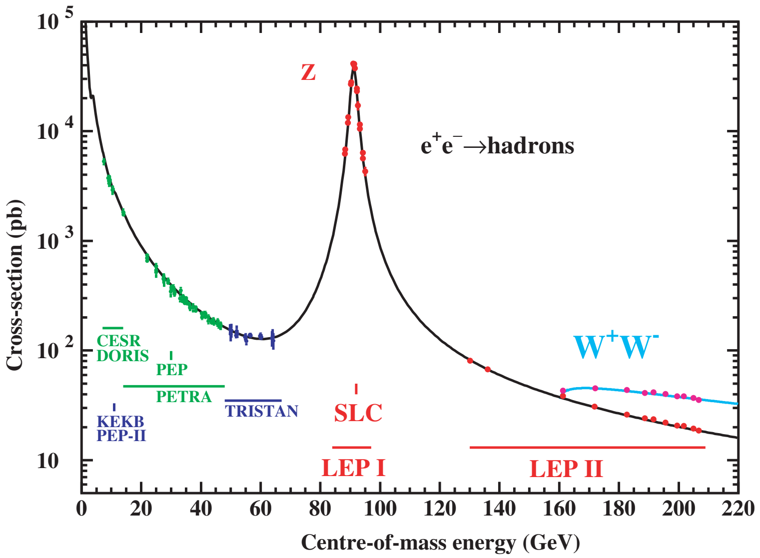
The Electroweak Fit of the Standard Model with a Higgs Boson at 126 GeV



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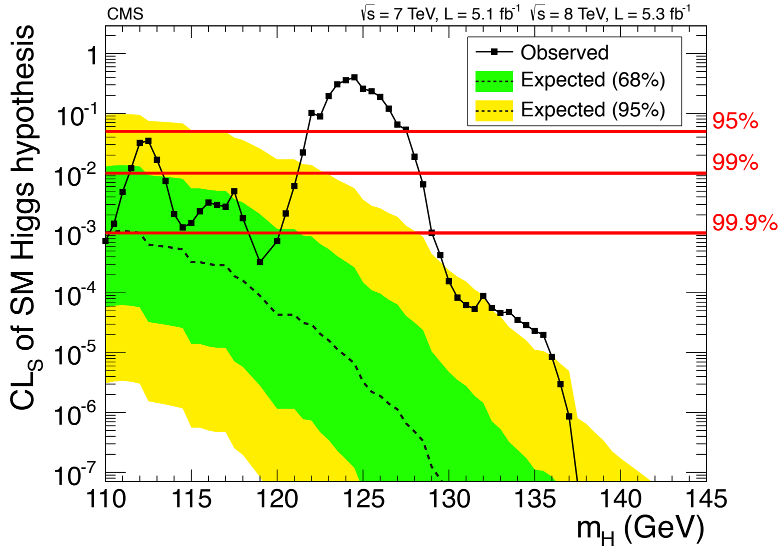
for the Gfitter group



LEXI Meeting

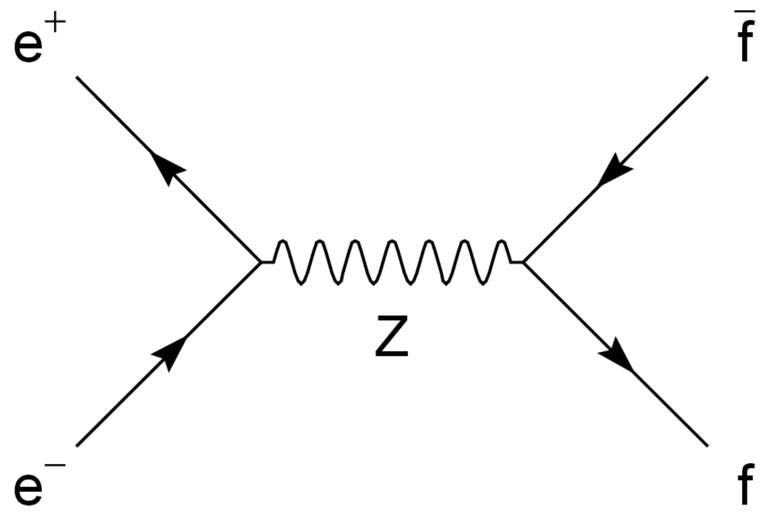
Hamburg, Oct 11, 2012

The Gfitter group: M. Baak (CERN), M. Göbel (Univ. Hamburg, DESY), J. Haller (Univ. Hamburg), A. Höcker (CERN), D. Kennedy (Univ. Hamburg, DESY), R. K. (Univ. Hamburg), K. Mönig (DESY), M. Schott (CERN) J. Stelzer (DESY)

