Scattering Amplitudes in AdS/CFT

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- Introduction: the particle-string connection
- Scattering amplitudes in N=4 SYM and String Theory
- Outlook: hopes

Introduction:
The particle-string connection
- view of a particle physicist

Duality conjecture:

certain QFT's in 4 dim Minkowski space are dual to string theories in 10 dim Anti-de Sitter space

Hope to calculate QFT (QCD?) beyond perturbation theory

N =4 SYM:
- from weak to strong coupling
- is it solvable?

This talk

Search for the duals of QCD (holography)
- Model building on the string side:
- success provide evidence for 'dual of QCD'

Ingo Kirsch et al

N=4 SYM SU(N): most symmetric gauge theory

- Particle content:
  gauge bosons, fermions, scalars
  (in the same representation of the gauge group)
- conformal invariance
- vanishing beta function (no running of coupling)

Different from QCD: maybe similar for high temperature (viscosity)

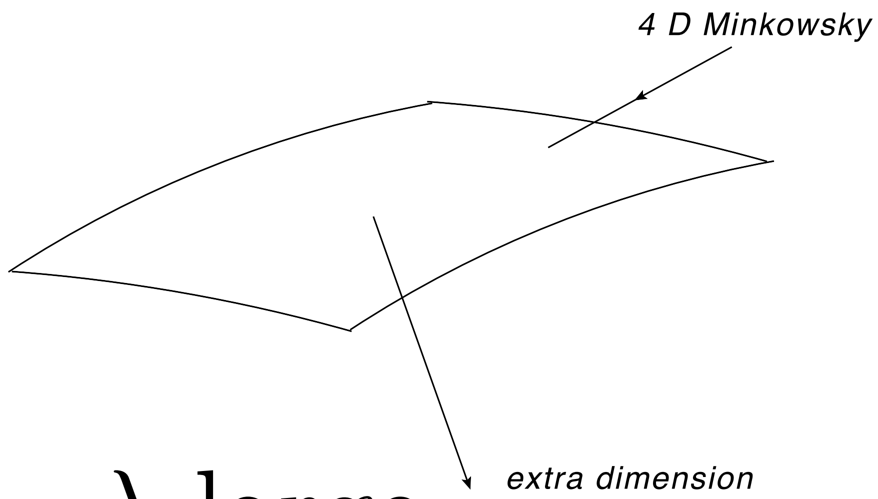
Quantities of interest:
- anomalous dimensions ✓
- scattering amplitudes   under study, this talk

AdS/CFT for N=4 SYM, planar amplitudes

$$\lambda \; finite; N_c \to \infty$$

N=4 gauge theory in D=4
perturbation theory

String theory in $AdS_5 \otimes S_5$



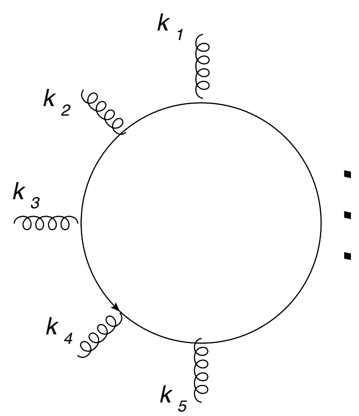
λ small

λ large

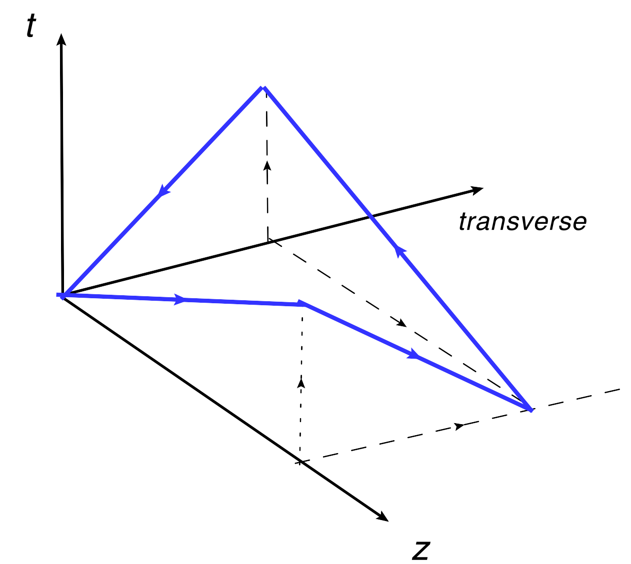
scattering amplitudes in planar limit,
e.g. multigluon $T(p_1, \ldots, p_n)$

