Interference Effects in the MSSM.

A Generalised Narrow-Width Approximation.

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in collaboration with
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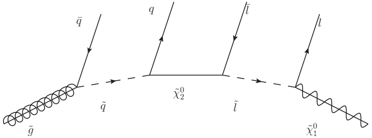
IRTG PhD Days 2012

HELMHOLTZ
GEMEINSCHAFT

Motivation

Useful approximation

- SUSY: extended spectrum $\rightarrow$ typical cascade decays
- many-particle final state not always technically feasible
- $\rightsquigarrow$ simplified by factorisation into **production**$\times$**decay**
- application in MC generators



$$\sigma_{production} \times BR_1 \times BR_2 \times BR_3 \times BR_4$$

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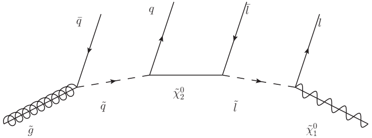
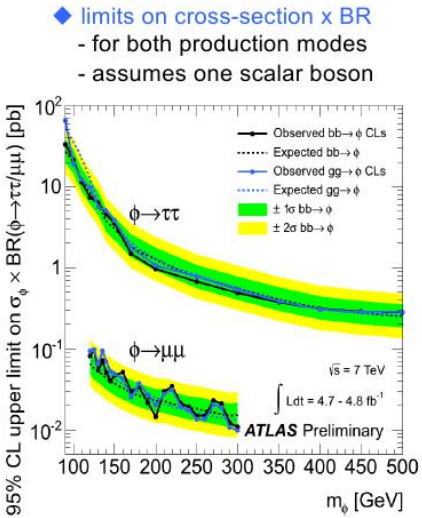


$$\sigma_{production} \times BR_1 \times BR_2 \times BR_3 \times BR_4$$

Extension necessary!

include interfering diagrams for intermediate particles with similar masses



Schumacher, DESY theory workshop 2012

Application of NWA

- useful:
  $\sigma(pp \to \phi \to \tau\tau/\mu\mu)$
  $\approx \sigma_\phi \cdot BR(\phi \to \tau\tau/\mu\mu)$

Limitations of $\sigma_\phi \cdot BR_\phi$

- several scalar bosons?
- similar masses
- interference possible

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> **Generalised Narrow-Width Approximation (NWA)**

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- Generalised NWA including interference term

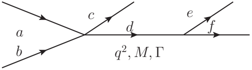
> **Tree-level results for example process** $\Gamma(\chi_4^0 \to \chi_1^0 \tau^+ \tau^-)$

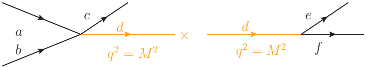
> **Towards a generalised NWA at 1-loop**

> **Conclusion**

Standard Narrow-Width Approximation (NWA)

generic example:
$ab \xrightarrow{d} cef$





Factorisation of the n-particle phase space $d\Phi_n$

- $d\Phi_n \equiv dlips\,(P; p_1, ..., p_f) = (2\pi)^4 \delta^{(4)}(P - \sum_{f=1}^{n} p_f) \prod_{f=1}^{n} \frac{d^3 p_f}{(2\pi)^3 2E_f}$
- here: kinematics of 3-body decay $\rightarrow$ 2-body
  $d\Phi = dlips(\sqrt{s}; p_c, p_e, p_f) = dlips(\sqrt{s}; p_c, q) \frac{dq^2}{2\pi} dlips(q; p_e, p_f)$

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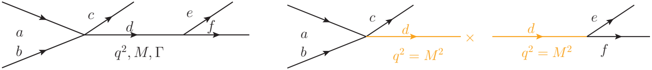

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Production $\times$ decay

- on-shell production of particle with mass M, and subsequent decay:
  $$\sigma_{ab\to cef} \approx \sigma_{ab\to cd}(q^2 = M^2) \cdot BR_{d\to ef}$$
- error of order $\mathcal{O}\left(\frac{\Gamma}{M}\right)$ [Uhlemann, Kauer '09]

Conditions and limitations for the NWA

Factorisation into production and decay IF

- narrow width $\Gamma \ll M$ (off-shell effects: s. Daniel Wiesler's talk)
- propagator separable from matrix element
- decaying particle more massive than daughter particles
- no threshold effects: $\sqrt{s} > M + m$
- no interference with other processes