Predictive Power of the SM

Tree level relations for $Z \rightarrow f\bar{f}$

$$g_{V,f}^{(0)} \equiv g_{L,f}^{(0)} + g_{R,f}^{(0)} = I_3^f - 2Q^f \sin^2\theta_W$$
$$g_{A,f}^{(0)} \equiv g_{L,f}^{(0)} - g_{R,f}^{(0)} = I_3^f\,,$$



with the weak mixing angle:

$$\sin^2\theta_W = 1 - \frac{M_W^2}{M_Z^2}$$

Electroweak unification connects the electromagnetic and the weak coupling strengths

$$G_F = \frac{\pi\alpha}{\sqrt{2}(M_W^{(0)})^2\left(1 - \frac{(M_W^{(0)})^2}{M_Z^2}\right)}$$

...and M_W can be expressed in terms of M_Z and G_F

$$M_W^2 = \frac{M_Z^2}{2}\left(1 + \sqrt{1 - \frac{\sqrt{8}\pi\alpha}{G_F M_Z^2}}\right)$$

Electroweak sector of SM is given by three free parameters, for example α, G_F and M_Z

Radiative Corrections

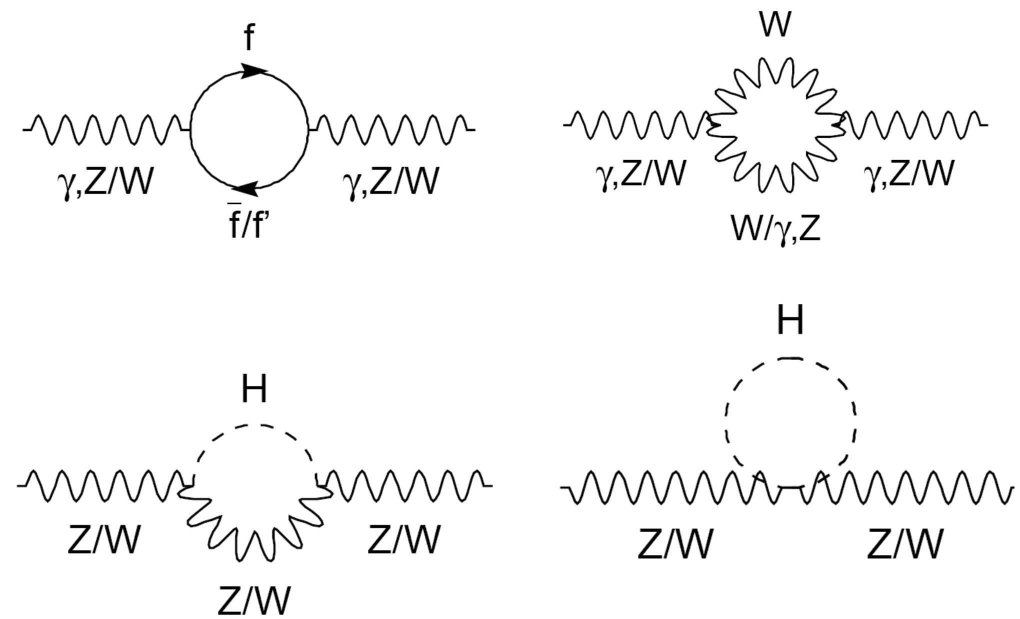
Modification of propagators and vertices

- Parametrisation of radiative corrections: electroweak form factors ρ, κ, Δr
- Effective couplings at the Z-pole:

$$g_{V,f} = \sqrt{\rho_Z^f}\left(I_3^f - 2Q^f \sin^2\theta_{\rm eff}^f\right)$$

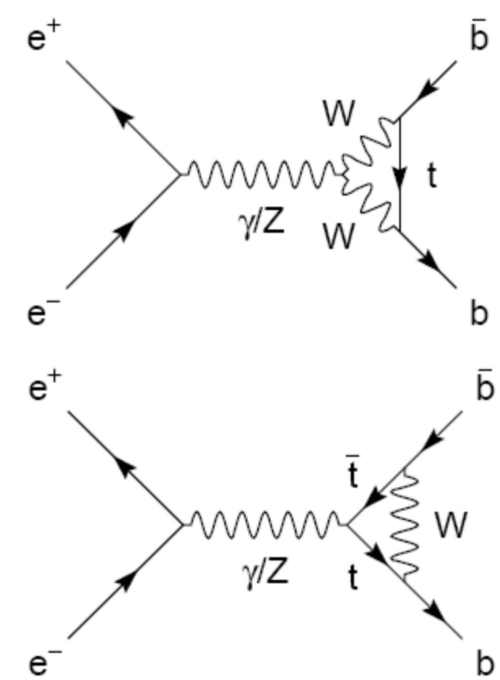
$$g_{A,f} = \sqrt{\rho_Z^f} I_3^f$$

$$\sin^2\theta_{\rm eff}^f = \kappa_Z^f \sin^2\theta_W$$



- Mass of the W boson:

$$M_W^2 = \frac{M_Z^2}{2}\left(1 + \sqrt{1 - \frac{\sqrt{8}\pi\alpha(1-\Delta r)}{G_F M_Z^2}}\right)$$



