

Higgs in Loops

G. Quast, A. Raspereza

Course “Higgs Physics”

Lecture 6, 31/05/2012

Recap and Summary

Reminder: The Lagrangian of EW interactions

$$\begin{aligned}
 \mathcal{L}_{EW} = & \underbrace{\bar{L}\gamma^\mu i\partial_\mu L + \bar{R}\gamma^\mu i\partial_\mu R}_{E_{kin}} - \\
 & \underbrace{\frac{g}{2}\bar{L}\gamma^\mu \tau_\alpha \cdot W_\mu^\alpha L - \frac{g'}{2}\bar{L}\gamma^\mu Y B_\mu L - \frac{g'}{2}\bar{R}\gamma^\mu Y B_\mu R}_{\text{Interaction with } W^\pm, Z, \gamma} - \\
 & \underbrace{\frac{1}{4}W_{\mu\nu}^\alpha \cdot W_\alpha^{\mu\nu} - \frac{1}{4}B_{\mu\nu} \cdot B^{\mu\nu}}_{E_{kin} \text{ of } W^\pm, Z, \gamma \text{ and self-interaction}} + \\
 & \underbrace{\left| \left(i\partial_\mu - g\frac{1}{2}\tau_\alpha \cdot W_\mu^\alpha - g'\frac{Y}{2}B_\mu \right) \Phi \right|^2 - V(\Phi)}_{W^\pm, Z, \gamma, H \text{ masses and couplings}} - \\
 & \underbrace{(G_1 \bar{L}\Phi R + G_2 \bar{L}\Phi_c R + \text{hermitian conjugate})}_{\text{lepton and quark masses and coupling to Higgs}}
 \end{aligned}$$

L: left-handed
 fermion doublet
 R: right-handed
 fermion singlet

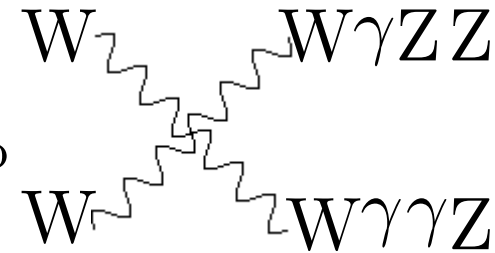
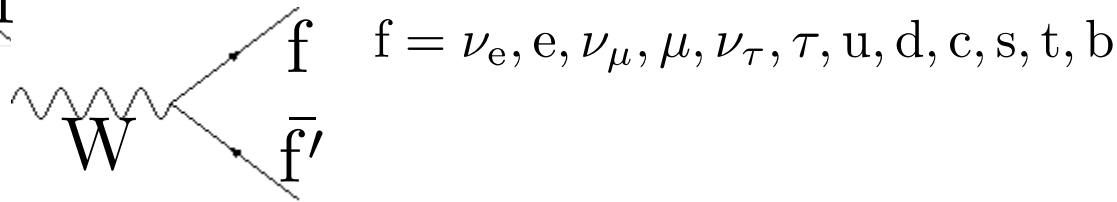
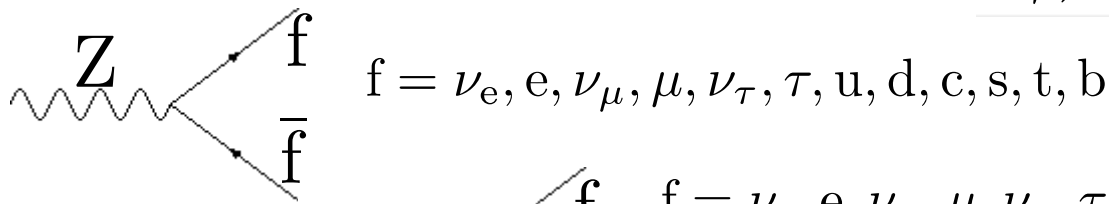
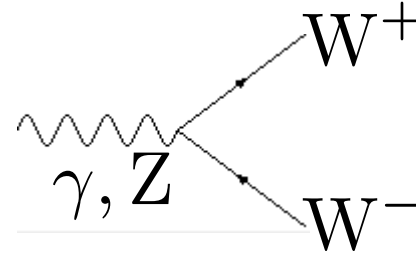
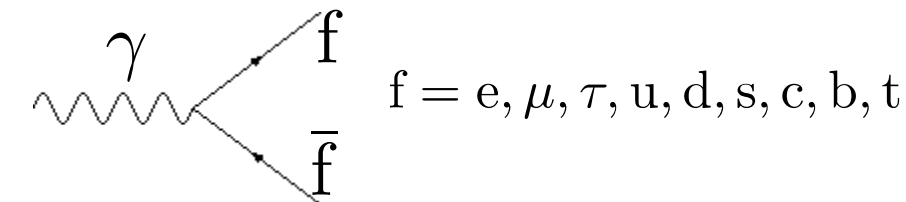
Structure of gauge boson couplings to fermions

$\gamma f f$	$ie\gamma^\mu$	vector coupling
$W f f$	$\frac{-ie}{\sqrt{2}\sin\Theta_W}\gamma^\mu\frac{1}{2}(1-\gamma^5)$	“V-A” coupling
$Z f f$	$\frac{-ie}{2\cos\Theta_W\sin\Theta_W}\gamma^\mu(g_V^f - g_A^f\gamma^5)$ $g_A^f = T_3^f, \quad g_V^f = T_3^f - 2q_f\sin^2\Theta_W$	vector and axial vector couplings
$H W W$	$\frac{ie}{\sin\Theta_W}M_W$	
$H Z Z$	$\frac{-ieM_Z}{\cos\Theta_W\sin\Theta_W}$	
$H f f$	$\frac{-ie}{2\sin\Theta_W}\frac{m_f}{M_W}$	

$$\hbar = c = 1; \quad e = \sqrt{4\pi\alpha} \text{ with } \alpha = \frac{1}{137, \dots}; \quad g\sin\Theta_W = e; \quad G_F = \frac{\sqrt{2}^2}{8M_W^2}$$

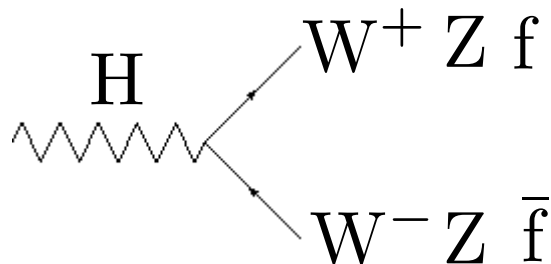
Quelle: Halzen, Martin

Reading Feynman Diagrams from Lagrangian



quartic couplings
(not seen yet)

Higgs couplings (not yet seen directly)



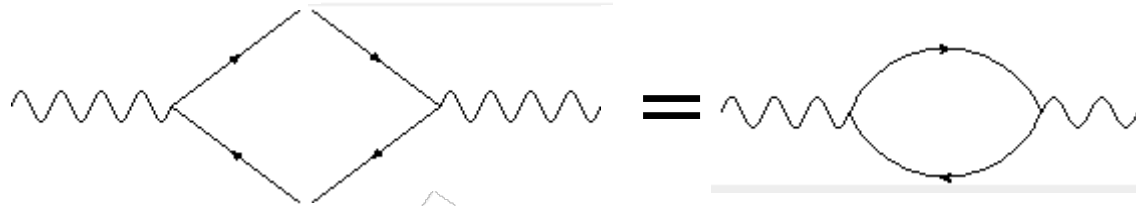
these are all “tree level couplings”

More complex diagrams constructed from these building blocks

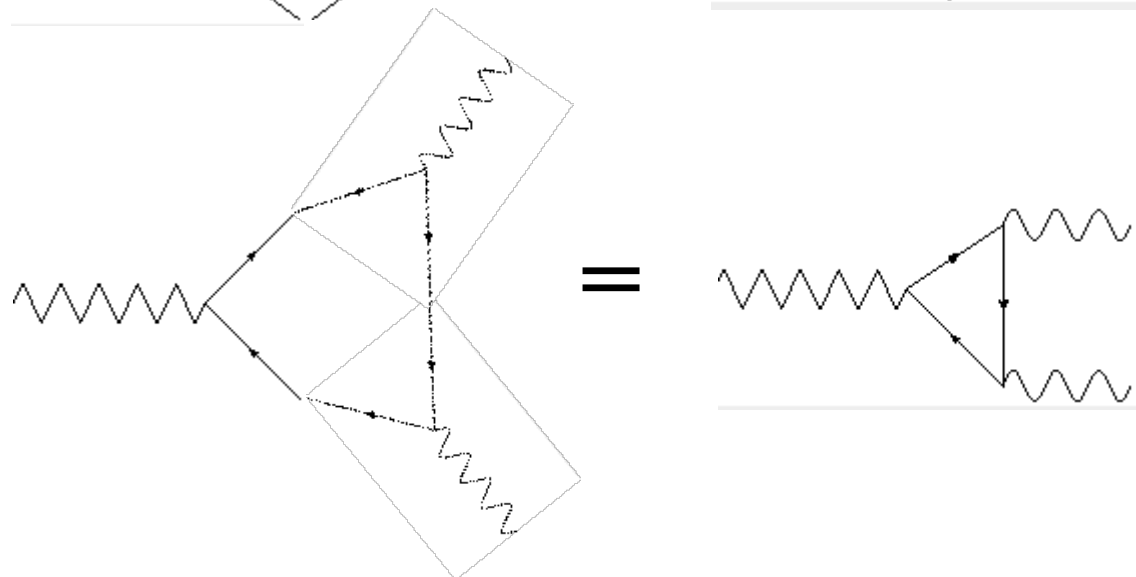
Feynman diagrams from building blocks - examples



$$f\bar{f} \rightarrow f'\bar{f}'$$



“fermion loop”
(modifies photon propagator)



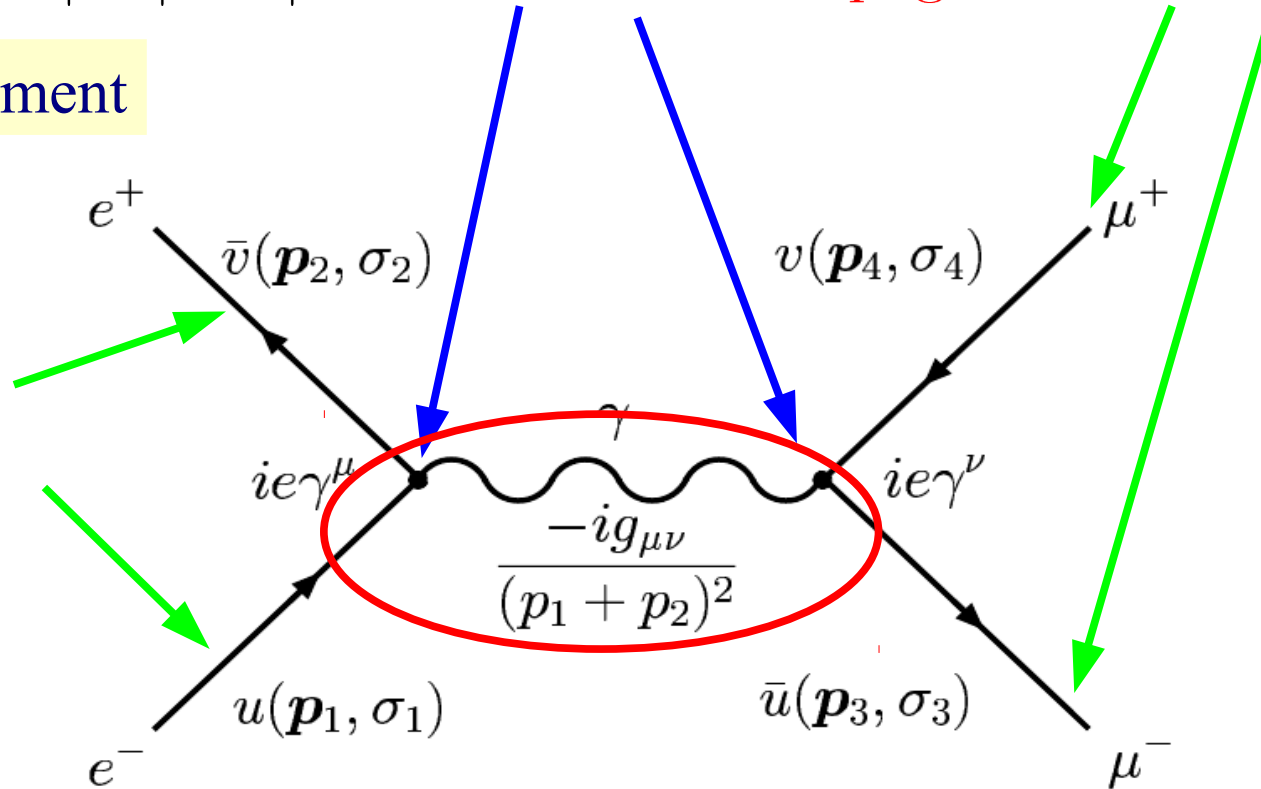
non-tree-level
coupling via loop,
here: $H \rightarrow \gamma \gamma$

Very many combinations possible, up to an infinite number of Loops
→ enormous number of diagrams contributing to a given process

Reminder: Calculation of differential cross sections (in lowest order)

$$|\mathcal{M}|^2 = |\text{Vertex factors} \cdot \text{Propagator} \cdot \text{External lines}|^2$$

Matrix element



Cross Section

$$\sigma = \frac{|\mathcal{M}|^2 \cdot [\text{Phase space}]}{[\text{Colliding particle flux}]}$$

Standard Model processes in higher order

Example: $e^+ e^- \rightarrow f \bar{f}$

$$|M|^2 = \left| \begin{array}{c} e^- \\ \searrow \\ \text{---} \gamma \text{---} \\ \nearrow \\ e^+ \end{array} \begin{array}{c} f \\ \nearrow \\ \text{---} e^- \\ \searrow \\ f \end{array} + \begin{array}{c} e^- \\ \searrow \\ \text{---} Z \text{---} \\ \nearrow \\ e^+ \end{array} \begin{array}{c} f \\ \nearrow \\ \text{---} e^- \\ \searrow \\ f \end{array} \right|^2$$

$$= \underbrace{\left| \begin{array}{c} e^- \\ \searrow \\ \text{---} \gamma \text{---} \\ \nearrow \\ e^+ \end{array} \begin{array}{c} f \\ \nearrow \\ \text{---} f \\ \searrow \\ f \end{array} \right|^2}_{\text{Photon}} + \underbrace{\left| \begin{array}{c} e^- \\ \searrow \\ \text{---} Z \text{---} \\ \nearrow \\ e^+ \end{array} \begin{array}{c} f \\ \nearrow \\ \text{---} f \\ \searrow \\ f \end{array} \right|^2}_{Z} + \underbrace{\begin{array}{c} e^- \\ \searrow \\ \text{---} \gamma \text{---} \\ \nearrow \\ e^+ \end{array} \begin{array}{c} f \\ \nearrow \\ \text{---} f \\ \searrow \\ f \end{array} * \begin{array}{c} e^- \\ \searrow \\ \text{---} Z \text{---} \\ \nearrow \\ e^+ \end{array} \begin{array}{c} f^* \\ \nearrow \\ \text{---} f \\ \searrow \\ f \end{array}}_{\text{Photon - Z Interference term}} + \text{CC}$$

Example: $e^+e^- \rightarrow f\bar{f}$ differential cross section

$$\frac{2s}{\pi} \frac{1}{N_c^f} \frac{d\sigma_{\text{ew}}}{d\cos\theta} (e^+e^- \rightarrow f\bar{f}) =$$

“color factor”;
3 for quarks,
1 for leptons

$$\underbrace{|\alpha|^2 (1 + \cos^2 \theta)}_{\gamma}$$

$$\underbrace{-8\alpha\chi(s) [g_{Ve}g_{Vf}(1 + \cos^2 \theta) + 2g_{Ae}g_{Af}\cos\theta]}_{\gamma - Z \text{ interference}}$$

$\gamma - Z$ interference

$$\underbrace{+16|\chi(s)|^2 [(g_{Ve}^2 + g_{Ae}^2)(g_{Vf}^2 + g_{Af}^2)(1 + \cos^2 \theta) + 8g_{Ve}g_{Ae}g_{Vf}g_{Af}\cos\theta]}_{Z}$$

Z

with $\chi(s) = \frac{m_Z^2}{8\pi\sqrt{2}} \frac{s}{s - m_Z^2 + i\Gamma_Z m_Z}$

Breit-Wigner Propagator

Use dimension-less (fine structure) coupling constant instead of e^2 :

$$\alpha = \frac{e^2}{4\pi\epsilon_0 \hbar c} \simeq \frac{1}{137}$$

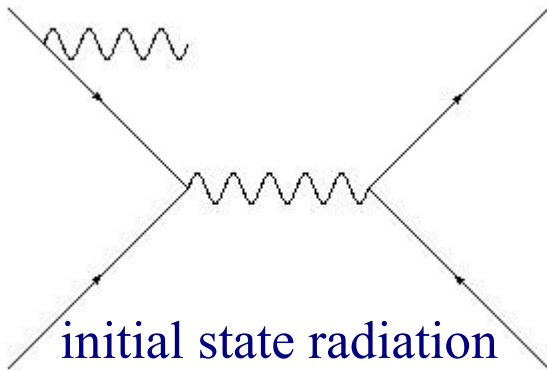
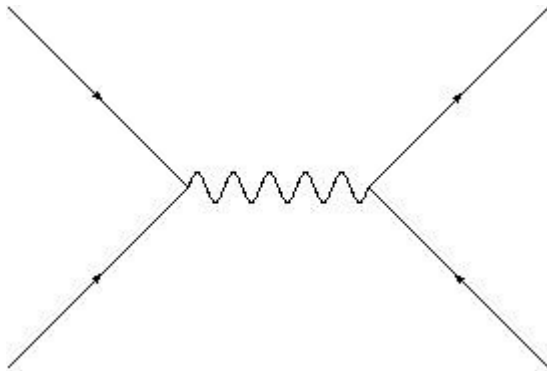
$2 \rightarrow 2$ process in higher orders

