Linear Colliders and the Furry Picture

how to deal with strong external fields

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Based on a project with A.F. Hartin and G.A. Moortgat-Pick

Outline

1. Future linear colliders
2. Strong field effects at the Interaction Point (IP)
3. Furry Picture (FP)
4. Some results of FP calculations
5. Conclusions and outlook

Future e^+e^- linear colliders

ILC and **CLIC**:

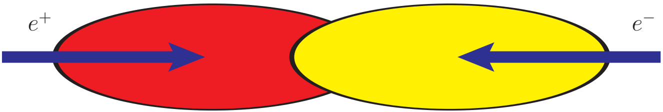
- ILC 90 GeV - 1.5 TeV
  CLIC 500 GeV - 3 TeV
- $\mathcal{L} \sim 10^{34} - 10^{35}$ cm^{-2}s^{-1}
- clean

Natural steps after LHC:

- For precision physics: Higgs, top, gauge bosons.
- Discovery of new physics BSM.

Future colliders: the Interaction Point (IP)

intense charge bunches



Future colliders: the Interaction Point (IP)

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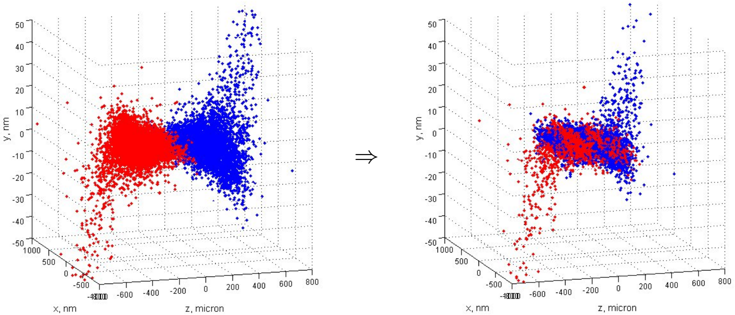
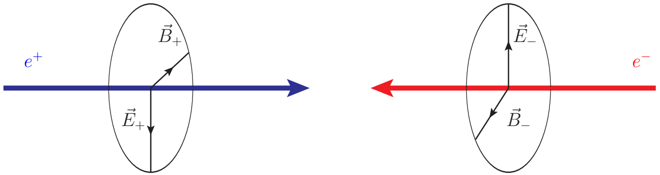


Walker'03

Future colliders: Strong fields at the IP

intense charge bunches $\longrightarrow$ strong field associated

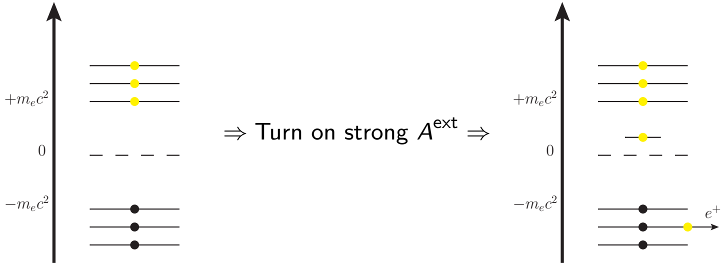


To a good approximation e^+ and e^- see 2 *almost* anticollinear **constant crossed fields**.

$$|\mathbf{E}| = |\mathbf{B}| \qquad \mathbf{E} \cdot \mathbf{B} = 0$$

static: em wave with infinite period of oscillations $\Rightarrow$ approximated as a classical field.

Schwinger critical field: $E_c = 1,3 \cdot 10^{18}$ V/m.



Vacuum is polarized.

χ **parameter**: $$\chi \equiv \gamma \frac{B}{B_c} = \frac{e|\vec{a}|}{m_e E_c}(k \cdot p)$$

CLIC-3TeV: $E = 10^{12}$ V/m, $\chi_{av} = 3.34$, $\chi_{max} = 10.9$.

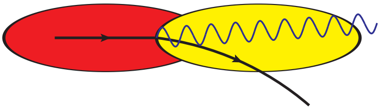
Quantum effects at the beam IP

Strong external fields

$\Downarrow$

quantum effects:

- Beamstrahlung
- Coherent Pair Production: int. with the collective field $\longrightarrow$ dominant at **CLIC**
- Incoherent Pair Production: int. with individual particles $\longrightarrow$ dominant at **ILC**



Usual treatment of strong field quantum effects

The previous quantum effects are presently estimated with approximations:

- Baĭer-Katkov approximation $\longrightarrow$ beamstrahlung & coherent pair production

The electron orbit is treated classically BUT the emission of a photon is a quantum process

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The previous quantum effects are presently estimated with approximations:

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The electron orbit is treated classically BUT the emission of a photon is a quantum process

- Equivalent photon approximation (EPA) $\longrightarrow$ incoherent pair production

Virtual photons are considered real.

