Factorization in QCD

S.Alekhin (*IHEP Protvino & DESY-Zeuthen)*

1. Basics and formalism

2. Factorization schemes

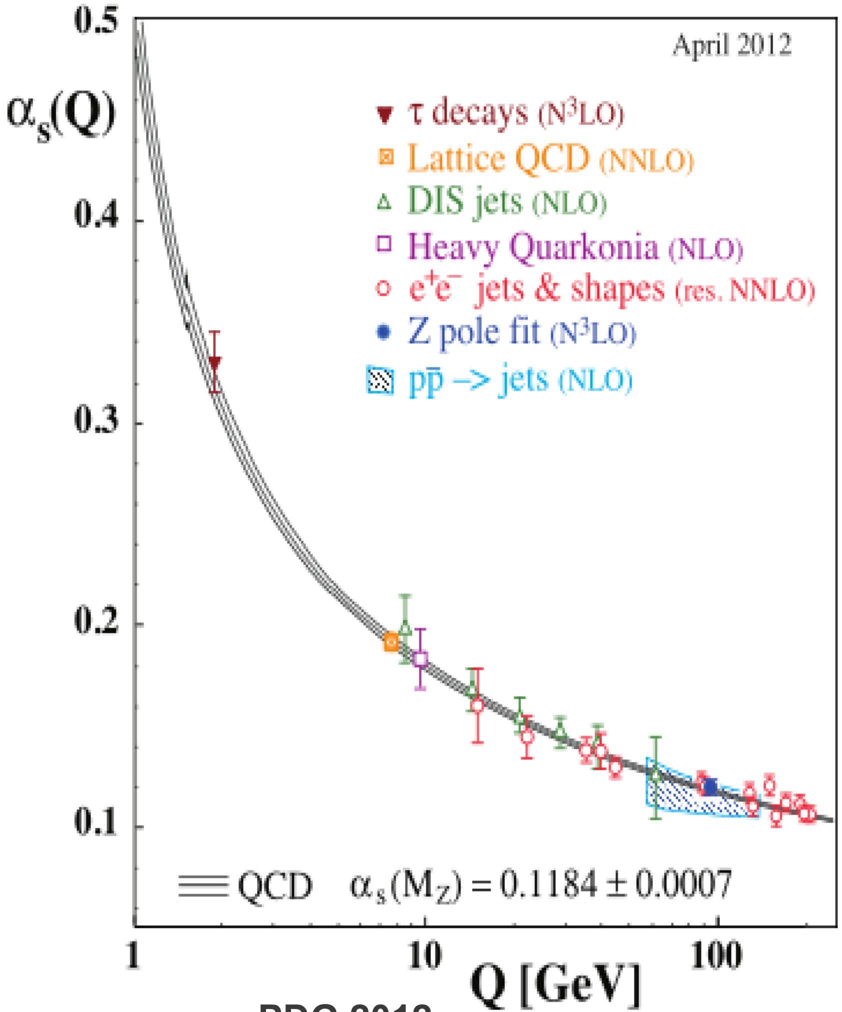
   2.1. Mass factorization and heavy-flavor PDFs

   2.2. GMVFN prescriptions

3. FFN scheme and big-log resummation

DESY, 22 Oct 2012

Basics

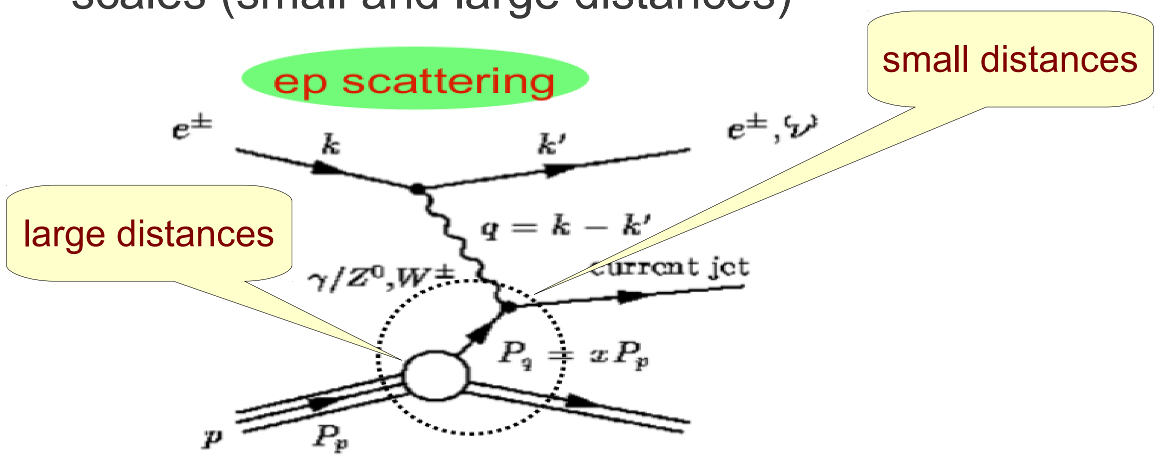


PDG 2012

Confinement: Quarks do not appear at large distances

Asymptotic freedom: Strong coupling vanishes at small distances

Factorization: Separation of the soft and hard scales (small and large distances)



courtesy of S.Magill

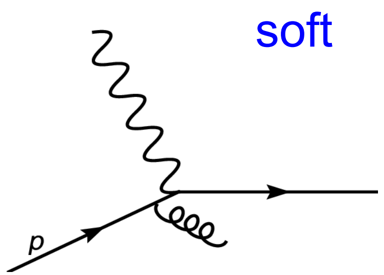
$$F_2(x, Q^2) = \sum_q e_q^2 f_q(x)$$

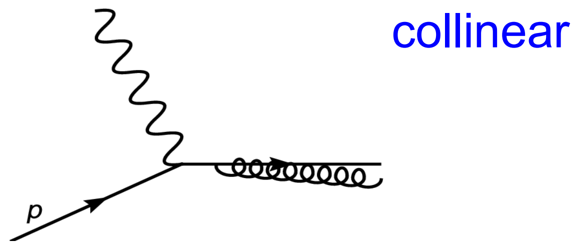
Example: deep-inelastic-scattering in the parton model

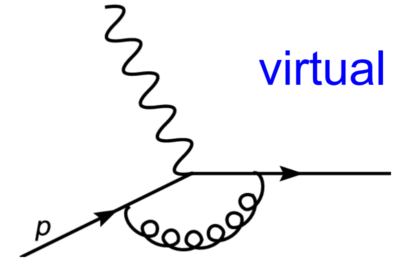
Probability to find parton(s) in the hadron with subsequent scattering of/off parton(s)

DIS in QCD

Seymour hep-ph 1010.2330







$$\Delta F_2(x,Q^2) = \sum_q \int_x^1 dx_p\, e_q^2 \frac{x}{x_p} f_q\left(\frac{x}{x_p}\right)\; J(x_P)$$

Contribution from lq → lqg is divergent for the soft/collinear radiation of gluon

$$J(x_P) = \frac{C_F \alpha_S}{2\pi} \int_0^1 dz \left(\frac{1+x_p^2}{1-x_p} \frac{1+z^2}{1-z} + 3 - z - x_p + 11 x_p z \right)$$

$$k_\perp^2 = Q^2 \left(\frac{1}{x_p} - 1 \right) z(1-z).$$

$$J(x_p)|_{k_\perp > \mu} = \frac{\alpha_S}{2\pi} \left(C_F \frac{1+x_p^2}{1-x_p} \log \frac{Q^2}{\mu^2} + R(x_p) \right)$$

Can be regularized with the $k_\perp$-cutoff

$$C_F \frac{1+x^2}{1-x} \to P^{(0)}(x) = C_F \left(\frac{1+x^2}{1-x} \right)_+$$

$$f(x)_+ = f(x) - \delta(1-x) \int_0^1 dx'\, f(x')$$

The virtual term exactly cancels the divergency (QCD is gauge theory) → regularized splitting function P(x), probability to find a quark within a quark (PDF evolution)

Putting everything together we still have divergency at μ=0

$$\Delta F_2(x,Q^2) = \sum_q \int_x^1 dx_p\, e_q^2 \frac{x}{x_p} f_q\left(\frac{x}{x_p}\right) \frac{\alpha_S}{2\pi}\left(P^{(0)}(x_p)\log\frac{Q^2}{\mu^2} + R(x_p)\right) \qquad k_\perp > \mu$$