- ρ, κ, Δr depend nearly quadratically on m_t and logarithmically on M_H

$\boxed{\text{Precision tests and constraints of the SM}}$

Electroweak Fits - History

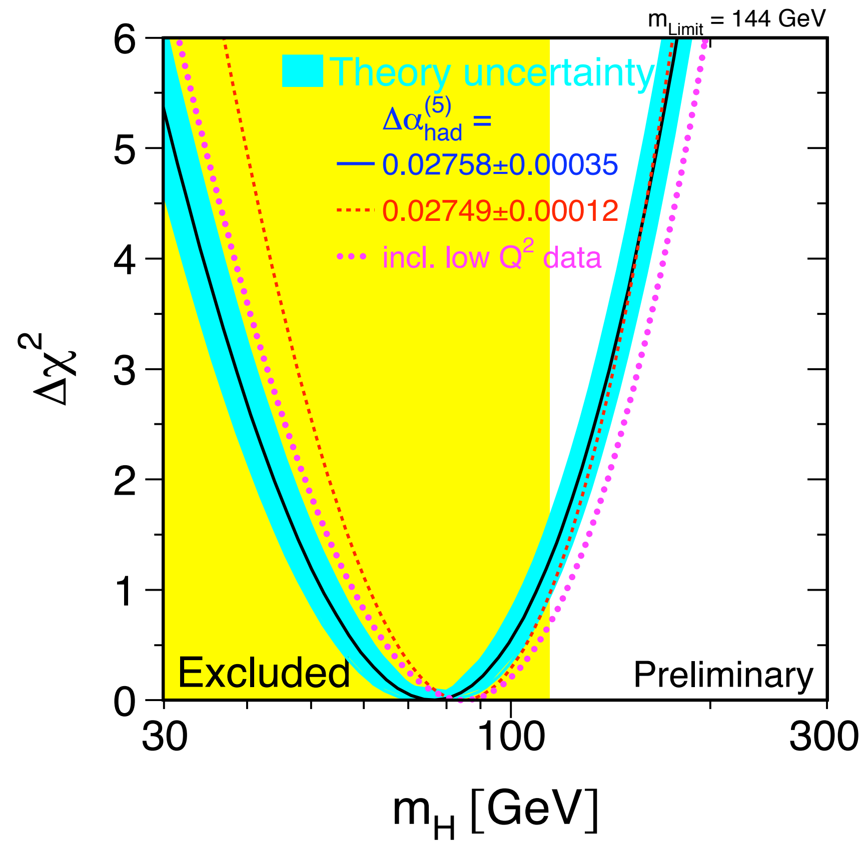
Electroweak Fits to precision data have a long history

- Huge amount of work to precisely understand loop corrections in the SM
- Precise SM predictions and measurements