Higher order corrections

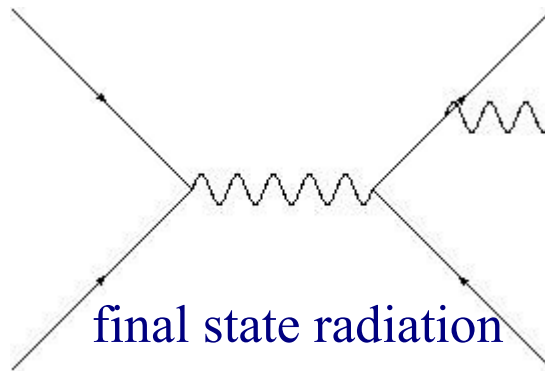
modify simple process $f\bar{f} \rightarrow f\bar{f}$

shown here: examples of

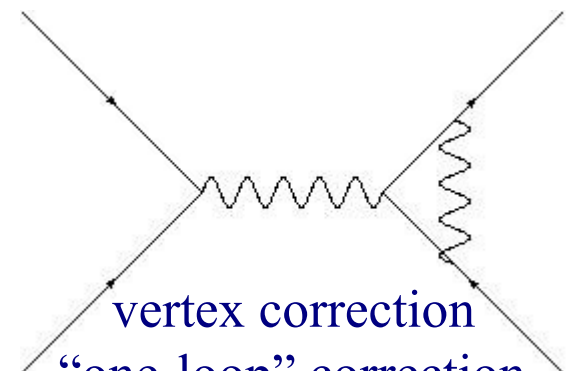
“next-to-leading order”- corrections



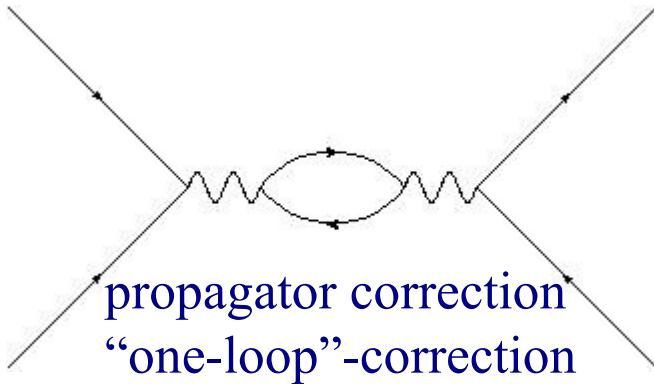
initial state radiation



final state radiation



vertex correction
“one-loop” correction

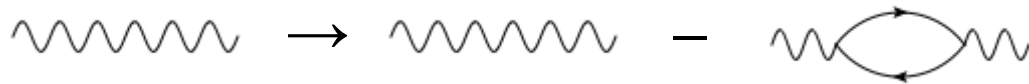


propagator correction
“one-loop”-correction

- all basic vertices can be inserted, involving all particles of the theory (fermions & bosons)
- integration over momenta of radiated particles or particles in loop

Higher-order amplitudes calculable with Feynman rules, with the following extensions:

- diagrams with n fermion loop receive factor $(-1)^n$
- integrate over 4-momenta in loops



Integrals lead to **divergences** at low and high momenta,
 $p \rightarrow 0$ or $p \rightarrow \infty$, respectively: infrared and ultraviolet divergences

handled by “regularisation” and “renormalisation” techniques

- “regularisation” of integrals to track infinities
cutoff-parameters, or finite photon mass, or “dimensional regularisation”
- reparametrisation of physical observables (mass, charge=couplings):
“**renormalisation**”
- exploit cancellations between contributions
(e.g. diagrams involving photon, Z and neutrino-exchange in WW cross section)

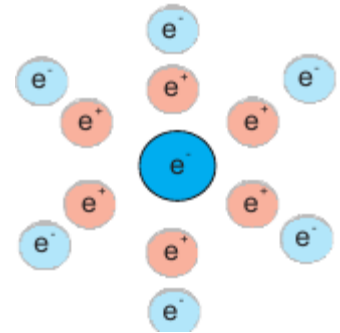
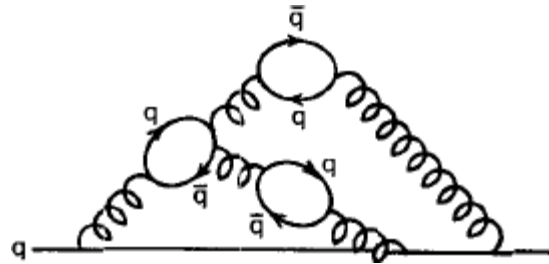
Remarks on higher order calculations - Renormalisation

Renormalisation:

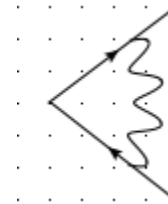
Example:

charge and magnetic moment of electron in QED

in pictures: electron is surrounded
by a cloud of virtual electrons,
existing at a scale $\Delta E \Delta t \geq \frac{\hbar}{2}$



virtual particles in loop
shield (bare) electric charge



virtual photon exchange
modifies magnetic moment

lead to experimentally well-established effects

Lamb shift

$g-2$ of electron $\neq 0$

Remarks on higher order calculations - Renormalisation (2)

Method: replace **bare electric charge** in lowest-order diagram by measured values in a process at a given energy scale μ^2 – re-parameterize perturbation series in terms of physical charge

Theory now depends on

- scale of the process, Q^2 ,
- and scale μ^2 , at which coupling was determined

$$\alpha(Q^2) = \frac{\alpha(\mu^2)}{1 - \frac{\alpha(\mu^2)}{3\pi} \log \frac{Q^2}{\mu^2}} \quad \text{“running coupling”}$$
$$\alpha(M_Z^2) \simeq \frac{1}{129}$$

Dependence on μ^2 diminishes if higher orders taken into account, and vanishes at infinite order (governed by “renormalisation group equation”).

“Running couplings” absorb effect of propagator loops

See Halzen, Martin, Chap. 7, for a brief introduction, or Theoretical Particle Physics II

Remarks on higher order calculations – running couplings

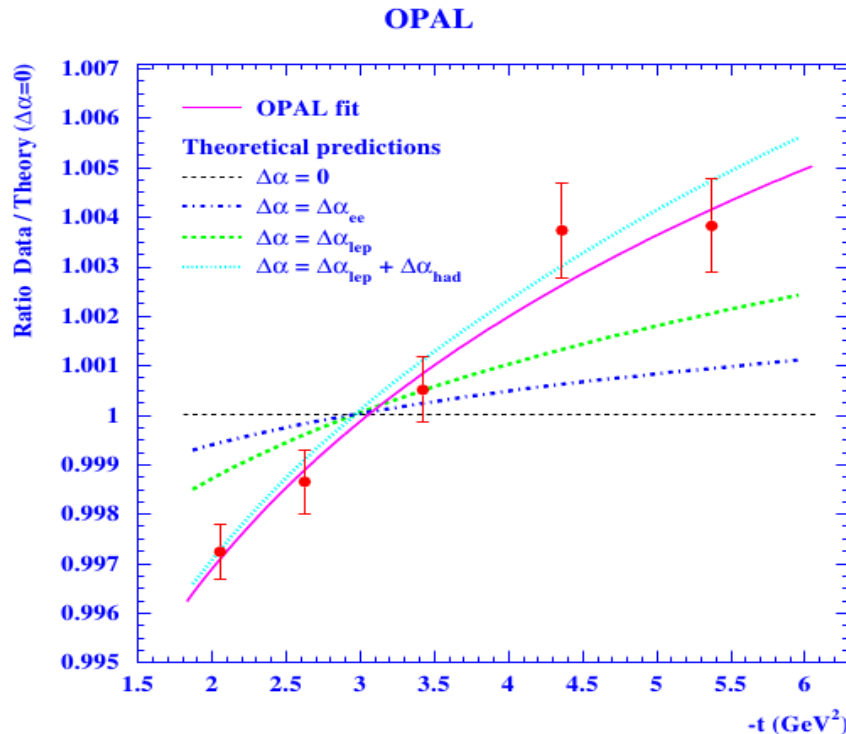
Examples:

Running of

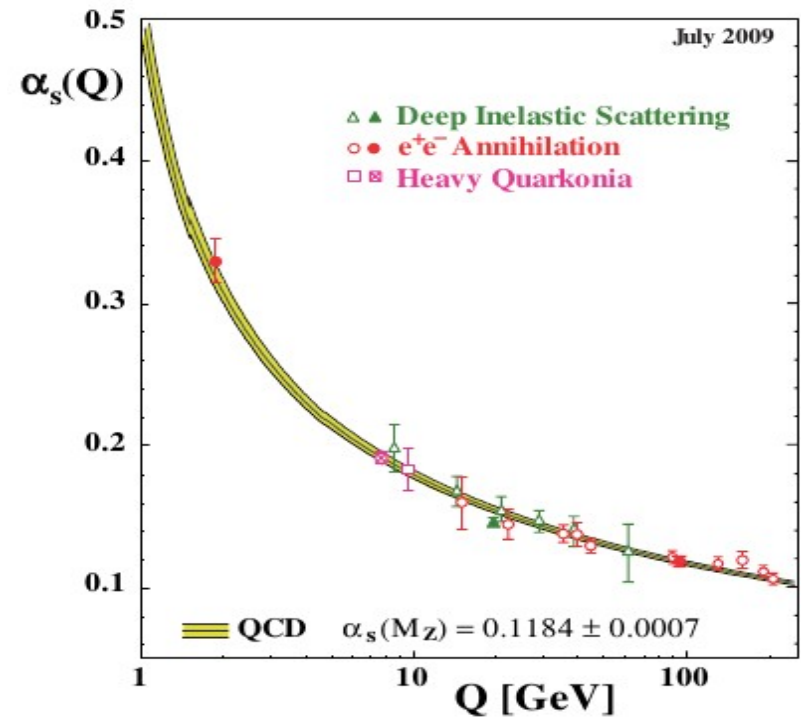
the electromagnetic

and

the strong coupling constants



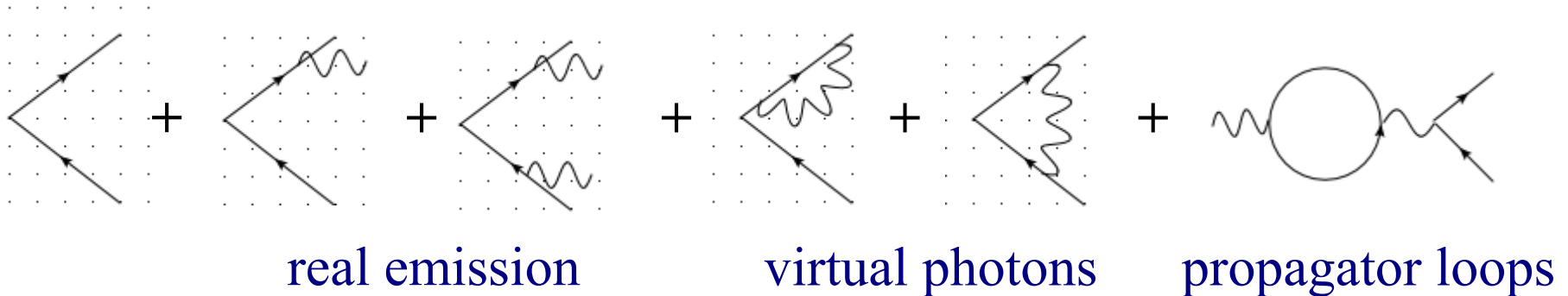
Measurement of the electromagnetic coupling constant as a function t-channel momentum transfer in $e^+e^- \rightarrow e^+e^-$ scattering



Strong coupling constant as measured at different energies and in different processes.

Remarks on higher orders - exploiting cancellations

Infrared-divergences in photon emission cancel out
if all diagrams up to given order are included

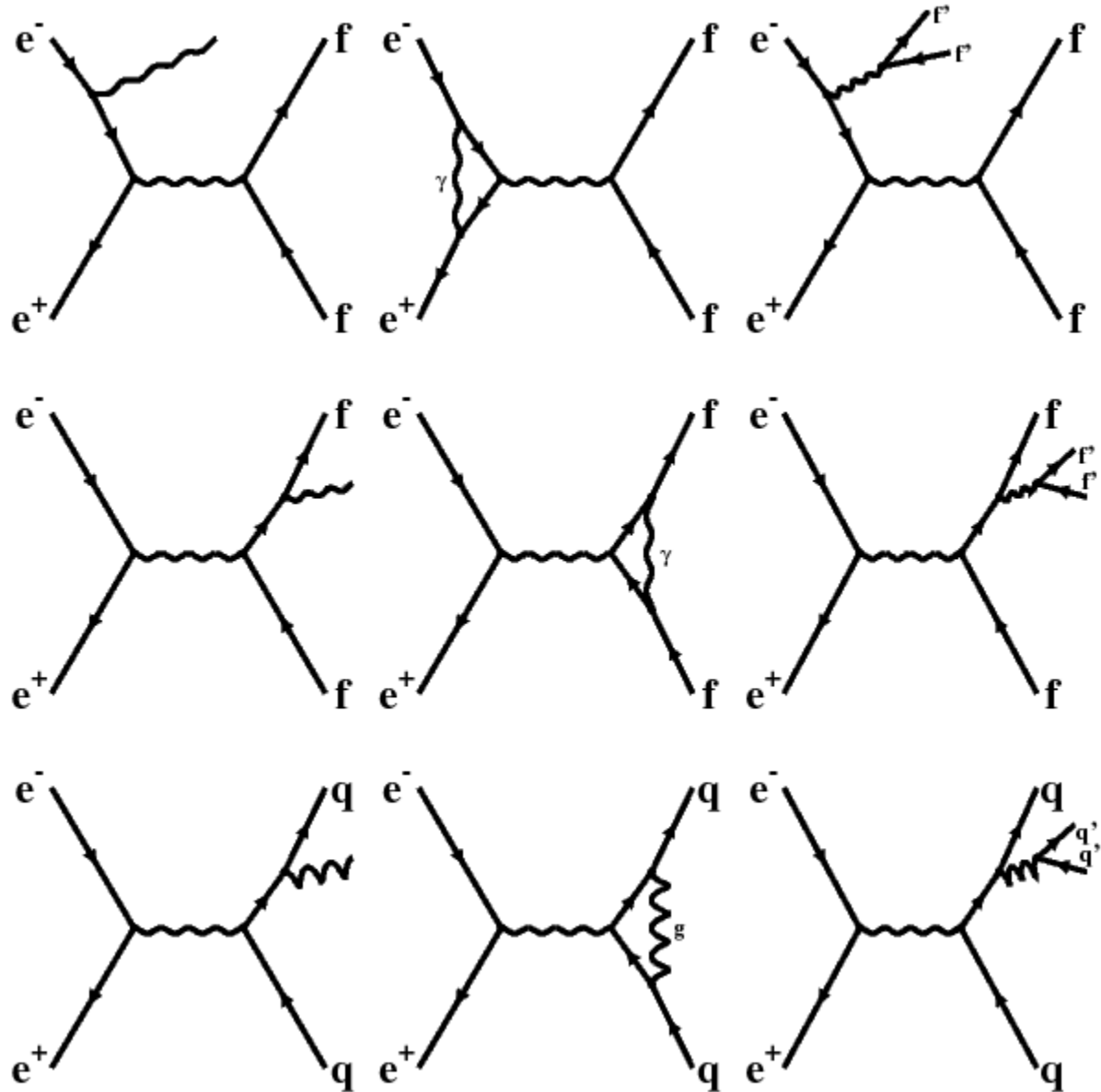


consequence of “Ward identities” in renormalizable theories

Examples of initial and final state radiation

QED corrections known to leading third order.

Corrections from strong interactions in quark final states (QCD) known to fourth order!



$$e^+e^- \rightarrow f\bar{f}$$

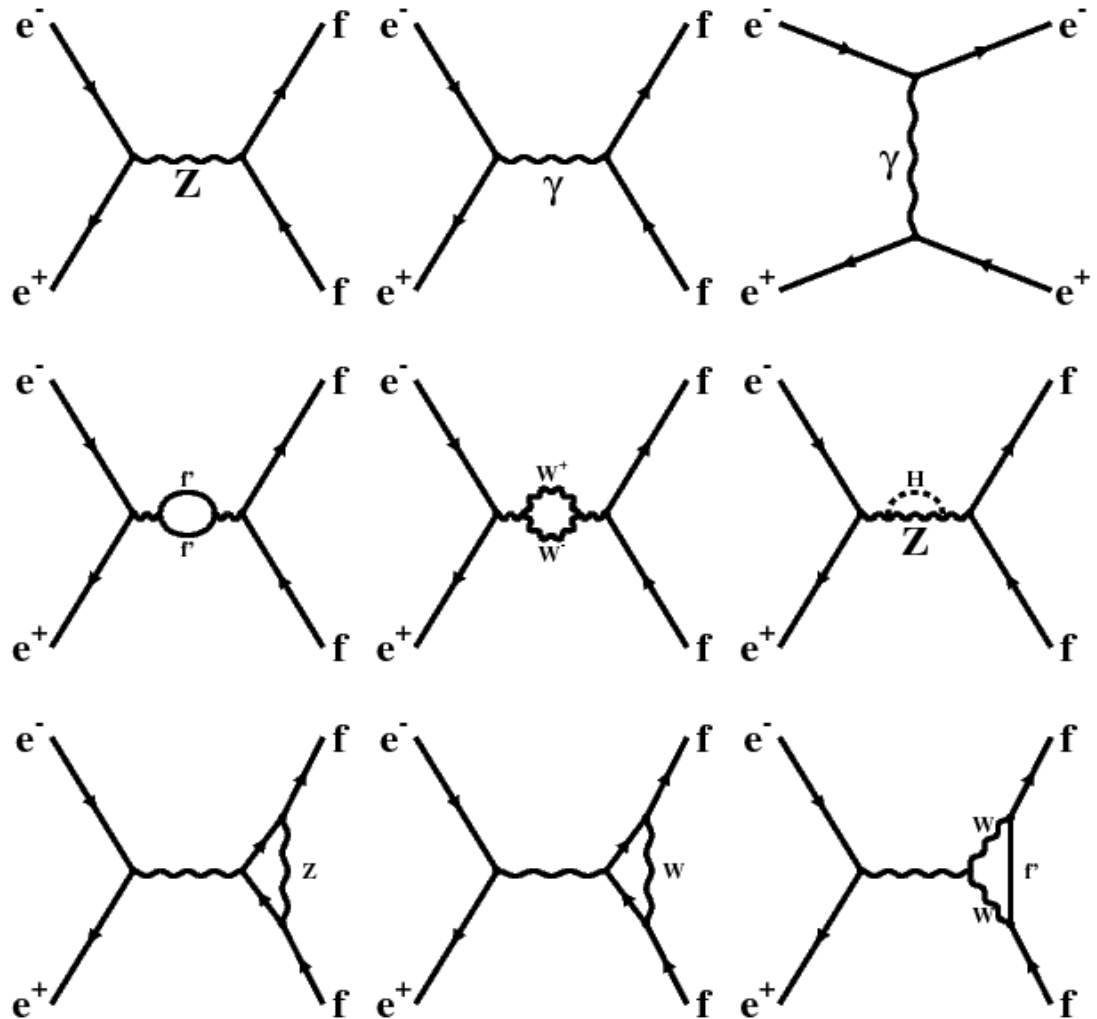
loop corrections

Examples of diagrams contributing to $e^+e^- \rightarrow f\bar{f}$

Via loops and radiation,
the simple $2 \rightarrow 2$ process
becomes sensitive to
all particles in the theory!