However this divergency can be absorbed into the PDFs, which anyway cannot be calculated in the pQCD and after redefinition contains all infrared singularities

$$\Delta F_2(x,Q^2) = \sum_q \int_x^1 dx_p\, \frac{x}{x_p} p_q\left(\frac{x}{x_p},\mu^2\right) C^{(1)}(x_p,Q,\mu^2)) \qquad \text{all } k_\perp$$

$$C^{(1)}(x_p,Q,\mu^2) = e_q^2\frac{\alpha_S}{2\pi}\left(P^{(0)}(x_p)\log\frac{Q^2}{\mu^2} + R(x_p)\right)$$

coefficient function

The soft scales (light-quark and nucleon masses, etc.) are localized in the PDFs $p(x,\mu^2)$ and the hard scales in the coefficient functions $C(x,Q,\mu^2)$

The cross sections should not depend on the factorization scale μ

$$\mu^2\frac{dF_2(x,Q^2)}{d\mu^2} = 0$$

$$\mu^2\frac{d}{d\mu^2}p_q\left(x,\mu^2\right) = \frac{\alpha_S}{2\pi}\int_x^1 \frac{dx_p}{x_p} p_q\left(\frac{x}{x_p},\mu^2\right) P^{(0)}(x_p)$$

Altarelli-Parisi evolution in $O(\alpha_S)$

The splitting functions P(x) define change of the PDFs with the factorization scale through the renormgroup evolution equation, which can be solved for the given boundary condition at $\mu=Q_0$

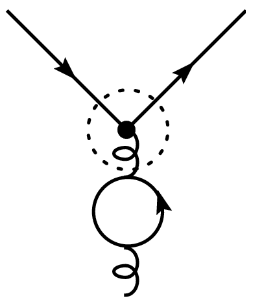
Factorization and high order corrections

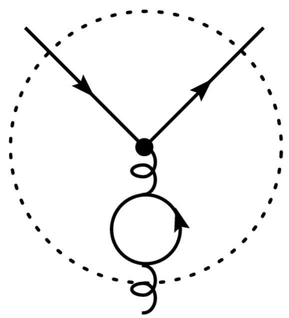
van Neerven, Vogt NPB 558, 345 (2000)

$$C(x) = c^{(0)}(x) + \sum_{l=1} a_s^l \left(c^{(l)}(x) + \sum_{m=1}^{l} c^{(l,m)}(x) \log^m \frac{Q^2}{\mu^2} \right)$$

massless coefficient functions

$$\begin{aligned}
c_a^{(1,1)} &= c_a^{(0)} \otimes P^{(0)} \\
c_a^{(2,1)} &= c_a^{(0)} \otimes P^{(1)} + c_a^{(1)} \otimes (P^{(0)} - \beta_0) \\
c_a^{(2,2)} &= \frac{1}{2} c_a^{(1,1)} \otimes (P^{(0)} - \beta_0) \\
c_a^{(3,1)} &= c_a^{(0)} \otimes P^{(2)} + c_a^{(1)} \otimes (P^{(1)} - \beta_1) + c_a^{(2)} \otimes (P^{(0)} - 2\beta_0) \\
c_a^{(3,2)} &= \frac{1}{2} \left\{ c_a^{(1,1)} \otimes (P^{(1)} - \beta_1) + c_a^{(2,1)} \otimes (P^{(0)} - 2\beta_0) \right\} \\
c_a^{(3,3)} &= \frac{1}{3} c_a^{(2,2)} \otimes (P^{(0)} - 2\beta_0)
\end{aligned}$$





courtesy of M.Seymour

$$\frac{da_s}{d \ln \mu^2} = \beta(a_s) = -\sum_{l=0} a_s^{l+2} \beta_l$$

strong coupling evolution

The logarithmic terms appear in the high-order correction recursively

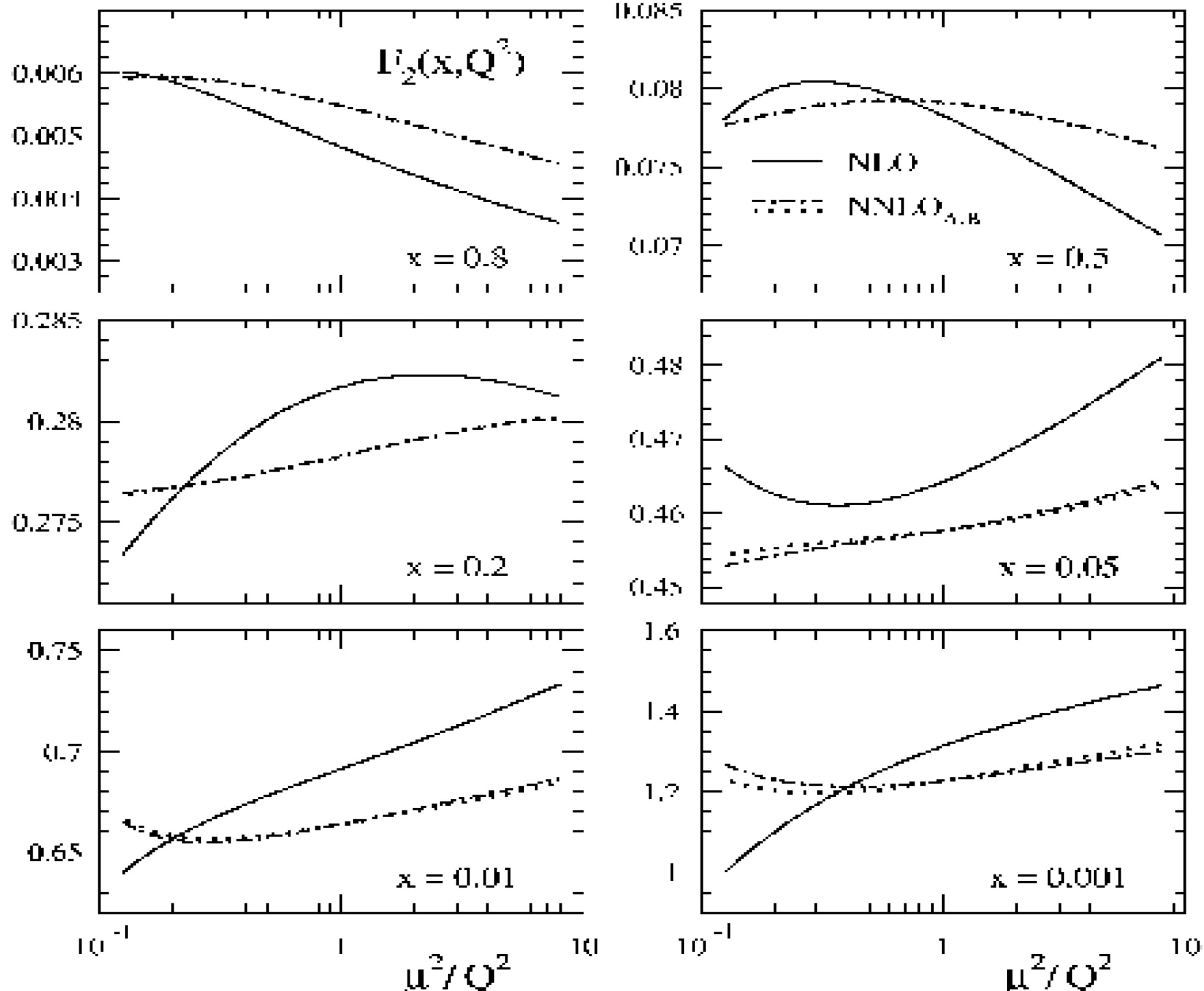
For dimensional regularization the phase space integrals are considered in (4-2ε) dimensions. The singularities appear as poles in ε and are absorbed into the PDFs in the same way as in the $k_\perp$ regularization. Dimensional regularization can be better automatized and has advantage in high-order calculations

Collins, Soper, Sterman hep-ph/0409313

Scale sensitivity in the fixed order

$$\mu^2 \frac{dF_2(x,Q^2)}{d\mu^2} = O(\alpha_s^n)$$

n is the order of calculations



van Neerven, Vogt NPB 558, 345 (2000)