In both the approximations e^+ and e^- only see one external field, the incoming one.

The next Linear Colliders programme requires highly precise knowledge of the processes at the IP:

- "2 *almost* anticollinear" external fields, due to deflection angle and other effects.
- EPA and Baĭer-Katkov approximations effective only if there is not significant transverse momentum.
- Then, analytically exact treatment of the external fields would be needed, even if time consuming.

Coming back,

Vacuum polarized by the strong "classical" external field $\Longrightarrow$ **Natural application for Furry Picture**

Furry Picture

[Furry51], [Moortgat-Pick09]

Interaction Picture: $\mathcal{H}_I = \mathcal{H}_0 + \mathcal{V}$

Furry Picture (FP): $\mathcal{H}_F = \mathcal{H}_0 + \mathcal{H}_{ext} + \mathcal{V} = \mathcal{H}_B + \mathcal{V}$

The external field is treated **classically**, not included in the interaction potential $\mathcal{V}$

Eigenstates in FP $\Rightarrow$ bound states of the electron in the external field.

Furry Picture

[Furry51], [Moortgat-Pick09]

Interaction Picture: $\mathcal{H}_I = \mathcal{H}_0 + \mathcal{V}$

Furry Picture (FP): $\mathcal{H}_F = \mathcal{H}_0 + \mathcal{H}_{ext} + \mathcal{V} = \mathcal{H}_B + \mathcal{V}$

related by a canonical transformation:

$$\Psi_F(x) = M^{-1}\Psi_I(x)M \qquad \Psi_F^\dagger(x) = M^{-1}\Psi_I^\dagger(x)M$$

Different basis system: $\lim_{A_\mu^{\mathrm{ext}} \to 0}\{\Psi_F, \Psi_F^\dagger\} = \{\Psi_I, \Psi_I^\dagger\}$

Furry Picture

In the F.P. the QED Lagrangian:

$$\mathcal{L}_F = \bar{\psi}(i\partial\!\!\!/ - eA\!\!\!/^{\text{ext}} - m)\psi - \frac{1}{4}FF - e\bar{\psi}A\!\!\!/\psi$$

modified Dirac equation:

$$(i\partial\!\!\!/ - eA\!\!\!/^{\text{ext}} - m)\psi = 0$$

Furry Picture

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Volkov solution [Volkov35]:

$$\Psi_p^V(k\cdot x) = \frac{1}{\sqrt{(2\pi)^3 2\epsilon_p}} E_p(k\cdot x) u_p$$

with

$$E_p(k\cdot x) \equiv \left(1 - \frac{eA\!\!\!/^{\text{ext}}k\!\!\!/}{2(k\cdot p)}\right)\exp\left[-i(p)\cdot x - i\int_0^{(k\cdot x)}\left[\frac{e(A^{\text{ext}}(\phi)\cdot p)}{(k\cdot p)} - \frac{e^2A^{\text{ext}}(\phi)^2}{2(k\cdot p)}\right]d\phi\right]$$

Furry Picture

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They constitue an orthogonal and complete system [Ritus72].

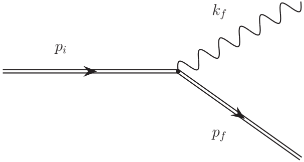
Furry Picture: QED Feynman Rules

- Fermion Green function:

$$G(x,x') = \frac{1}{(2\pi)^4}\int_{-\infty}^{+\infty} d^4p\, E_p(k\cdot x)\frac{\not{p}+m}{p^2-m^2}\overline{E}_p(k\cdot x')e^{ip\cdot(x'-x)}$$

- Photon propagator unchanged.

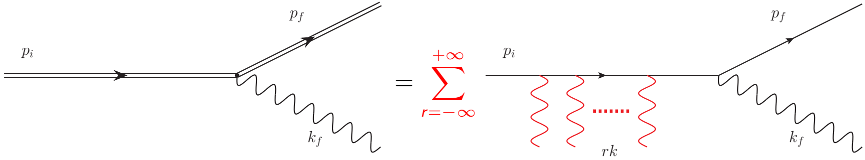
- QED vertex in momentum space:



$$\Gamma^e_\mu = (2\pi)^4 \sum_{r=-\infty}^{+\infty} \overline{E}_{p_f}(r)\gamma_\mu E_{p_i}(r)\delta^4(p_f + k_f - p_i - r\,k)$$

Interpretation of FP 1-vertex process

Momentum conservation encoded in $\delta^4(p_f + k_f - p_i - rk)$ allows 1-vertex processes, not permitted in absence of an external field:



Each term of the sum over r can be seen as the absorption or the emission of r photons of the external field [Nikishov64].