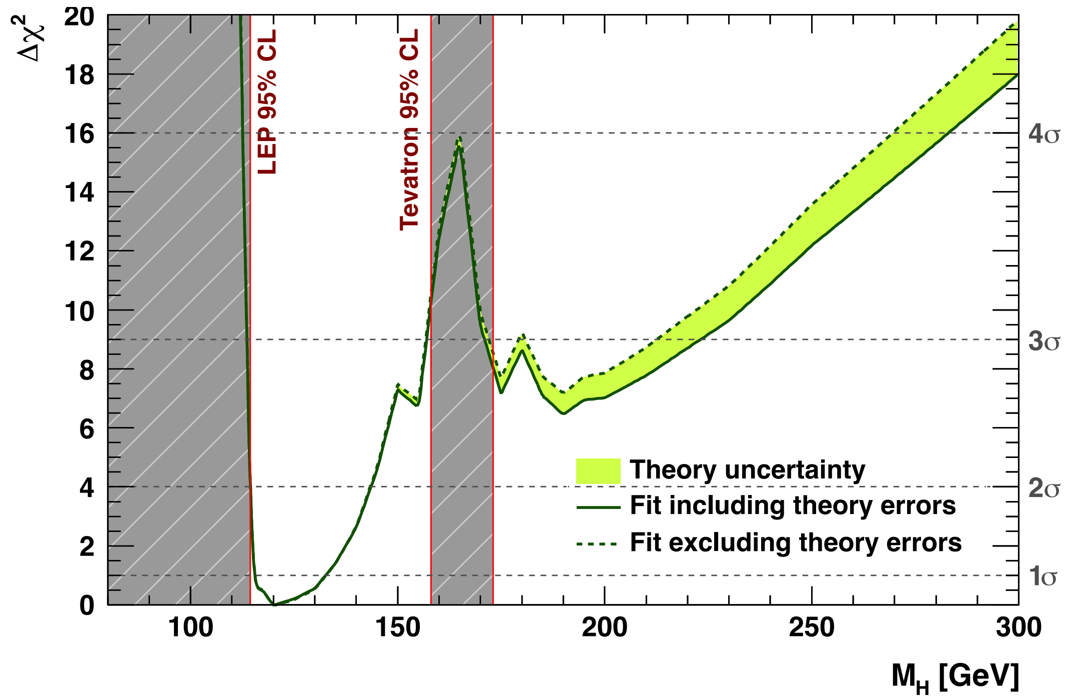
Electroweak Fits routinely performed by many groups

- D. Bardin et al. (ZFITTER), G. Passarino et al. (TOPAZ0), M. Grünewald et al. (LEP EWWG), J. Erler (GAPP), M. Baak et al. (Gfitter),...
- Many important results obtained, e.g. constraints on the mass of the Higgs boson

The LEP EWWG, arXiv: 0712.9029



Gfitter group, EPJC 72, 2003 (2012)



The Gfitter Project

G fitter

A Generic Fitter Project for HEP Model Testing

- Modular framework for involved fitting problems in the LHC era
- Coherent treatment of statistical, systematic and theoretical uncertainties together with possible correlations
- Different packages/plug-ins possible

G fitter SM

A Gfitter package for the global electroweak fit

- Complete implementation of SM predictions of precision observables
- State of the art calculations used, in particular:
  - Full calculation of the QCD Adler function (massless and massive terms) in N^3LO [P.A. Baikov et al., Phys. Rev. Lett. 101 (2008) 012022, Phys. Rev. Lett. 108, 222003 (2012)]
  - Full two-loop correction (NNLO) to R^0_b [A. Freitas et al., JHEP 1208, 050 (2012)]