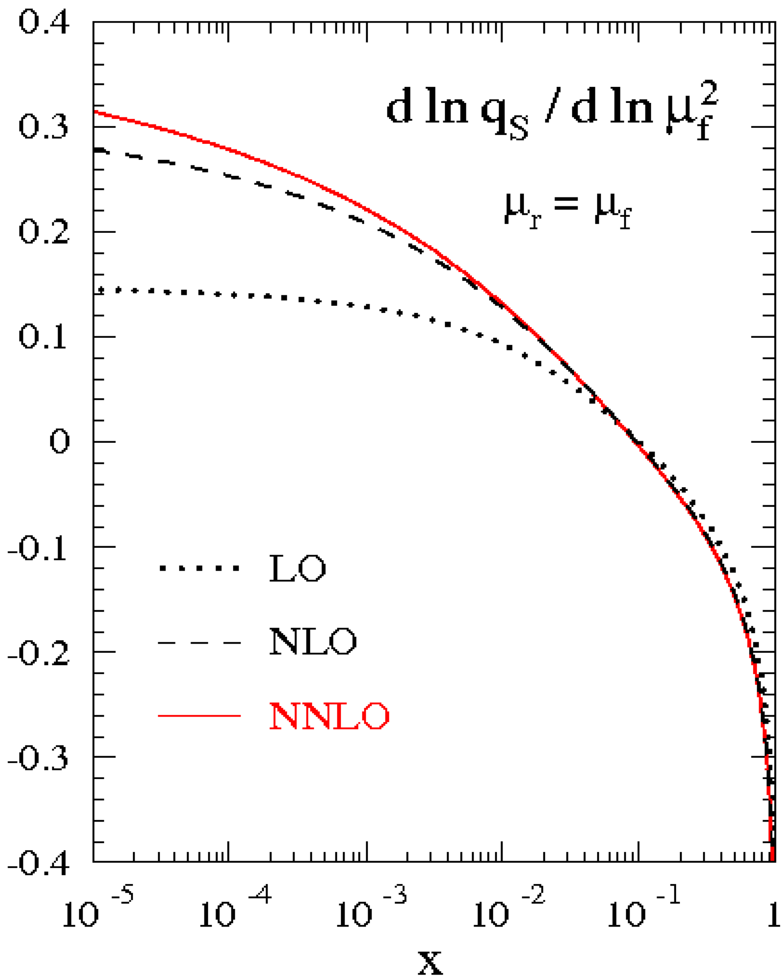
The theoretical error is commonly estimated from the scale variation from $\mu_0^2/2$ to $2\mu_0^2$

To achieve theoretical accuracy of O(%) the NNLO corrections are necessary for typical kinematics of the hadronic processes currently studied at LHC.

PDFs in perturbative QCD

$$P(x) = \quad a_s P^{(0)}(x) + a_s^2 P^{(1)}(x) + a_s^3 P^{(2)}(x) + \ldots$$

splitting functions up to NNLO



Nice perturbative convergence

Moch, Vermasseren, Vogt NPB 688, 101 (2004)

Vogt, Moch, Vermasseren NPB 691, 129 (2004)

- In QCD the PDFs ***are not*** probabilities. They cannot be not neither measured nor calculated, only parametrized
- The PDFs are scheme-dependent

$$F_2(x,Q^2) = \sum_q e_q^2 \int_x^1 dx_p \frac{x}{x_p} p_q\left(\frac{x}{x_p},\mu^2\right)\left\{\delta(1-x_p) + \frac{\alpha_S}{2\pi}\left(P^{(0)}(x_p)\log\frac{Q^2}{\mu^2} + R(x_p) - K(x_p)\right)\right\}$$

i.e. defined up to the finite term K(x) (equal to 0 in the MSbar scheme), *however once the scheme is fixed an arbitrary modification of the coefficients functions violates factorization*

Ingredients of the global PDF fit

$$\sigma_h(p_h) = \sum_{q,g} \int d\eta\, p_{q,g}\left(\eta, \mu^2\right) \left\{ \sigma_q(\eta p_h) + \frac{\alpha_s}{2\pi} \log\frac{Q^2}{\mu^2} \int dz\, P^{(0)}(z)\, \sigma_{q,g}(z\eta p_h) \right\}$$

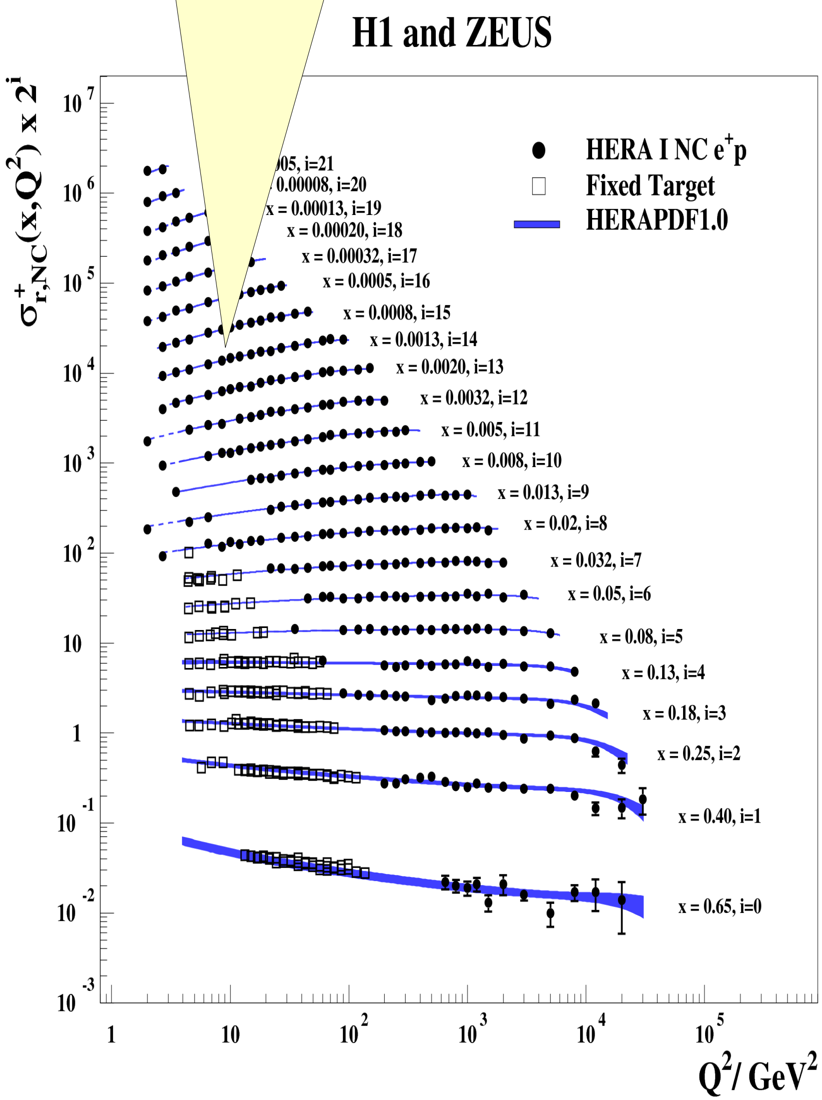
- The factorization should be proved for each particular process
- The form of factorized hadronic cross sections is universal
- The splitting functions P(x) are universal as well → *global PDF fit is possible*

Deep-inelastic scattering	*l h → l X*	NNLO
Drell-Yan process	*h h →* $l^+ l^-$ *X*	NNLO
Heavy-quark DIS production	*l h → Q X*	NLO
Jet DIS production	*l h → l jet X*	NLO
Hadronic jet production	*h h → jet X*	NLO

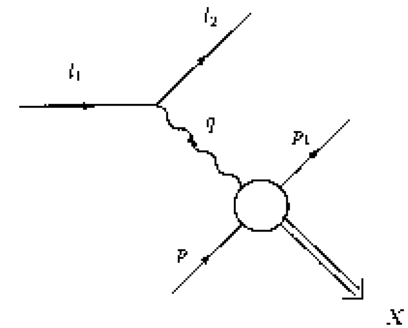
The current PDF accuracy is limited due to the missing NNLO corrections to several relevant processes

Heavy-quark lepto-production

c-quark contribution up to 30%



H1 and ZEUS JHEP 1001, 109 (2010)



- Phenomenological importance:
  - – semi-inclusive and inclusive DIS → constraint on the small-x gluons and sea
  - – dimuon neutrino-nucleon DIS production → constraint on the strange sea

  (cf. Mandy's talk)

- Theoretical difficulties:

  Two hard scales are relevant, lepton momentum transfer Q and the heavy-quark mass, m_h