Strong field quantum effects with Furry Picture

Beamstrahlung and coherent pair production are 1^{st} order Furry Picture processes:

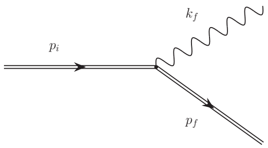


Figure: Beamstrahlung.

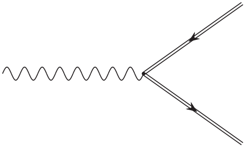


Figure: Coherent pair production.

The incoherent pair production instead is a 2^{nd} order (2-vertices) process:

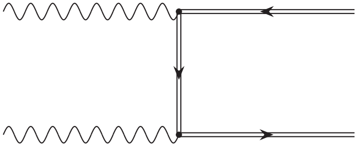


Figure: Incoherent pair production, Breit-Wheeler process.

Straightforward and powerful method

But analytical more complex and lenghty

Results (tree-level): photon emission and pair production

The photon emission by an electron and pair production were studied:

- in a polarized plane electromagnetic wave and in a constant field [Nikishov64].

$$W_{\text{cost}} = -\frac{e^2 m_e}{2} \int_0^{+\infty} \frac{du}{(1+u)^2} \Big[\int dz + \frac{1+(1+u)^2}{z(1+u)} \frac{d}{dz} \Big] \mathrm{Ai}(z) \quad \text{with } z \equiv \Big(\frac{u m_e^2}{\nu (k \cdot p_i)} \Big)^{2/3}$$

- in two collinear, linearly and orthogonally polarized waves [Lyul'ka74].
- in N collinear fields [Hartin11].
- in two constant crossed fields of any orientation [Hartin12].

Observed a dependence of the energy of the radiated photon on the intensity of external field ($\nu = ea/m_e$).

Results (tree-level): W leptonic decays

Volkov solution for W_μ boson:

$$W_\mu(x) = E^W_{p\,\mu\nu}\, e^{-ip\cdot x}\, \epsilon^{W\,\nu}_p$$

with:

$$E^W_{p\,\mu\nu} = \Big(g_{\mu\nu} + \frac{e}{k\cdot p}\int F_{\mu\nu} - \frac{e^2}{2(k\cdot p)^2} A^{\text{ext}\,2} k_{\mu\nu}\Big)\exp\Big[-\frac{i}{2(k\cdot p)}\Big(2e\Big(A^{\text{ext}}\cdot p\Big) - e^2 A^{\text{ext}\,2}\Big)\Big]$$

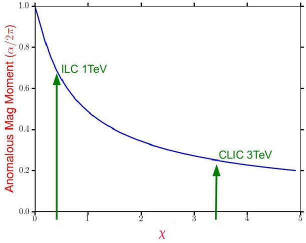
- The partial decay width $\Gamma(W^- \to l^- \bar{\nu}_l)$ in strong external fields has been considered [Kurilin03], revealing important correction $\mathcal{O}(10)$ for $\chi \gg 1$.
- Viceversa a light lepton $(m_l < m_W)$ can decay $l^- \to W^- \nu_l$, taking the necessary energy from the external field.

One-loop results

A few one-loop effects in a costant field at the IP regions have been studied [Ritus70], [Ritus72]:

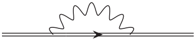
- anomalous magnetic moment:

$$\frac{\Delta\mu}{\mu_0} = \frac{\alpha}{2\pi}\int_0^{+\infty} \frac{dx\, 2\pi}{(1+x)^3}\left(\frac{x}{\chi}\right)^{\frac{1}{3}} \mathrm{Gi}\left(\frac{x}{\chi}\right)^{\frac{1}{3}}$$



Hartin'11

- Photon mass. 

- Shift in the electron mass. 

We want to study systematically one-loop effects and renormalization.

Conclusions and outlook

- In the next e^+e^- linear colliders, the em fields associated to the charge bunches are so strong that their effect at the IP are not negligible.
- Precision physics and search for BSM require these processes to be known as precisely as possible.
- The **Furry Picture** take entirely into account of the effects of the strong external field at the IP.

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Work in progress:

- Analytically calculate 2-vertices Furry Picture processes, ex. $e^+e^- \to W^+W^-$.
- Extend these calculation to the field of two bunches; extend most general shape of external fields, for applications in other contexts, ex. laser or plasma physics.
- Study loop corrections and development of a coherent treatement of renormalization.

Thank for your attention!

Go office 506!