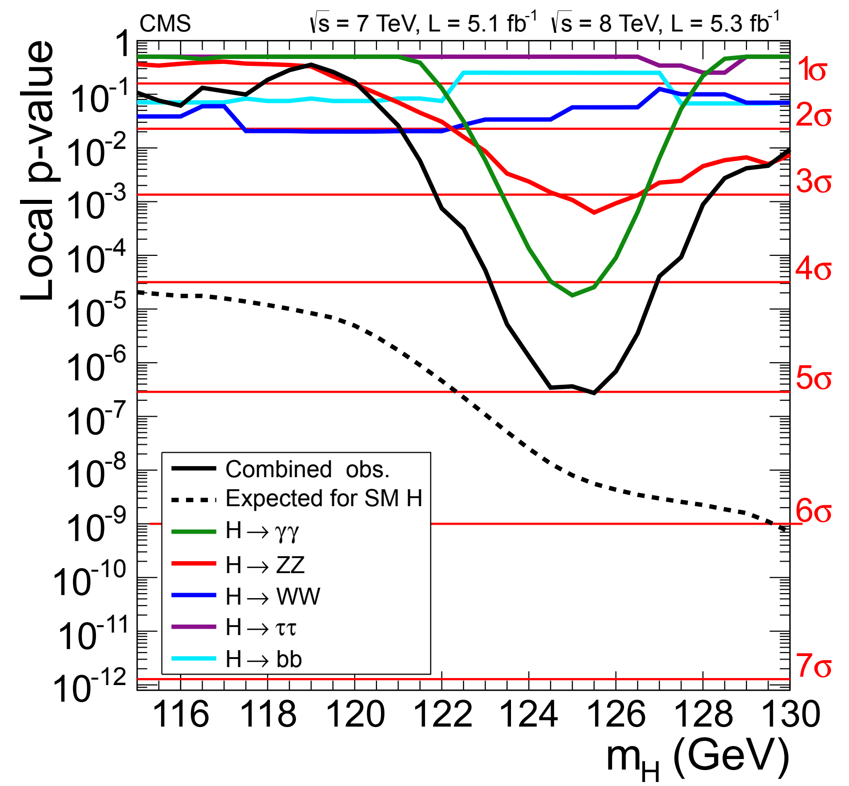
www.cern.ch/gfitter

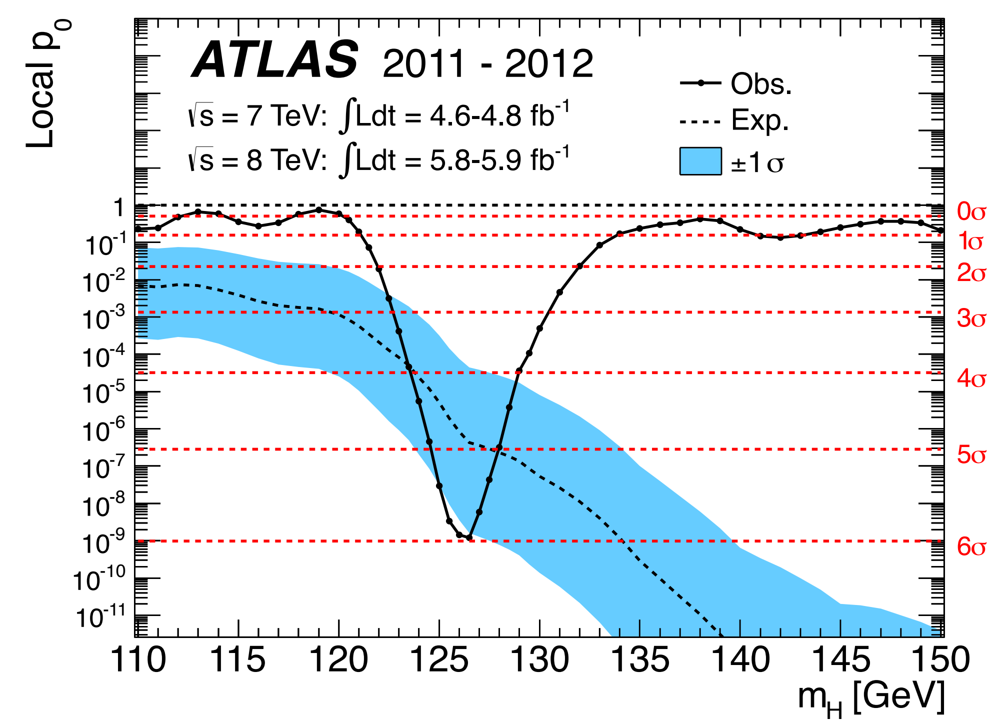
This Year's Discovery

ATLAS and CMS have reported the discovery of a new boson

- The cross section and branching ratios are compatible with the SM Higgs boson
- Measured mass:
  ATLAS: 126.0 ± 0.4 (stat) ± 0.4 (sys) GeV
  CMS: 125.3 ± 0.4 (stat) ± 0.4 (sys) GeV
- Assume that it is the Higgs boson, then
  M_H = 125.7 ± 0.4 GeV
- Difference between fully uncorrelated and fully correlated systematic uncertainties: uncertainty on M_H 0.4 → 0.5 GeV

The SM is for the first time fully overconstrained → test its consistency





Experimental Input

Observables:

- Z-pole observables: LEP/SLD results
  [ADLO+SLD, Phys. Rept. 427, 257 (2006)]
- M_W and Γ_W: LEP/Tevatron [arXiv:1204:0042]
- m_t : Tevatron [arXiv:1207:1069]
- $\Delta\alpha_{had}^{(5)}(M_Z)$ [M. Davier et al., EPJC 71, 1515 (2011)]
- $\overline{m}_c$, $\overline{m}_b$: world averages
  [PDG, J. Phys. G33, 1 (2006)]
- M_H: LHC [arXiv:1207.7214 , arXiv:1207.7235]

Free fit parameters:

- M_Z, M_H, $\Delta\alpha_{had}^{(5)}(M_Z)$, $\alpha_s(M_Z)$, $\overline{m}_c$, $\overline{m}_b$, m_t
- Scale parameters for theoretical uncertainties
  ΔM_W (4 MeV), $\Delta\sin^2\theta^l_{eff}$ ($4.7 \cdot 10^{-5}$)