Minimal area of polygon

$$T \sim e^{-\frac{\sqrt{\lambda}}{2\pi} \cdot A_{polygon}}$$



Integrability



Importance of integrability - classical mechanics:

Equations of motions:

- In general cannot be integrated analytically, sensitivity to initial conditions.
- Exceptions: integrability (existence of sufficiently many constants of motion) allows closed analytic expressions. Prominent example: Kepler potential

Lesson:

- look for integrable structures inside the scattering amplitudes
- should contain physical observables

Work from both sides: 'bridge'

small λ,
pert.theory

integrability

large λ ,
minimal area

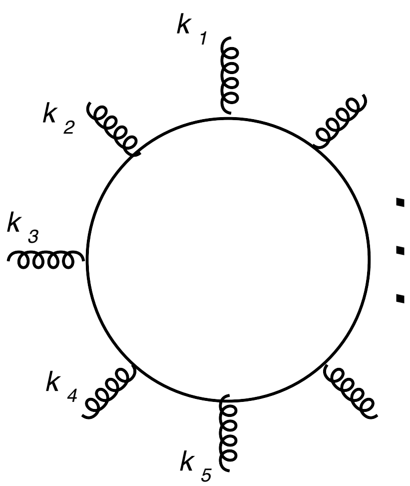
- very intense work in recent years,
- new insight into perturbation theory

much less explored

The weak coupling limit: perturbation theory

The BDS conjecture for the color-stripped, MHV planar scattering amplitude:

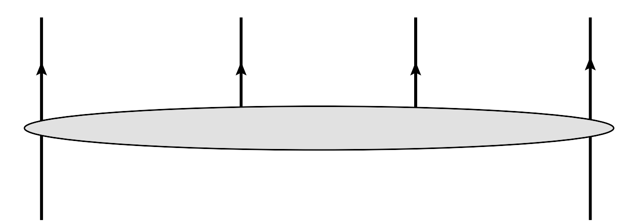
Bern, Dixon, Smirnow



$$T_n = T_n^{tree} \cdot e^{A^{BDS}}, \quad A^{BDS} = A_{sing} + A_{reg}$$

correct for n=4, 5; needs corrections for $n \geq 6$

$$T_n = T_n^{tree} \cdot e^{A^{BDS} + R^{(n)}}$$



Search for the remainder function $R^{(6)}$:

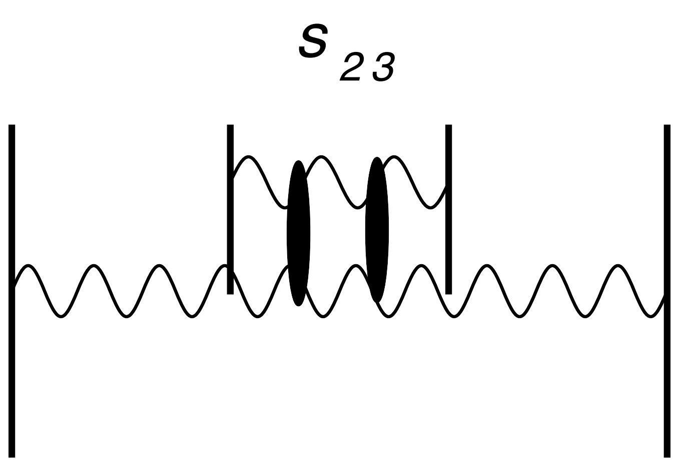
- all-order prediction for multi-Regge limit (LL,NLL)