Known to leading
two-loop contributions!

Must calculate a large
number of diagrams
contributing to same
(experimental) final
states.



From Standard Model in lowest order ...

Model parameters:

- e electron charge, or fine structure constant $\alpha = \frac{e^2}{4\pi\epsilon_0 \hbar c} \simeq \frac{1}{137}$
- M_W } or, alternatively G_F Fermi constant
- M_Z } $\sin^2 \Theta_W$ weak mixing angle
- 12 fermion masses
 \leftrightarrow 12 Higgs-Fermion Yukawa couplings
- quark-mixing (CKM) matrix, 4 angles
 \leftrightarrow Higgs Yukawa Couplings
- Lepton mixing matrix $m_\nu \neq 0$!(?)
- Strong coupling constant α_s

in total: 25 parameters

Relations in lowest order:

$$\frac{G_F}{\sqrt{2}} = \frac{\alpha}{2 \sin^2 \Theta_W M_W^2}$$

$$\sin^2 \Theta_W = 1 - \frac{M_W^2}{M_Z^2}$$

... to Standard Model in higher order

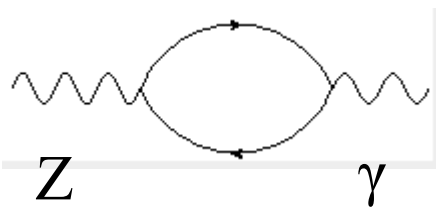
Introduce running couplings: $\alpha(M_Z)$, $G_F(M_Z)$

keep $\sin^2 \Theta_W := 1 - \frac{M_W^2}{M_Z^2}$ as definition “on-shell weak angle”

with “radiative corrections” $\Delta r = \Delta\alpha + r_W$ 2nd relation becomes:

$$M_Z^2 \sin^2 \Theta_W \cos^2 \Theta_W = \frac{\pi \alpha^{(0)}}{\sqrt{2} G_F} \cdot \frac{1}{1 - \Delta r} \quad \alpha(M_Z) = \frac{\alpha^{(0)}}{1 - \Delta\alpha}$$

There are other variants of an “effective weak angle”:



+ vertex corrections:

$$\sin^2 \Theta_W^{\text{eff},f} = \frac{1}{4} \left(1 - \frac{g_v^{\text{eff},f}}{g_a^{\text{eff},f}} \right)$$

from couplings
measured in experiment

Observables in higher order

Results in modifications of couplings:

$$g_a^f = T_3 \rightarrow g_a^{\text{eff},f} = \sqrt{\rho^{\text{eff},f}} T_3 \quad \rho^{\text{eff}} = 1 + \Delta\rho$$

$$g_v^f = T_3 - 2q_f \sin^2 \Theta_W \rightarrow g_v^{\text{eff},f} = \sqrt{\rho^{\text{eff},f}} (T_3 - 2q_f \sin^2 \Theta_W^{\text{eff},f})$$

$$\Delta r_w, \Delta\rho \text{ and } \sin^2 \Theta_W^{\text{eff}}$$

absorb higher orders to very good approximation

Radiative corrections depend

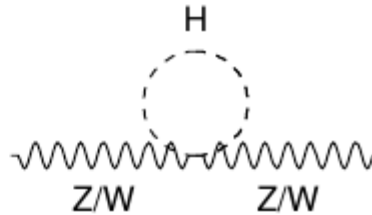
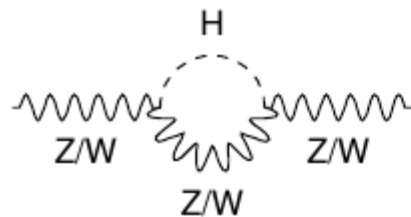
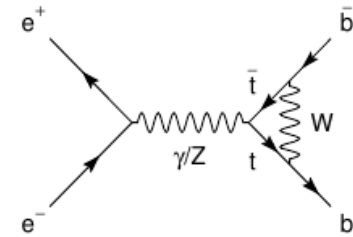
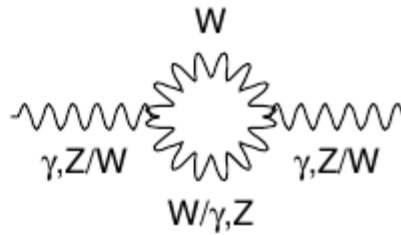
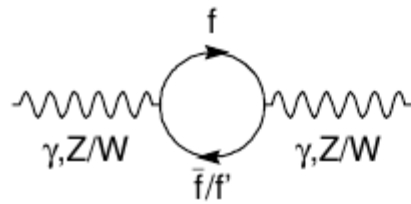
- **quadratically** on **top mass**
- **only logarithmically** on M_H

$$\text{e. g. } \Delta r_W(M_t, M_H) = \frac{3\alpha \cos^2 \Theta_W}{16\pi \sin^4 \Theta_W} \frac{M_t^2}{M_W^2} + \frac{11\alpha}{48\pi \sin^2 \Theta_W} \log \frac{M_H^2}{M_W^2} + \dots$$

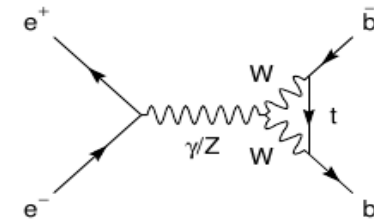
Software implementations of higher-order calculations

exist to calculate Observables $\mathcal{O}(\alpha, G_F, M_Z, M_{\text{top}}, M_H, \dots)$

Differential cross section near Z resonance in higher orders



Corrections



to boson propagators and vertex-corrections,
as shown above, and many others, absorbed into

- modified Breit-Wigner with s-dependent width
- “running” electromagnetic and strong coupling constants
- effective Z-couplings: $\mathcal{G}_{Vf} = \sqrt{\mathcal{R}_f} (T_3^f - 2Q_f K_f \sin^2 \theta_W)$

$$\mathcal{G}_{Af} = \sqrt{\mathcal{R}_f} T_3^f.$$

with complex form factors R_f, K_f

dominant real parts can be extracted from
measurements as “effective Z couplings”

Effective Parametrization of differential cross section near Z resonance

$$\begin{aligned}
 \frac{2s}{\pi} \frac{1}{N_c^f} \frac{d\sigma_{\text{ew}}}{d\cos\theta} (e^+e^- \rightarrow f\bar{f}) = & \quad \text{Form of tree-level formula retained !} \\
 & \underbrace{|\alpha(s)|^2 (1 + \cos^2 \theta)}_{\gamma} \\
 & \underbrace{-8\Re \left\{ \alpha^*(s) \chi(s) \left[\mathcal{G}_{Ve} \mathcal{G}_{Vf} (1 + \cos^2 \theta) + 2\mathcal{G}_{Ae} \mathcal{G}_{Af} \cos \theta \right] \right\}}_{\gamma - Z \text{ interference}} \\
 & \underbrace{+16|\chi(s)|^2 \left[(|\mathcal{G}_{Ve}|^2 + |\mathcal{G}_{Ae}|^2)(|\mathcal{G}_{Vf}|^2 + |\mathcal{G}_{Af}|^2)(1 + \cos^2 \theta) + 8\Re \left\{ \mathcal{G}_{Ve} \mathcal{G}_{Ae}^* \right\} \Re \left\{ \mathcal{G}_{Vf} \mathcal{G}_{Af}^* \right\} \cos \theta \right]}_{Z}
 \end{aligned}$$

with $\chi(s) = \frac{m_Z^2}{8\pi\sqrt{2}} \frac{s}{s - m_Z^2 + is\Gamma_Z/m_Z}$ Breit-Wigner with s-dependent width

Literature: “Z Pole Report” by LEP EW Working group and the LEP collaborations

furthermore:

partial decay widths

$$\Gamma_f^{(ew)} = N_c^f \frac{G_F m_Z^3}{6\sqrt{2}\pi} \left(g_V^{\text{eff}^2} + g_A^{\text{eff}^2} \right)$$

receive radiative corrections for real photon and gluon emission:

$$\Gamma_f = \Gamma_f^{(ew)} \cdot \left(1 + \frac{3}{4\pi} q_f^2 \alpha(m_Z) + \dots \right) \cdot \left(1 + \frac{\alpha_s(m_Z)}{\pi} + \dots \right) \quad \begin{array}{l} \text{last factor} \\ \text{for quarks only} \end{array}$$

Important to remember:

Via radiative and loop corrections

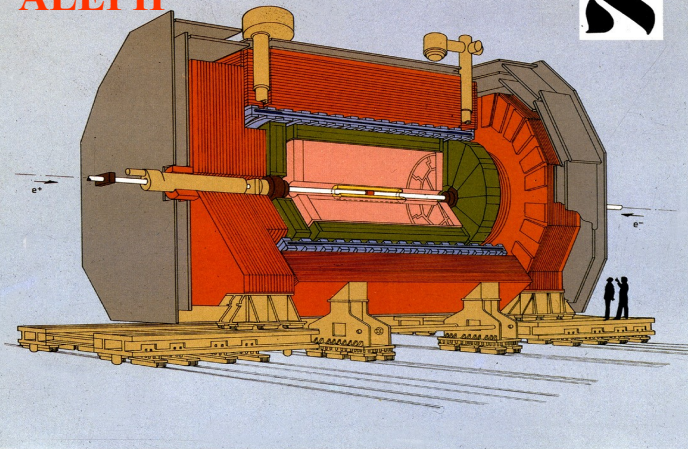
effective couplings (i.e. corrections to tree-level couplings)

depend on all parameters of the theory, in particular:

- **top quark mass** (\rightarrow prediction of top mass before top quark discovery)
- **Higgs boson mass**

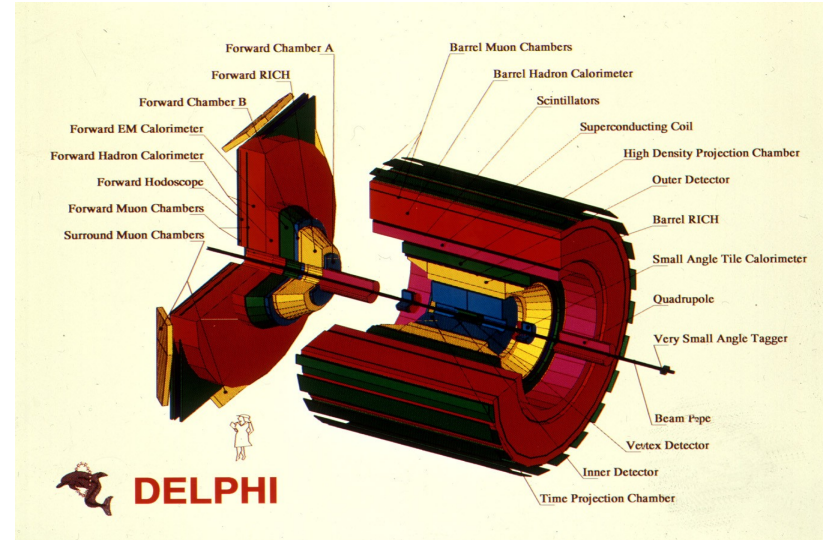
Measurements in $e^+ e^-$ at LEP – the four experiments

ALEPH



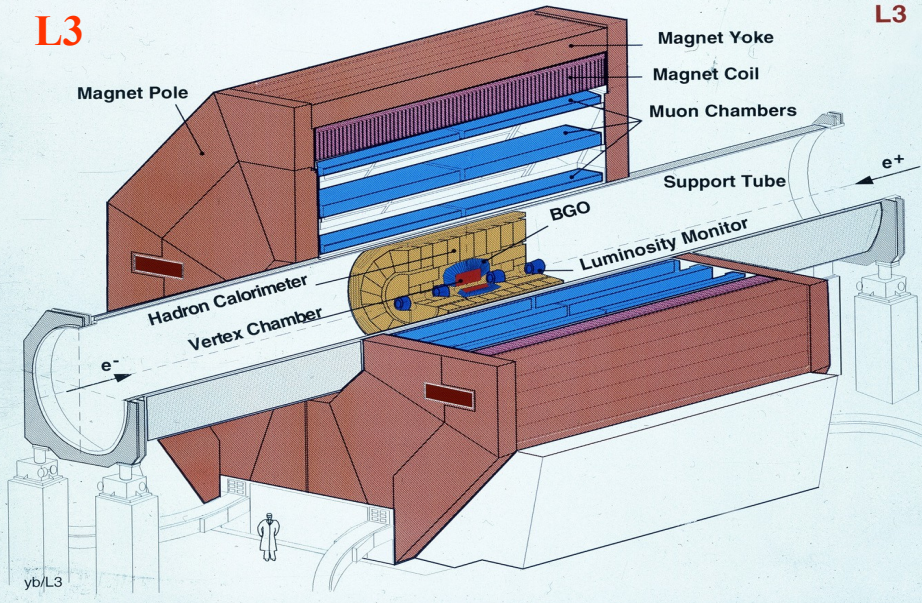
- Vertex Detector
- Inner Track Chamber
- Time Projection Chamber
- Electromagnetic Calorimeter
- Superconducting Magnet Coil
- Hadron Calorimeter
- Muon Detection Chambers
- Luminosity Monitors