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Breakdown for mass degeneracy: Breit-Wigner overlap

- NWA not applicable for $|M_i - M_j| \leq \Gamma_i, \Gamma_j$
- MSSM: for some parameters, h^0, H^0, A^0 have similar masses
- also relevant for other models
- extension of NWA required for interference term [Fowler, PhD Thesis '10]

Generalised NWA with interference term

$$\sigma(ab \to cef) = \frac{1}{F} \int d\Phi \left(\frac{|\mathcal{M}(ab \to ch)|^2 |\mathcal{M}(h \to ef)|^2}{(q^2 - M_h^2)^2 + M_h^2 \Gamma_h^2} + \frac{|\mathcal{M}(ab \to cH)|^2 |\mathcal{M}(H \to ef)|^2}{(q^2 - M_H^2)^2 + M_H^2 \Gamma_H^2} \right.$$

$$\left. + \, 2Re \left\{ \frac{\mathcal{M}(ab \to ch) \mathcal{M}^*(ab \to cH) \mathcal{M}(h \to ef) \mathcal{M}^*(H \to ef)}{(q^2 - M_h^2 + iM_h\Gamma_h)(q^2 - M_H^2 - iM_H\Gamma_H)} \right\} \right)$$

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3 steps for approximation of interference term

- matrix elements on-shell $\mathcal{M}(q^2 = M^2)$, but phase space $\Phi(q^2)$
- phase space on-shell $\Phi(q^2 = M^2)$
- approximation $M_h \approx M_H$: interference term as R-factors

$$\sigma \approx \sigma_{P_1} BR_1 \cdot (1 + R_1) + \sigma_{P_2} BR_2 \cdot (1 + R_2)$$
$$R_i = R_i(M, \Gamma, \sigma_P, BR, couplings, I)$$
$$I = \int \frac{dq^2}{2\pi} \Delta_1^{BW}(q^2) \cdot \Delta_2^{*BW}(q^2)$$

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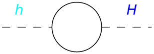
Intermezzo: MSSM Higgs sector

Higgs mixing

- MSSM: 2 complex Higgs doublets $\mathcal{H}_1, \mathcal{H}_2 \quad \to \underbrace{h^0, H^0}_{\mathcal{CP}+}, \underbrace{A^0}_{\mathcal{CP}-}, H^\pm$
- complex parameters: at loop level also $\not{\mathcal{CP}}$ mixing
- correct on-shell properties of external Higgs bosons: $\hat{Z}_{ij}$



$$\Gamma_{h_i}^{(Z)} = \hat{Z}_{h_i h}\Gamma_h + \hat{Z}_{h_i H}\Gamma_H + \ldots$$

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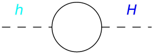

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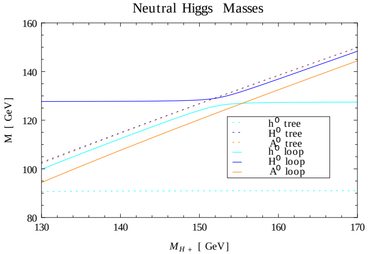
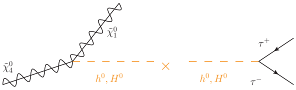


higher orders in the Higgs sector very relevant
program: `FeynHiggs`

Example process: $\Gamma(\tilde{\chi}_4^0 \to \tilde{\chi}_1^0 \tau^+ \tau^-)$



- $\mathbb{R}$MSSM ($\mathcal{CP}$):
  $h^0 - H^0$
- high $\tan\beta = 50$

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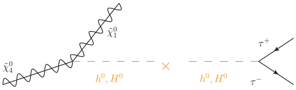

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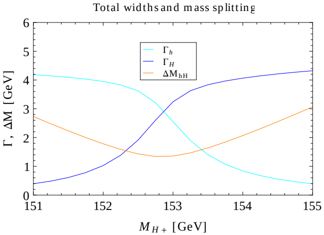

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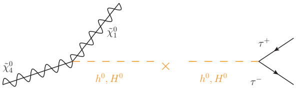

