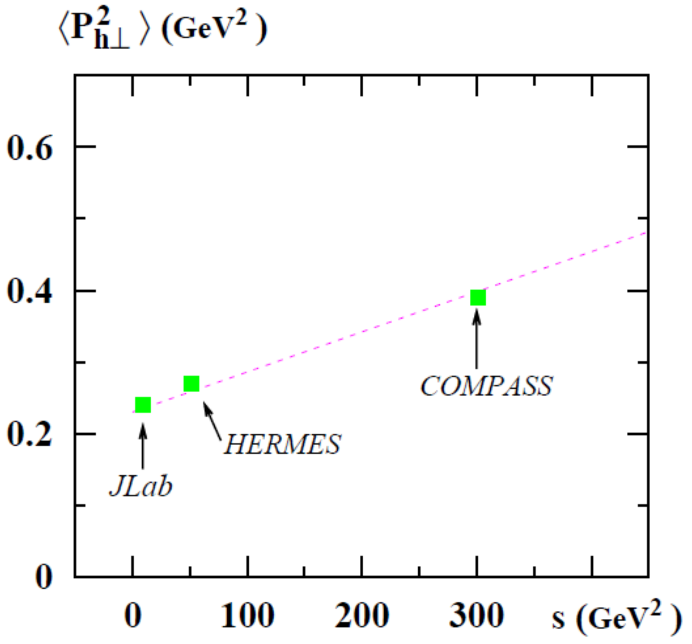
- ✔Cahn effect (Twist-4) comparable to BM effect
- ✔Same sign of Cahn contribution for positive and negative pion
- ✔Different average transverse momenta are preferred
- ✔BM contribution opposite in sign for positive and negative pions

V. Barone, S. Melis and A. Prokudin Phys. Rev. D81, 114026 (2010)

Extraction of the Boer-Mulders function

✔Different average transverse momenta are preferred

Schweitzer, Teckentrup, Metz (2010)

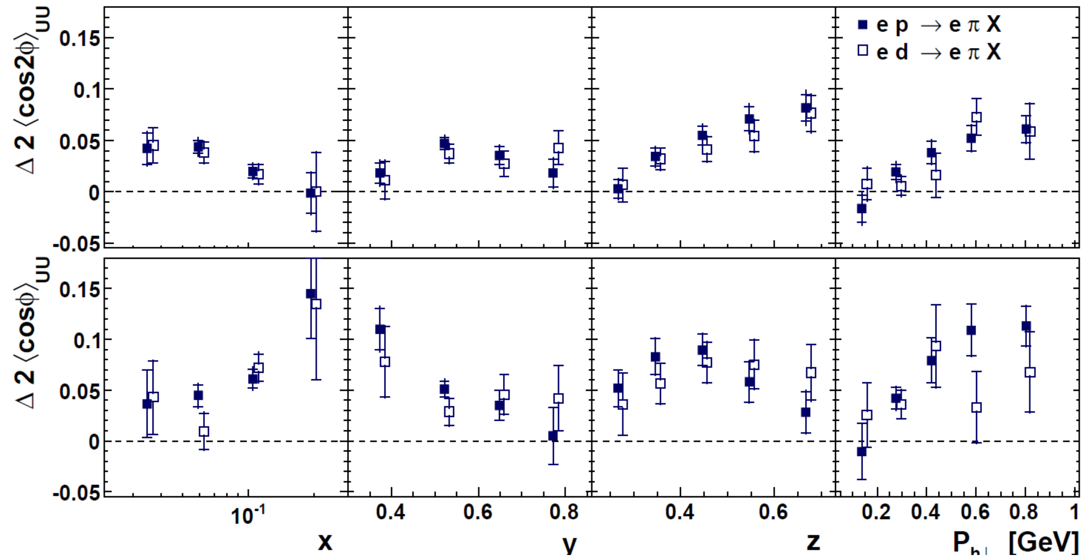


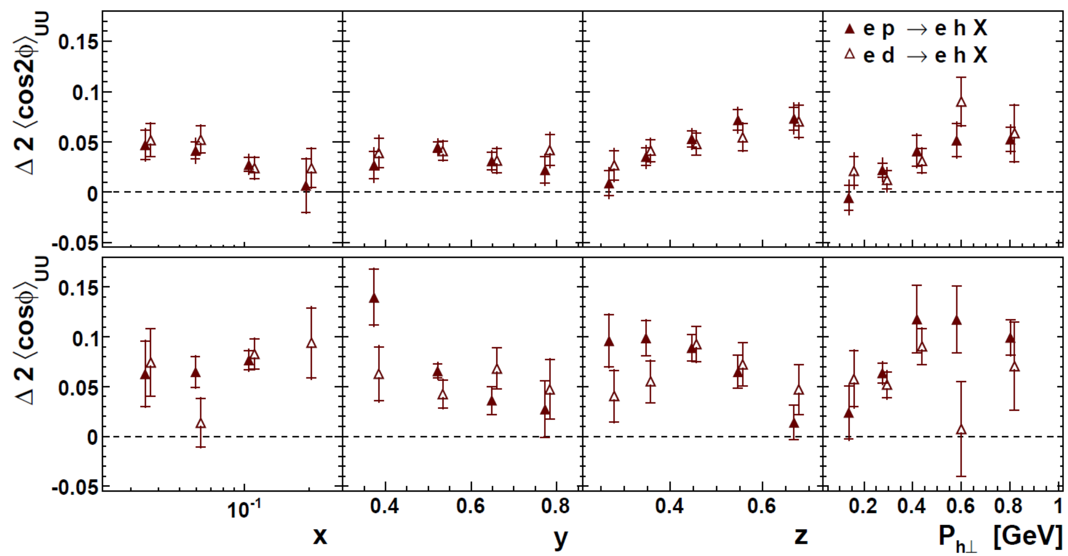
Extraction of the Boer-Mulders function

- ✔Same sign of Cahn contribution for positive and negative pion
- ✔BM contribution opposite in sign for positive and negative pions

$$\langle \cos 2\phi \rangle \propto h_1^{\perp} H_1^{\perp} + \text{Cahn}$$

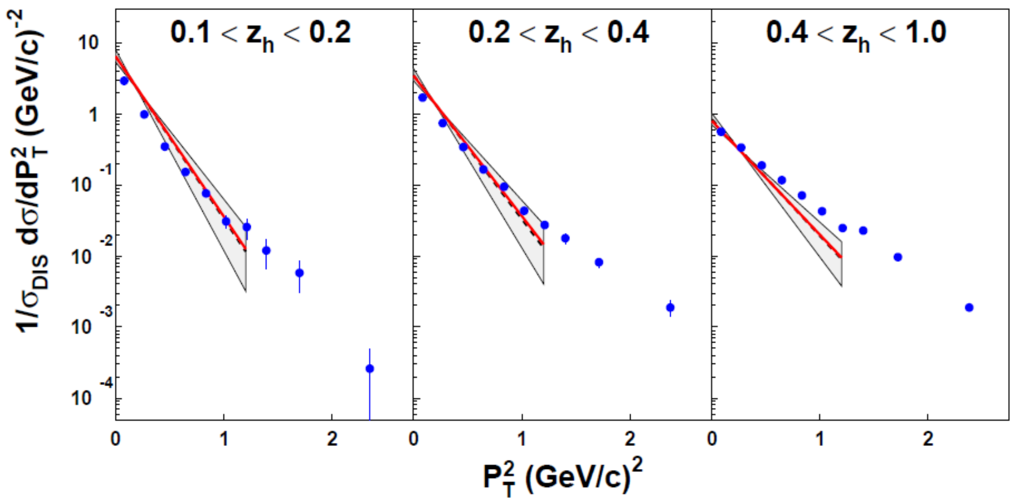
$$\langle \cos \phi \rangle \propto -h_1^{\perp} H_1^{\perp} - \text{Cahn}$$

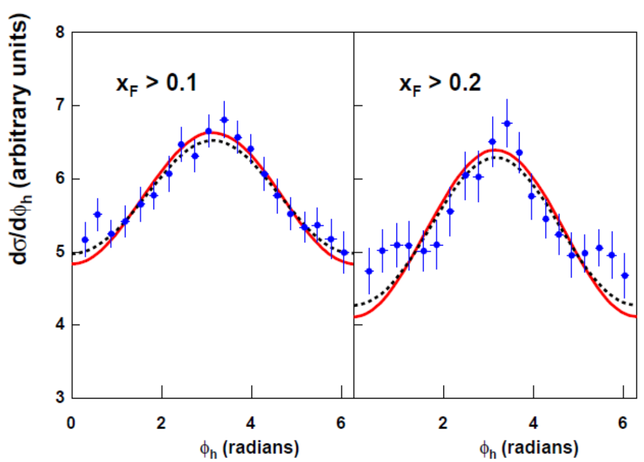




Extraction of the Boer-Mulders function

✔.. large cahn effect!



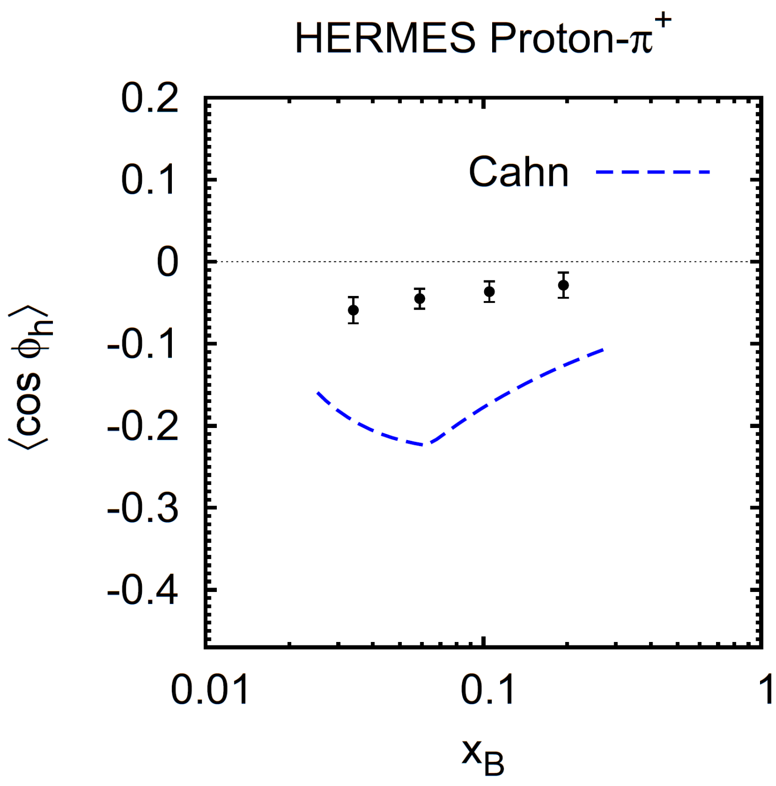


Fit of EMC data: Anselmino et al (2005)

...but...

Extraction of the Boer-Mulders function

✔... large cahn effect!



Why such a large Cahn effect?

➢ The Cahn effect is suppressed by powers of Q:

$$d\sigma = A + B\cos\phi + C\cos 2\phi$$

- $A = \propto f_1 \otimes D_1$ is the usual ϕ-independent contribution
- $B \propto \frac{1}{Q}\left(f_1 \otimes D_1 + h_1^\perp \otimes H_1^\perp\right)$ subleading Cahn+Boer-Mulders effect
- $C \propto h_1^\perp \otimes H_1^\perp + \frac{1}{Q^2} f_1 \otimes D_1$ BM effect+Twist-4 Cahn effect

$$\frac{k_\perp}{Q} \ll 1 \text{ ??}$$

Why such a large Cahn effect?