Heavy-quark electro-production in the FFNS

- Only 3 light flavors appear in the initial state
- The dominant mechanism is photon-gluon fusion
- The coefficient functions are known up to the NLO

Witten NPB 104, 445 (1976)

Laenen, Riemersma, Smith, van Neerven NPB 392, 162 (1993)

- Involved high-order calculations: The full NNLO corrections are missing
  - NNLO term due to threshold resummation

    Laenen, Moch PRD 59, 034027 (1999)

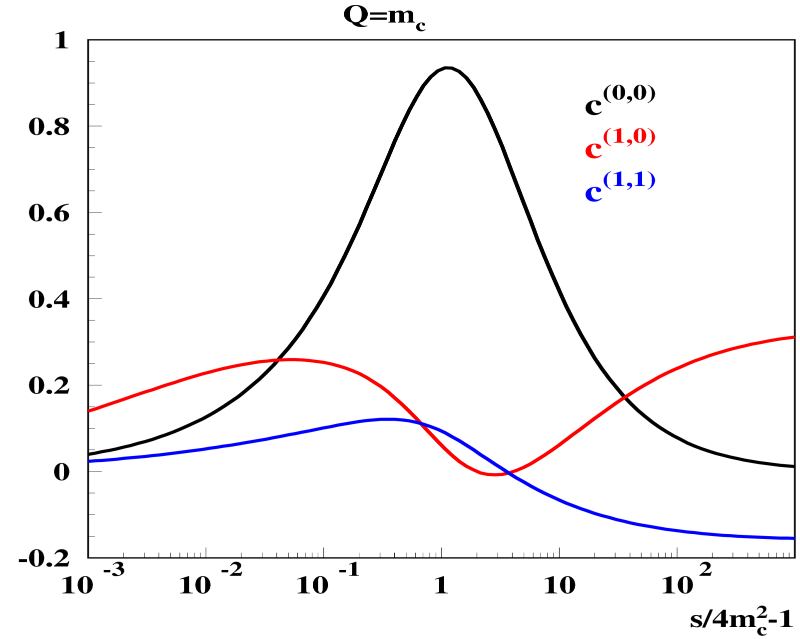
    Lo Presti, Kawamura, Moch, Vogt [hep-ph 1008.0951]
  - limited set of the NNLO Mellin moments

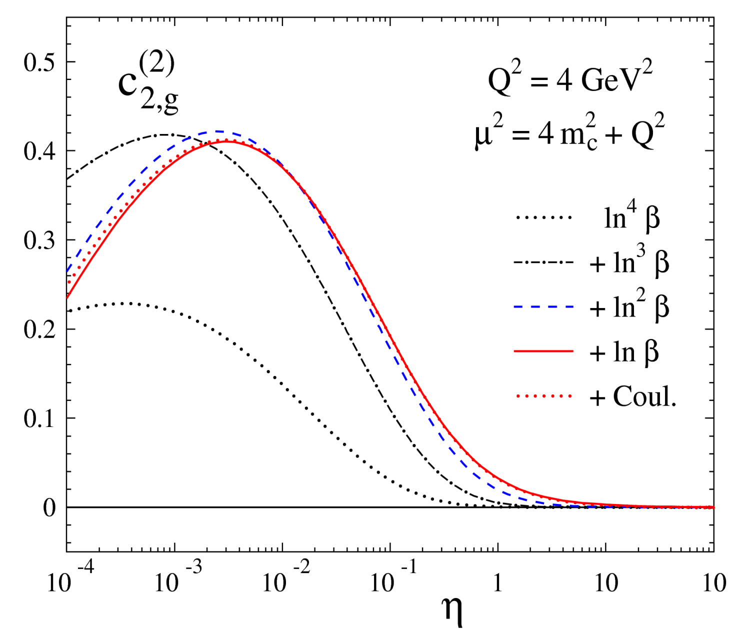
    Ablinger at al. NPB 844, 26 (2011)

    Bierenbaum, Blümlein, Klein NPB 829, 417 (2009)

- At large Q the leading-order coefficient → *ln(Q/m_h)* and may be quite big despite the suppression by factor of α_s and should be resummed

Shifman, Vainstein, Zakharov NPB 136, 157 (1978)





The 4-flavor scheme

At $Q \gg m_h$ the massive theory is dominated by terms $\sim \ln(Q/m_h)$ → the mass effects may be completely ignored once these mass singularities are absorbed into the PDFs .

Collins, Tung NPB 278, 934 (1986)

The factorization formalism is based on the 3-flavor scheme results, however it is somewhat involved for high orders due to the collinear and mass singularities are mixed

Buza, Matiounine, Smith, van Neerven EPJC 1, 301 (1998)

$$H_{g,2}^{\mathrm{asymp}} = a_s(N_f)\left[A_{hg}^{(1)} + \frac{1}{N_f}C_{g,2}^{(1)}(N_f)\right] + a_s^2(N_f)\left\{A_{hg}^{(2)} + A_{hg}^{(1)} \otimes C_{q,2}^{(1),\mathrm{NS}} + A_{gg,h}^{(1)} \otimes \frac{1}{N_f}C_{g,2}^{(1)}(N_f) + \frac{1}{N_f}C_{g,2}^{(2)}(N_f)\right\}$$

Asymptotic 3-flavor coefficient function

$$A_{ij}^{(1)}\left(z, \frac{m_h^2}{\mu^2}\right) = a_{ij}^{(1,1)}(z)\ln\left(\frac{\mu^2}{m_h^2}\right) + a_{ij}^{(1,0)}(z)$$

$$A_{ij}^{(2)}\left(z, \frac{m_h^2}{\mu^2}\right) = a_{ij}^{(2,2)}(z)\ln^2\left(\frac{\mu^2}{m_h^2}\right) + a_{ij}^{(2,1)}(z)\ln\left(\frac{\mu^2}{m_h^2}\right) + a_{ij}^{(2,0)}(z)$$

Massive operator matrix elements (OMEs) – probabilities to find heavy quark in other partons, similar to massless splitting functions

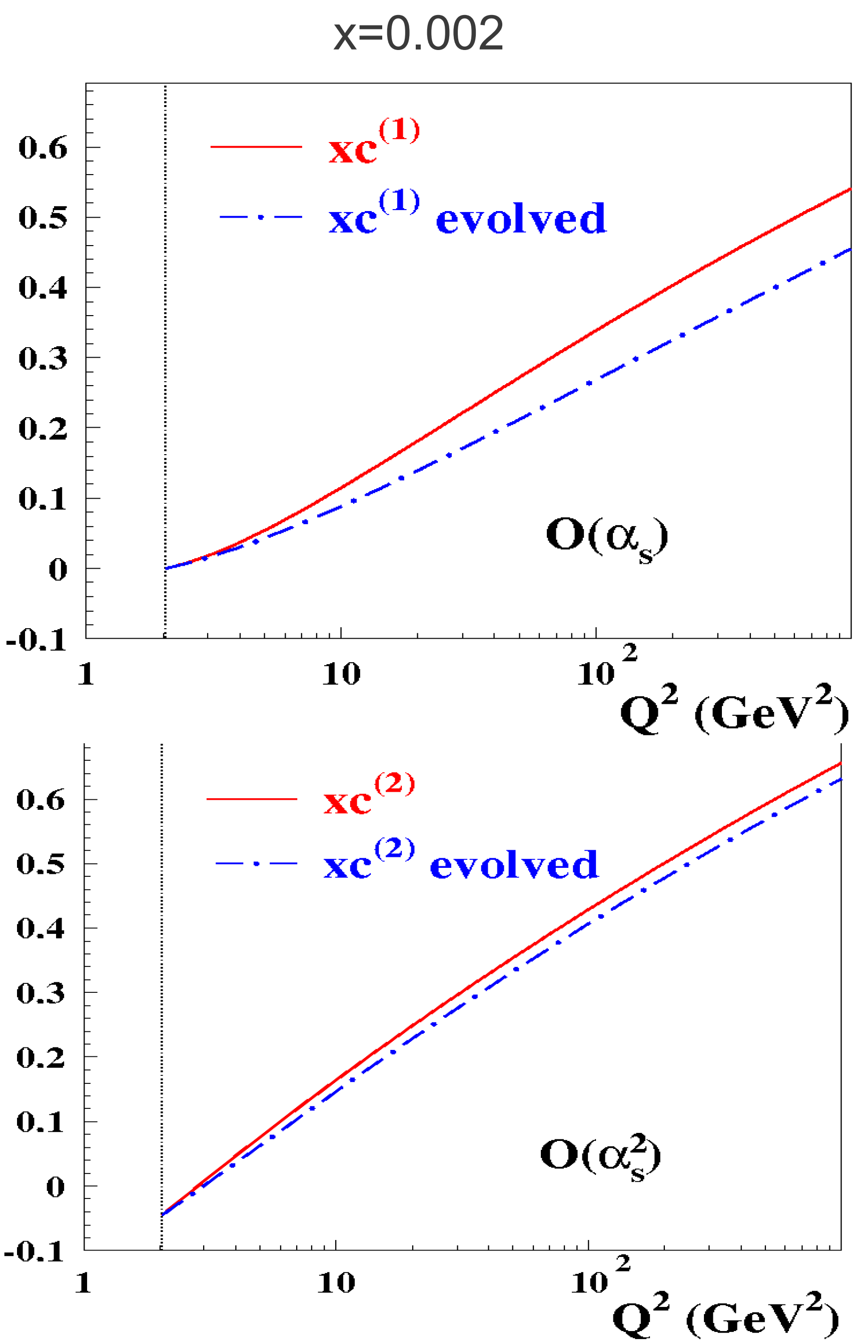
$$h^{(1)}(x,\mu^2) + \bar{h}^{(1)}(x,\mu^2) = a_s(N_f+1,\mu^2)\left[A_{hg}^{(1)}\left(\frac{m_h^2}{\mu^2}\right) \otimes G^{(3)}\left(N_f,\mu^2\right)\right](x)$$