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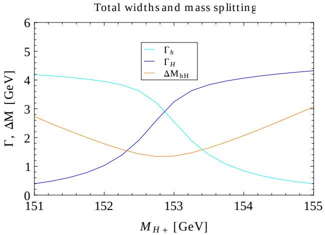

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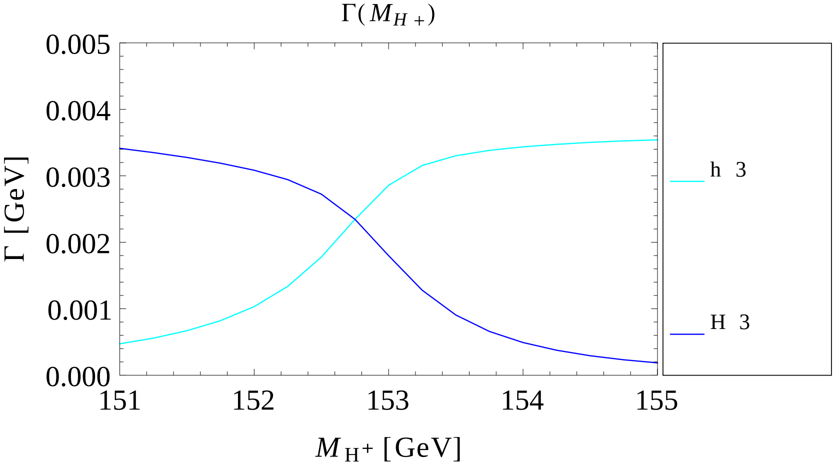
Calculation

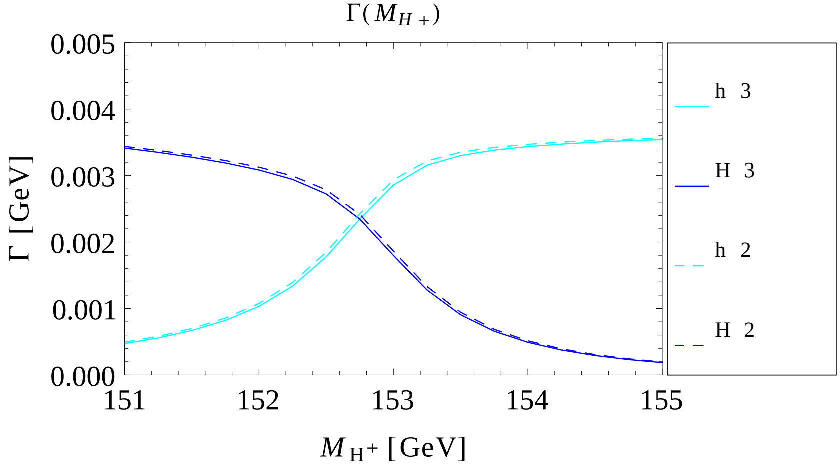
- 2-/3-body decays with FeynArts/ FormCalc
- interference term implemented in different approximations
- tree-level amplitudes with Breit-Wigner propagators
- 2-loop Higgs masses and widths from FeynHiggs

Decay width $\Gamma(\tilde{\chi}_4^0 \to \tilde{\chi}_1^0 \tau^+ \tau^-)$ depending on M_{H^+}



3-body decays with h^0 and H^0

Decay width $\Gamma(\tilde{\chi}_4^0 \to \tilde{\chi}_1^0 \tau^+ \tau^-)$ depending on M_{H^+}