- HERMES and COMPASS: $\langle Q^2 \rangle \simeq 2 \text{ GeV}^2$

$$Q^2 > 1 \text{ GeV}^2$$

- Analytical integration of the transverse momenta

$$f_{q/p}(x, k_\perp) = f(x) \frac{e^{-k_\perp^2/\langle k_\perp^2 \rangle}}{\pi \langle k_\perp^2 \rangle}$$

$$\langle k_\perp^2 \rangle \simeq 0.25 \ (\text{GeV}/c)^2$$

$$\int d^2\boldsymbol{k}_\perp \Rightarrow \int_0^{2\pi} d\varphi \int_0^\infty dk_\perp \, k_\perp$$

Bounds on the intrinic transverse momenta

- ✔The integration from 0 to infinity can be a crude assumption
- ✔The parton model provides kinematical limits on the transverse momentum size

➢By requiring the energy of the parton to be smaller than the energy of its parent hadron, we have

$$k_\perp^2 \leq (2 - x_B)(1 - x_B)Q^2 \quad , \quad 0 < x_B < 1$$

➢By requiring the parton not to move backward with respect to its parent hadron, we find

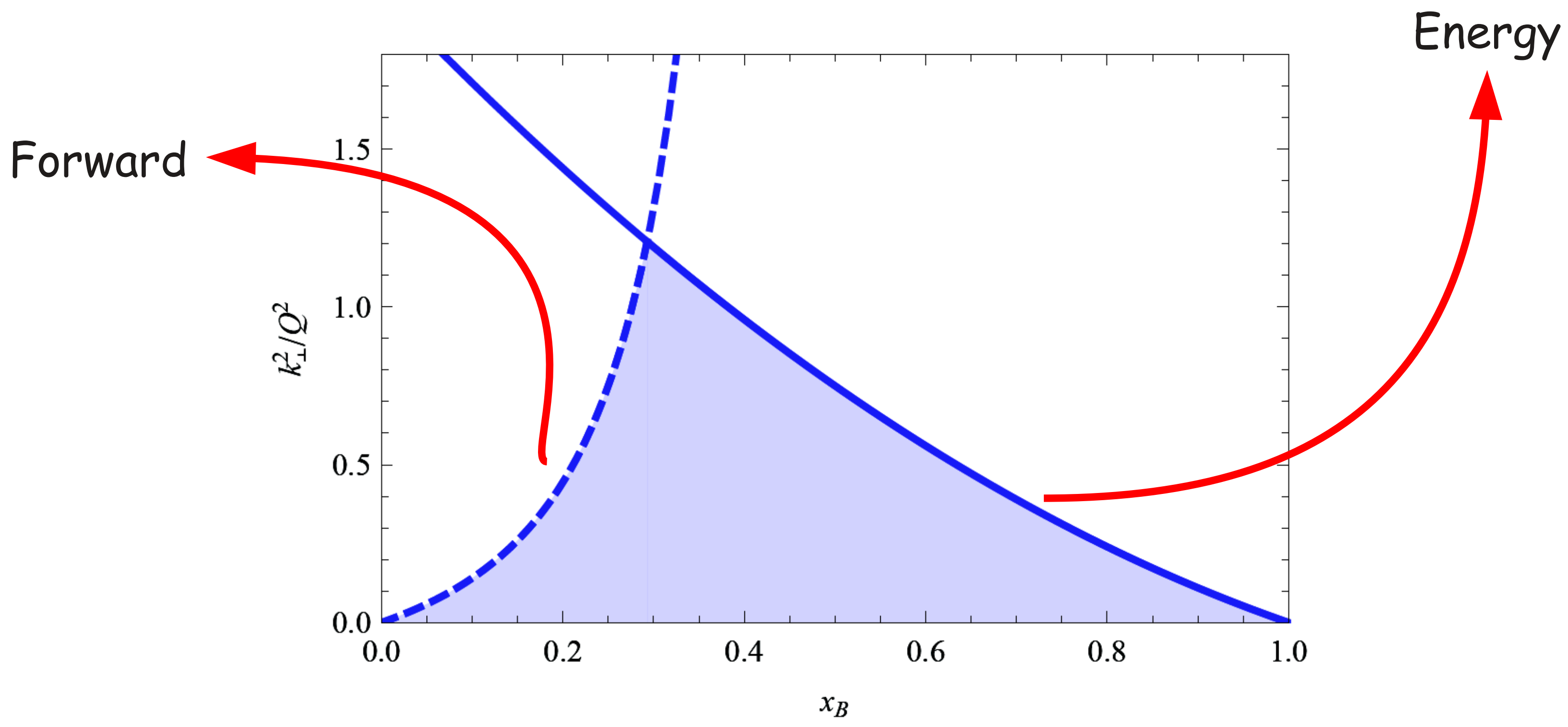
$$k_\perp^2 \leq \frac{x_B(1 - x_B)}{(1 - 2x_B)^2}Q^2 \; , \quad x_B < 0.5$$

Boglione, Melis, Prokudin, Phys. Rev. D 84, 034033 (2011)

Bounds on the intrinic transverse momenta

✔The integration from 0 to infinity can be a crude assumption

✔The parton model provides kinematical limits on the transverse momentum size



Bounds on the intrinic transverse momenta

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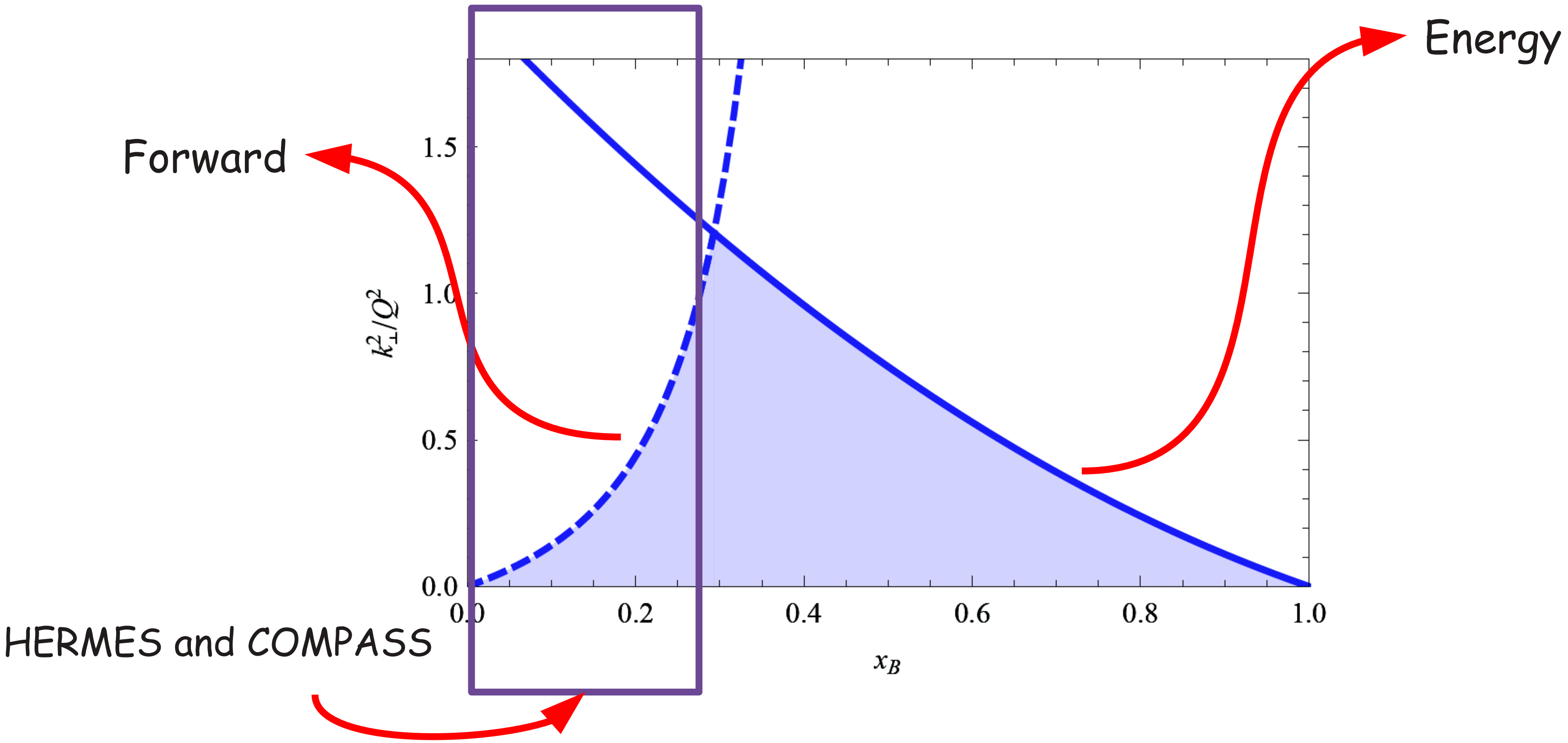
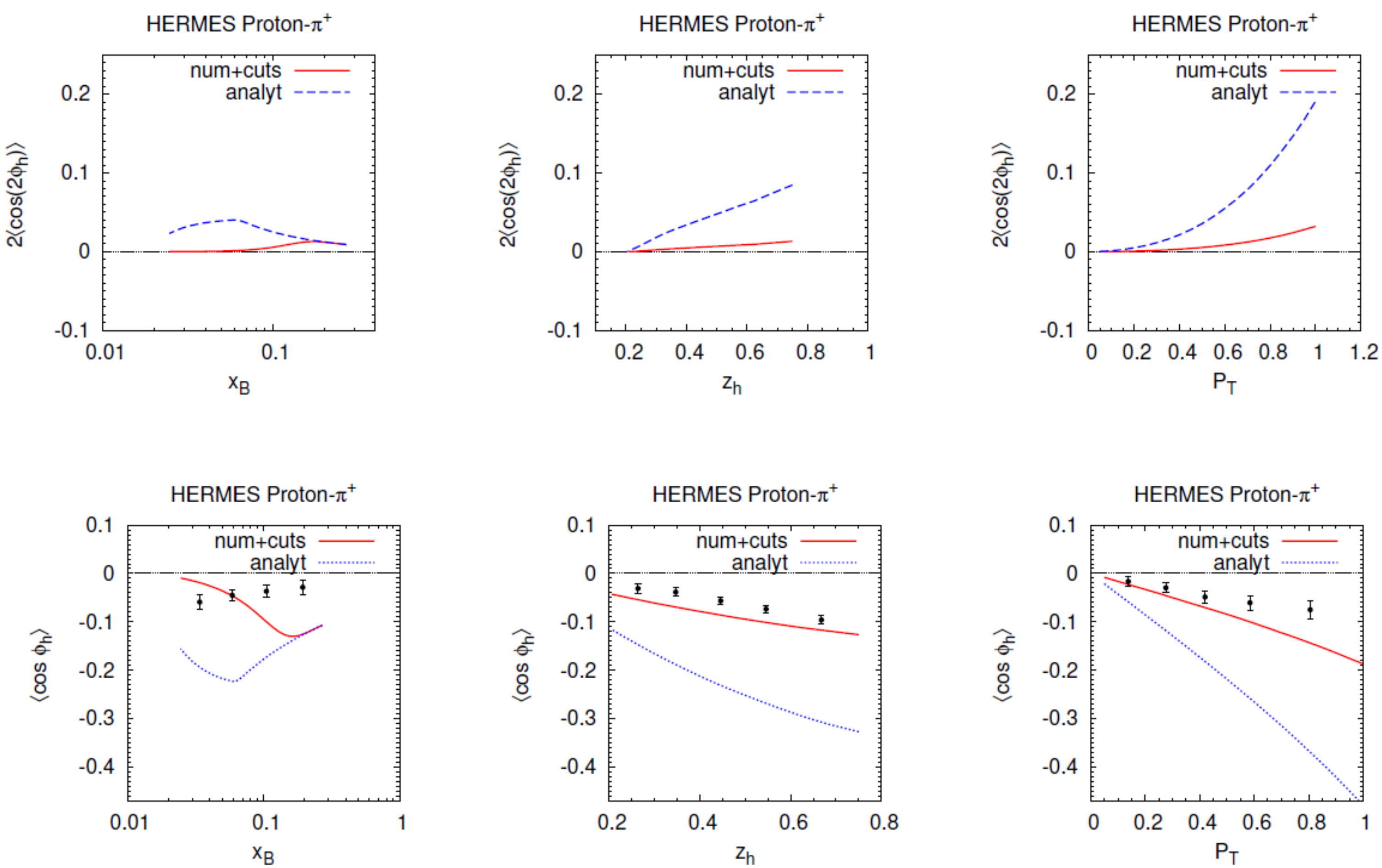
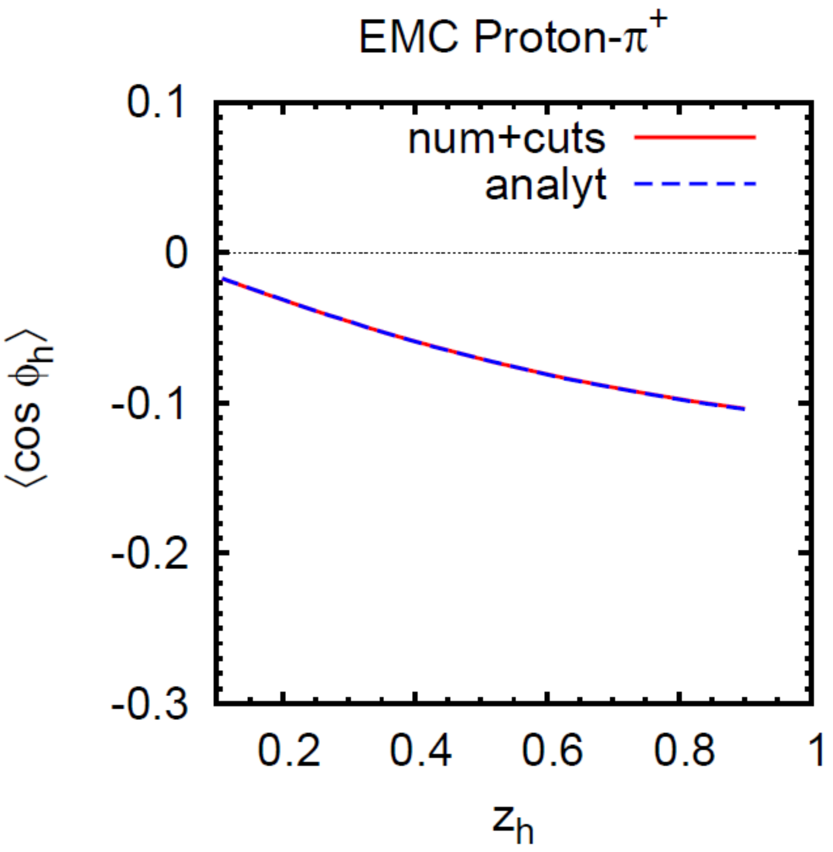


Smaller Cahn effect...