Fig. 1 - The ALEPH Detector



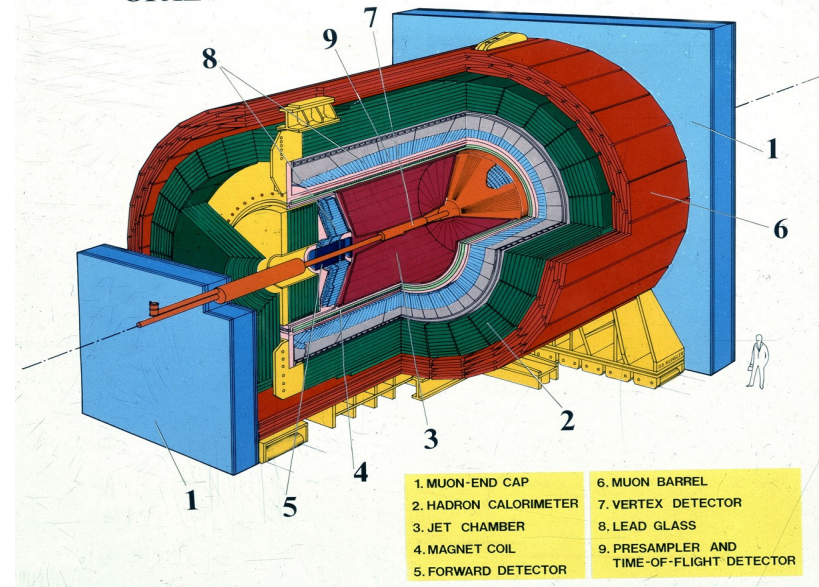
L3

L3

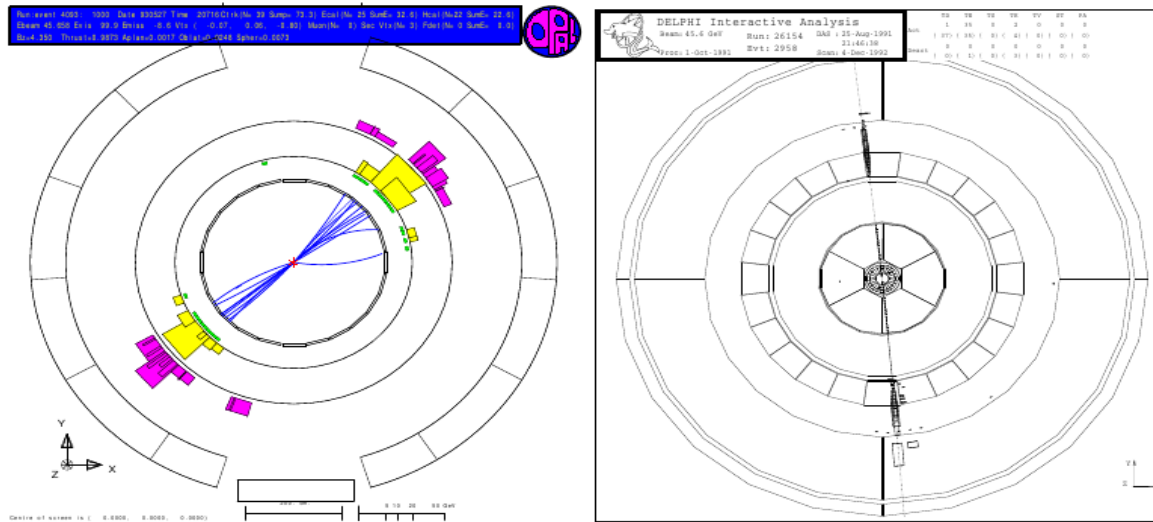


yb/L3

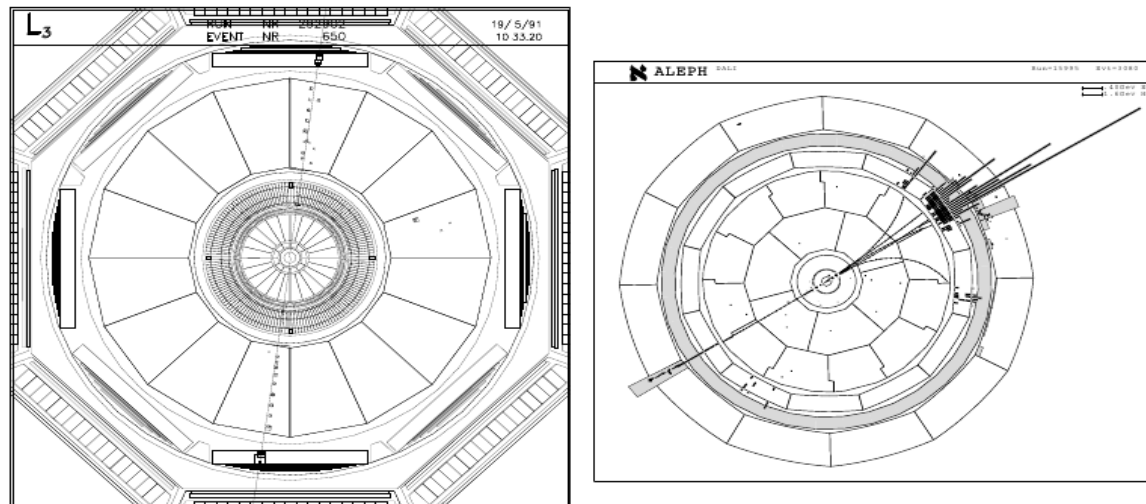
OPAL



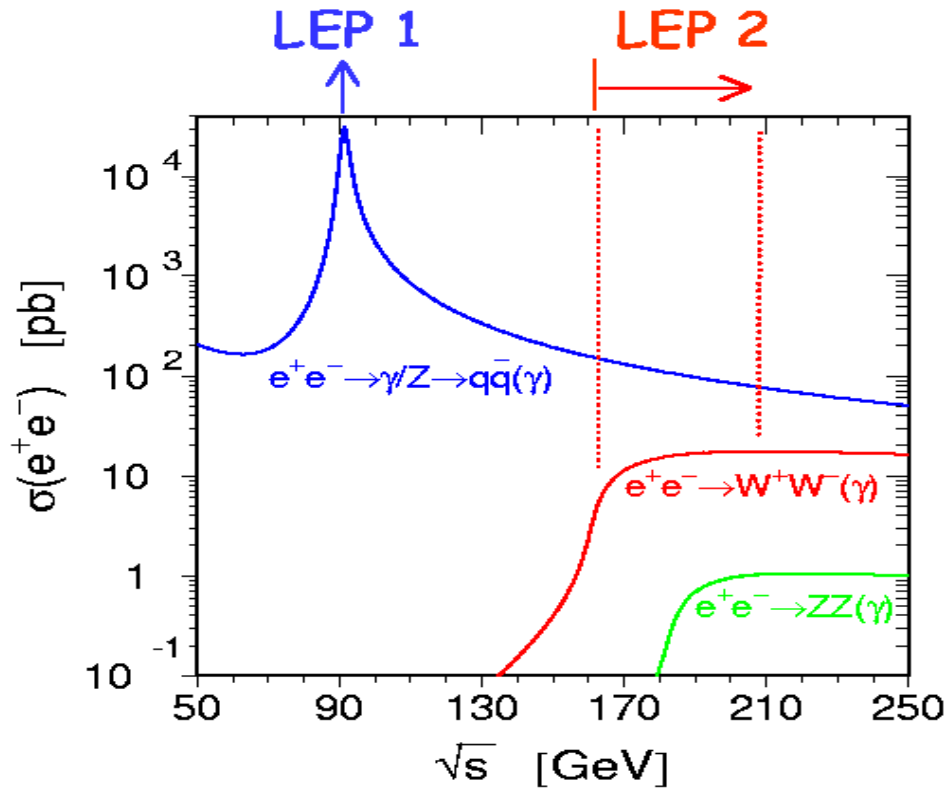
Measurements in $e^+ e^-$ at LEP



Event Displays from LEP experiments



Measurements in $e^+ e^-$ at LEP



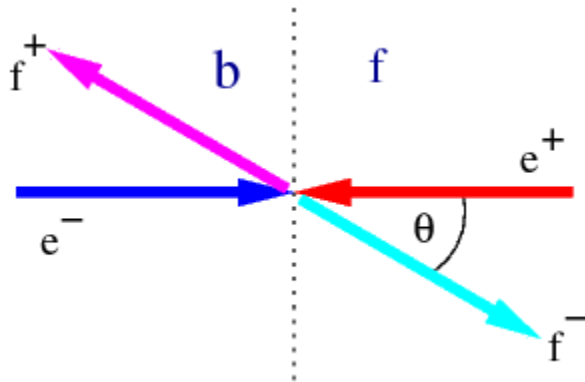
LEP 1: measurement of Z boson parameters (16 million Z's)

LEP 2: measurement of W and Z boson pair production,
W boson Parameters (40'000 W pairs)

total cross section

$$\sigma_{tot} \equiv \int_{-1}^1 \frac{d\sigma}{d\cos\theta} d\cos\theta$$

depends on $(1+\cos^2 \Theta)$ terms only



Forward-backward asymmetry

$$A_{FB} \equiv \frac{\int_0^1 \frac{d\sigma}{d\cos\theta} d\cos\theta - \int_{-1}^0 \frac{d\sigma}{d\cos\theta} d\cos\theta}{\sigma_{tot}}$$

depends on $\cos^2 \Theta$ terms only

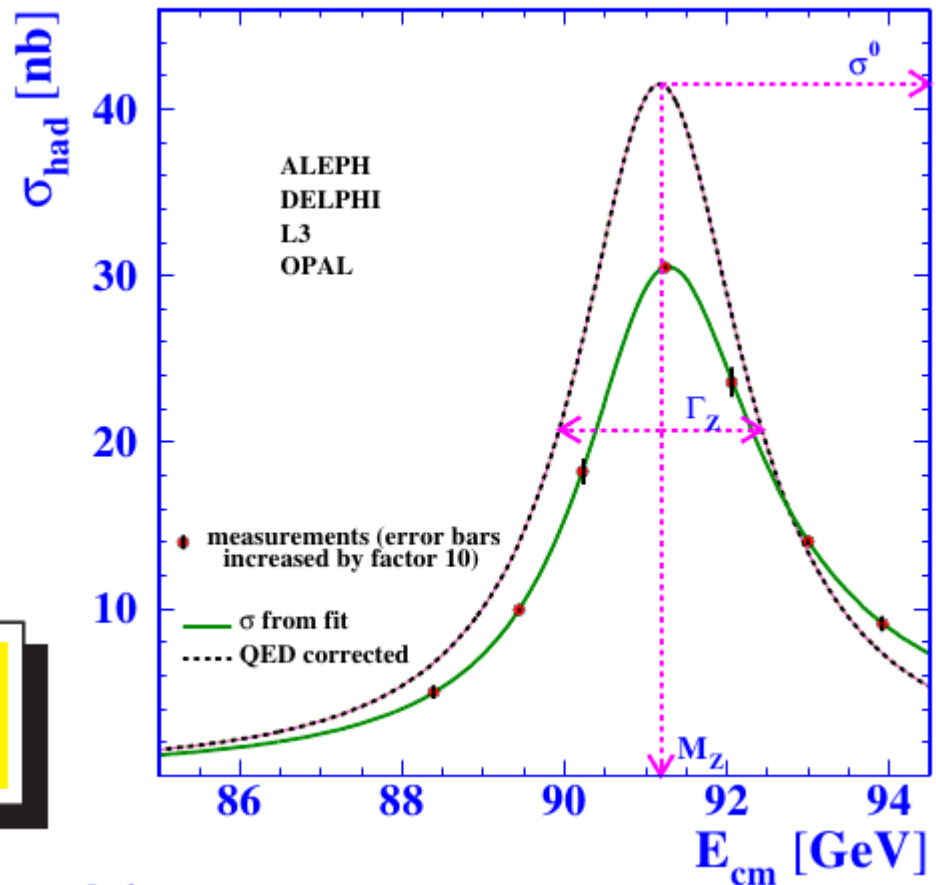
Principle:

Extract combinations of Z-couplings to fermions from measurements of σ_{tot} and A_{FB} in different channels and at different energies

Measurement of cross sections:

- Determine candidate events for signal process
- Subtract background passing selection criteria
- Determine selection efficiency: from Monte- Carlo simulation or using “data-driven” methods
- Normalize corrected number of signal events to luminosity

$$\sigma(E_i) = \frac{N_{\text{ff}}^{\text{cand}}(E_i) - N_{\text{ff}}^{\text{bkg}}(E_i)}{\epsilon_{\text{ac}}(E_i)} \frac{1}{\int L(E_i)},$$



Luminosity determination:

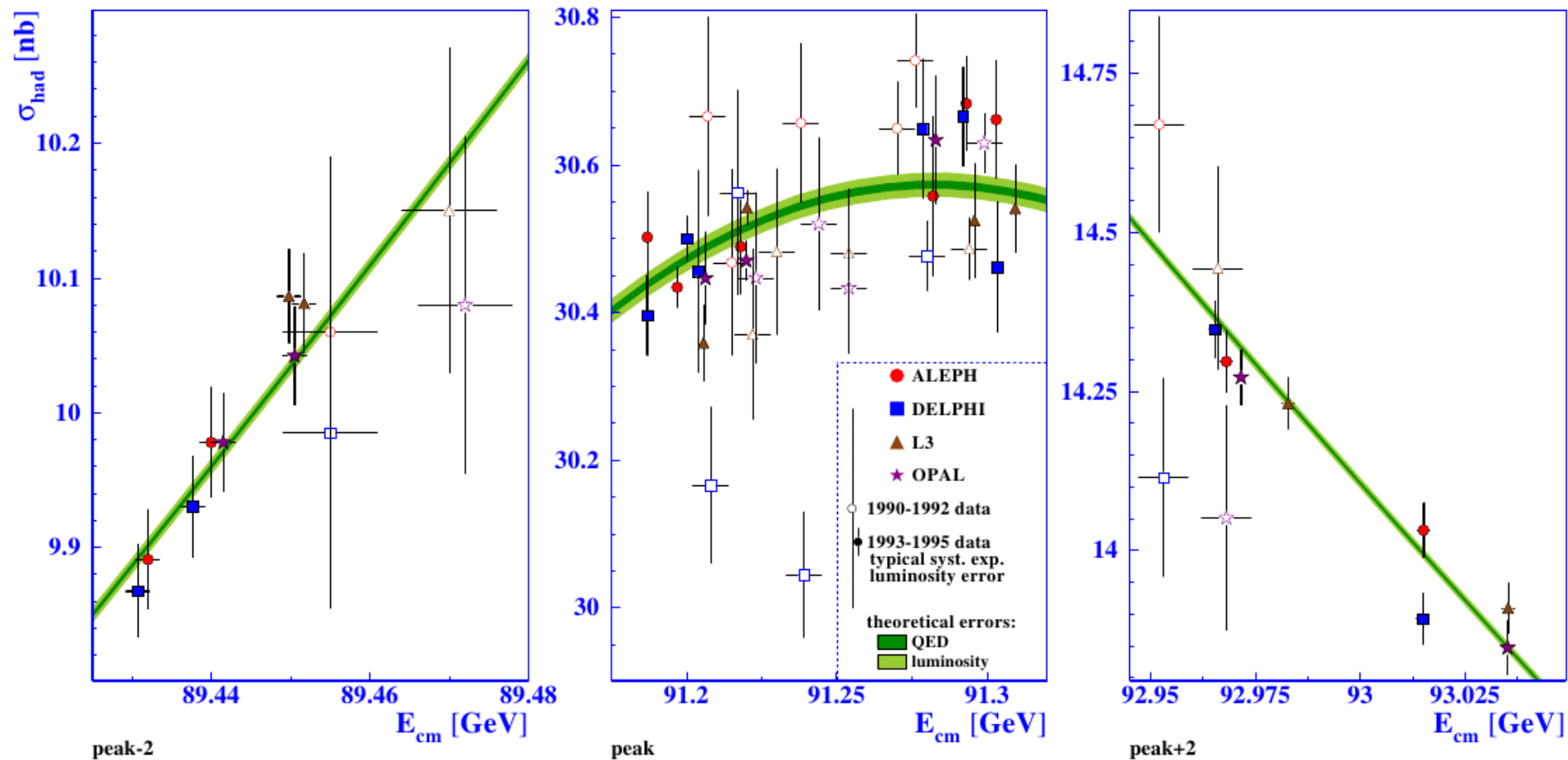
count number of events in reference reaction

with well-known cross section: $\int L = n_{\text{ref}} / \sigma_{\text{ref}}$

reference reaction @ LEP: $e^+e^- \rightarrow e^+e^-$ at small angles (“Bhabha scattering”)

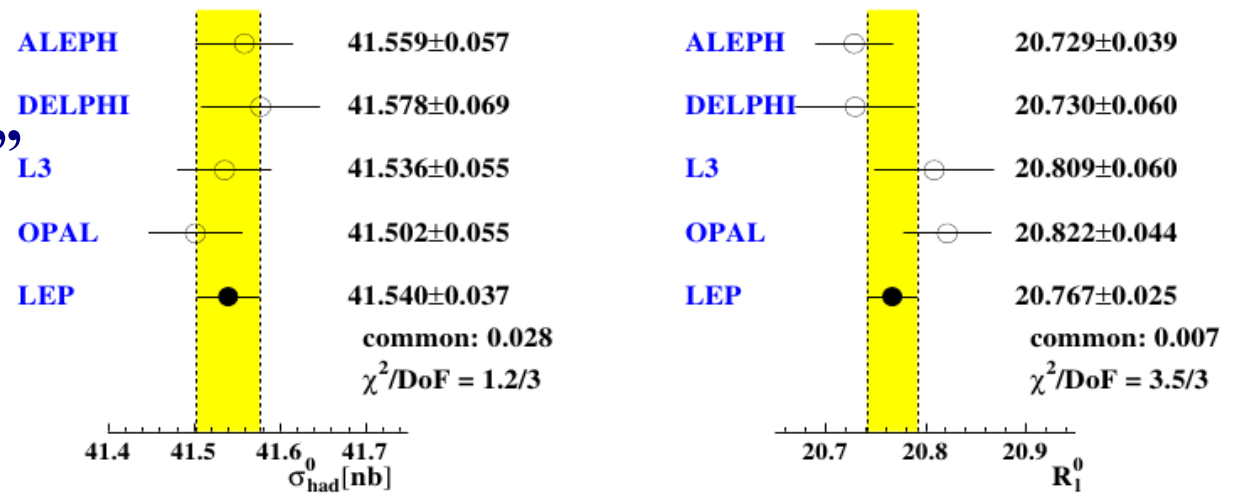
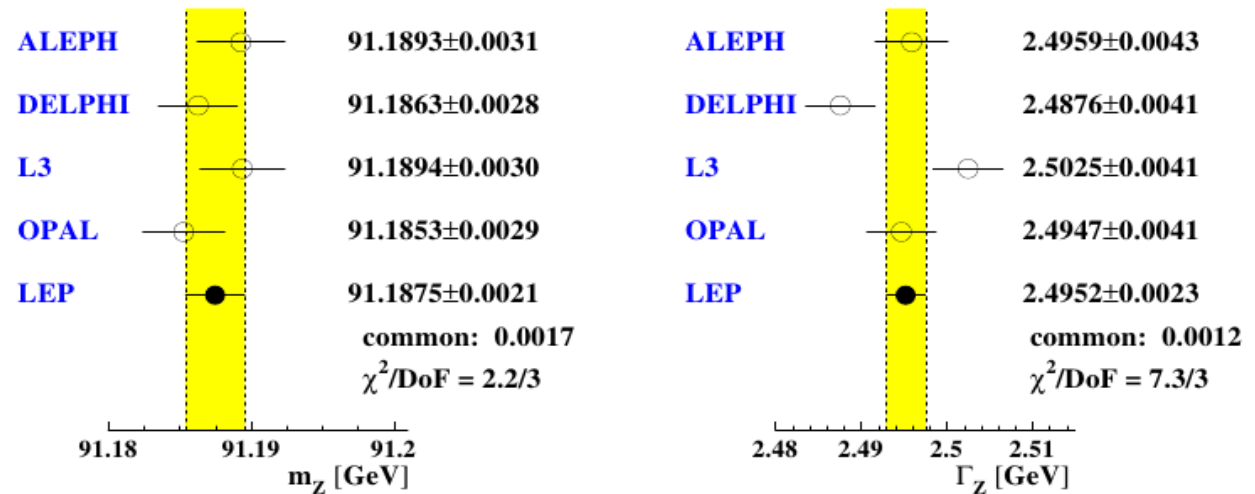
*photonic corrections are large,
but well known
theoretically*

Precision measurements: Z line shape



The challenge: combining more than 800 individual measurements (different channels, CM-energies and data taking periods)

Precision measurements: Z line shape for leptons and hadrons



Parametrization
of cross section
using
“pseudo-observables”

- m_Z
- Γ_Z
- $\sigma_{\text{had}}^o = \frac{12\pi}{m_Z^2} \frac{\Gamma_{ee}\Gamma_{\text{had}}}{\Gamma_Z^2}$
- $R_e = \Gamma_{\text{had}}/\Gamma_{ee}$
- $R_\mu = \Gamma_{\text{had}}/\Gamma_{\mu\mu}$
- $R_\tau = \Gamma_{\text{had}}/\Gamma_{\tau\tau}$

partial decay width

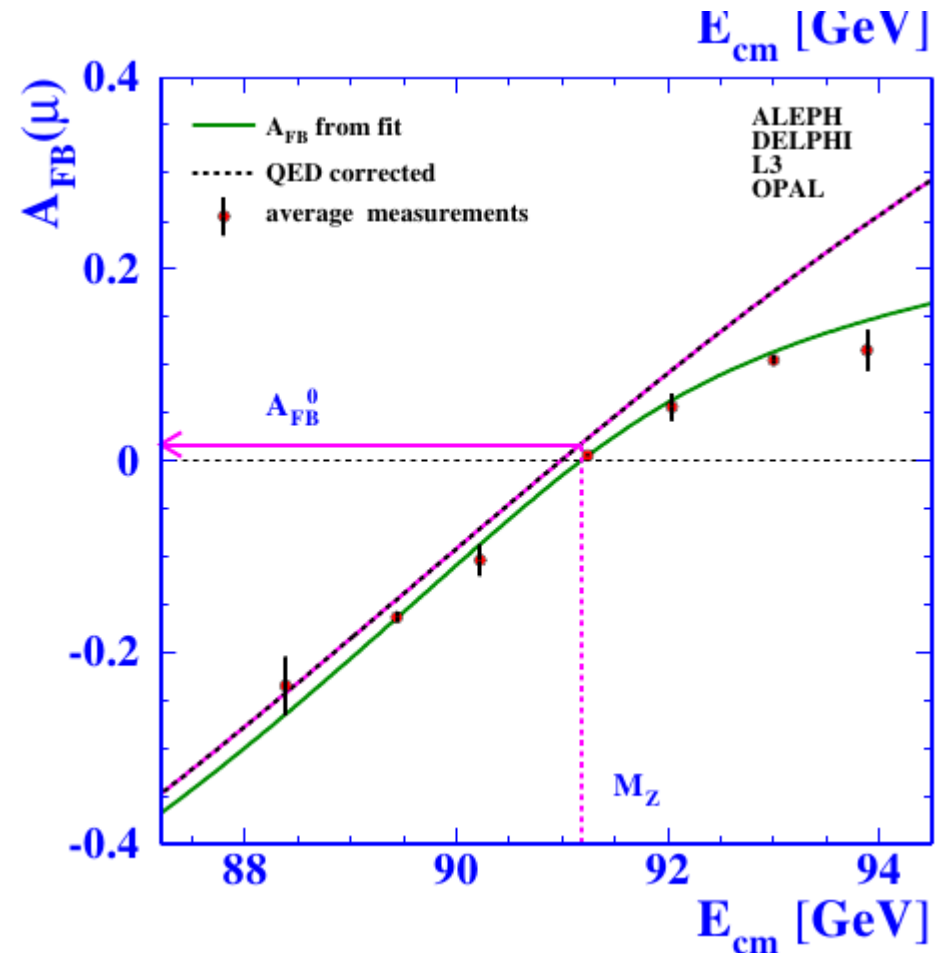
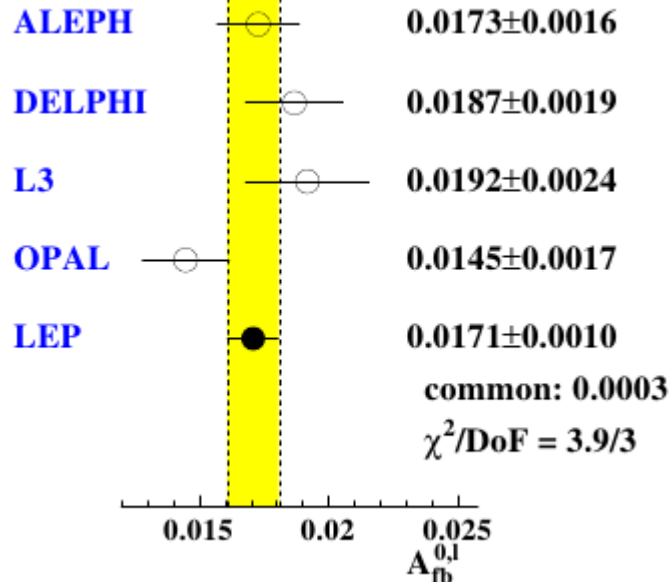
$$\Gamma_{ff} \propto (g_{Vf}^2 + g_{Af}^2) \text{ for } f=e, \mu, \tau$$