$$h^{(2)}(x,\mu^2) + \bar{h}^{(2)}(x,\mu^2) = h^{(1)}(x,\mu^2) + \bar{h}^{(1)}(x,\mu^2) + a_s^2(N_f+1,\mu^2)\left\{\left[A_{hg}^{(2)}\left(\frac{m_h^2}{\mu^2}\right) \otimes G^{(3)}\left(N_f,\mu^2\right)\right](x) + \left[A_{hq}^{(2),\mathrm{PS}}\left(\frac{m_h^2}{\mu^2}\right) \otimes \Sigma^{(3)}\left(N_f,\mu^2\right)\right](x)\right\}$$

Fixed-oder matching condition for the heavy-quark PDFs

The heavy-quark PDFs obey the evolution equation with the MSbar splitting functions

The 4- and 5-flavor PDFs



sa, Blümlein, Klein, Moch PRD 81, 014032 (2010)

- *$h(x,m_h)=0$ in $O(\alpha_s)$* – accidental, due to constant term in OME *$a^{(1,0)}=0$*
- *$h(x,m_h)$* is negantive *in $O(\alpha_s^2)$* – it is not a particular problem since anyway 4-flavor PDFs make sence only at $\mu>>m_h$

Two options are possible to produce heavy-quark PDFs

– use the fixed-order matching conditions in the whole range of μ

– match the PDFs at $\mu=m_h$ and use it as a boundary In the massless AP evolution

The difference between these two is due to big-log resummation. *It reduces with the order of α_s*

Transition from 4 to 5-flavors is similar, **but**

– The scales of m_c and m_b and are not too different

→ the 4-flavor PDFs may be irrelevant at $\mu=m_b$

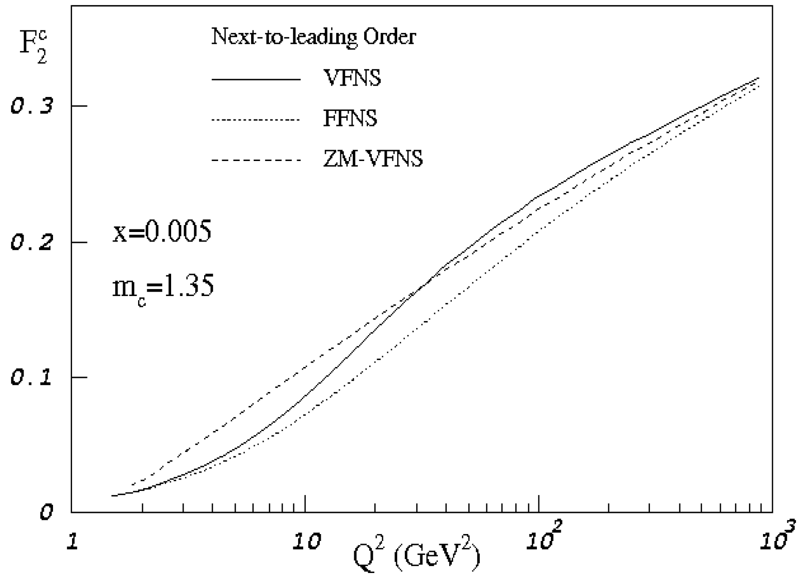
– In high order the with c- and b-quarks are mixed

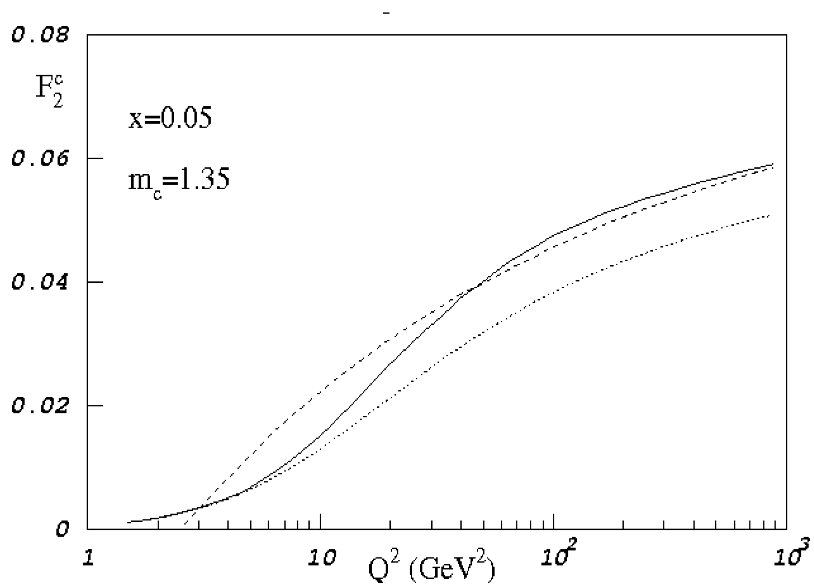
→ mass factorization is not evident

ZMVFN and GMVFN schemes

ZMVFN (zero-mass variable-flavor-number) scheme

- The PDFs, including the the heavy-quark one are convoluted with the massless coefficient functions
- The corrections up to N^3LO are available
- The big logs $\sim ln^n(Q/m_c)$ can be in a natural way resummed in the massless QCD evolution
- Irrelevant outside the asymptotic region $Q>>m_h$





Thorne, Roberts PLB 421, 303 (1998)