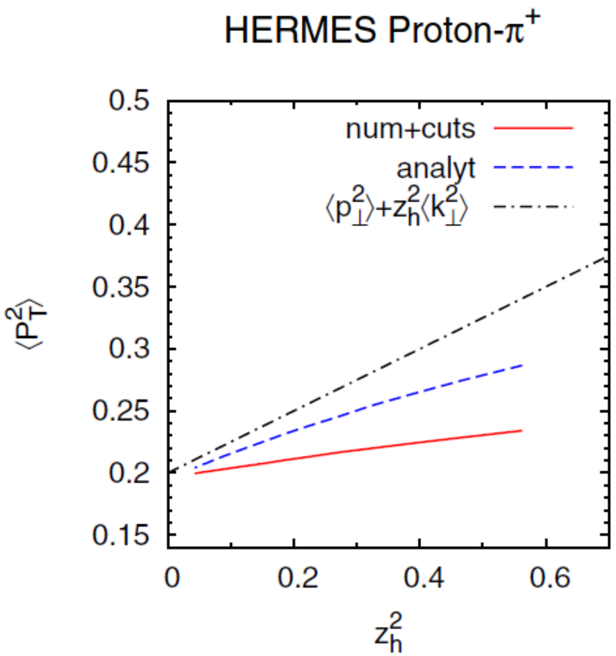
No effects in "true" DIS regime...



EMC like kinematics:

$$Q^2 \geq 5 \text{ GeV}^2$$

$\langle P_T^2 \rangle$



Very often the relation

$$\langle P_T^2 \rangle = \langle p_\perp^2 \rangle + z_h^2 \langle k_\perp^2 \rangle$$

is used in phenomenological analysis
But is wrong unless you integrate from
0 to infinity P_T which is never
the case experimentally

$$f_1(x,\mathbf{k}_\perp^2) = N\, f_1(x)\, \mathrm{e}^{-\mathbf{k}_\perp^2/\overline{\mathbf{k}_\perp^2}} \qquad D_1(z,\mathbf{p}_\perp^2) = N\, D_1(z)\, \mathrm{e}^{-\mathbf{p}_\perp^2/\overline{\mathbf{p}_\perp^2}}$$

$$\langle \mathbf{k}_\perp^2 \rangle \equiv \int \mathrm{d}^2\mathbf{k}_\perp\, \mathbf{k}_\perp^2\, f_1(x,\mathbf{k}_\perp^2) \qquad \langle \mathbf{p}_\perp^2 \rangle \equiv \int \mathrm{d}^2\mathbf{p}_\perp\, \mathbf{p}_\perp^2\, D_1(z,\mathbf{p}_\perp^2)$$

If you integrate from 0 to infinity! $\langle \mathbf{k}_\perp^2 \rangle = \overline{\mathbf{k}_\perp^2}$ $\langle \mathbf{p}_\perp^2 \rangle = \overline{\mathbf{p}_\perp^2}$

$$F_{UU} = \sum_a e_a^2 \int \mathrm{d}^2\mathbf{k}_\perp \int \mathrm{d}^2\mathbf{p}_\perp\, \delta^2(\mathbf{p}_\perp + z_h\mathbf{k}_\perp - \mathbf{P}_{h\perp})\, f_1^a(x_B,\mathbf{k}_\perp^2)\, D_1^a(z_h,\mathbf{p}_\perp^2)$$

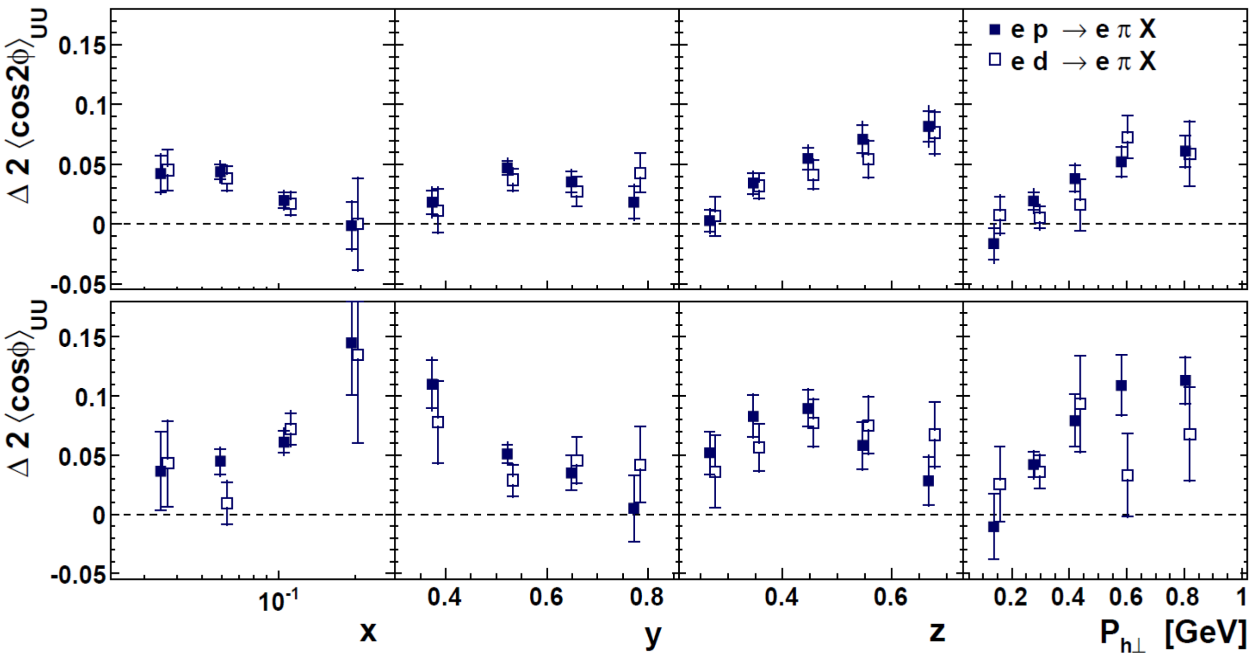
$$F_{UU} = \sum_a e_a^2\, f_1^a(x_B)\, D_1^a(z_h)\, \frac{\mathrm{e}^{-\mathbf{P}_{h\perp}^2/\overline{\mathbf{P}_{h\perp}^2}}}{\pi\, \overline{\mathbf{P}_{h\perp}^2}}$$

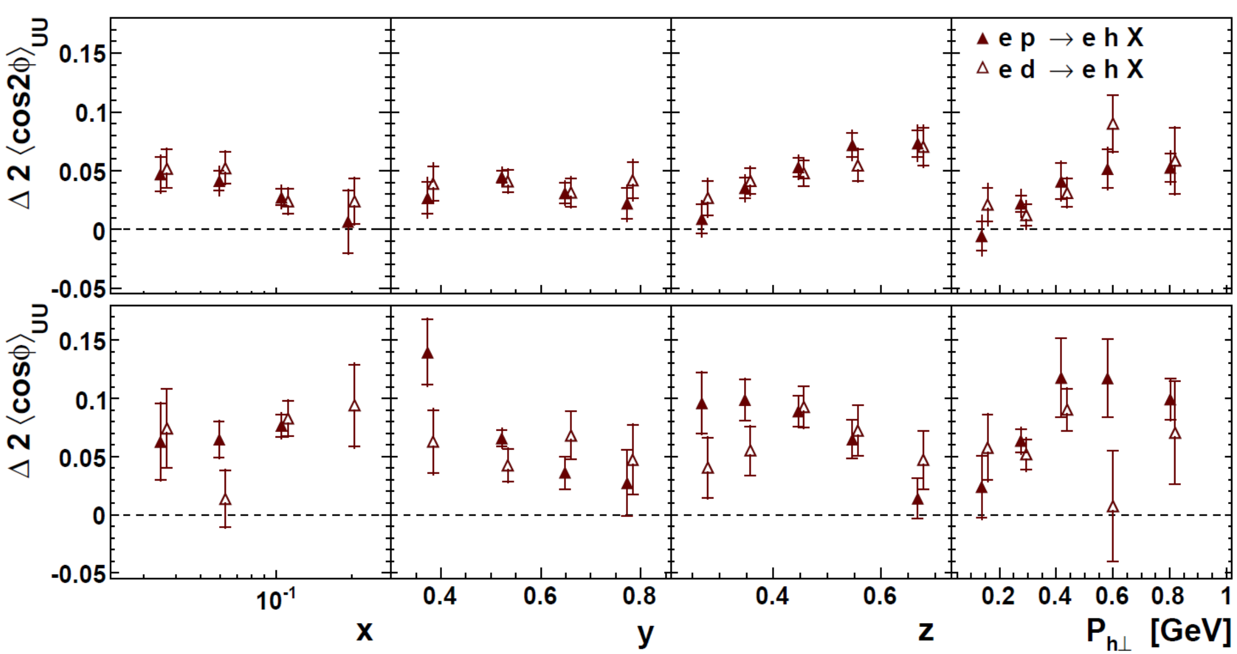
$$\overline{\mathbf{P}_{h\perp}^2} = \overline{\mathbf{p}_\perp^2} + z_h^2\, \overline{\mathbf{k}_\perp^2}$$

$\langle \mathbf{P}_{h\perp}^2 \rangle = \overline{\mathbf{P}_{h\perp}^2}$ <u>Only if you integrate from 0 to infinity!</u>

Conclusions III

- From $\langle \cos 2\varphi \rangle$ analysis BM compatible with models
- Large Cahn effect
- Too large for $\langle \cos\varphi \rangle$...
- The parton model provides constraints on the intrinsic transverse momenta
- Better description of $\langle \cos\varphi \rangle$ and $\langle \cos 2\varphi \rangle$ data
- Impact in the calculation of $\langle P_T^2 \rangle$


e p → e π X
e d → e π X
Δ 2 ⟨cos2φ⟩UU
Δ 2 ⟨cosφ⟩UU
x
y
z
Ph⊥ [GeV]


e p → e h X
e d → e h X
Δ 2 ⟨cos2φ⟩UU
Δ 2 ⟨cosφ⟩UU
x
y
z
Ph⊥ [GeV]

Extraction of the transversity and the Collins function

- Parametrization of Transversity function:

$$\Delta_T q(x, k_\perp) = \frac{1}{2}\mathcal{N}_q^T(x)[f_{q/p}(x) + \Delta q(x)]\,\frac{1}{\pi\langle k_\perp^2\rangle}\,e^{-k_\perp^2/\langle k_\perp^2\rangle}$$

Unpolarized PDF

Helicity PDF

$$\mathcal{N}_q^T(x) = N_q^T\, x^{\alpha_q}(1-x)^{\beta_q}\,\frac{(\alpha_q+\beta_q)^{(\alpha_q+\beta_q)}}{\alpha_q^{\alpha_q}\,\beta_q^{\beta_q}}$$

N_q^T, α, β free parameters

Extraction of the transversity and the Collins function

- Parametrization of the Collins function:

$$\Delta^N D_{\pi/q^\uparrow}(z,p_\perp) = 2\mathcal{N}_q^C(z)\, h(p_\perp)\, D_{\pi/q}(z,p_\perp)$$

- $\mathcal{N}_q^C(z) = N_q^C\, z^\gamma (1-z)^\delta \dfrac{(\gamma+\delta)^{(\gamma+\delta)}}{\gamma^\gamma \delta^\delta}$