Precision measurements: forward-backward asymmetries of lepton pairs

$$A_{FB} = \frac{N_{\text{forw}} - N_{\text{back}}}{N_{\text{tot}}} = \frac{\int_0^1 \frac{d\sigma}{d\cos\theta} d\cos\theta - \int_{-1}^0 \frac{d\sigma}{d\cos\theta} d\cos\theta}{\sigma_{\text{tot}}}$$

pseudo-observables:

$$A_{FB}^{0,f} \equiv \frac{3}{4} \mathcal{A}_e \mathcal{A}_f \quad \text{with} \quad \mathcal{A}_f = \frac{2g_{Vf}/g_{Af}}{1 + (g_{Vf}/g_{Af})^2}$$



More Precision measurements – tau lepton polarisation

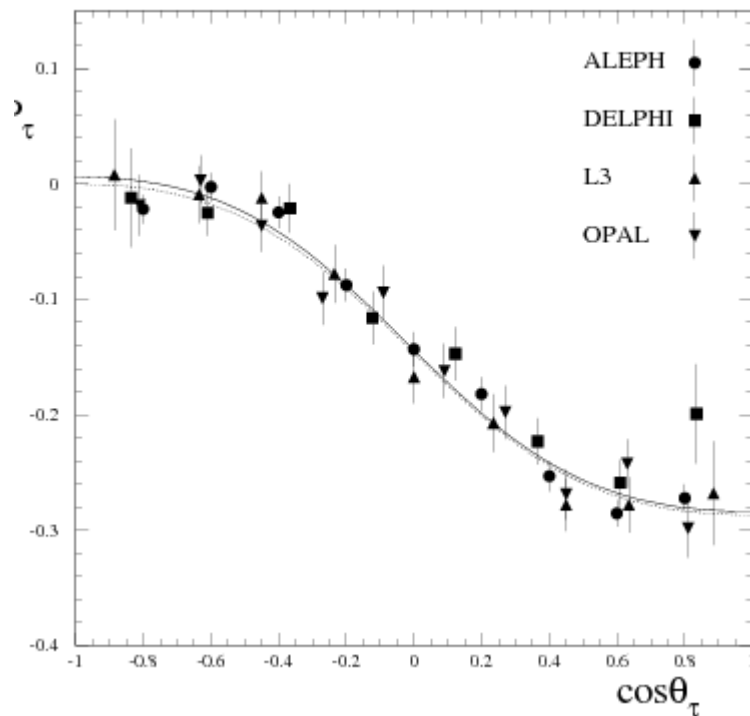
in $e^+e^- \rightarrow \tau^+\tau^-$:

Spin in final state can be measured assuming V-A structure in τ decay

average τ polarization depends on e and τ couplings

$$\mathcal{P}_\tau(\cos\theta) = -\frac{\mathcal{A}_\tau(1 + \cos^2\theta) + 2\mathcal{A}_e\cos\theta}{1 + \cos^2\theta + 2\mathcal{A}_e\mathcal{A}_\tau\cos\theta}$$

Measured P_τ vs $\cos\theta_\tau$



allows precise measurement
of vector and axial vector
couplings of τ to Z !

More Precision measurements – with polarized e-beam

Measurements at SLAC linear collider:

polarized e^- colliding with unpolarized e^+ at $\sqrt{s}=M_Z$

measurements analogous to LEP, but

can **determine σ** and A_{FB} for left- and right-handed e^-

observables at SLAC:

$$A_{LR} = \frac{1}{\mathcal{P}_e} \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R} \propto \mathcal{A}_e$$

$$A_{fb,LR} = \frac{1}{\mathcal{P}_e} (A_{fb,L} - A_{fb,R}) \propto \mathcal{A}_f$$

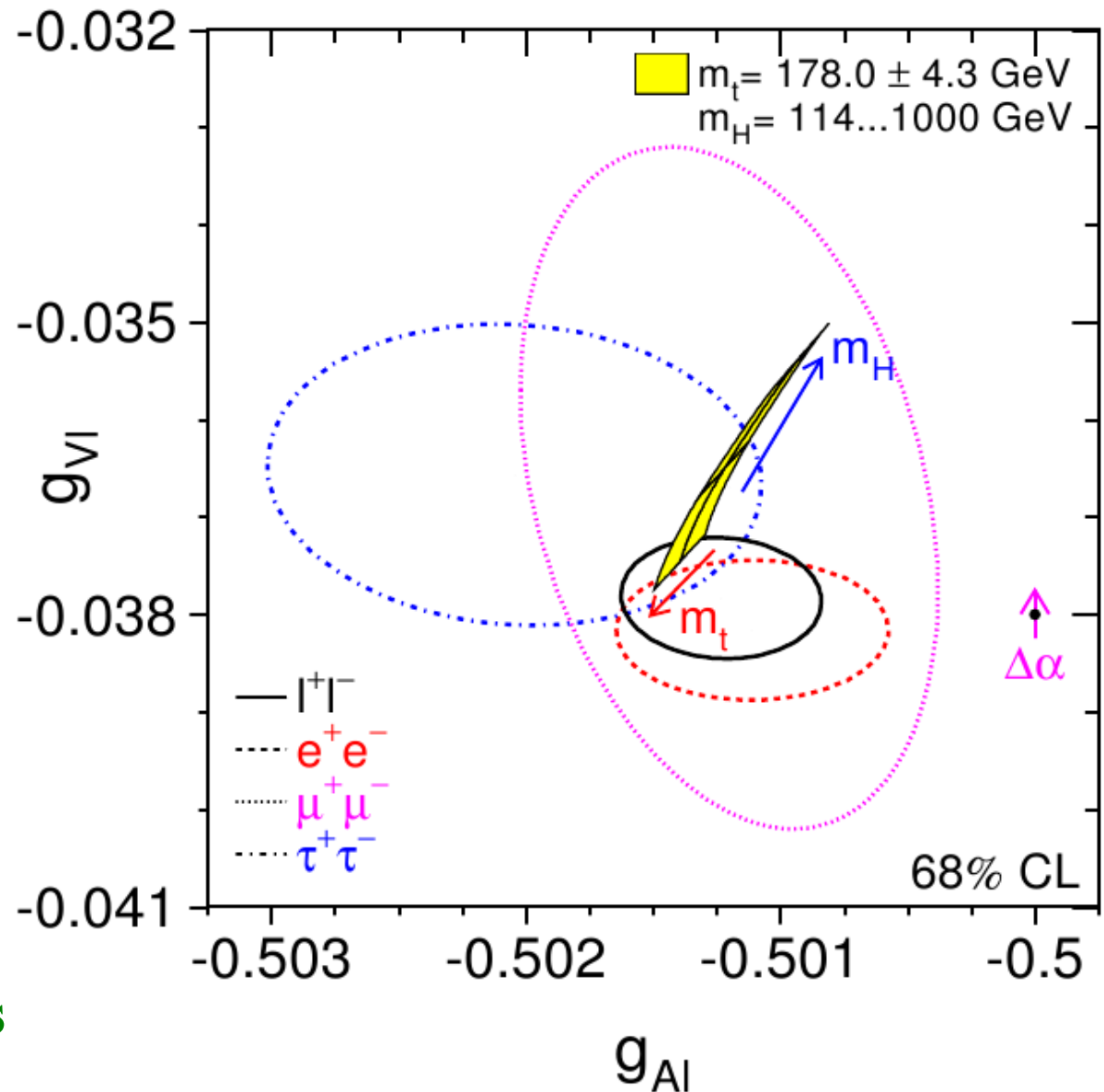
A_{LR} is most sensitive single measurement to $\sin^2\theta_W^{\text{eff}}$

Determination of lepton couplings to Z boson

Combination of previously shown measurements gives most precise values of lepton couplings.

Consistent with “lepton universality”
→ combine all into leptonic g_V and g_A

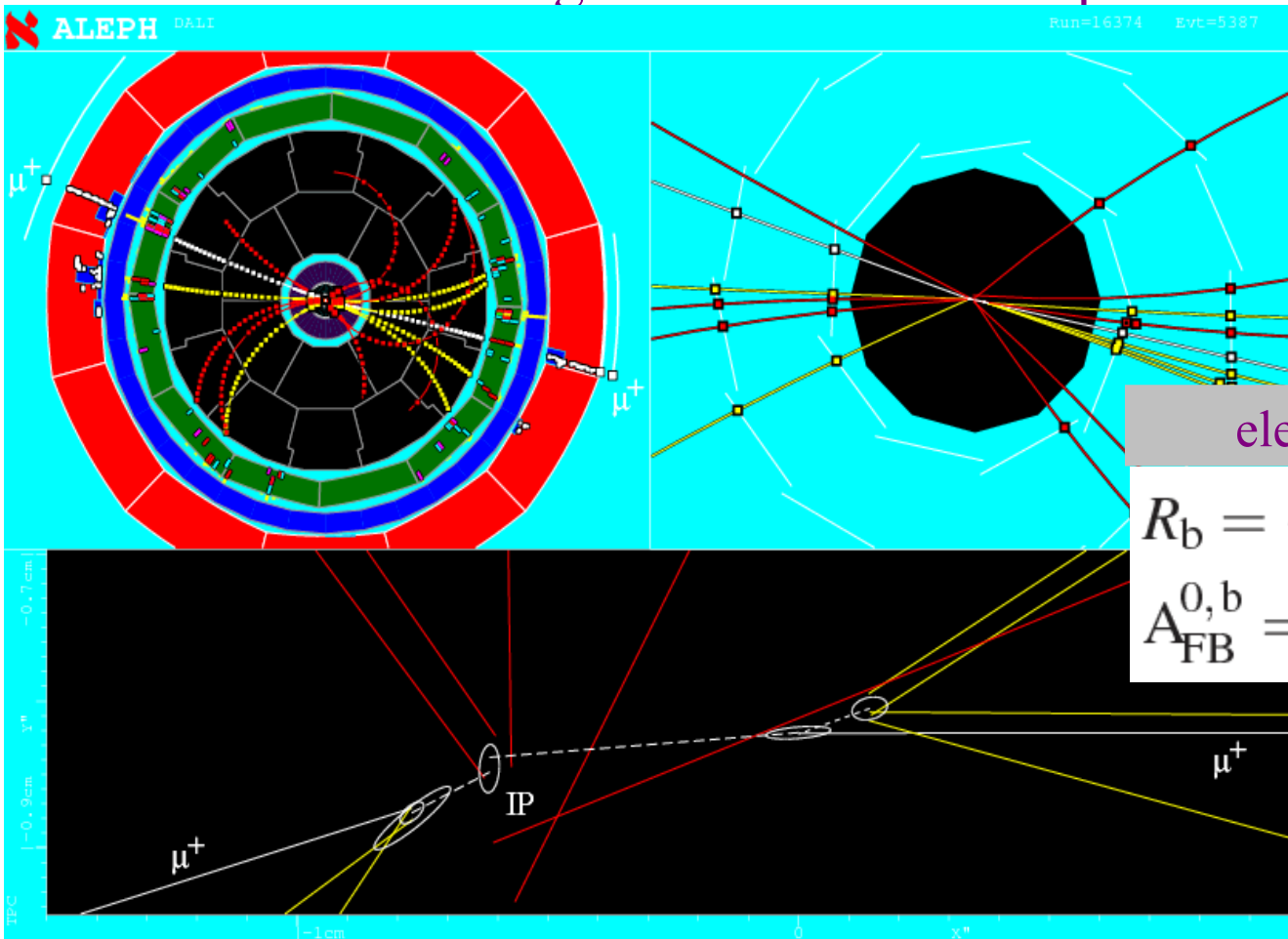
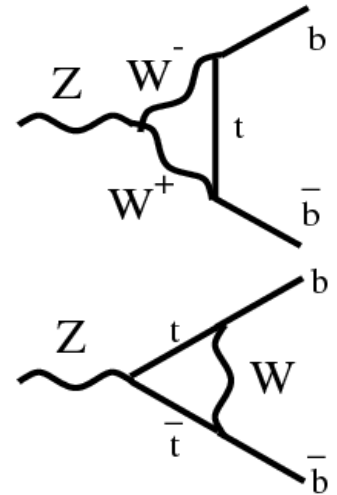
Measurements are sensitive to top and Higgs boson masses



More Precision measurements – heavy quarks

- b-quarks** – are (relatively) long-lived
 – are heavy
 – decay in cascades → can be distinguished from other quarks

special relation
 to top quark:



electroweak observables

$$R_b = \frac{\Gamma_{b\bar{b}}}{\Gamma_{\text{had}}} \Rightarrow \rho_b$$

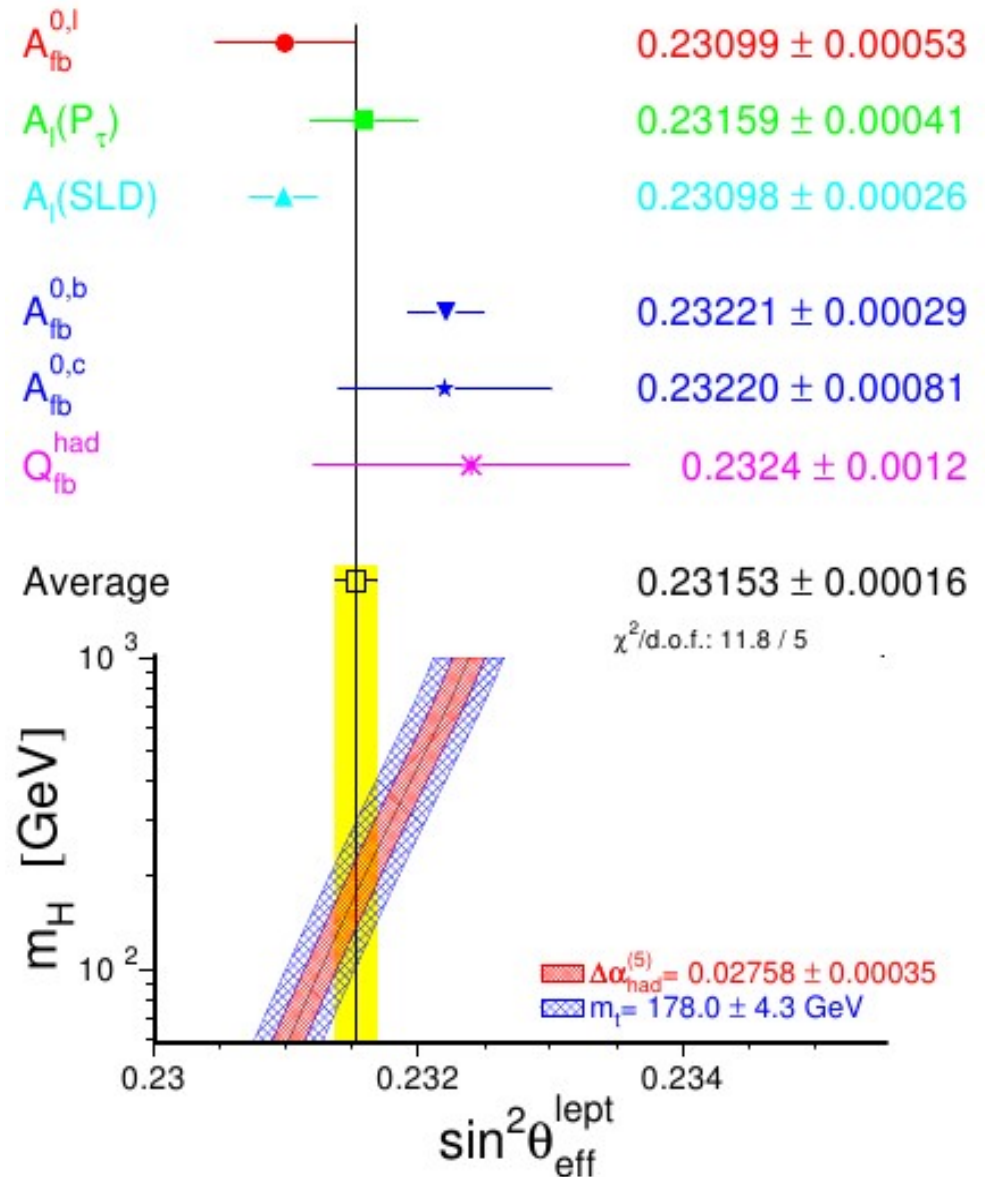
$$A_{\text{FB}}^{0,b} = \frac{4}{3} \mathcal{A}_e \mathcal{A}_b \Rightarrow \sin^2 \theta_{\text{eff}}^{\text{lept}}$$