dashed: 2-body decays×BR with h^0 and H^0

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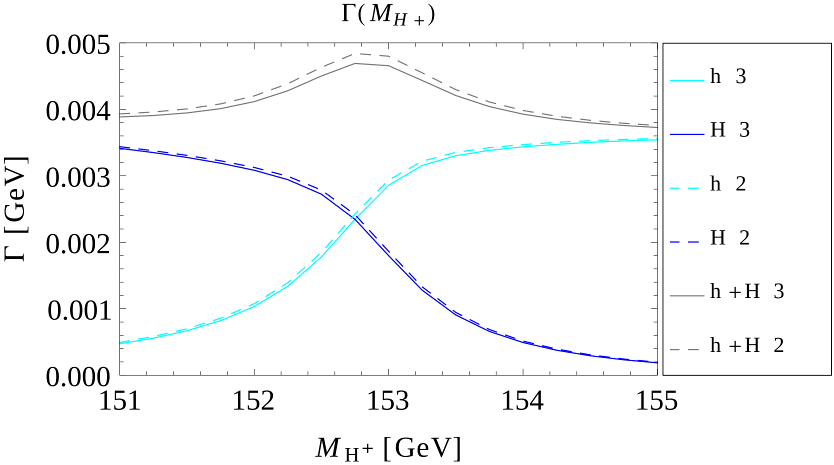
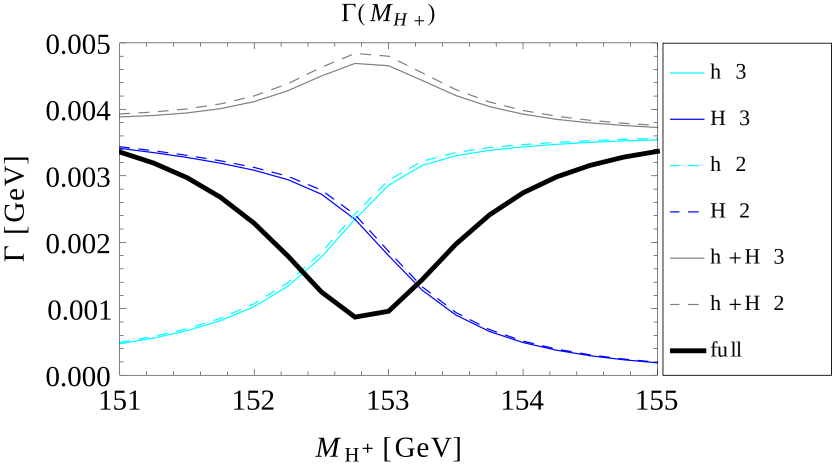


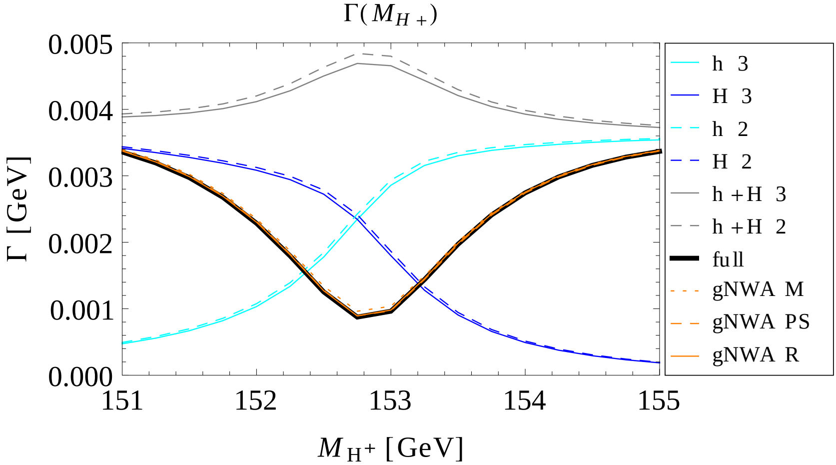
incoherent sum $|h^0|^2 + |H^0|^2$ of 3-/2-body decays

Decay width $\Gamma(\tilde{\chi}_4^0 \to \tilde{\chi}_1^0 \tau^+ \tau^-)$ depending on M_{H^+}