GMVFN (general-mass variable-flavor-number) scheme

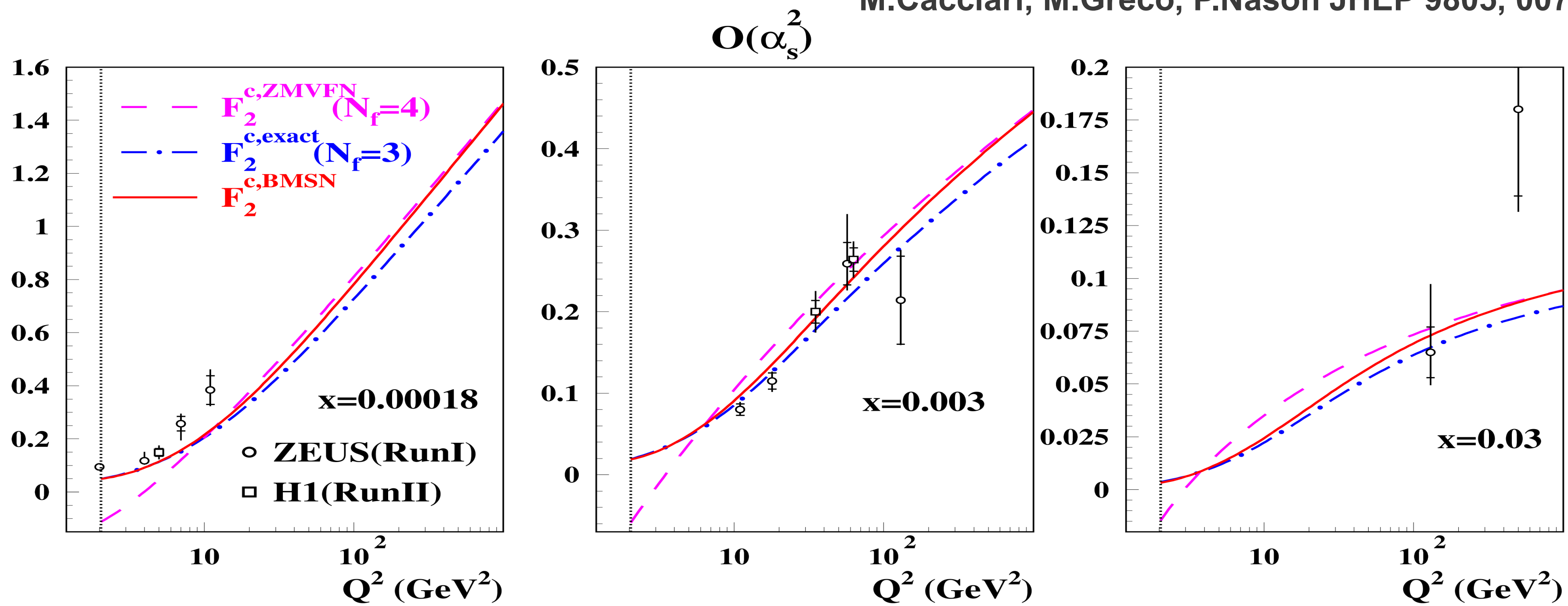
- Provides matching with the FFNS in the limit of $Q \rightarrow m_h$
- Modeling at small Q cannot be based on the solid footing; many prescriptions available that causes theoretical uncertainty

BMSN prescriptions

Buza, Matiounine, Smith, van Neerven EPJC 1, 301 (1998)

$$F_2^{h,\mathrm{BMSN}}(N_f+1,x,Q^2)=F_2^{h,\mathrm{exact}}(N_f,x,Q^2)+F_2^{h,\mathrm{ZMVFN}}(N_f+1,x,Q^2)-F_2^{h,\mathrm{asymp}}(N_f,x,Q^2)$$

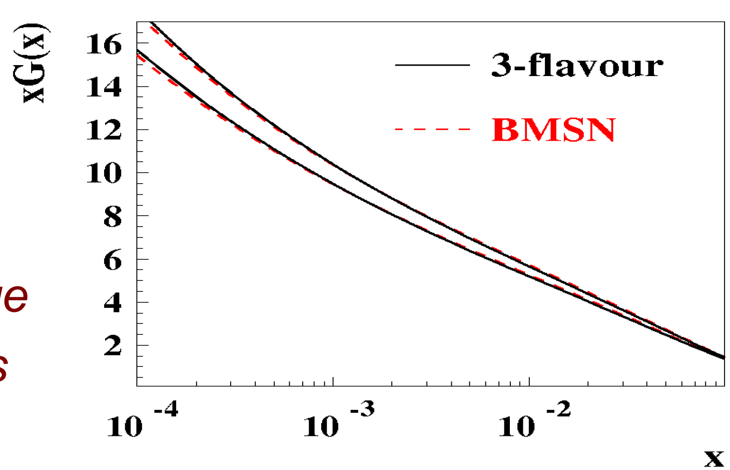
M.Cacciari, M.Greco, P.Nason JHEP 9805, 007 (1998)



sa, Blümlein, Klein, Moch PRD 81, 014032 (2010)

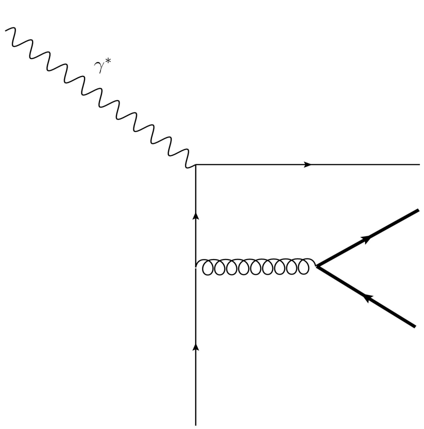
- Very smooth matching with the FFNS at $Q \to m_h$
- Renormgroup invariance is conserved; the PDFs in MSbar scheme

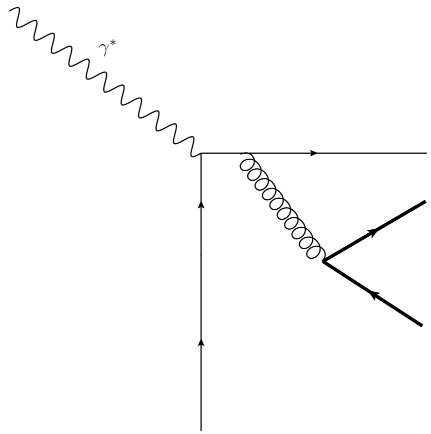
In the $O(\alpha_s^2)$ the FFNS and GMVFNS are comparable at large scales since the big logs appear in the high order corrections to the massive coefficient functions



Glück, Reya, Stratmann NPB 422, 37 (1994)

Definition of the charm structure functions





Forte, .Laenen, Nason, Rojo NPB 834, 116 (2010)

At $Q \gg m_h$ the nonsinglet contribution to the charm electro-production is $\sim ln^3(Q/m_h)$. For the massless case it is canceled by the vertex loop diagram, however now it does not appear due to the charm SF definition

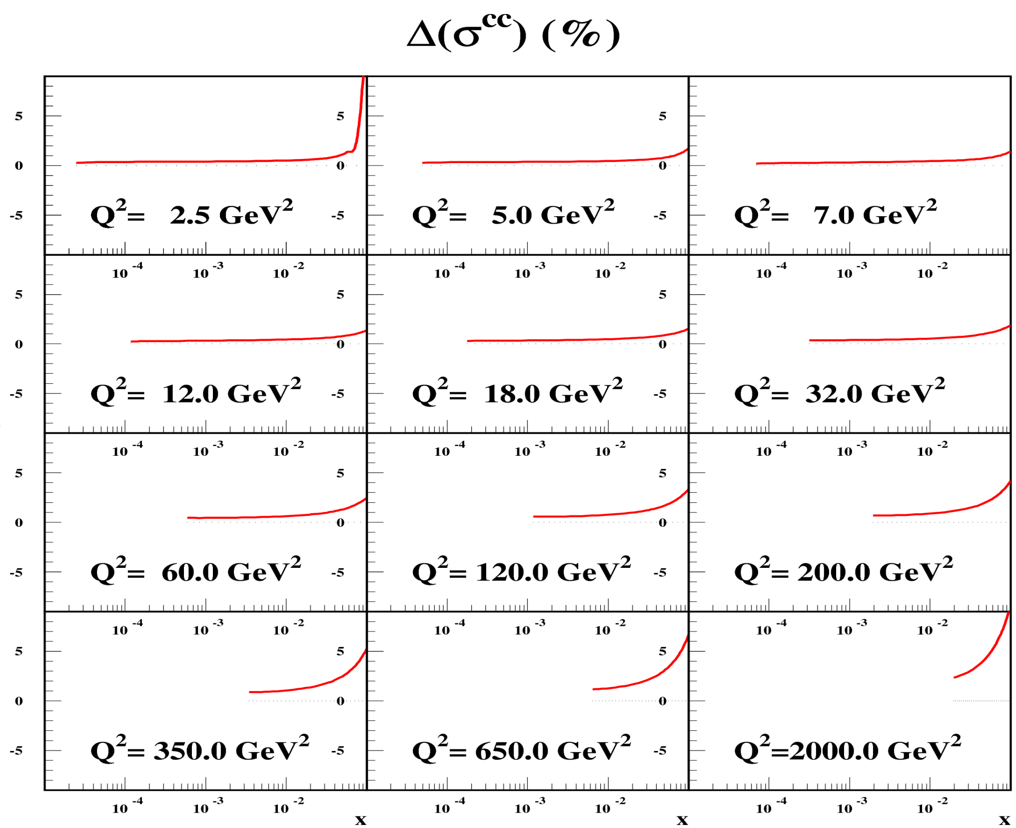
Buza, Matiounine, Smith, Mingeron van Neerven NPB 472, 611 (1996)