Summary of asymmetry-type measurements

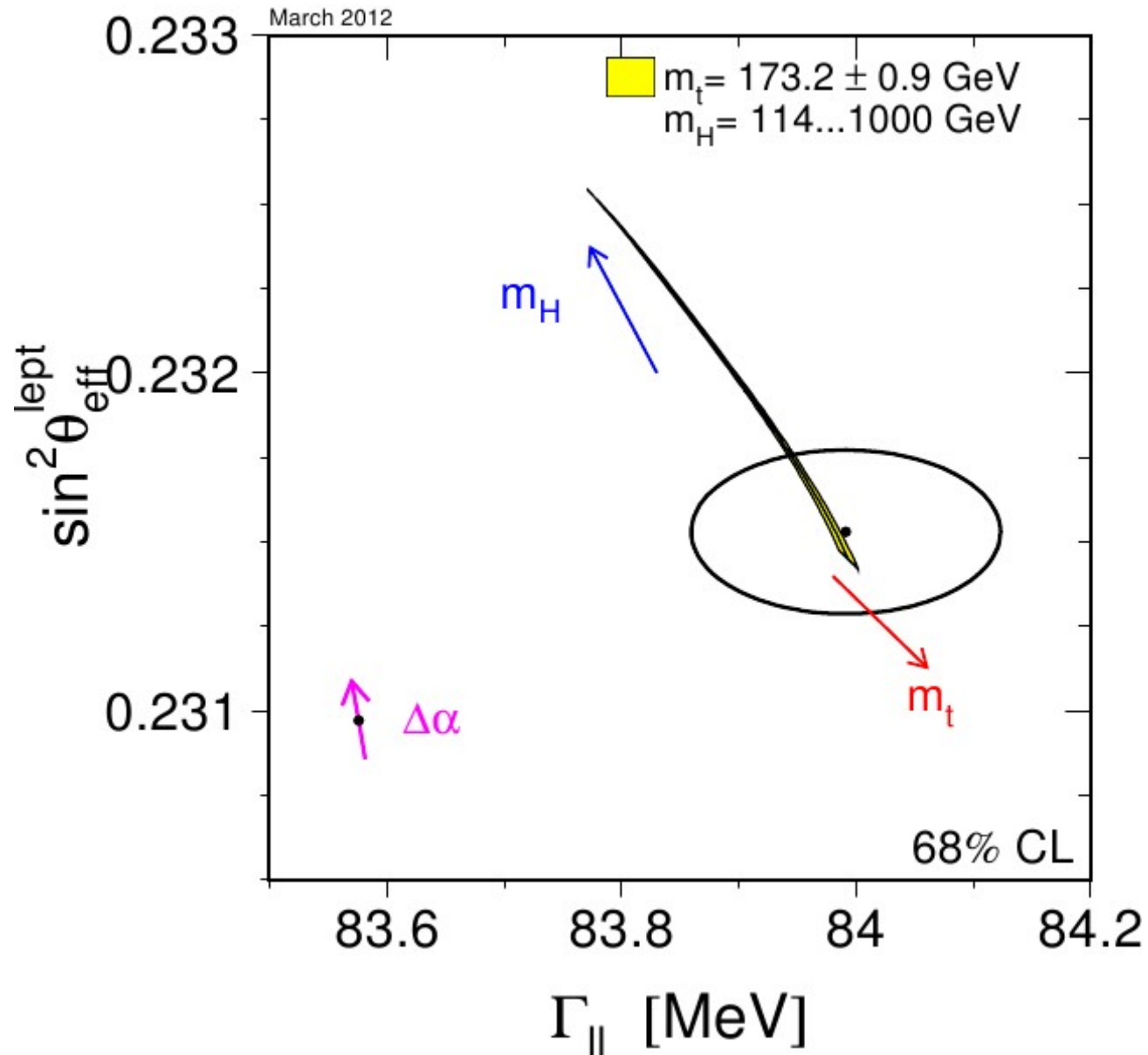
Asymmetry-type measurements depend on vector and axial vector couplings, g_V and g_A , and hence on $\sin^2\theta_W^{\text{eff}}$

Average of $\sin^2\theta_W^{\text{eff}}$ very sensitive to Higgs mass

There is, however, some “tension” between the most precise measurements !

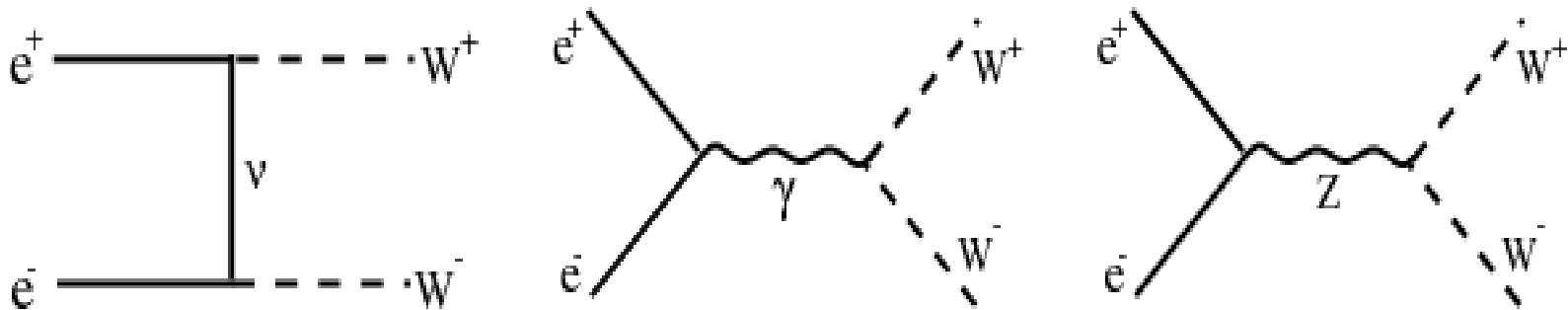


Combination of precision measurements: Γ_{lept} and $\sin^2\theta_W^{\text{eff}}$



Γ_{lept} and $\sin^2\theta_W^{\text{eff}}$
together provide
strong constraint
on Higgs mass !

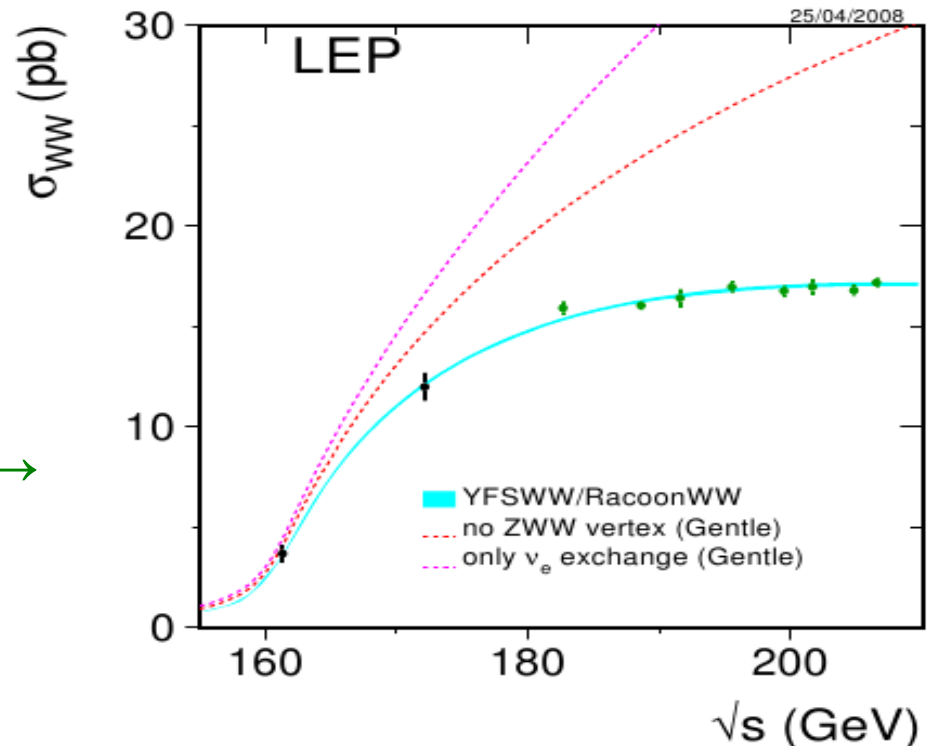
W pair production at LEP II



$e^+e^- \rightarrow W^+W^-$ cross section in EW theory remains finite only if photon, Z and neutrino exchange are all taken into account -

**experimentally
well confirmed** →

Remark: there is also a Higgs diagram, but negligible in e^+e^- !



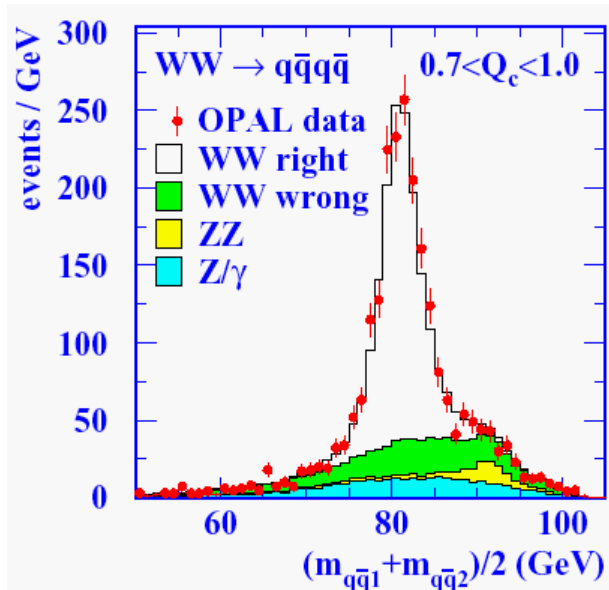
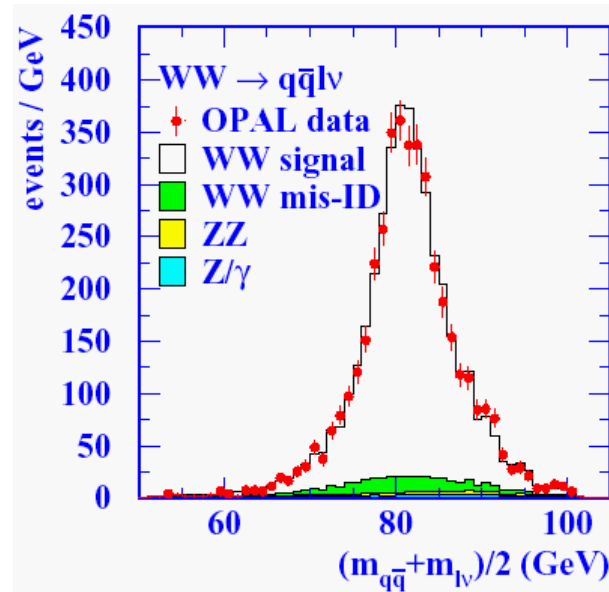
W Boson mass

W-Boson mass

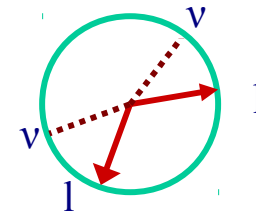
from reconstructed
objects in detectors
by LEP experiments
ALEPH, DELPHI,
L3 and OPAL and
by Tevatron expe-
riments CDF & D0

(LHC not yet ...)

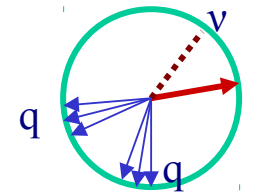
very difficult,
dominated by
systematic errors



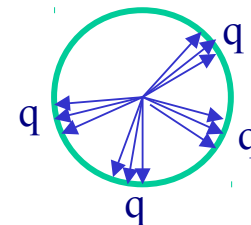
@LEP: W pair events:



BR \sim 10%



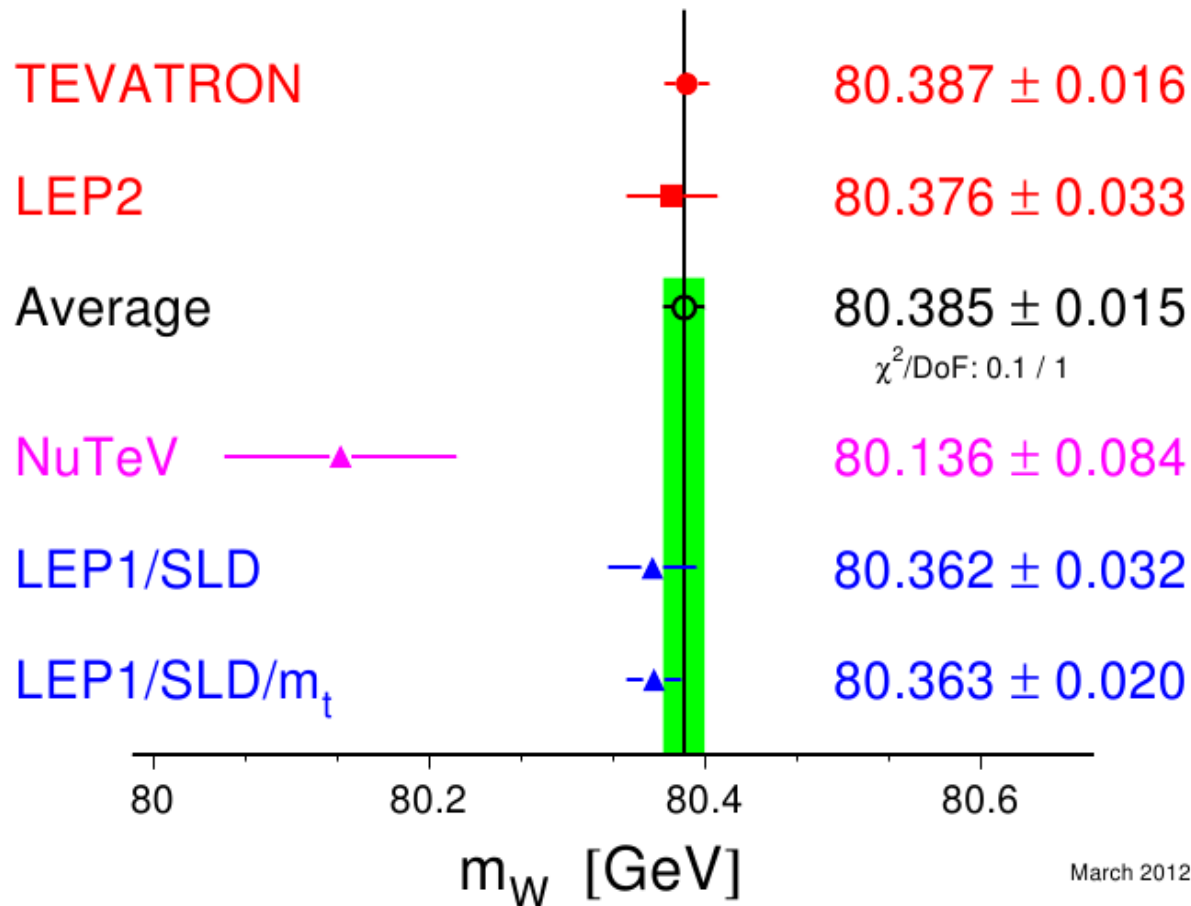
BR \sim 44%



BR \sim 46%

W Boson mass – world average 2012

W-Boson Mass [GeV]



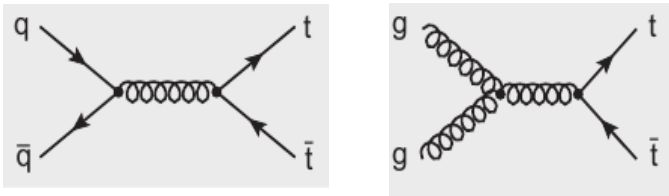
Remember: Relevance of W mass:
determines on-shell weak angle

One more ingredient: need the top quark mass

top quark mass

measured from reconstructed
objects in detectors by
Tevatron experiments
CDF & D0 (LHC on the way ...)