- $h(p_\perp) = \sqrt{2e}\,\dfrac{p_\perp}{M_h}\, e^{-p_\perp^2/M_h^2}$

Unpolarized FF

N_q^C, γ, δ, M_h free parameters

✔Bound:

$$\Delta^N D_{\pi/q^\uparrow}(z,p_\perp) = \frac{2p_\perp}{zM} H_1^\perp(z,p_\perp)$$

✔Torino vs Amsterdam notation

$$\Delta^N D_{\pi/q^\uparrow}(z,p_\perp) = \frac{2p_\perp}{zM} H_1^\perp(z,p_\perp)$$

Extraction of the transversity and the Collins function

- Evolution of the Collins function: an exercize

$$\Delta^N D_{\pi/q^\uparrow}(z, p_\perp) = 2\mathcal{N}_q^C(z)\, h(p_\perp)\, D_{\pi/q}(z, p_\perp)$$

Evolved as the unpolarized FF at BELLE scale

The exercize: Let us evolve D with a transversity like kernel in DGLAP eq. at BELLE scale

Extraction of the transversity and the Collins function

The exercize: D evolved with a "transversity like" kernel in DGLAP eq. at BELLE scale

Unpolarized like evolution

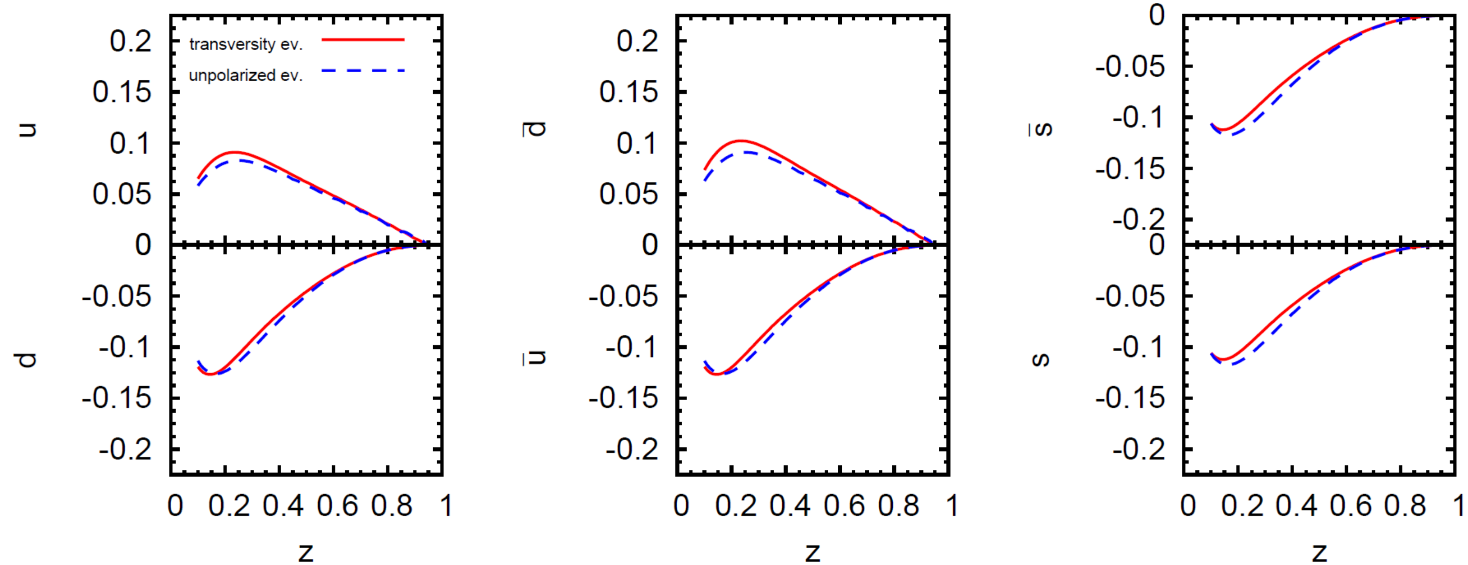
$$\chi^2_{d.o.f} = 1.22$$

Transversity like evolution

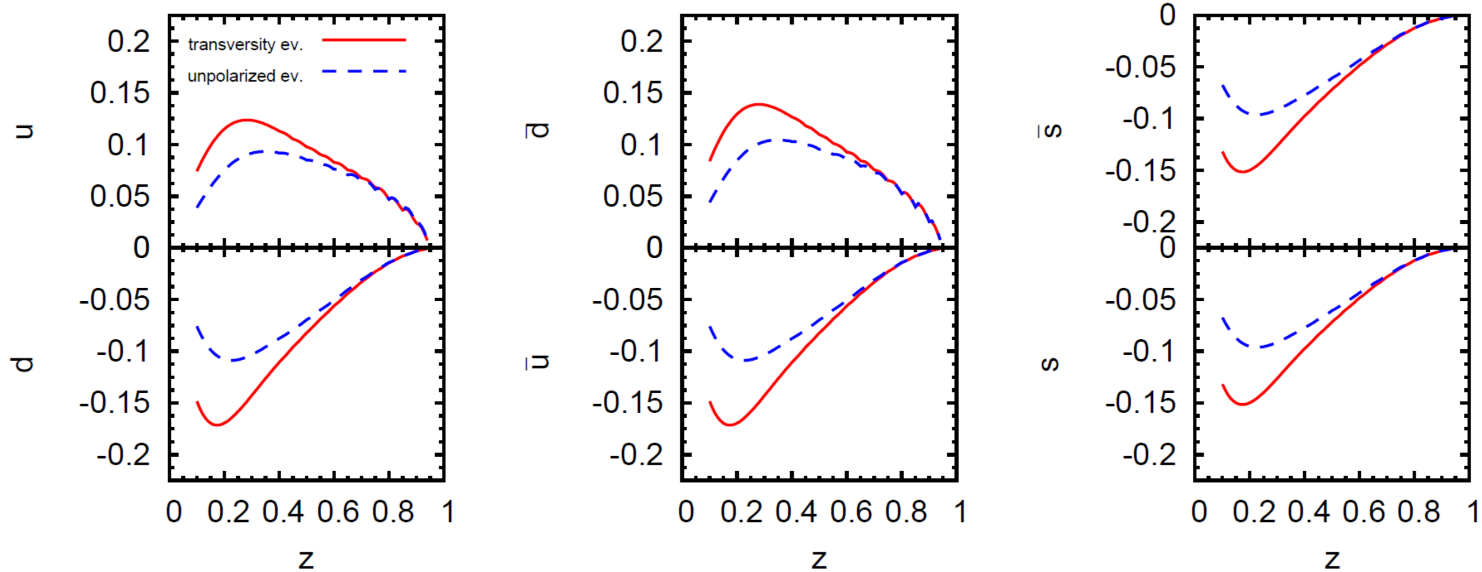
$$\chi^2_{d.o.f} = 1.20$$

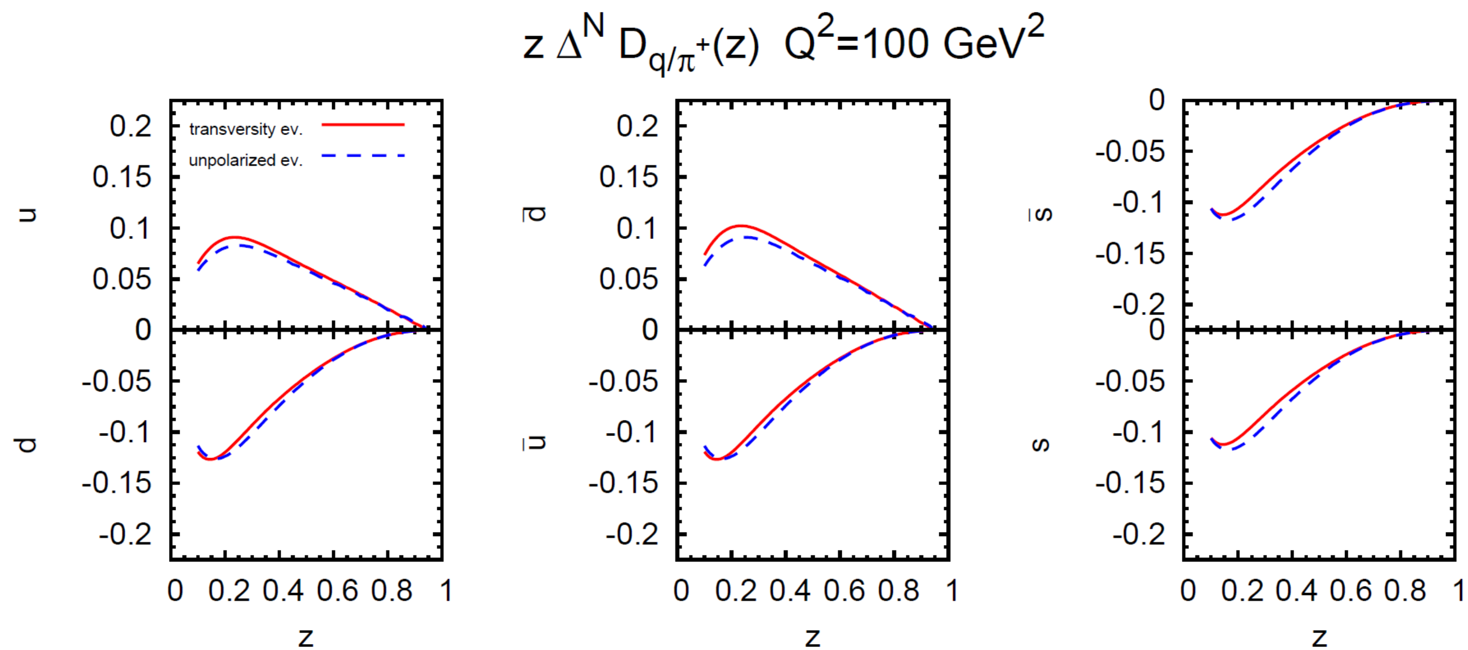
HERMES DATA 2009+ COMPASS D+ BELLE DATA

$z\,\Delta^N D_{q/\pi^+}(z)\ \ Q^2=100\ \mathrm{GeV}^2$

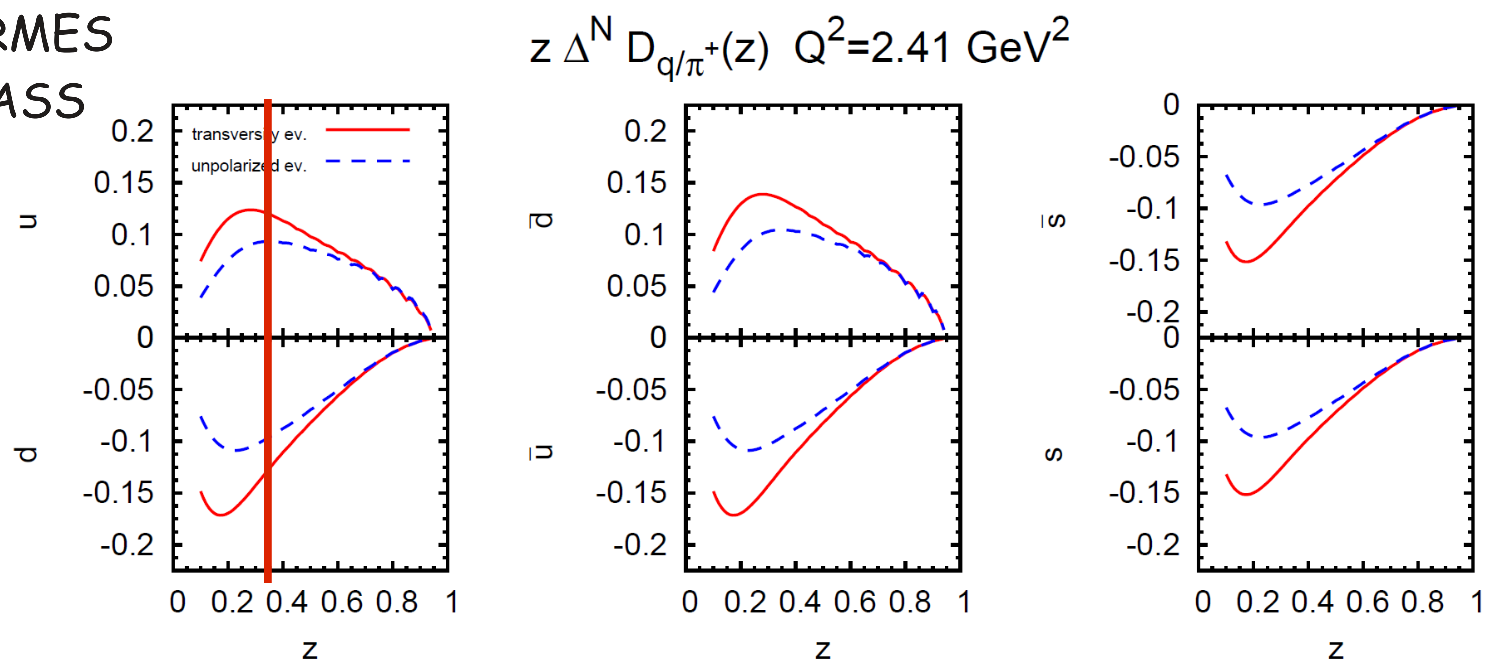


$z\,\Delta^N D_{q/\pi^+}(z)\ \ Q^2=2.41\ \mathrm{GeV}^2$

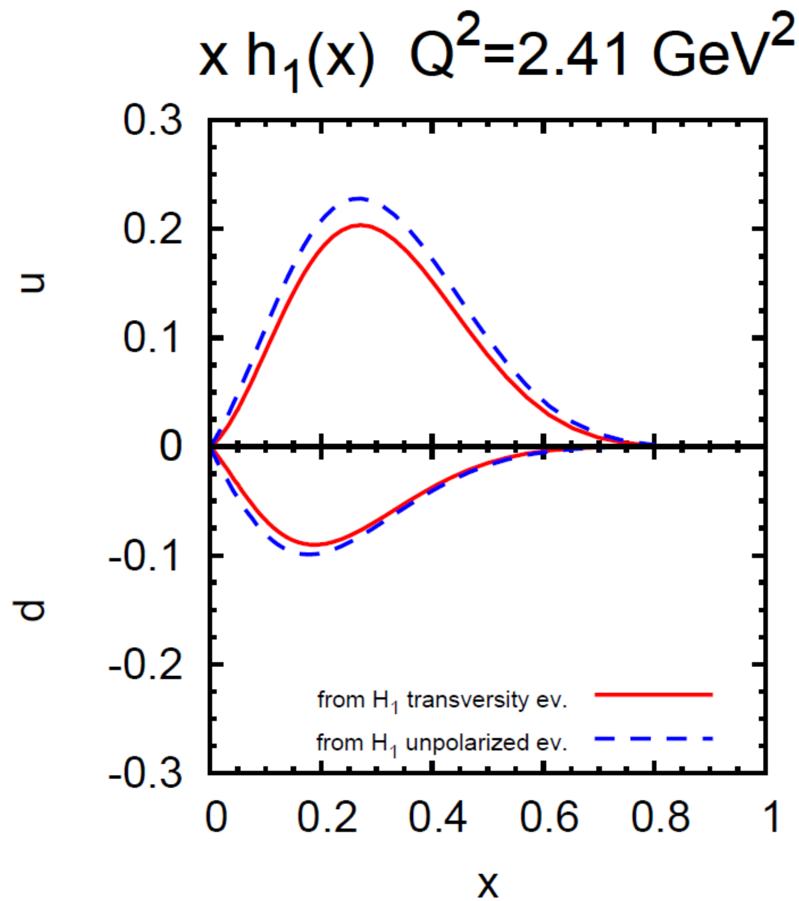




<z> @HERMES &COMPASS



Extraction of the transversity and the Collins function

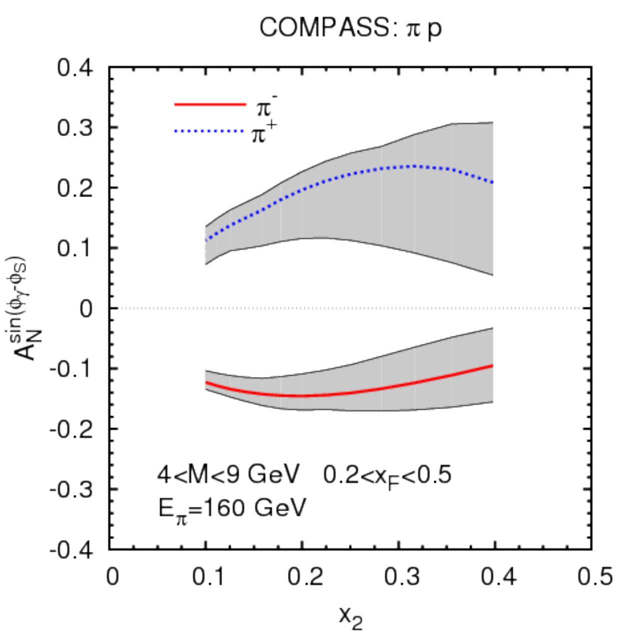


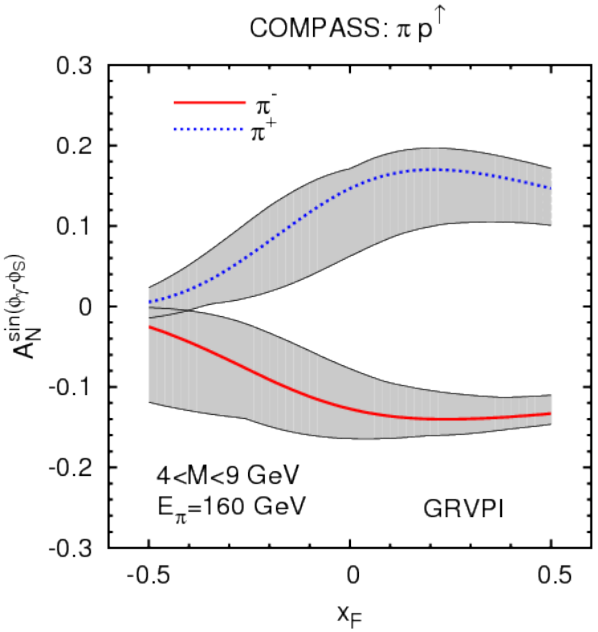
Conclusions II

- u and d transversity functions are opposite in signs
- Favored and unfavored are opposite in signs
- BELLE data sets are not symmetric in z_1 <-> z_2 exchange
- The transversity function does not change dramatically changing evolution in our simple exercise.

Predictions for COMPASS DY

- Polarized NH_3
- Pion beam
- Valence region for the Sivers function



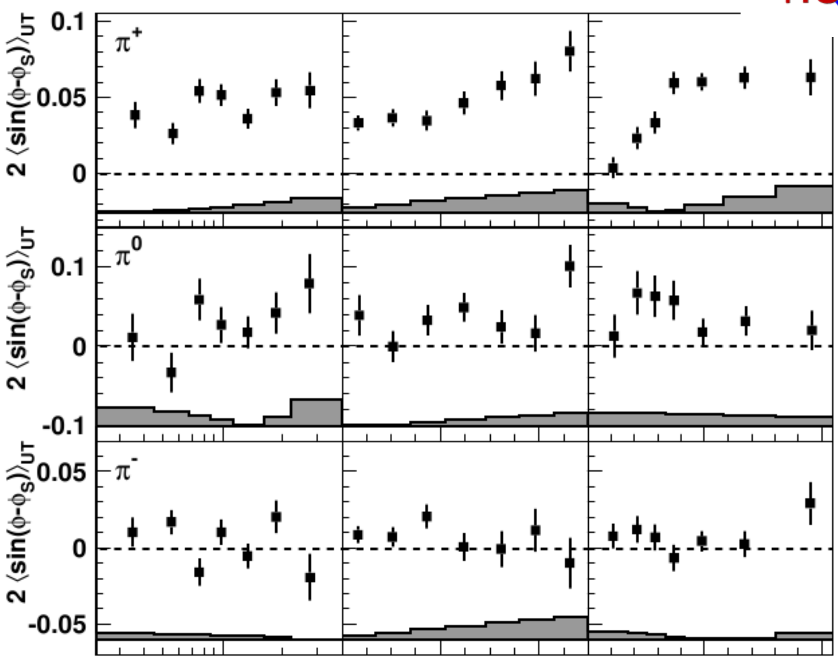


Large measurable asymmetry

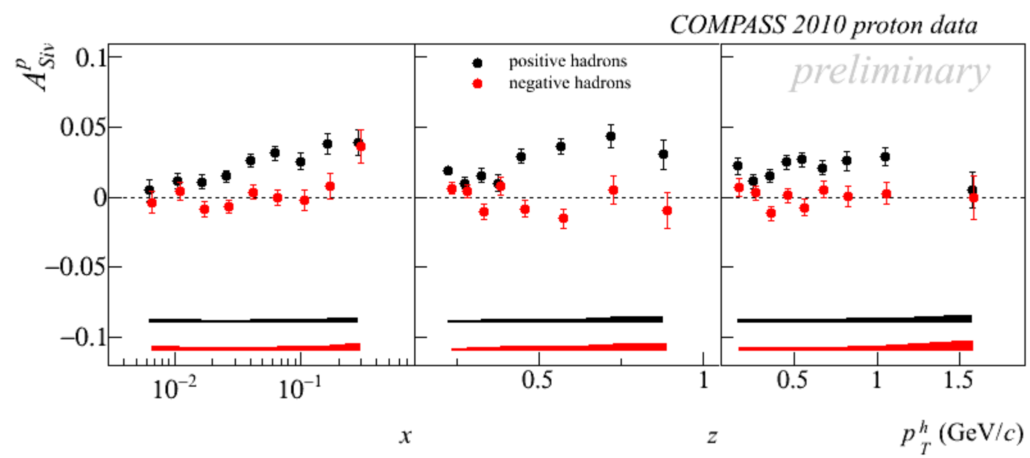
- Anselmino et al. Phys. Rev. D79,054010

Sivers function in SIDIS