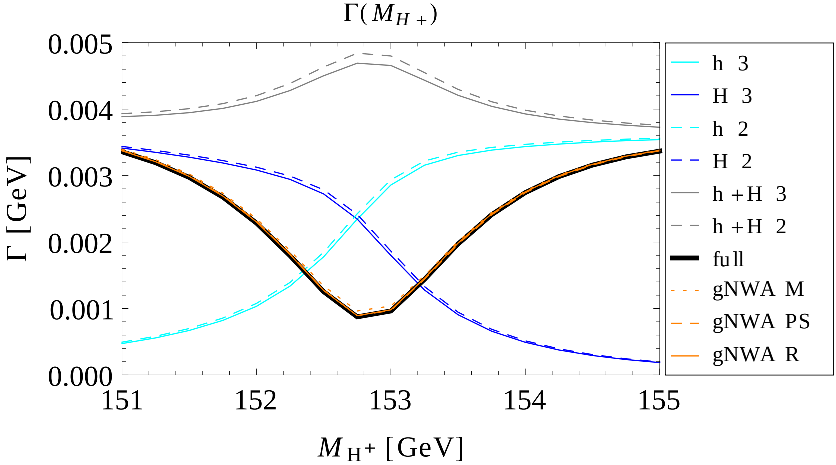
full 3-body decay $|h^0 + H^0|^2$ including interference term

Decay width $\Gamma(\tilde{\chi}_4^0 \to \tilde{\chi}_1^0 \tau^+ \tau^-)$ depending on M_{H^+}



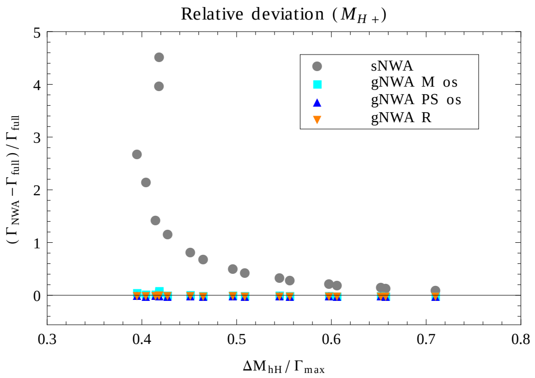
generalised NWA including interference term

Decay width $\Gamma(\tilde{\chi}_4^0 \to \tilde{\chi}_1^0 \tau^+ \tau^-)$ depending on M_{H^+}



large interference effect neglected in **sNWA**, but well approximated by **gNWA**

Evaluation of the interference term

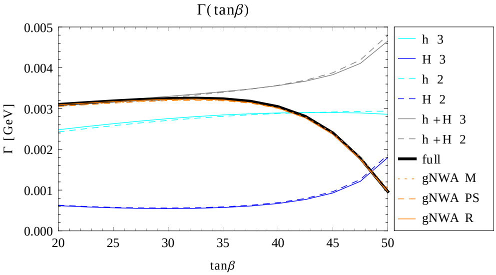


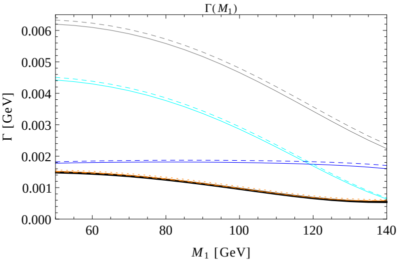
$$\frac{\Delta M_{hH}}{\Gamma_{max}} \equiv \frac{M_H - M_h}{max\{\Gamma_h, \Gamma_H\}}$$

Improvement of accuracy

- standard NWA: overestimation of full width by up to factor 5
- generalised NWA: approximation of interference term to $2-3\%$

Dependence on $\tan\beta$ and M_1





Parameter dependence of the interference term

- more sensitive to parameters from Higgs sector than from neutralino sector
  - interference effect large only at high $\tan\beta$ (smaller mass difference)
  - for fixed $M_{H^+}, \tan\beta$: large effect throughout M_1 interval
- good performance of the generalised NWA in the analysed parameter space

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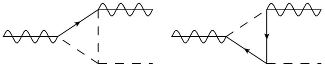
Vertex corrections

Motivation

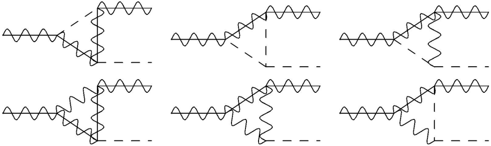
radiative corrections to sub-processes possibly relevant

vertex corrections to $\Gamma(\tilde{\chi}_4^0 \to \tilde{\chi}_1^0 h^0/H^0)$:

Triangles with (s)fermions



Triangles with Higgs and gauge bosons and their superpartners



1-loop integrals with `LoopTools`

Renormalisation scheme

Higgs sector: $\overline{DR}$ [Frank, Hahn, Heinemeyer, Hollik, Rzehak, Weiglein '07]