- top quarks (mostly) produced
in pairs via $q\bar{q}$ or gg



- dominant decay:

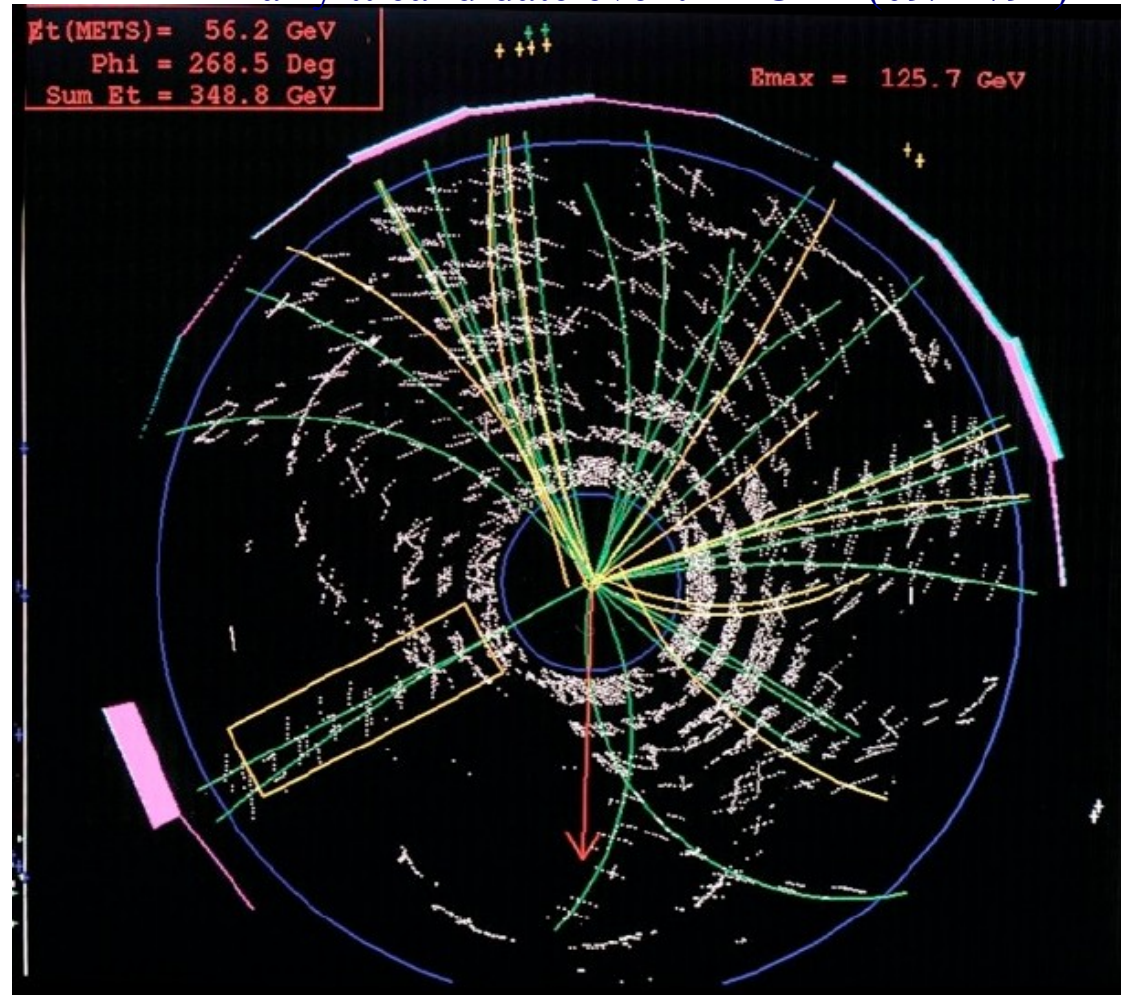
$$t \rightarrow b + W^+$$

$$W \rightarrow \bar{q}q' \text{ or } l\nu$$

signatures studied:

- fully hadronic
- lepton + jets
- di-lepton

Early $t\bar{t}$ candidate event in CDF (09/24/92)



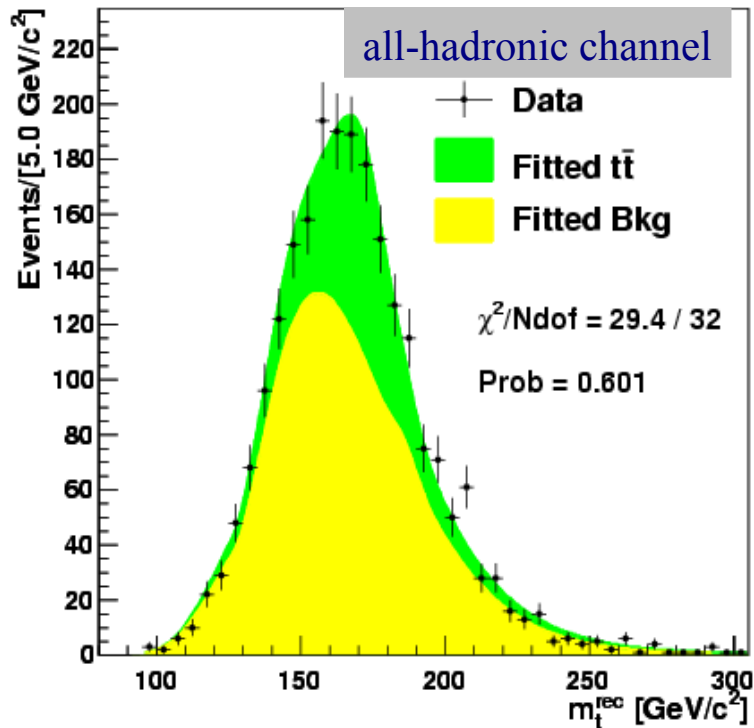
Signature: “lepton + jets”

- 1 lepton
- four jets
- missing transverse energy

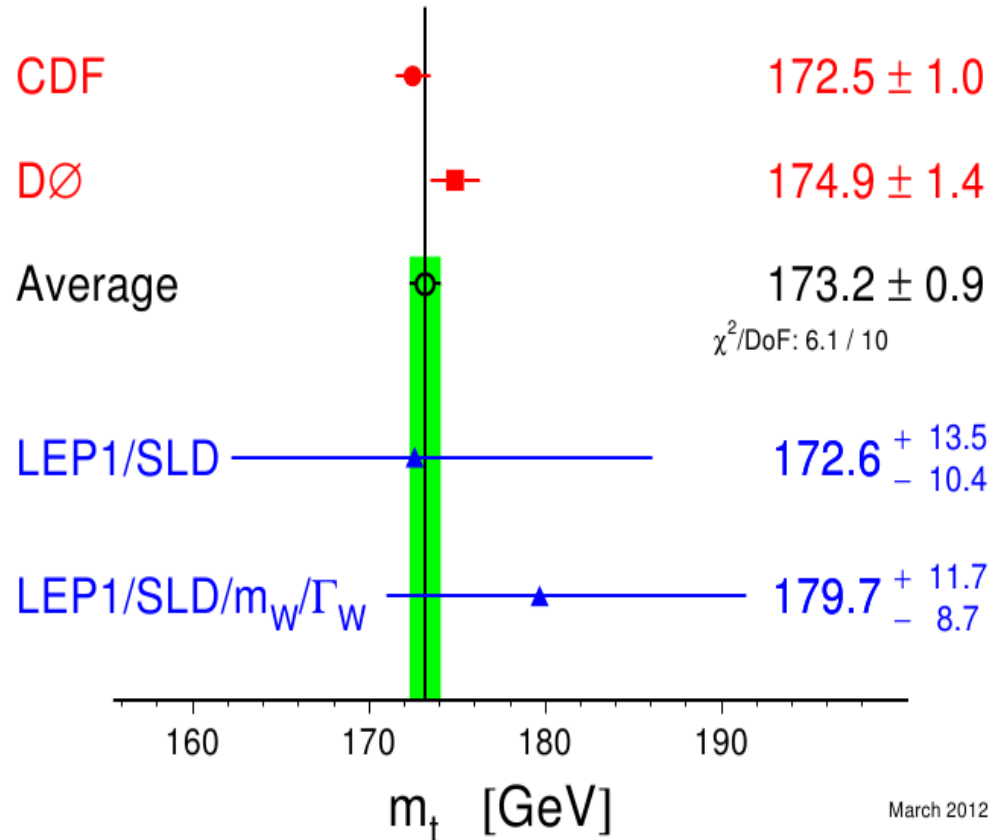
One more ingredient: top quark mass

Example of observed mass distribution:

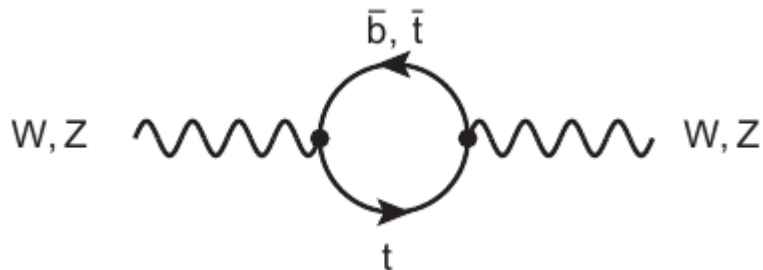
CDF Run II Preliminary (5.8 fb⁻¹)



Top-Quark Mass [GeV]



March 2012



Remember: dominant radiative corrections arise from top ($\sim G_F m_t^2$) !

Top quark mass from loop corrections ...

... at Moriond conference
in March 1994:

direct search for top at Tevatron:

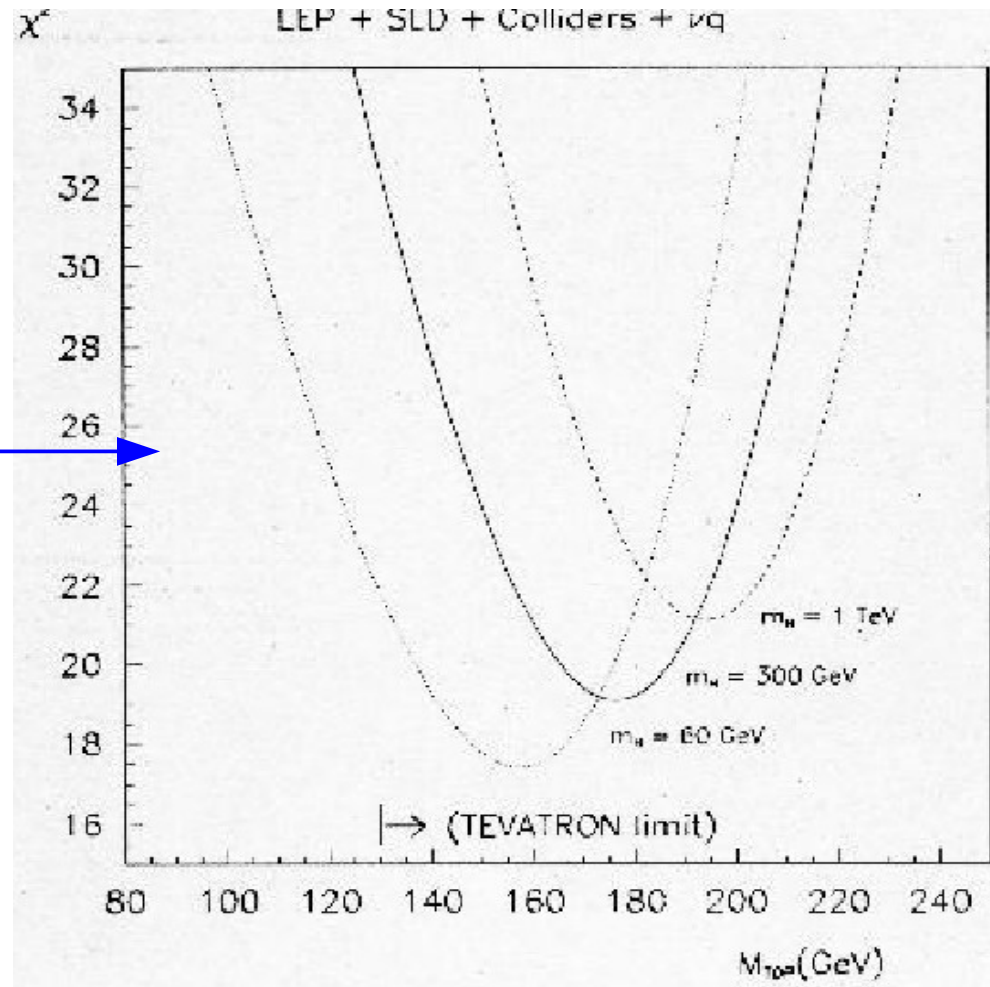
$$m_t > 130 \text{ GeV}/c^2$$

from **radiative corrections**:

$$m_t = 177 \pm 11^{+18}_{-19}_{\text{Higgs}} \text{ GeV}/c^2$$

a little later in summer, direct
observation of top quark (CDF):

$$m_t = 174 \pm 10^{+13}_{-12}_{\text{syst}} \text{ GeV}/c^2$$



Excellent agreement between
top from loops and from direct measurement!

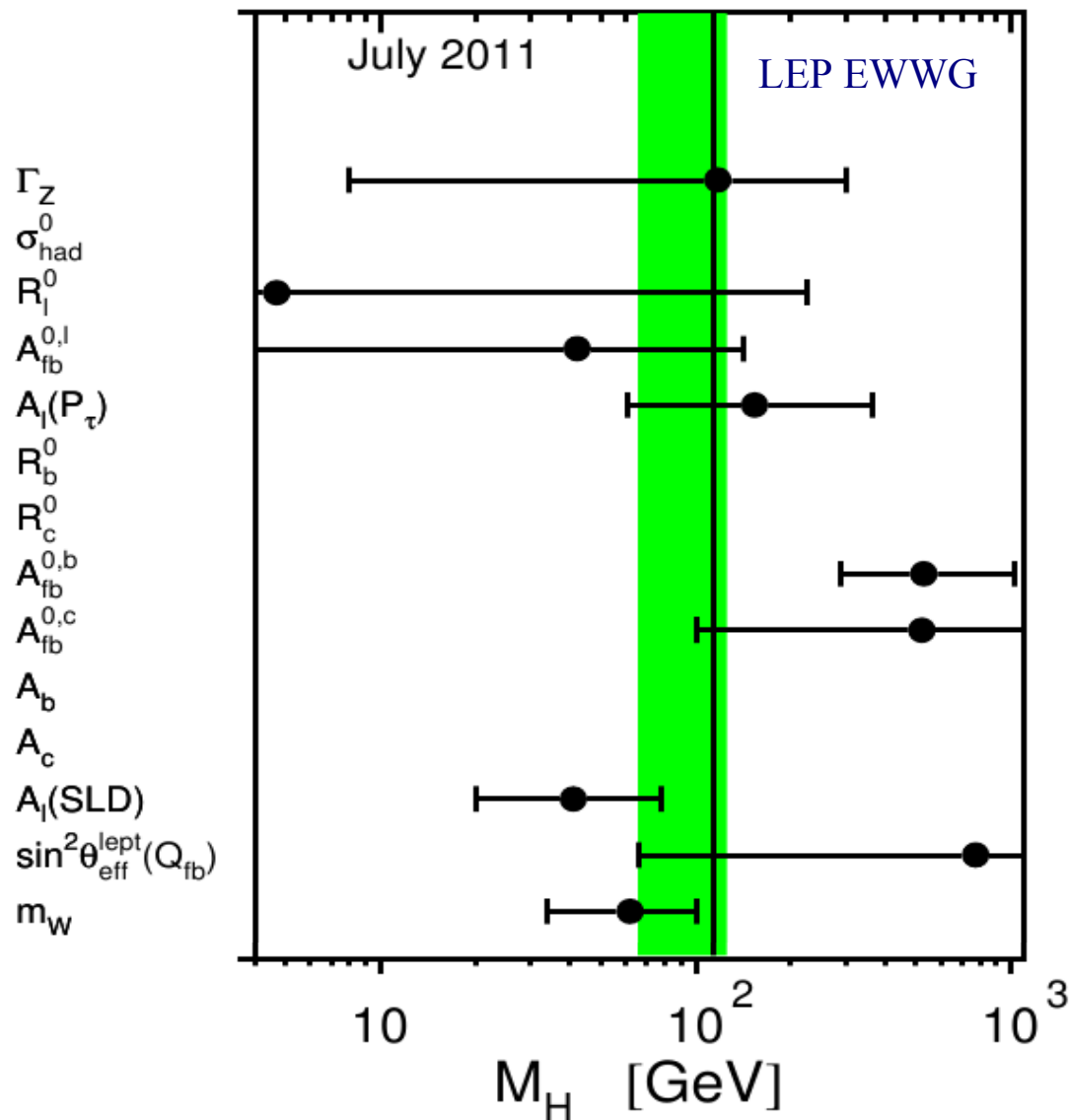
M_H from individual precision measurements

With known top mass,
can now disentangle
radiative effects from
top and Higgs

(provided nothing else
is in the loops ...)

→

M_H from precision
measurements
with large errors



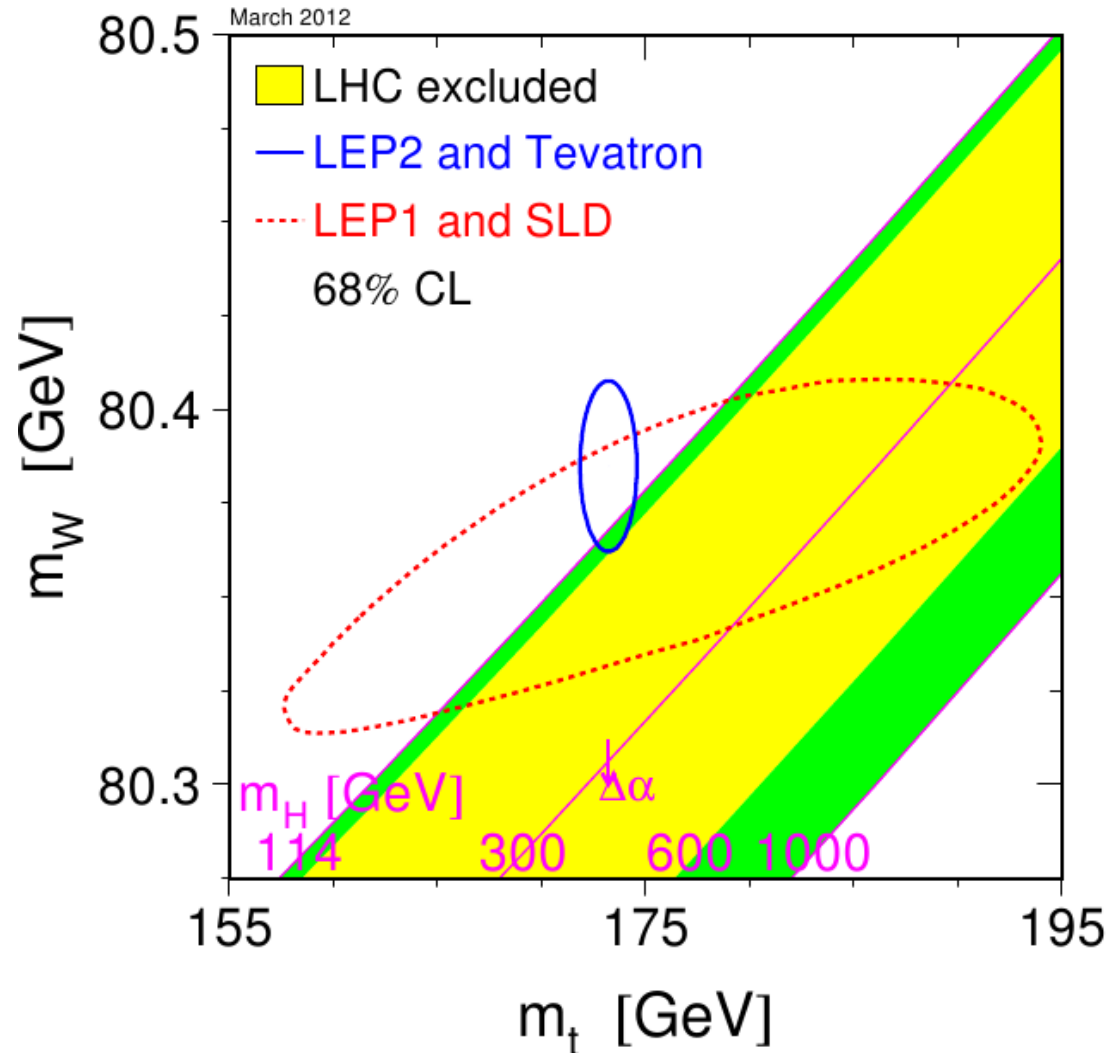
Putting it all together

... in a rather complicated plot:

top quark vs. W boson mass

1. direct measurements
2. indirect determination
from precision variables:
 $O := O(\alpha, G_F, M_Z, m_t, M_H)$
solved numerically
(“Newton's method”) for
 m_t and M_H (using SM)
3. W mass predicted for
different values of M_H

nice consistency check,
prefers low value for
Higgs mass !