- New SIDIS data from HERMES and COMPASS



Phys.Rev.Lett.103:152002,2009



Bradamante, Transversity 2011

Sivers function in SIDIS

- New theoretical tools: TMD evolution!

- *J.C. Collins, Foundation of Perturbative QCD, Cambridge Monographs on Particle Physics, Nuclear Physics and Cosmology, No. 32, Cambridge University Press, 2011.*
- *S. M. Aybat and T. C. Rogers, Phys. Rev. D83, 114042 (2011), arXiv:1101.5057 [hep-ph]*
- *S. M. Aybat, J. C. Collins, J.-W. Qiu and T.C. Rogers, arXiv:1110.6428 [hep-ph]*

- What are the consequences from the phenomenological point of view??

Turin standard approach (DGLAP)

Turin standard approach (DGLAP)

- Unpolarized TMDs are factorized in x and $k_\perp$. Only the collinear part evolves with DGLAP evolution equation. No evolution in the transverse momenta:

$$\widehat{f}_{q/p}(x, k_\perp; Q) = f_{q/p}(x; Q) \frac{e^{-k_\perp^2/\langle k_\perp^2 \rangle}}{\pi \langle k_\perp^2 \rangle}$$

Collinear PDF (DGLAP evolution)

Normalized Gaussian: no evolution

Turin standard approach (DGLAP)

- The Sivers function is factorized in x and $k_\perp$ and proportional to the unpolarized PDF.

$$\begin{aligned}
\Delta^N \hat{f}_{q/p^\uparrow}(x, k_\perp; Q) &= 2\mathcal{N}_q(x) h(k_\perp) \hat{f}_{q/p}(x, k_\perp; Q) \\
&= 2\mathcal{N}_q(x) f_{q/p}(x; Q) \sqrt{2e}\,\frac{k_\perp}{M_1} \frac{e^{-k_\perp^2/\langle k_\perp^2\rangle_S}}{\pi\langle k_\perp^2\rangle}
\end{aligned}$$

Collinear PDF (DGLAP)

$$\mathcal{N}_q(x) = N_q\, x^{\alpha_q}(1-x)^{\beta_q} \frac{(\alpha_q+\beta_q)^{(\alpha_q+\beta_q)}}{\alpha_q^{\alpha_q}\,\beta_q^{\beta_q}}$$

$$\langle k_\perp^2\rangle_S = \frac{M_1^2\,\langle k_\perp^2\rangle}{M_1^2 + \langle k_\perp^2\rangle}$$

$$\Delta^N \hat{f}_{q/p^\uparrow}(x, k_\perp) = -\frac{2k_\perp}{m_p} f_{1T}^\perp(x, k_\perp)$$

TMD evolution formalism

- One can get the TMD in the momentum space by Fourier trasforming:

$$\widehat{f}_{q/p}(x, k_\perp; Q) = \frac{1}{2\pi} \int_0^\infty \mathrm{d}b_T \, b_T \, J_0(k_\perp b_T) \, \widetilde{f}_{q/p}(x, b_T; Q)$$