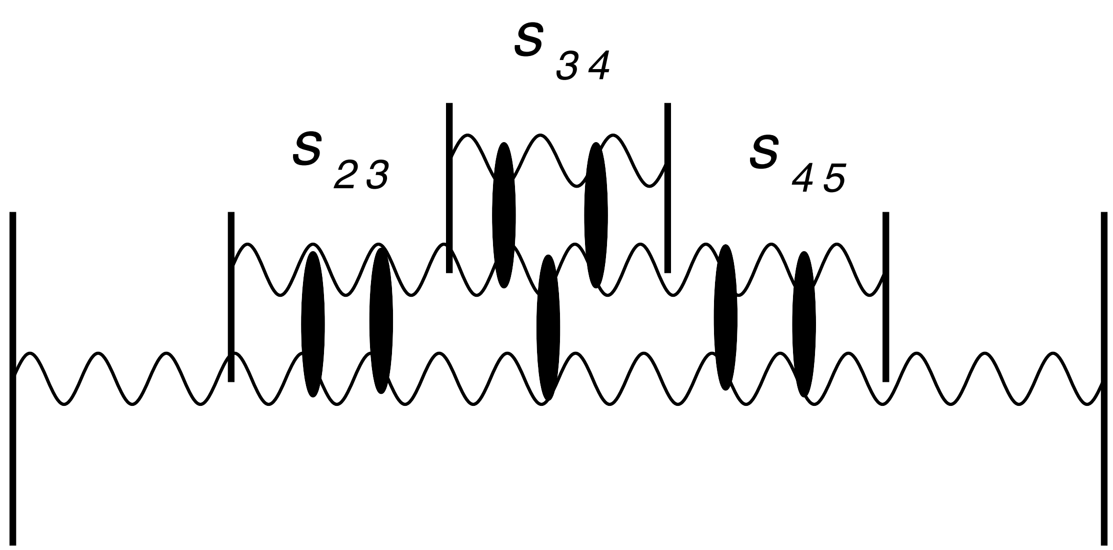
JB, Lipatov, Sabio-Vera, Fadin, Prygarin, Vacca, Kormilitzin

- 2 loop, 3 loop exact calculations

Dixon,..., Goncharov,...

An important detail of the remainder functions $R^{(n)}$: contains Regge cuts (special kinematic region)





BFKL

BKP

$$T_6 \sim s_{23}^{\omega(0,1)}$$

$$T_8 \sim s_{23}^{\omega(0,1)} \; s_{34}^{\omega_3(\nu_1,n_1..)} \; s_{45}^{\omega(0,1)}$$

$$\omega(\nu, n) = \lambda \omega^{(1)}(\nu, n) + \lambda^2 \omega^{(2)}(\nu, n) + \ldots$$

Hamiltonian for n-gluon system is integrable!

Lipatov

Current work on the weak coupling side:

- NLO corrections for the function $\omega(\nu, n)$

B,Fadin, Lipatov, Vacca, Sabio-Vera, Kormilitzin, Prygarin

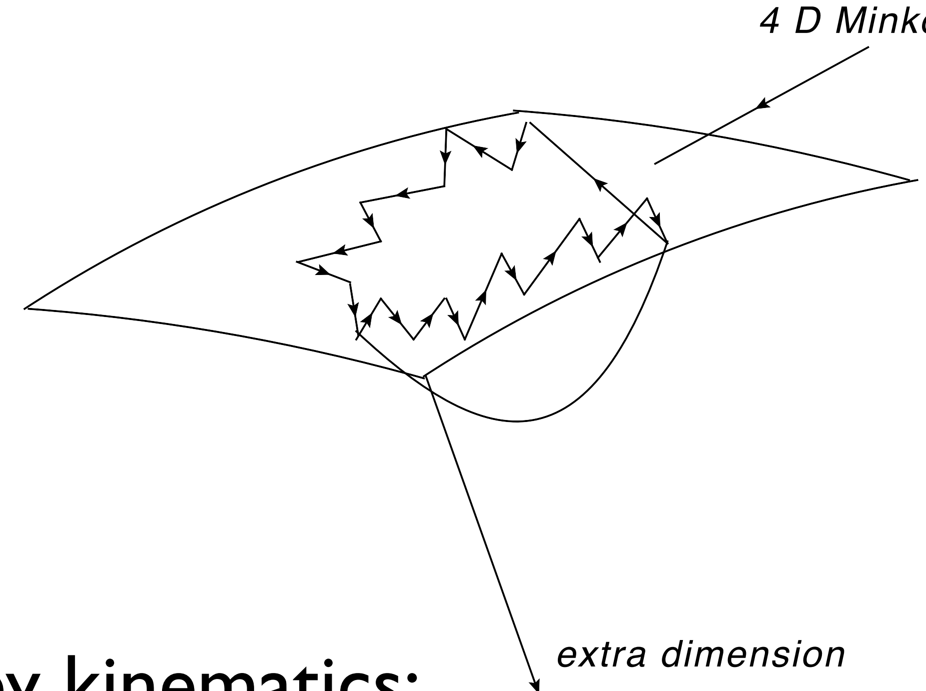
- NLO Hamiltionian for 3 gluon system:
  integrability beyond leading order in N=4 SYM
  evolution of n-gluon systems (3 body kernel)
  (Odderon, spin structure functions,
  unitarization of BFKL Pomeron)