Putting it all together

... in an overall fit

of Higgs boson mass
within the minimal
Standard Model

“pull plot” and overall
value of χ^2 to identify
potential problems

good overall consistency,
but there is one “outlier”;
the measurement of the
b-quark forward backward
asymmetry.



Putting it all together

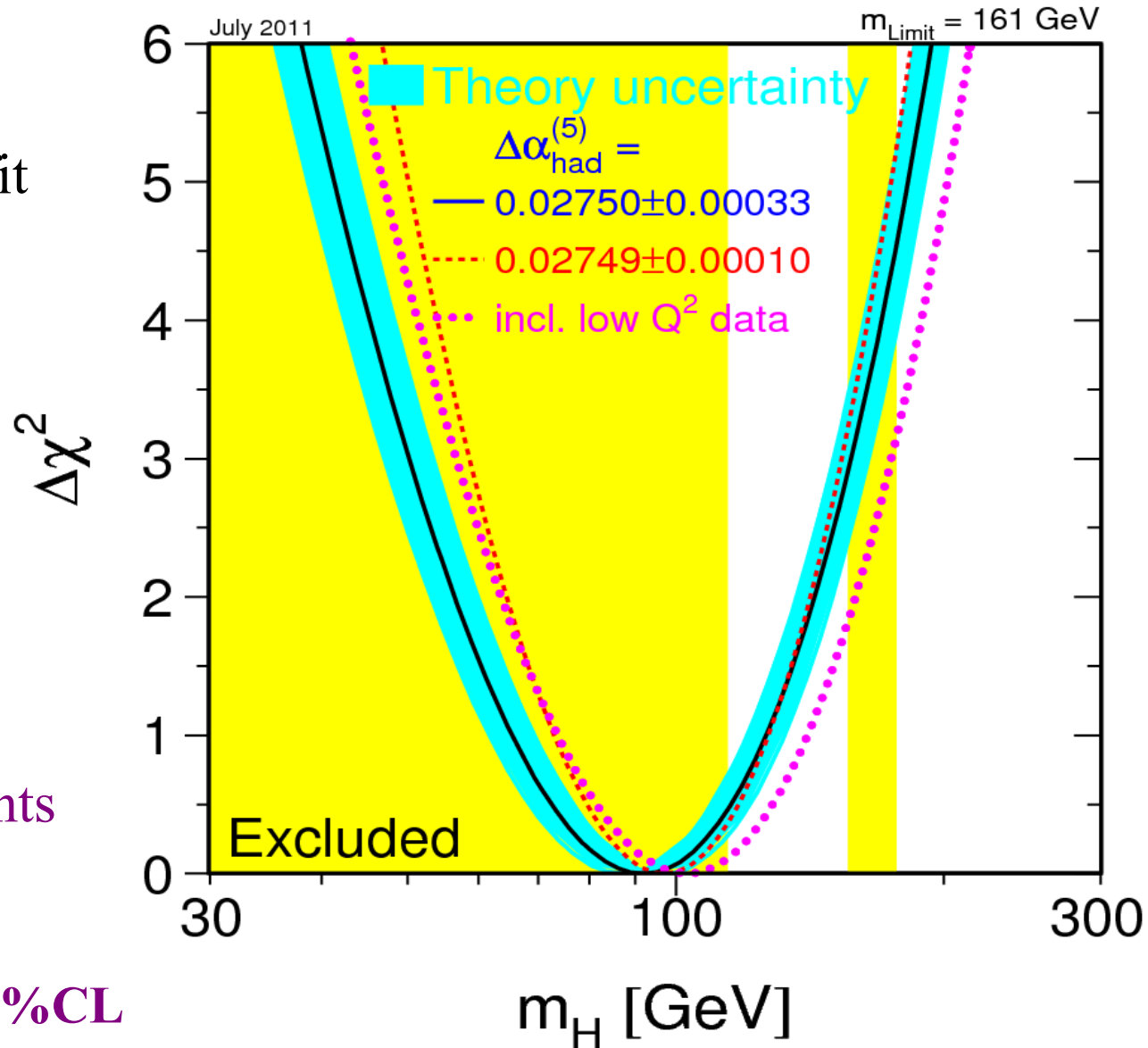
... profiled χ^2

for M_H from overall fit
within the minimal
Standard Model

“Blue Band Plot”

Precision measurements
predict mSM Higgs
boson to be light,

$M_H < 161 \text{ GeV}/c^2 @ 95\% \text{CL}$



Conclusions on results from precision measurements

Standard Model established as a renormalizable field theory:

- in agreement with (almost) all experimental results, prec. $\sim 0.1\%$
- quantum corrections seen and well established
- indirect determination of top quark mass
- triple gauge boson couplings confirmed with prec. 1%
- precision measurement constrain Higgs to be light

Theoretical work on proof
of renormalizability awarded
with Nobel prize in 1999



Gerardus 't Hooft



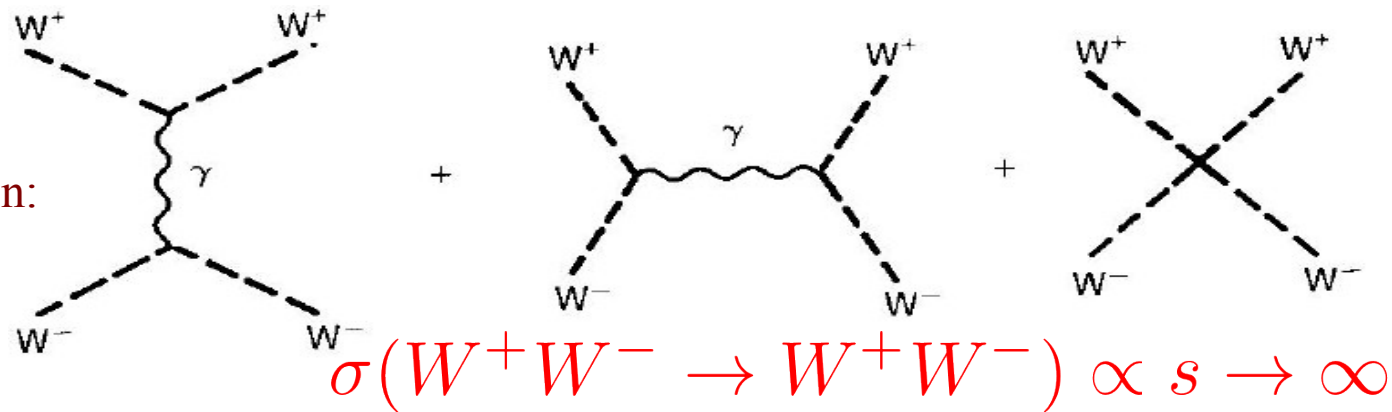
Martinus J.G. Veltman

The Nobel Prize in Physics 1999 was awarded jointly to Gerardus 't Hooft and Martinus J.G. Veltman "for elucidating the quantum structure of electroweak interactions in physics"

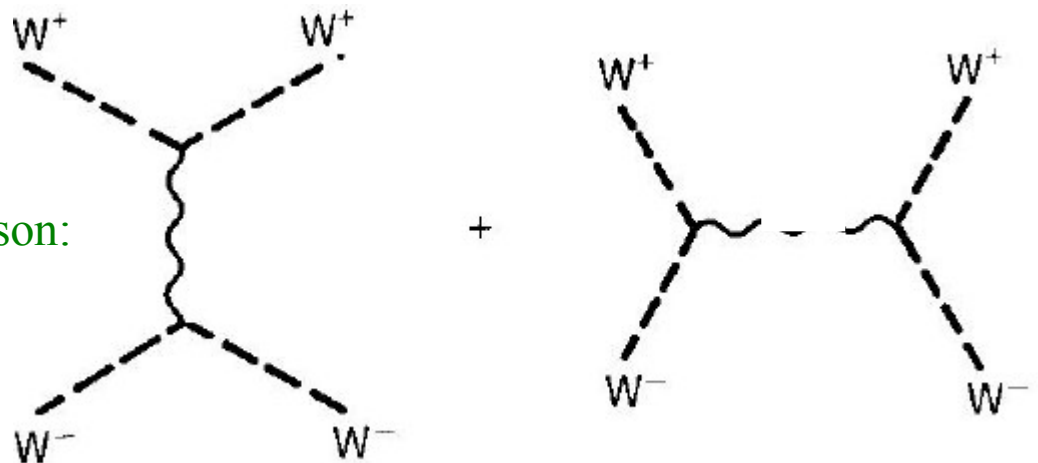
Theoretical constraints on Higgs boson mass

Diagrams with Higgs-Boson prevent divergencies:

Without Higgs boson:



additional diagrams involving Higgs boson:

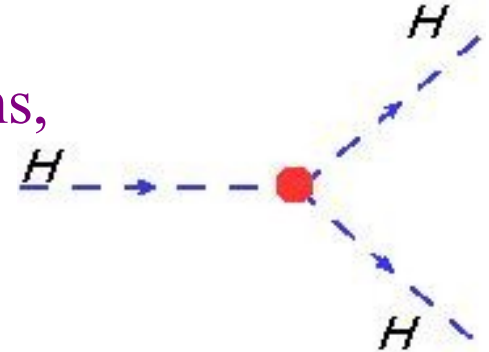


Cross section for WW scattering remains finite

Higgs must not be too heavy:: $< \sim 1 \text{ TeV}/c^2$

Theoretical constraints M_H loop corrections

Higgs propagator affected by higher order corrections,
in particular from Higgs self couplings



$$m^2(p^2) = m_0^2 + \underbrace{\text{triangle loop}}_{J=1, \text{H}} + \underbrace{\text{bubble loop}}_{J=1/2} + \underbrace{\text{self-energy loop}}_{J=0, \text{H}}$$

Higgs in loops couples to itself with strength proportional to Higgs mass \rightarrow

large corrections to Higgs mass and Higgs potential

- high Higgs mass: Higgs self-coupling becomes strongly interacting,
breakdown of perturbation theory
- small Higgs mass: Higgs self-coupling becomes negative
 \rightarrow unstable vacuum
(in contradiction to age of universe)

Theoretical constraints on M_H – vacuum stability

Example of renormalized Higgs Potential

Casas, Espinosa, Quiros (1996)

Loops (in particular top) drive quartic coupling negative above some scale Λ for small M_H

→ renormalized **Higgs potential**

is either **unbounded** (inconsistent with the existence of the universe)

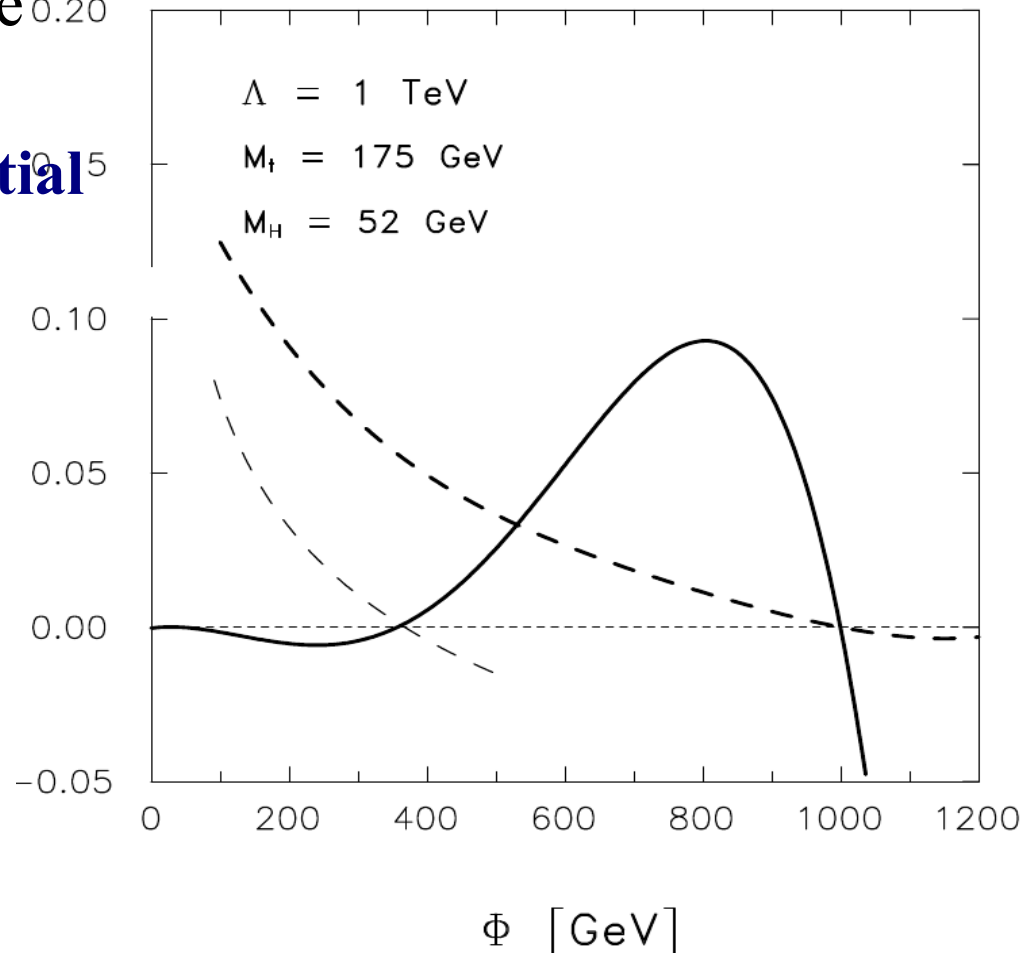
or in local minimum

“false vacuum”,

(assume tunneling time

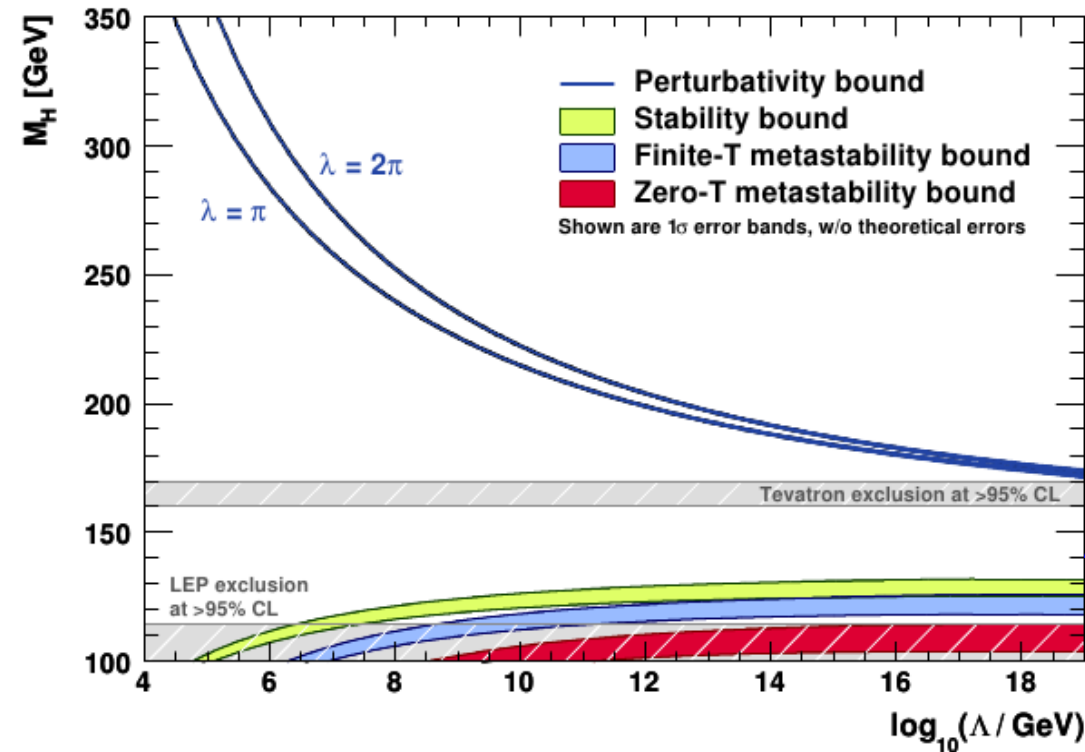
> 14 billion years to

derive limit on M_H)

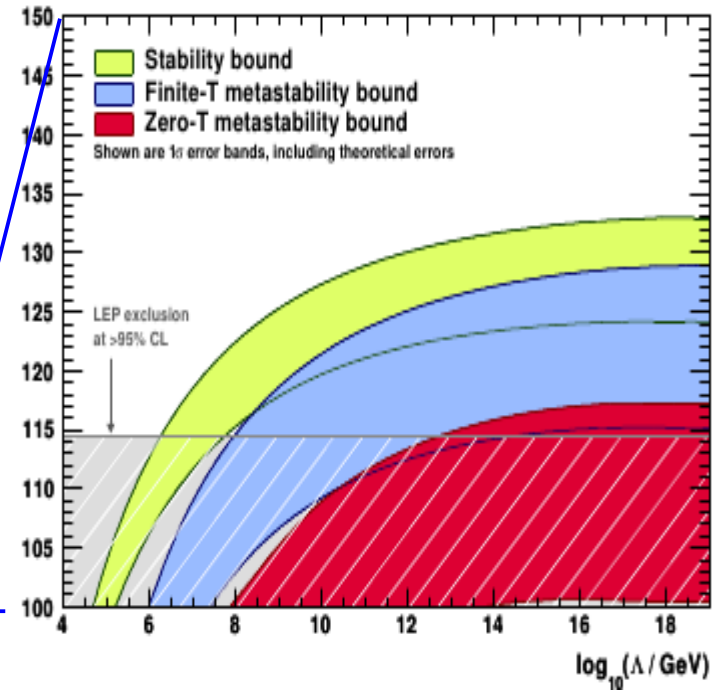


Theoretical constraints on M_H – vacuum stability

→ beyond energy scale Λ new Physics must be present



J. Ellis et al. CERN-PH-TH/2009-058



→ corridor of allowed values of Higgs mass within Standard Model

Final remarks

We know that the Standard Model is incomplete:

- no candidate for dark matter
(would have to be heavy, stable and only weakly interacting)
- no explanation of “dark energy”
- inclusion of gravity causes problems
(Why are Higgs mass and Boson masses so “small” compared to the relevant scale, the Planck mass?)

There must be new physics, even if the Higgs boson will be found one day at a mass making the Standard Model self-consistent).

Summary:

Standard Model looks fine and gauge couplings and Higgs mechanism appear to work, although the model is clearly incomplete.

To be done:

- discovery of Higgs boson
- determination of Yukawa couplings to fermions
- test Higgs potential via measurement of Higgs self couplings

Will the model show inconsistencies and thus point to way to extend it, or will it remain to be a self-consistent theory for the part of reality it describes ?