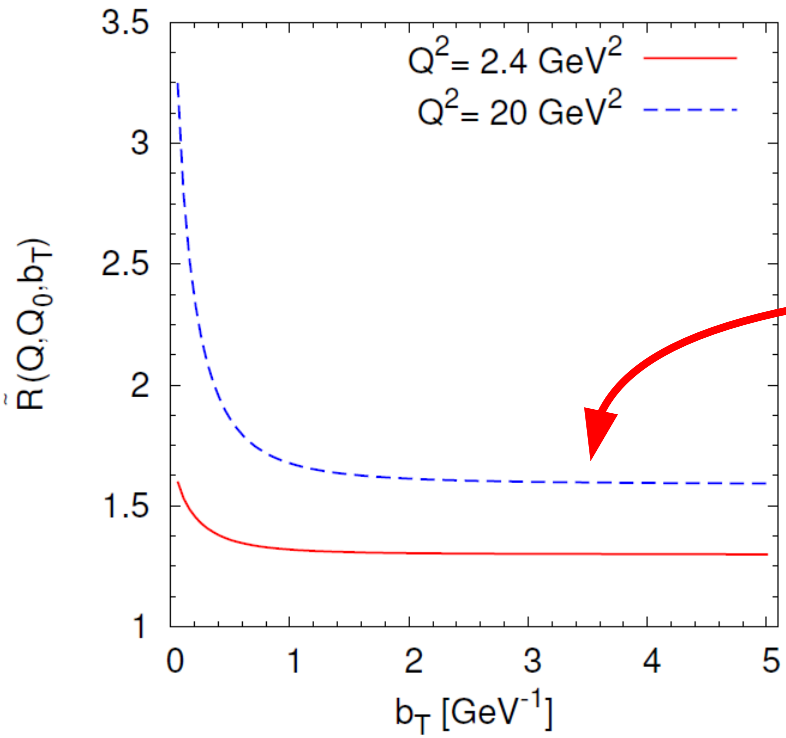
$$\widehat{D}_{h/q}(z, p_\perp; Q) = \frac{1}{2\pi} \int_0^\infty \mathrm{d}b_T \, b_T \, J_0(\mathrm{k}_T b_T) \, \widetilde{D}_{h/q}(z, b_T; Q)$$

$$\widehat{f}_{1T}^{\perp f}(x, k_\perp; Q) = \frac{-1}{2\pi k_\perp} \int_0^\infty \mathrm{d}b_T \, b_T \, J_1(k_\perp b_T) \, \widetilde{f}'^{\perp q}_{1T}(x, b_T; Q)$$

$$\begin{aligned} f_{q/p^\uparrow}(x, \boldsymbol{k}_\perp, \boldsymbol{S}; Q) &= f_{q/p}(x, k_\perp; Q) - f_{1T}^{\perp q}(x, k_\perp; Q) \frac{\epsilon_{ij} \, k_\perp^i \, S^j}{M_p} \\ &= f_{q/p}(x, k_\perp; Q) + \frac{1}{2} \Delta^N f_{q/p^\uparrow}(x, k_\perp; Q) \frac{\epsilon_{ij} \, k_\perp^i \, S^j}{k_\perp} \end{aligned}$$

Analytical (approximated) solution of the TMD evolution equation

- $\tilde{R}(Q,Q0,b_T)$ exhibits a non trivial dependence on b_T that prevents any analytical integration



$\tilde{R}(Q,Q0,b_T)$ becomes **constant** for $b_T > 1$ GeV $^{-1}$

We can therefore neglect the $\tilde{R}$ dependence on b_T and define:

$$R(Q, Q_0) \equiv \widetilde{R}(Q, Q_0, b_T \rightarrow \infty)$$

Good approximation for large b_T i.e. small $k_\perp$

Analytical (approximated) solution of the TMD evolution equation

- For instance, replacing $\tilde{R}$ with R in the unpolarized, we get:

$$\widetilde{f}_{q/p}(x, \boldsymbol{b}_T; Q) = f_{q/p}(x, Q_0)\ R(Q, Q_0)\ \exp\left\{ -b_T^2 \left(\alpha^2 + \frac{g_2}{2} \ln \frac{Q}{Q_0} \right) \right\}$$

Which is Gaussian in b_T, and will then Fourier-transform into a Gaussian in $k_\perp$

$$\widehat{f}_{q/p}(x, k_\perp; Q) = f_{q/p}(x, Q_0)\ R(Q, Q_0)\ \frac{e^{-k_\perp^2/w^2}}{\pi w^2}$$

$$w^2(Q, Q_0) = \langle k_\perp^2 \rangle + 2\, g_2 \ln \frac{Q}{Q_0}$$

Analytical (approximated) solution of the TMD evolution equation

- Similarly, for the unpolarized TMD fragmentation function, we have

$$\widehat{D}_{h/q}(z, p_\perp; Q) = D_{h/q}(z, Q_0)\, R(Q, Q_0)\, \frac{e^{-p_\perp^2/w_F^2}}{\pi w_F^2}$$

$$w_F^2 \equiv w_F^2(Q, Q_0) = \langle p_\perp^2 \rangle + 2z^2 g_2 \ln \frac{Q}{Q_0}$$

Analytical (approximated) solution of the TMD evolution equation

➢ For the Sivers distribution function, we find:

$$\Delta^N \widehat{f}_{q/p^\uparrow}(x, k_\perp; Q) = \frac{k_\perp}{M_1} \sqrt{2e} \, \frac{\langle k_\perp^2 \rangle_S^2}{\langle k_\perp^2 \rangle} \, \Delta^N f_{q/p^\uparrow}(x, Q_0) \, R(Q, Q_0) \, \frac{e^{-k_\perp^2/w_S^2}}{\pi w_S^4}$$

$$w_S^2(Q, Q_0) = \langle k_\perp^2 \rangle_S + 2 g_2 \ln \frac{Q}{Q_0}$$

$$\Delta^N \widehat{f}_{q/p^\uparrow}(x, k_\perp) = -\frac{2 k_\perp}{m_p} f_{1T}^\perp(x, k_\perp)$$

$$\langle k_\perp^2 \rangle_S = \frac{M_1^2 \, \langle k_\perp^2 \rangle}{M_1^2 + \langle k_\perp^2 \rangle}$$

Consequences on DY data and warnings

- Numerator of the asymmetry in analytical approximation for a SIDIS process

$$N_{SIDIS} \propto \Delta^N f(x,Q_0) D(z,Q_0) \sqrt{2e} \frac{P_T}{M_1} \frac{z \langle k_\perp^2 \rangle_{Siv}^2}{\langle k_\perp^2 \rangle \langle P_T^2 \rangle_{Siv}^2} e^{-P_T^2/\langle P_T^2 \rangle_{Siv}}$$

$$\langle P_T^2 \rangle_{Siv}^{SIDIS} = z^2 \omega_{Siv}^2 + \omega_{FF}^2$$

$$w_S^2(Q,Q_0) = \langle k_\perp^2 \rangle_S + 2 g_2 \ln \frac{Q}{Q_0}$$

$$w_F^2 \equiv w_F^2(Q,Q_0) = \langle p_\perp^2 \rangle + 2 z^2 g_2 \ln \frac{Q}{Q_0}$$

- Here it is squared, strongly suppresses the asymmetry as it becomes larger and larger

- $0.2 < z < 0.8$

Consequences on DY data and warnings

- Numerator of the asymmetry in analytical approximation for a DY process

$$N_{DY} \propto \Delta^N f(x_1,Q_0) f(x_2,Q_0) \sqrt{2e} \frac{P_T}{M_1} \frac{\langle k_\perp^2 \rangle_{Siv}^2}{\langle k_{\perp 1}^2 \rangle \langle P_T^2 \rangle_{Siv}^2} e^{-P_T^2/\langle P_T^2 \rangle_{Siv}}$$

$$\langle P_T^2 \rangle_{Siv}^{DY} = \omega_{Siv}^2 + \omega_2^2$$

$$w_S^2(Q,Q_0) = \langle k_\perp^2 \rangle_S + 2 g_2 \ln \frac{Q}{Q_0}$$

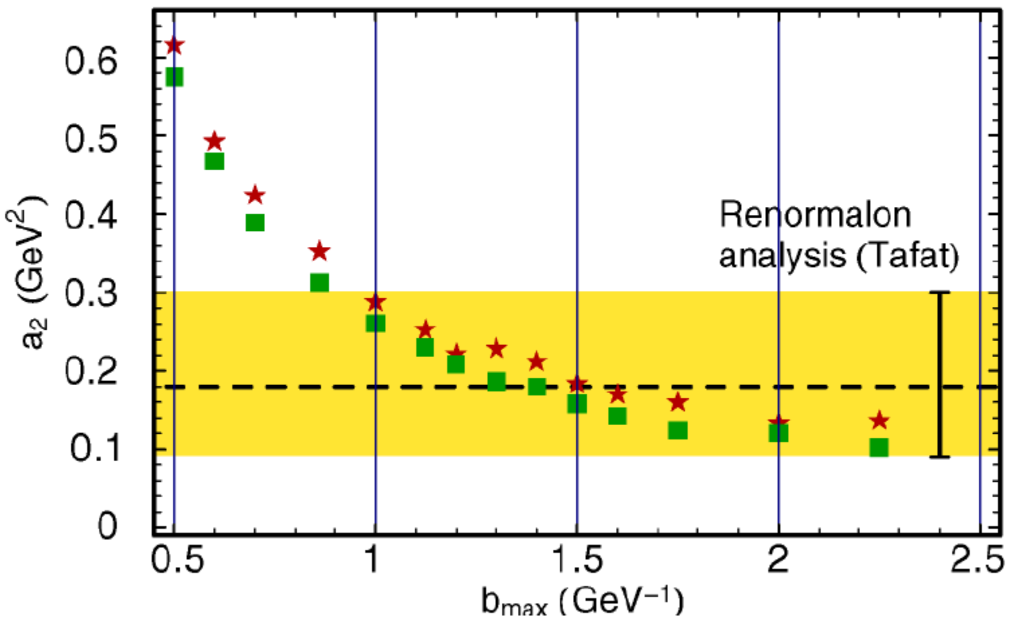
$$w^2(Q,Q_0) = \langle k_\perp^2 \rangle + 2\, g_2 \ln \frac{Q}{Q_0}$$

- Here it is squared, strongly suppresses the asymmetry as it becomes larger and larger

- g_2 is more crucial for DY processes than for the present SIDIS data (because of a wider kinematical range in Q^2)

Consequences on DY data and warnings

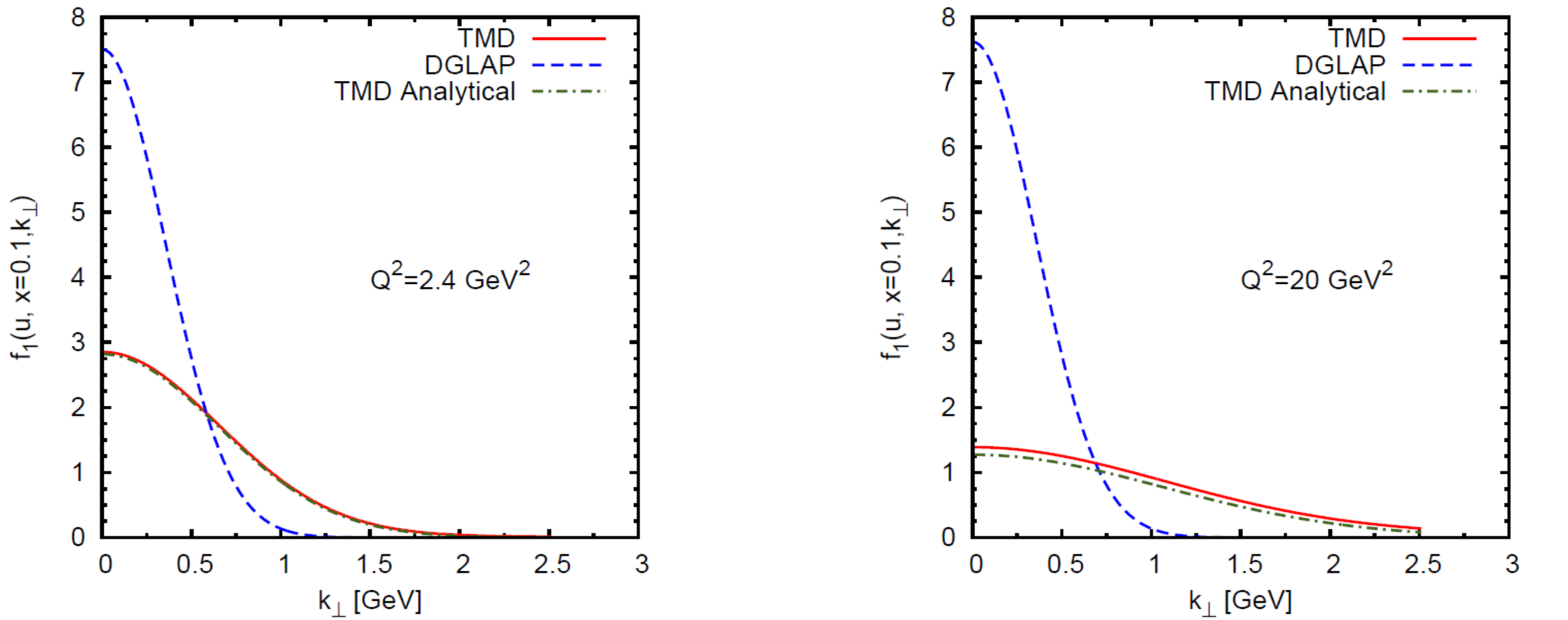
➢g_2 depends on the prescription for the separation of the perturbative region from the non -perturbative one. Depends also on the "order" at which you stop in the perturbative expansion.