Boels

- High energy behavior in gravity

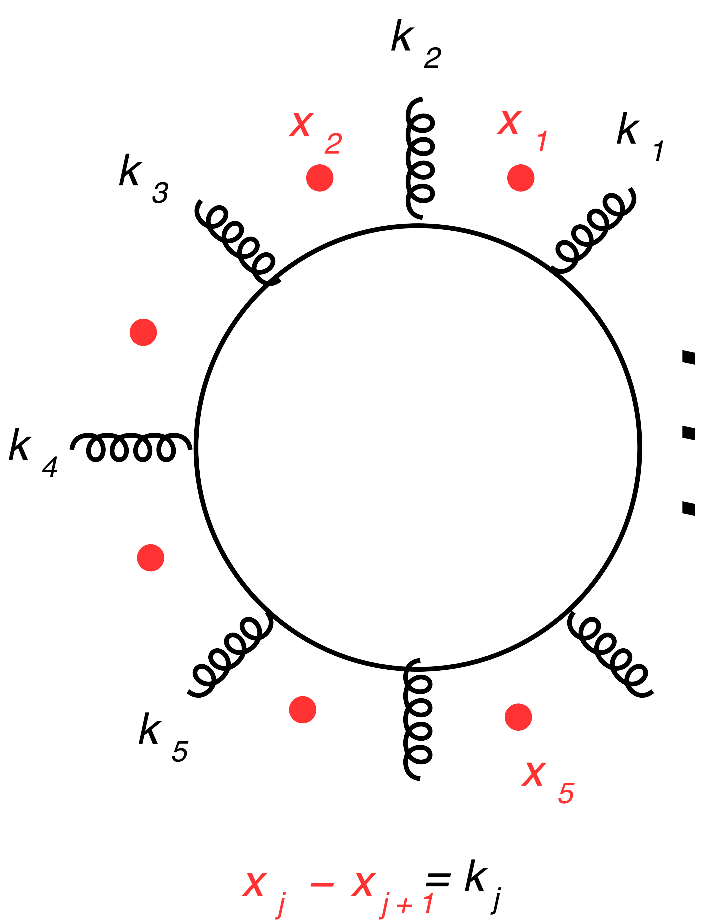
The strong coupling limit: compute minimal areas

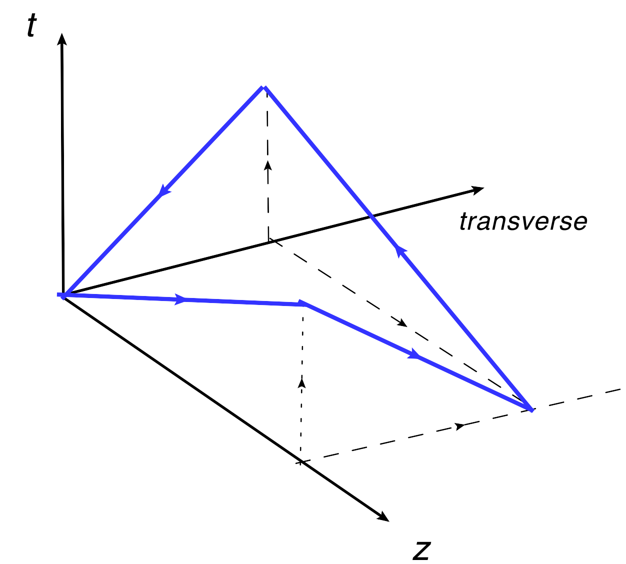
$$AdS_5 \otimes S_5$$



$$T \sim e^{-\frac{\sqrt{\lambda}}{2\pi} \cdot A_{polygon}}$$

Polygon is determined by kinematics:
dual space, conformal invariance





How to find minimal area: solve

- Euler-Lagrange equations (with boundary conditions) or
- Y-equations: integrable quantum system (Maldacena et al)