The charm SFs are infrared unsafe. The problem can be solved with an additional cut-off imposed on the c-quark pair invariant mass

Chuvakin, Smith, van Neerven PRD 61, 096004 (2000)

(cf. e.g. infrared safe kT algoritm for the hadronic jet definition), however this recipe Is difficult in practice

The numerical effect of the infrared unsafety is marginal for the realistic kinematics

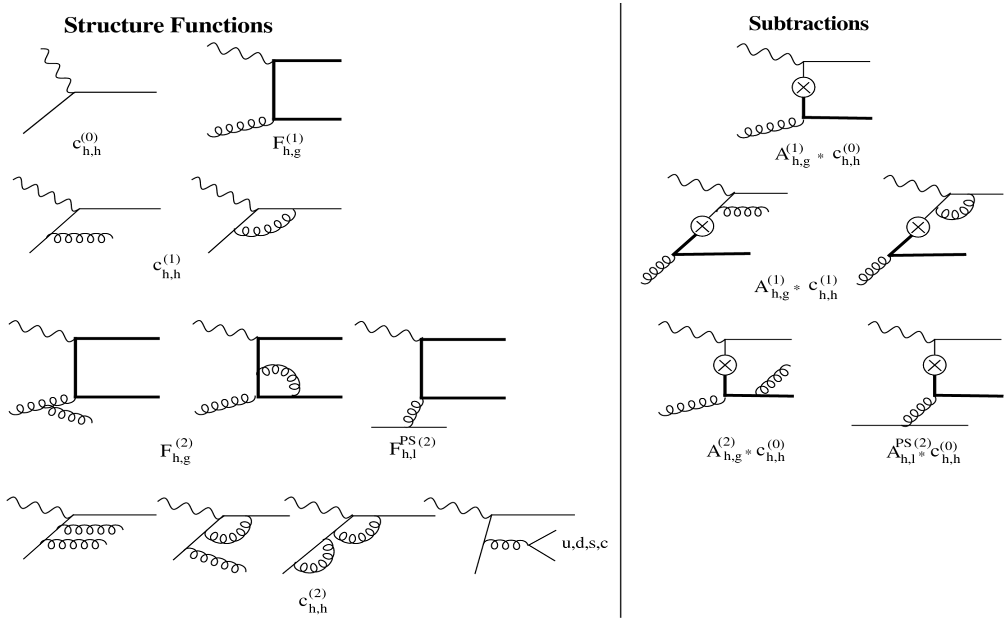


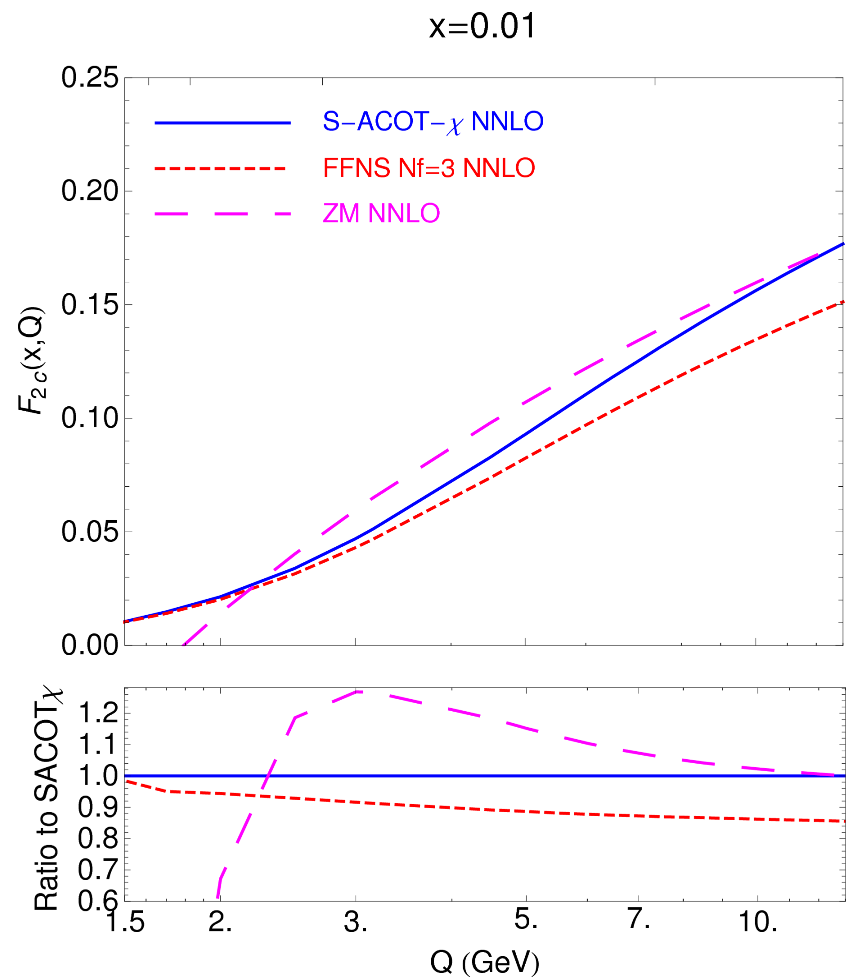
produced with OPENQCDRAD

ACOT prescription

Guzzi, Nadolsky, Lai, Yuan PRD 86, 053005 (2012)

The prescription is based on the subtractions, similarly to the BMSN one





Extrapolation to $Q = m_h$ is based on the assumption for the coefficient function of heavy-quark initiated processes

$$C_{h,h}^{(k)}\left(\frac{x}{\xi}, \frac{Q}{\mu}, \frac{m_h}{Q}\right) = c_{h,h}^{(k)}\left(\frac{\chi}{\xi}, \frac{Q}{\mu}, m_h = 0\right)$$

$$\chi = x\left(1 + \frac{(\sum_{f_s} m_h)^2}{Q^2}\right)$$

$$x = \frac{\zeta}{1 + \zeta^\lambda \cdot (4m_c^2)/Q^2},$$

- The “slow-rescaling” is consistent with the QCD factorization
- A variety of rescaling forms gives different prescription: SACOT, ACOT-χ, …..
- Matching with FFNS $Q = m_h$ is not very smooth

Thorne's prescription

Thorne hep-ph/1201.6180

Based on the ACOT *(different from the Thorne-Roberts prescription)*

Thorne, Roberts PLB 421, 303 (1998)

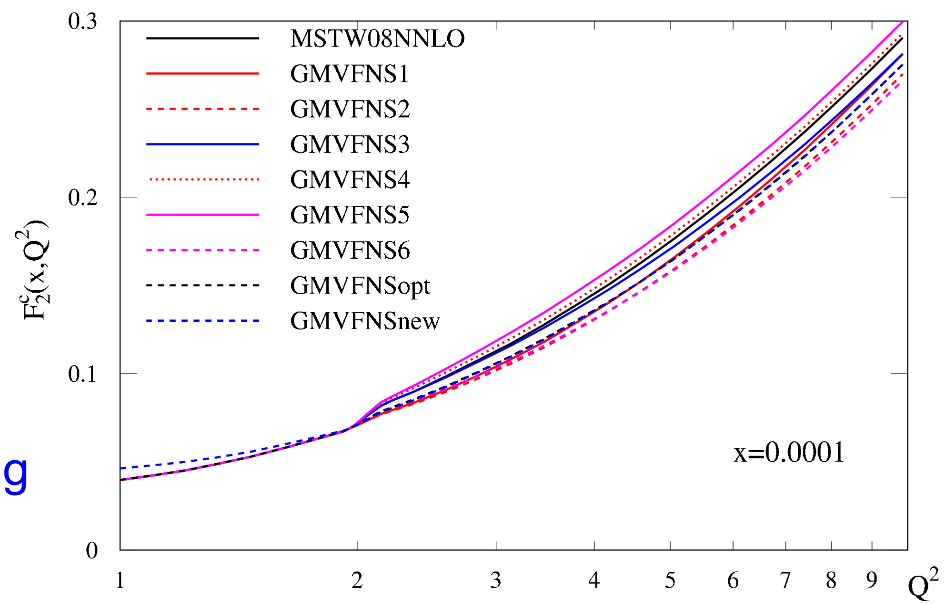
$$C^{\mathrm{GMVF},(0)}_{2,h\bar{h}}(Q^2/m_h^2, z) \to (1+b(m_h^2/Q^2)^c)\delta(z-x_{\max})$$