a_2=g_2, stars correspond to the choice C1=2 exp(-γ_e), squares to C1=4 exp(-γ_e)

Konychev and Nadolsky Phys. Lett. B633 (2006)

Comparative analysis of TMD evoultion equations

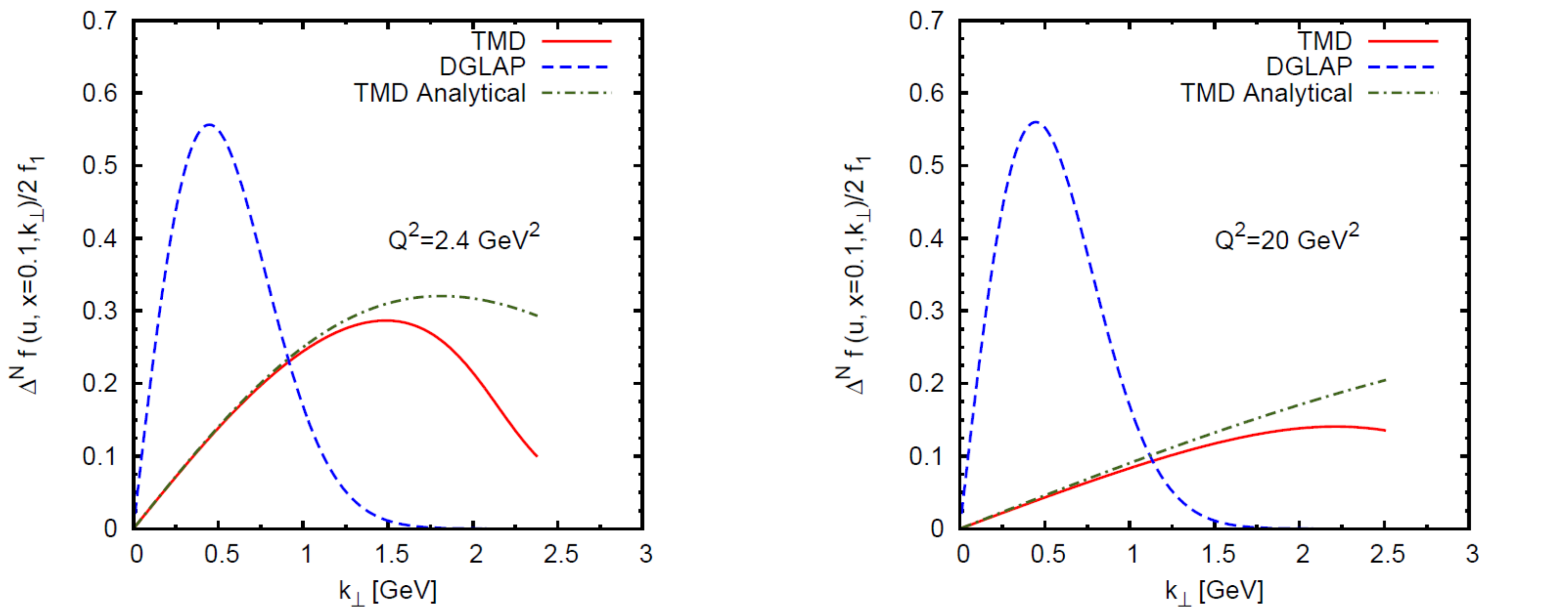


Starting scale Q_0=1 GeV
Same function at Q_0

DGLAP evolution is slow at moderate x and in this range of Q^2

For the unpolarized PDF, the analytical approximation holds up to large $k_\perp$

Comparative analysis of TMD evoultion equations



Starting scale Q_0=1 GeV

Same function at Q_0

For the Sivers function, the analytical approximation breaks down at large $k_\perp$ values

Fit of HERMES and COMPASS SIDIS data

- We perform 3 different fits:
  - TMD-fit (computing TMD evolution equations numerically)
  - TMD-analytical fit (solving TMD evolution equations in the analytical approx.)
  - DGLAP fit (using DGLAP evolution equation for the collinear part of the TMD)

- Data sets:
  - HERMES (2009) $\pi+$ $\pi-$ π^0 K+ K-
  - COMPASS Deuteron (2004) $\pi+$ $\pi-$ K+ K-
  - COMPASS Proton (2011) h+ h-

Fit of HERMES and COMPASS SIDIS data

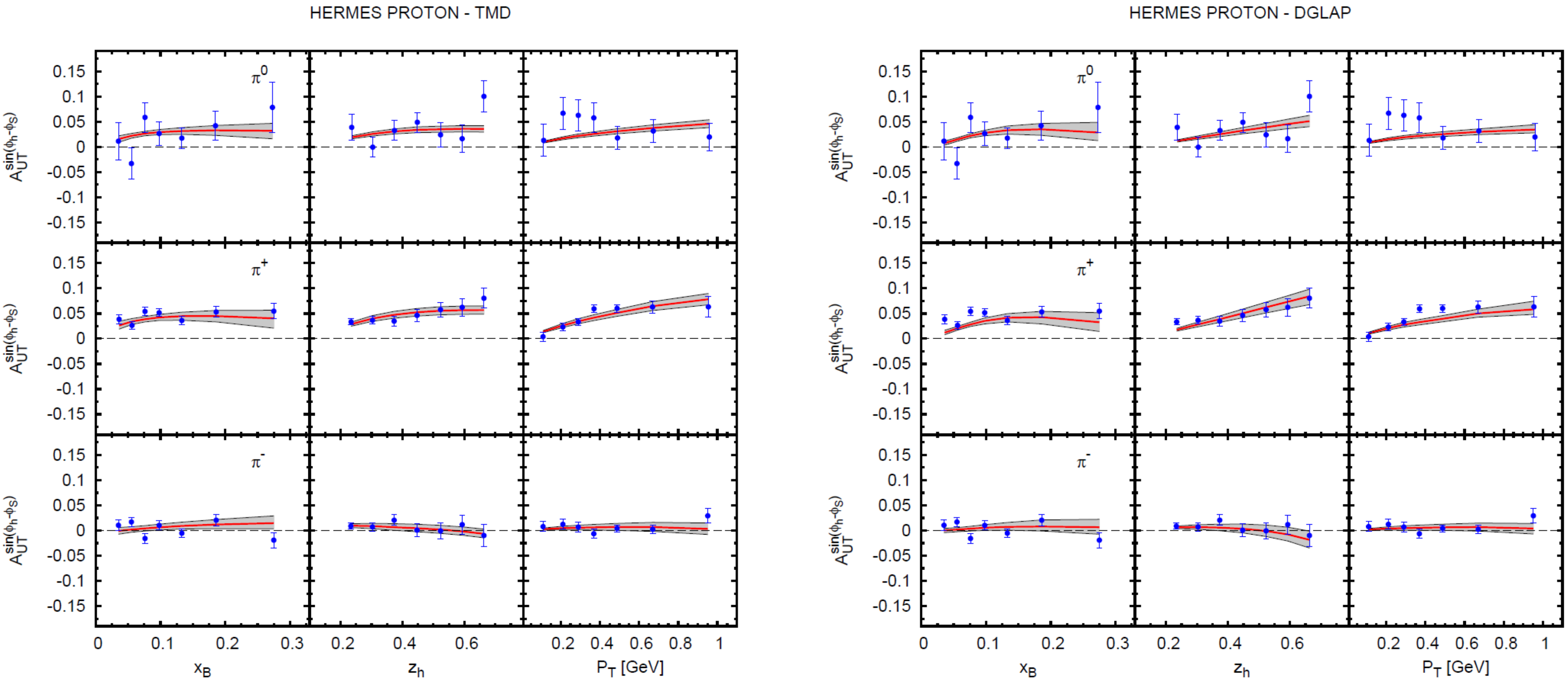
χ^2 tables

11 free parameters, 261 points

	TMD Evolution (Exact)		TMD Evolution (Analytical)	DGLAP Evolution
	$\chi^2_{tot} = 255.8$ $\chi^2_{d.o.f} = 1.02$		$\chi^2_{tot} = 275.7$ $\chi^2_{d.o.f} = 1.10$	$\chi^2_{tot} = 315.6$ $\chi^2_{d.o.f} = 1.26$
HERMES π^+	$\chi^2_x = 10.7$ $\chi^2_z = 4.3$ $\chi^2_{P_T} = 9.1$	7 points	$\chi^2_x = 12.9$ $\chi^2_z = 4.3$ $\chi^2_{P_T} = 10.5$	$\chi^2_x = 27.5$ $\chi^2_z = 8.6$ $\chi^2_{P_T} = 22.5$
COMPASS h^+	$\chi^2_x = 6.7$ $\chi^2_z = 17.8$ $\chi^2_{P_T} = 12.4$	9 points	$\chi^2_x = 11.2$ $\chi^2_z = 18.5$ $\chi^2_{P_T} = 24.2$	$\chi^2_x = 29.2$ $\chi^2_z = 16.6$ $\chi^2_{P_T} = 11.8$

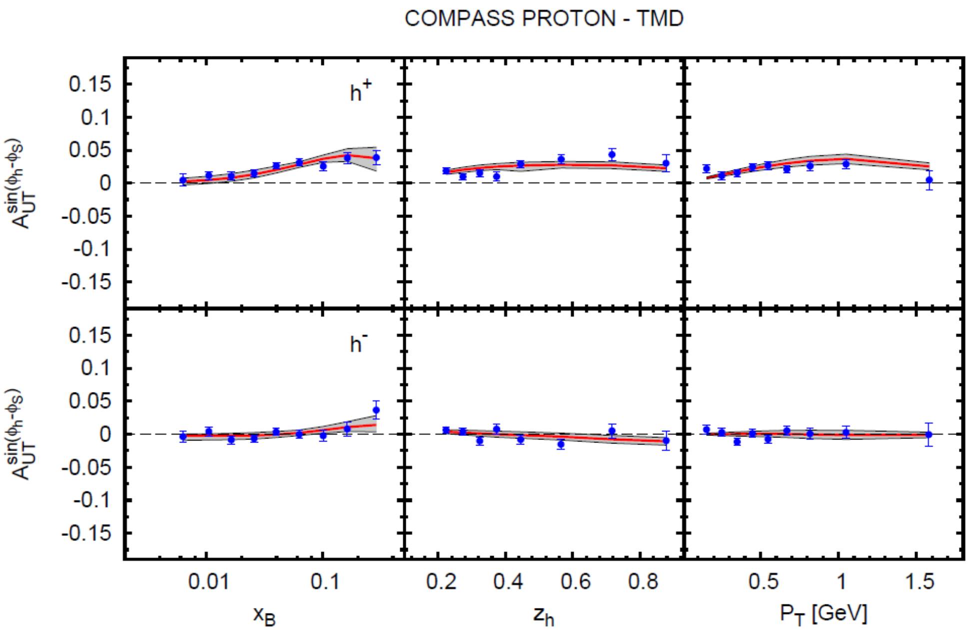
Fit of HERMES and COMPASS SIDIS data

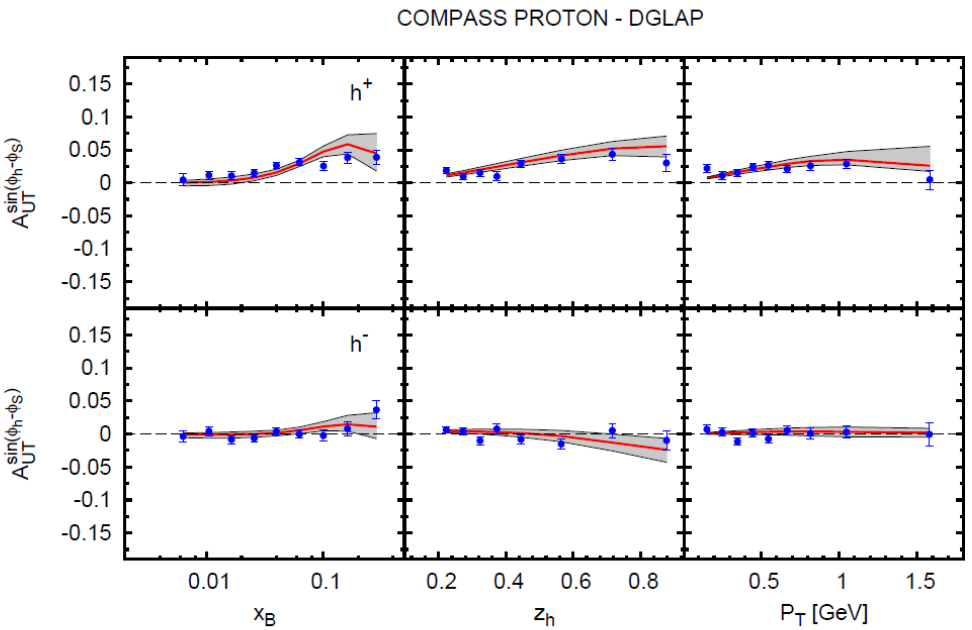
A. Airapetian et al., Phys. Rev. Lett. 103, 152002 (2009), arXiv:0906.3918 [hep-ex]