M_H [GeV]$^{(\circ)}$	125.7 ± 0.4	LHC
M_W [GeV]	80.385 ± 0.015	Tevatron
Γ_W [GeV]	2.085 ± 0.042	Tevatron
M_Z [GeV]	91.1875 ± 0.0021	LEP
Γ_Z [GeV]	2.4952 ± 0.0023	LEP
$\sigma^0_{\rm had}$ [nb]	41.540 ± 0.037	LEP
R^0_ℓ	20.767 ± 0.025	LEP
$A^{0,\ell}_{\rm FB}$	0.0171 ± 0.0010	LEP
A_ℓ $^{(\star)}$	0.1499 ± 0.0018	LEP, SLC
$\sin^2\theta^\ell_{\rm eff}(Q_{\rm FB})$	0.2324 ± 0.0012	LEP
A_c	0.670 ± 0.027	SLC
A_b	0.923 ± 0.020	SLC
$A^{0,c}_{\rm FB}$	0.0707 ± 0.0035	LEP
$A^{0,b}_{\rm FB}$	0.0992 ± 0.0016	LEP
R^0_c	0.1721 ± 0.0030	LEP, SLC
R^0_b	0.21629 ± 0.00066	LEP, SLC
$\overline{m}_c$ [GeV]	$1.27^{+0.07}_{-0.11}$	
$\overline{m}_b$ [GeV]	$4.20^{+0.17}_{-0.07}$	
m_t [GeV]	173.18 ± 0.94	Tevatron
$\Delta\alpha^{(5)}_{\rm had}(M^2_Z)$ $^{(\triangle\bigtriangledown)}$	2757 ± 10	