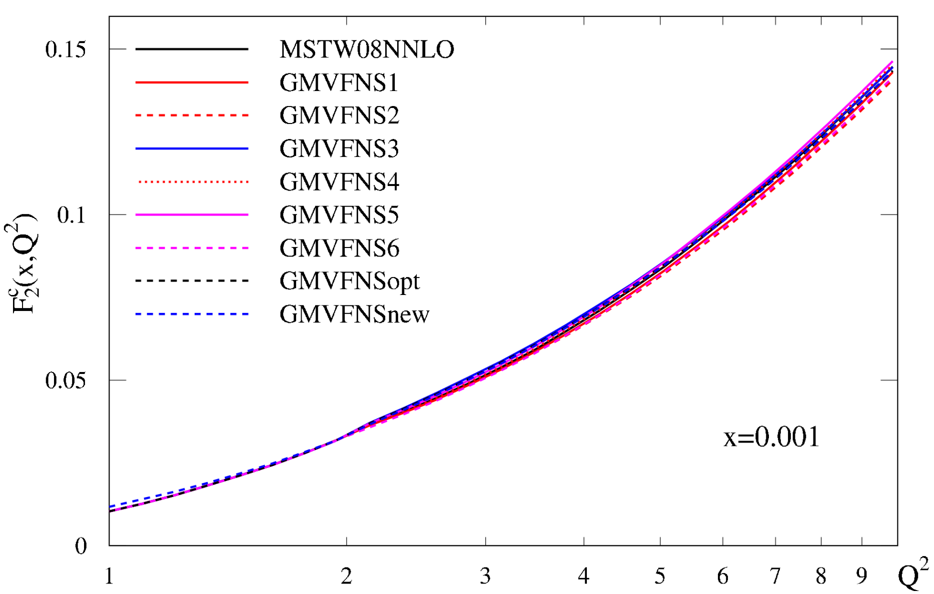
$$\xi = x/x_{\max} \to x\left(1+\left(x(1+4m_h^2/Q^2)\right)^d 4m_h^2/Q^2\right)$$

Additional parameters b and c improved matching with FFNS and the NNLO term stemming from the threshold resummation added

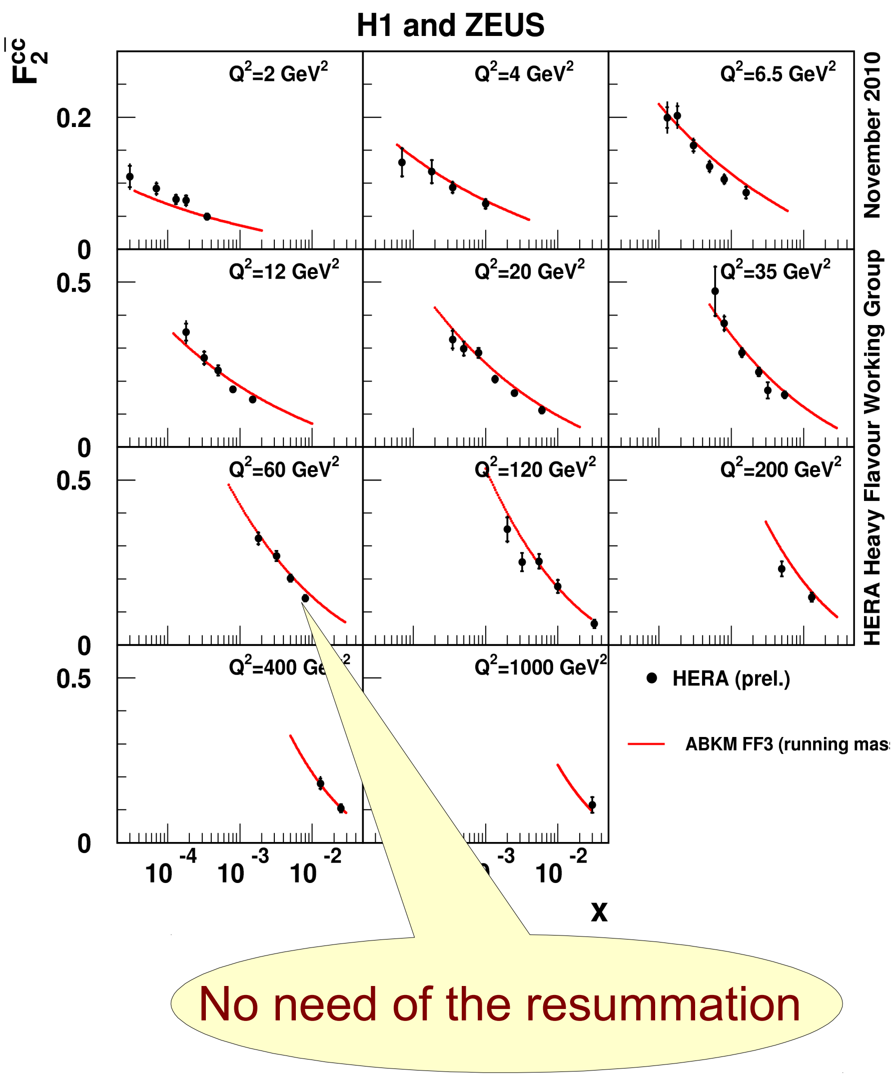
$$A(Q^2/m_h^2)(1-z/x_{\max})^{\tilde{a}}(\ln(1/z)-\tilde{b})/z,$$

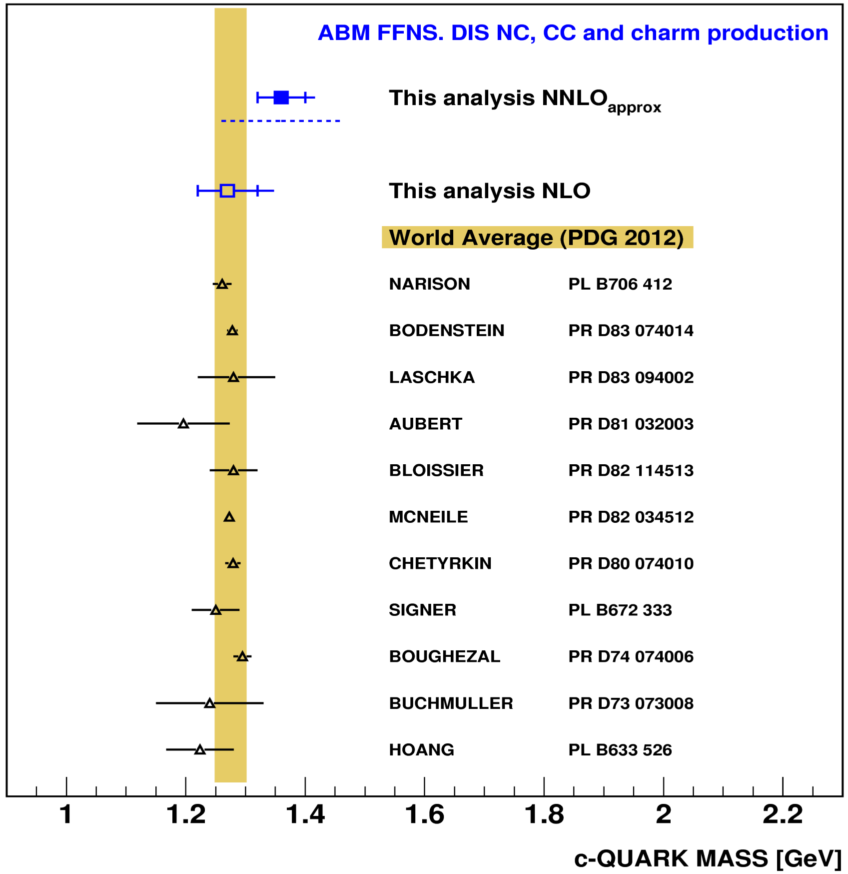
- With the variety of parameters smooth matching is achieved
- Does the MSbar scheme persist?
- With a smooth matching to FFNS provided at $Q = m_h$ the Thorne's prescription in NNLO does not differ very much from FFNS elsewhere





NNLO corrections in the FFNS





The NNLO FFNS predictions based on the running mass definition are in a good agreement with the recent HERA data

The value of m_c determined in the FFNS from the DIS data is in agreement with the world average

sa, Daum, Lipka, Moch hep-ph/1209.0436

More details tomorrow in the OPENQCDRAD tutorial