Fit of HERMES and COMPASS SIDIS data

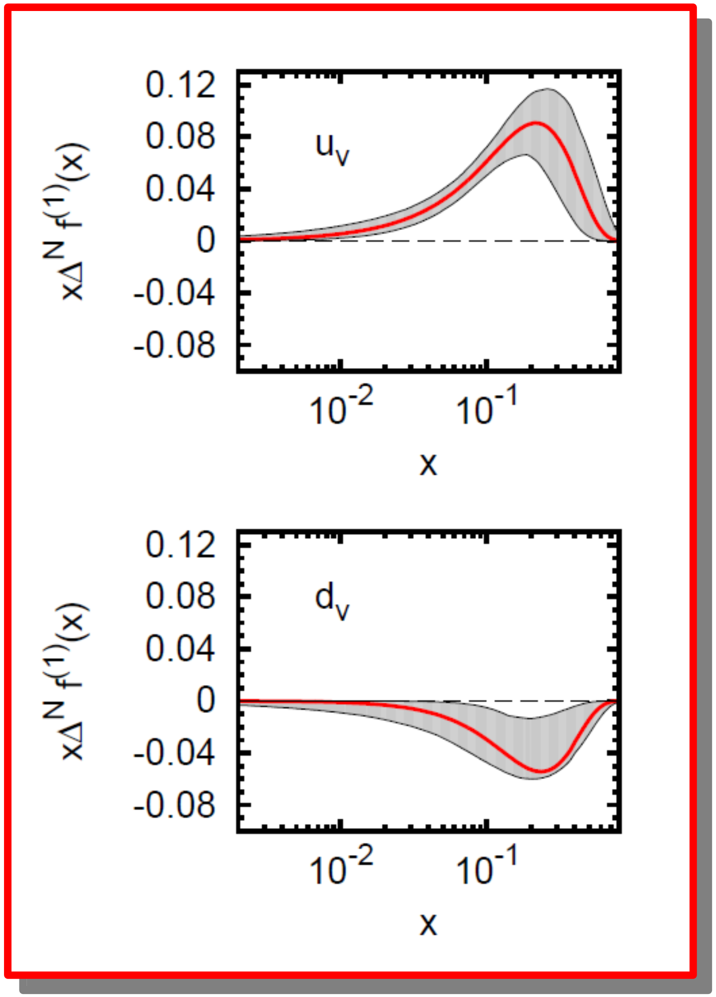
F. Bradamante, arXiv:1111.0869 [hep-ex]





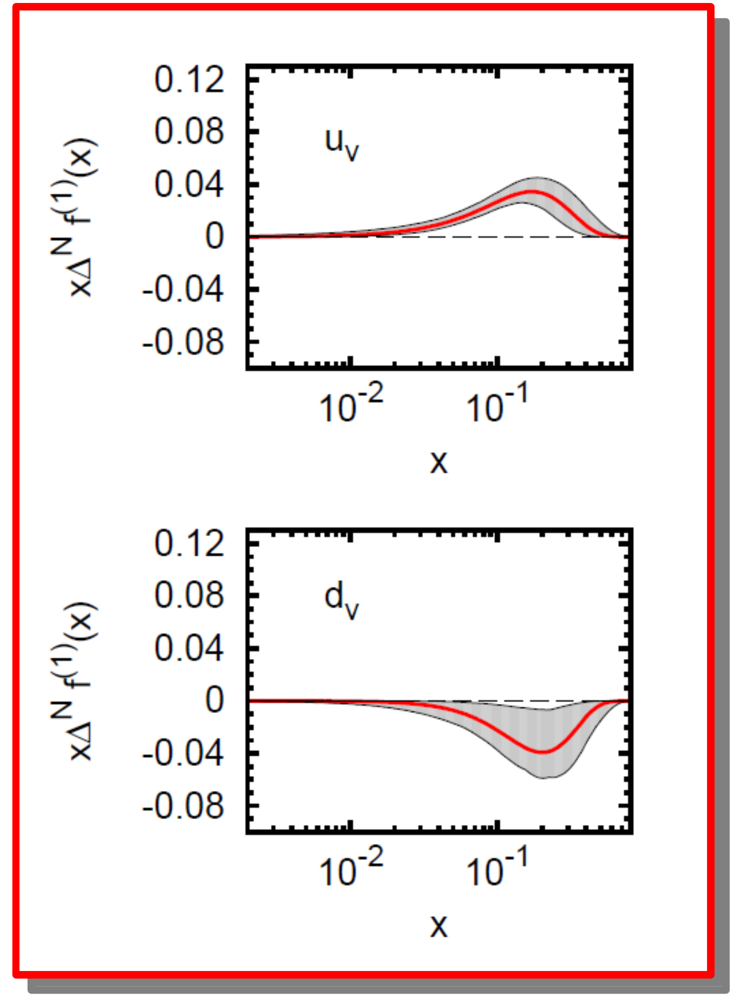
Fit of HERMES and COMPASS SIDIS data

TMD Evolution



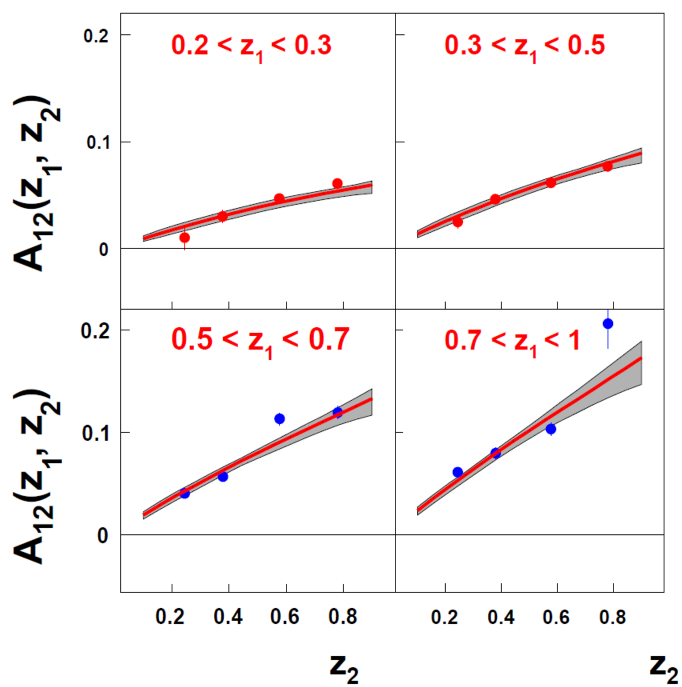
Q_0=1 GeV

DGLAP Evolution



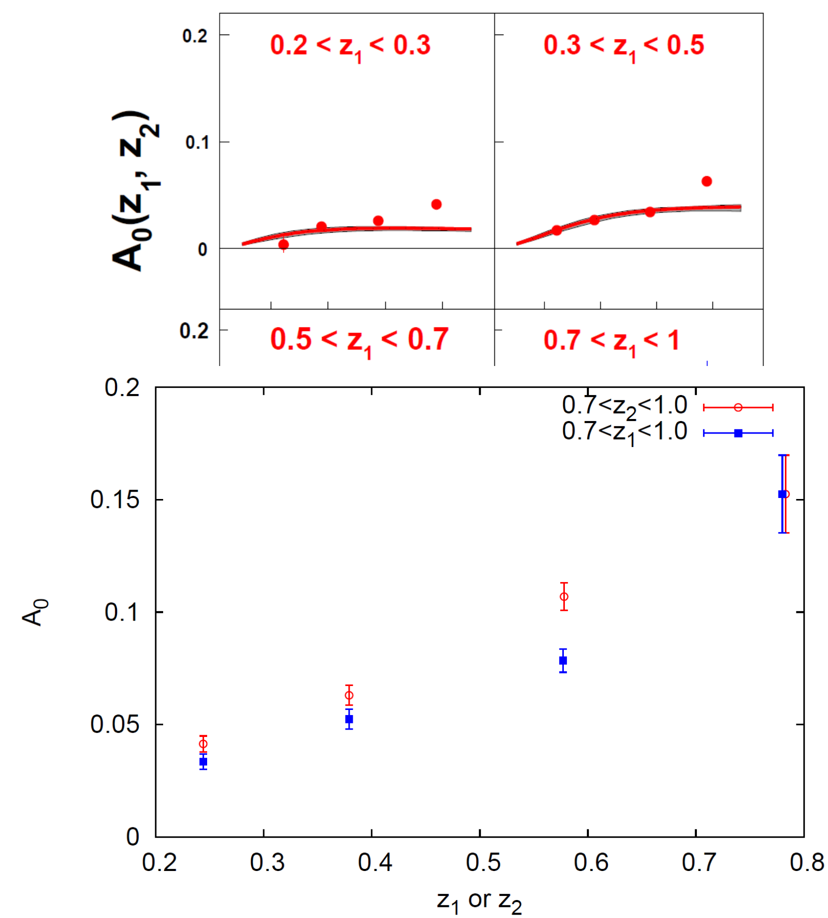
Extraction of the transversity and the Collins function

BELLE A_{12} (FIT)



◇ R. Seidl et al., Phys. Rev. D78

BELLE A_0 (Predicted)



- Anselmino et. al arXiv: 0812.4366v1

Fit of HERMES and COMPASS SIDIS data

- Data sets:
  - HERMES (2009) $\pi+$ $\pi-$ π^0 K+ K-
  - COMPASS Deuteron (2004) $\pi+$ $\pi-$ K+ K-
  - COMPASS Proton (2011) h+ h-

- DGLAP and TMD fits
  - TMD-fit (computing TMD evolution equations numerically)
  - DGLAP fit (using DGLAP evolution equation for the collinear part of the TMD)

Extraction of the Boer-Mulders function

- The Cahn effect is a crucial ingredient

✔Gaussians: $\langle k_\perp^2 \rangle = 0.25\ (GeV/c)^2$
$\langle p_\perp^2 \rangle = 0.20\ (GeV/c)^2$

From Ref.[*]: analysis of Cahn cosϕ effect from EMC data

COMPASS	HERMES
$\langle k_\perp^2 \rangle = 0.25\ (GeV/c)^2$	$\langle k_\perp^2 \rangle = 0.18\ (GeV/c)^2$
$\langle p_\perp^2 \rangle = 0.20\ (GeV/c)^2$	$\langle p_\perp^2 \rangle = 0.20\ (GeV/c)^2$
~EMC	~HERMES MC

[*] Anselmino et al. Phys. Rev. D71 074006 (2005)

Extraction of the Boer-Mulders function

- FIT II

COMPASS

$\langle k_\perp^2 \rangle = 0.25\ (GeV/c)^2$

$\langle p_\perp^2 \rangle = 0.20\ (GeV/c)^2$

~EMC

HERMES

$\langle k_\perp^2 \rangle = 0.18\ (GeV/c)^2$

$\langle p_\perp^2 \rangle = 0.20\ (GeV/c)^2$

~HERMES MC

FIT II

◇ $\chi^2/d.o.f. = 2.41$

• $\lambda_u = 2.1 \pm 0.1$

• $\lambda_d = -1.11^{+0.00}_{-0.02}$

Better description of HERMES but the BM is unchanged