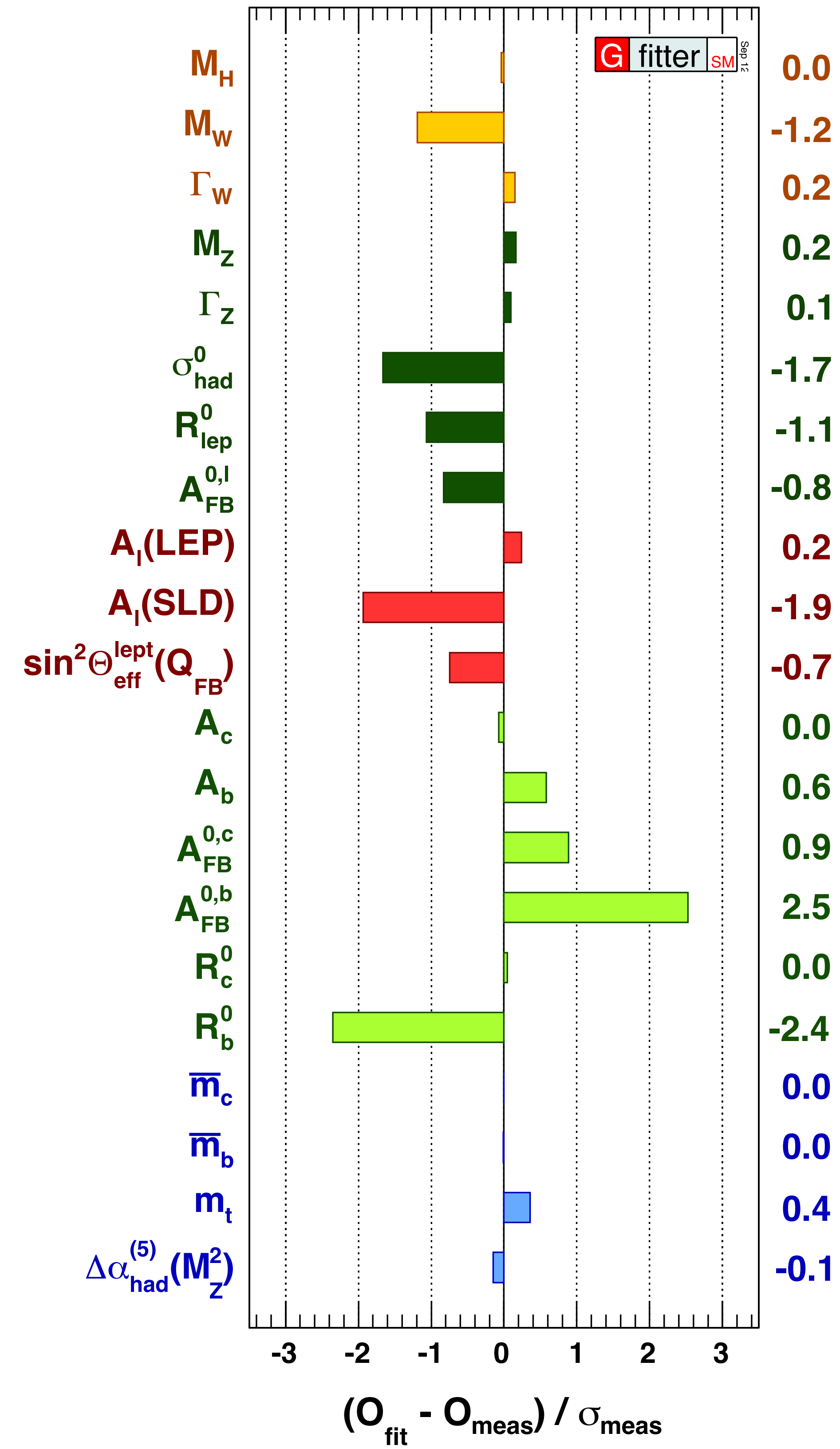
Global Fit: Results

χ^2_{min}/ndf = 21.8/14 → p-value = 0.08

- large value of χ^2_{min} not due to inclusion of M_H measurement
- without M_H measurement:
  χ^2_{min} /ndf = 20.3/13 → naive p-value = 0.09

Pull values after the fit

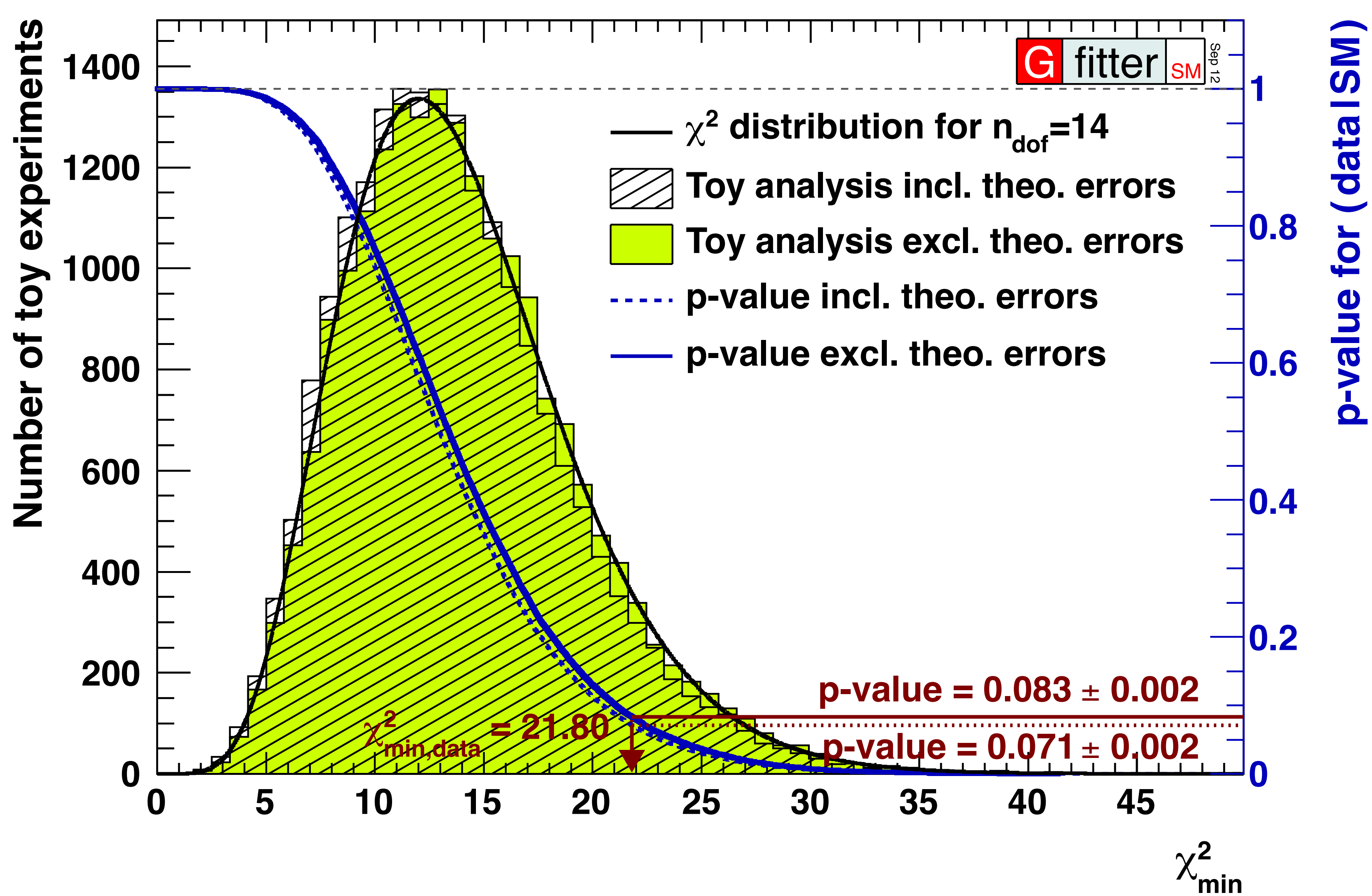
- Pull defined as $P = \dfrac{O_{\mathrm{fit}} - O_{\mathrm{meas}}}{\sigma_{\mathrm{meas}}}$
- No pull value exceeds deviations of more than 3σ (good consistency of SM)
- Small values for M_H, A_c, R^0_c, m_c and m_b indicate that their input accuracies exceed the fit requirements
- Largest deviations in the b-sector: $A^{0,b}_{FB}$ and R^0_b with 2.5σ and -2.4σ