Y-equations: coupled set of nonlinear equations,
e.g. 6-point remainder function

$$\begin{aligned}\log Y_2(\theta) = & \quad -m\sqrt{2}\cosh(\theta - i\phi) - 2\int_{-\infty}^{\infty} d\theta' K_1(\theta-\theta')\log(1+Y_2(\theta')) \\ & -\int_{-\infty}^{\infty} d\theta' K_2(\theta-\theta')\log((1+Y_1(\theta'))(1+Y_3(\theta'))) \\ \log Y_{2\pm1}(\theta) = & \quad -m\cosh(\theta - i\phi) \pm C - \int_{-\infty}^{\infty} d\theta' K_2(\theta-\theta')\log(1+Y_2(\theta')) \\ & -\int_{-\infty}^{\infty} d\theta' K_1(\theta-\theta')\log((1+Y_1(\theta'))(1+Y_3(\theta'))) \ .\end{aligned}$$

Constants m, C, ϕ contain kinematic variables:
area (A_{free}) is obtained from the Y-functions

Current work on the strong coupling side:

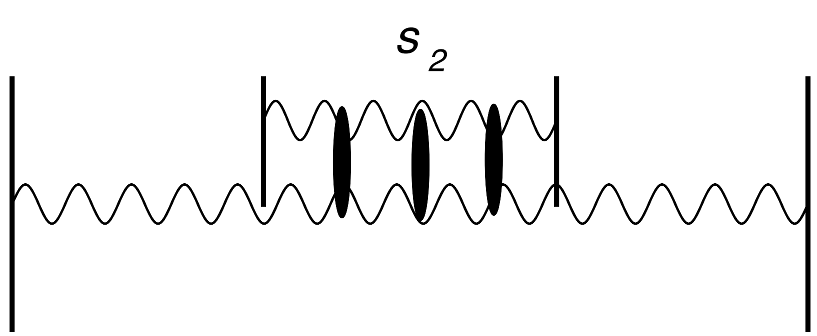
B, Schomerus,Kotanski,Sprenger,Kormelitzin

- solve the Y-equations

Results for n-point R-function:

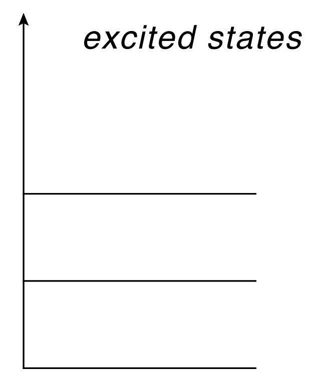
- Multiregge limit allows enormous simplification, approximate solution
- appearance of integrable structure
- result for n=6:

weak coupling,LL



$$\Delta_{2\to 4} \sim \frac{a}{2} s_2^{\omega(0,1)} \cosh C.$$

strong coupling



$$T \sim e^{\frac{\sqrt{\lambda}}{2\pi} R'} \sim s_2^{-\frac{\sqrt{\lambda}}{2\pi} e_2}$$
$$\times \left[\left(\frac{\tilde{u}_1}{\tilde{u}_2}\right)^{-\frac{\sqrt{\lambda}}{\sqrt{2}\pi}} \theta\left(\frac{\tilde{u}_1}{\tilde{u}_2} - 1\right) + \left(\frac{\tilde{u}_2}{\tilde{u}_1}\right)^{-\frac{\sqrt{\lambda}}{\sqrt{2}\pi}} \theta\left(\frac{\tilde{u}_2}{\tilde{u}_1} - 1\right) \right]$$

precise connection between weak and strong coupling result under investigation
generalization to n-point amplitudes

Outlook: what can we expect?

Compute quantities (at large N_c) for small and large coupling:

- anomalous dimensions $\gamma_n(\omega; \lambda)$
- spectrum of BKP states $\omega(\nu, n; \lambda)$
- scattering amplitudes $T(p_1, p_2, \ldots, p_n; \lambda)$ in Regge limit
- $\ldots$

How to get to QCD?