- problem: dimensional regularisation violates USY
- dimensional reduction: SUSY-preserving modification with: spacetime, momenta in D; fields, γ's in 4 dimensions

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neutralino sector: on-shell [Fowler, Weiglein '09] [Bharucha, Heinemeyer, Pahlen, Schappacher '12]

- 3 parameters $|M_1|, |M_2|, |\mu|$
  $\rightarrow$ fix 3 out of 6 neutralino and chargino masses
- loop-corrected masses: $M_{\tilde{\chi}_i} = m_{\tilde{\chi}_i} + \Delta m_{\tilde{\chi}_i}$

added counterterms to `FeynArts`

Higher-order corrections in the generalised NWA

> separate calculation of loop corrections to production and decay

> approximation of interference term based on NLO matrix elements

> calculate Γ, M and couplings at high precision (e.g. FeynHiggs)

$\Longrightarrow$

combination of higher-order corrections to sub-processes in **generalised NWA**

$\Longrightarrow$ applicable also for processes for which the complete calculation of higher-order corrections is difficult or impossible

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Summary: significant improvement of accuracy by generalised NWA

- example: decay $\tilde{\chi}_4^0 \overset{h^0,H^0}{\rightarrow} \tilde{\chi}_1^0 \tau^+ \tau^-$
  - standard NWA overestimates Γ at $\Delta M_{hH} \leq \Gamma_{h,H}$ by up to a factor of 5
  - generalised NWA approximates full width to a few percent accuracy
- approximation extended to include loop-corrections

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Outlook: $\mathcal{CP}$-violating mixing, more complicated processes

- $\mathbb{C}$MSSM, $\mathcal{CP}$-violation $\Rightarrow H^0 - A^0$ interference
- relevant for $\sigma_{H^0} + \sigma_{A^0} \xrightarrow{\not{\mathcal{CP}}} \sigma_{H^0+A^0} \approx (2\sigma_{H^0})_{\mathcal{CP}} + \sigma_{int,\not{\mathcal{CP}}}$
- combination of most advanced results for production and branching ratios with appropiate prediction for the interference term

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$$\left. + 2Re\left\{ \frac{\mathcal{M}(ab \to ch)\mathcal{M}^*(ab \to cH)\mathcal{M}(h \to ef)\mathcal{M}^*(H \to ef)}{(q^2 - M_h^2 + iM_h\Gamma_h)(q^2 - M_H^2 - iM_H\Gamma_H)} \right\} \right)$$

$$\overset{\mathcal{M}\ on-shell}{\approx} \sigma_{ab\to ch} BR_{h\to ef} + \sigma_{ab\to cH} BR_{H\to ef}$$

$$+ \frac{2}{F}\mathrm{Re}\left\{ \int \frac{dq^2}{2\pi} \left(\Delta_1^{BW}(q^2)\Delta_2^{*BW}(q^2) \left[\int d\Phi_P(q^2)\mathcal{M}_{P_1}(M_1^2)\mathcal{M}^*_{P_2}(M_2^2) \right] \right.\right.$$

$$\left.\left. \left[\int d\Phi_D(q^2)\mathcal{M}_{D_1}(M_1^2)\mathcal{M}^*_{D_2}(M_2^2) \right] \right) \right\}$$

$$\overset{M_h \simeq M_H}{\approx} \sigma_{P_1}\, BR_1 \cdot (1 + R_1) + \sigma_{P_2}\, BR_2 \cdot (1 + R_2)$$

$$R_i := 2M_i\Gamma_i w_i \cdot 2\mathrm{Re}\{x_i I\}$$

$$w_i := \frac{\sigma_{P_i}\, BR_i}{\sigma_{P_1}\, BR_1 + \sigma_{P_2}\, BR_2}$$

$$x_i := \frac{g_{P_i} g^*_{P_j} g_{D_i} g^*_{D_j}}{|g_{P_i}|^2 |g_{D_i}|^2} \qquad (g_{P/D} : \text{couplings in production/ decay})$$

Masses and widths

Higgs masses and widths depending on $\tan\beta$ and M_1