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Backup: ILC and CLIC parameters

	ILC (1 TeV)	CLIC (3 TeV)
$\mathcal{L}$	$4 \cdot 10^{34}$	$3.6 \cdot 10^{34}$
N_{coh}	0	$6.8 \cdot 10^{8}$
N_{inc}	$3.9 \cdot 10^{5}$	$3.8 \cdot 10^{5}$
χ_{av}	0.27	3.34
χ_{max}	0.94	10.9

χ estimated taking into account small bunch dimensions (with `CAIN`).

$$\chi \equiv \frac{2}{3}\frac{\hbar\omega_c}{\epsilon_e} = \gamma\frac{B}{B_c} = \frac{e|\vec{a}|}{m_e E_c}(k \cdot p) = \frac{e}{m_e^3}\sqrt{|(F_{\mu\nu}p^{\nu})^2|}$$

$$\chi_{av} \approx \frac{2Nr_e^2\gamma}{\alpha\sigma_z(\sigma_x + 1.85\sigma_y)} \qquad \chi_{max} \approx \frac{5}{6}\frac{Nr_e^2\gamma}{\alpha\sigma_z(\sigma_x + \sigma_y)}$$

$$E_c = m_e^2/e = 1.32 \cdot 10^{18} \text{ V/m}.$$

Backup: Baier-Katkov or quasiclassical approximation

If

- Ultra-relativistic initial state fermions.

$$+$$

- Energy levels of the fermion states in the external field extremely close together.

$$+$$

- Fermion final states also ultra-relativistic.

$$\Downarrow$$

fermion motion can be considered classical in the external field.

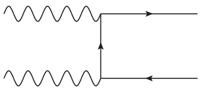
The transition probability is obtained after allowing the operators of the electron motion to commute.

The recoil of the emission of a photon on the electron is taken into account: commutation relations between photon and electron variables are preserved.

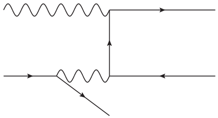
Backup: Equivalent photon approximation (EPA)

The two photons involved in each process:

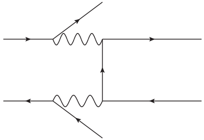
- Breit-Wheeler ($\gamma\gamma \to e^+e^-$):



- Bethe-Heitler ($e^\pm\gamma \to e^\pm e^+e^-$):



- Landau-Lifshitz ($e^+e^- \to e^+e^-e^+e^-$):



Are considered real.

Backup: Furry Picture

Furry Picture is related to the Dirac Picture by a canonical transformation:

$$\Psi_F(x) = M^{-1}\Psi_I(x)M \qquad \Psi_F^\dagger(x) = M^{-1}\Psi_I^\dagger(x)M$$

Ψ_F and Ψ_I spanned by a different basis system so that

$$\{\Psi_F, \Psi_F^\dagger\} \neq \{\Psi_I, \Psi_I^\dagger\}$$

The usual commutation relations are recovered in the limit $A_\mu^{\text{ext}} \to 0$.

Gauge transformation:

$$A^\mu(x) \to A^\mu(x) - \frac{\delta\Lambda(x)}{\delta x^\mu}, \qquad A^\mu_{ext}(x) \to A^\mu_{ext}(x) - \frac{\delta\Lambda_{ext}(x)}{\delta x^\mu}, \qquad \Psi(x) \to e^{-ie\Lambda(x) - ie\Lambda_{ext}(x)}\Psi(x)$$

Backup: dependence on $\nu = ea/m_e$

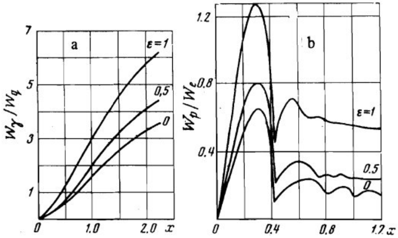


FIG. 1. The probability W_γ of emission of a photon, (a), and the probability W_p of pair production, (b); $\chi = 1$.

Lyul'ka'74

Backup: BSM processes

Strong external em fields affect BSM physics processes as well, opening new channels that allow particles to decays into heavier particles, see [Kurilin99].

$$P(1 \to 2,\, 3) \sim \exp\left[-\frac{2}{3} Z^{3/2}\right]$$

- $A^{\pm} \to B^{\pm} C^{0}$ with $Z_{\pm} = \frac{m_B^2\, u + m_C^2 (1-u) - m_A^2\, u(1-u)}{m_e^2 [\chi\, u^2 (1-u)]^{2/3}}$
- $A^{0} \to B^{+} C^{-}$ with $Z_{0} = \frac{m_B^2\, u + m_C^2 (1-u) - m_A^2\, u(1-u)}{m_e^2 [\chi\, u (1-u)]^{2/3}}$