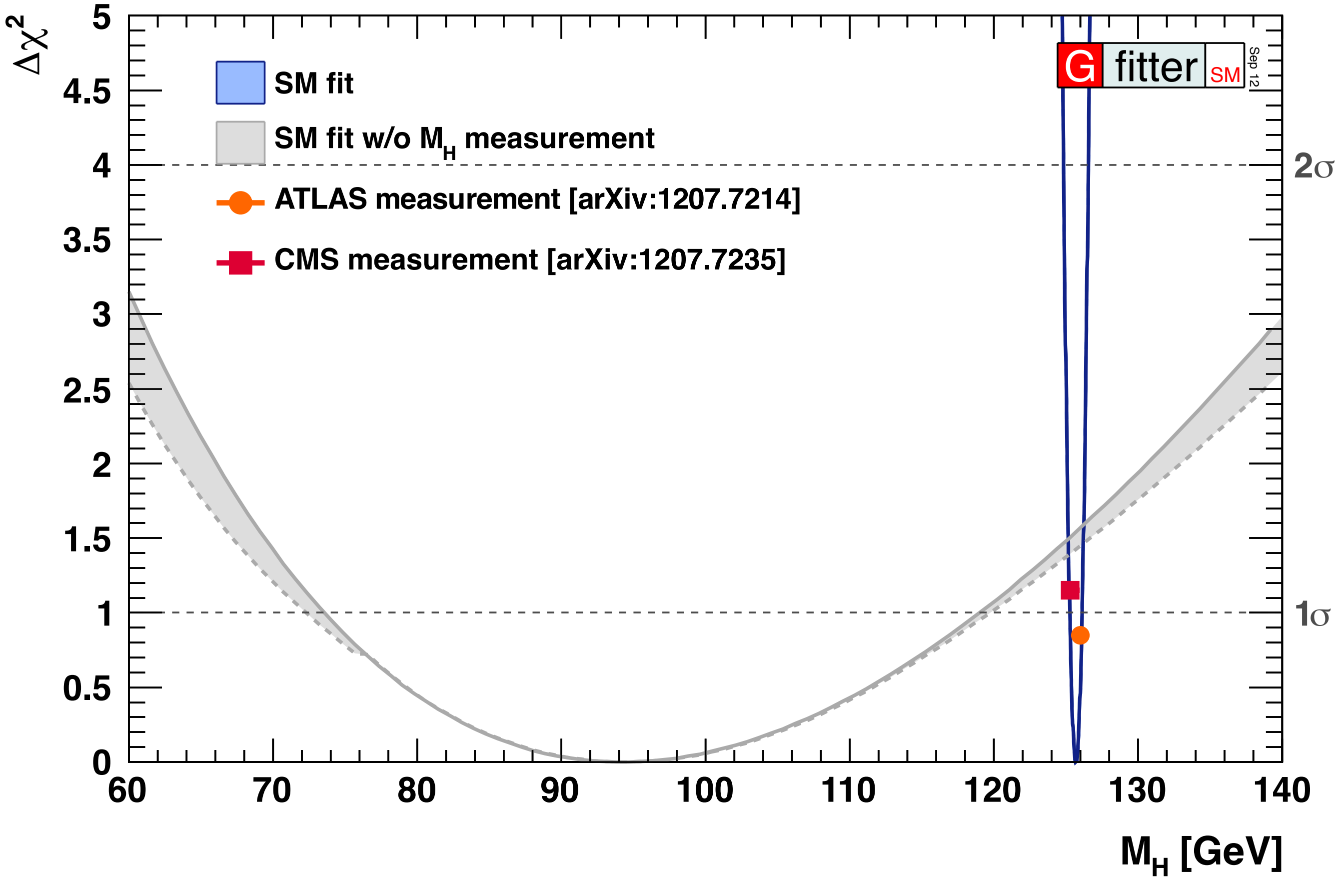
Goodness of Fit

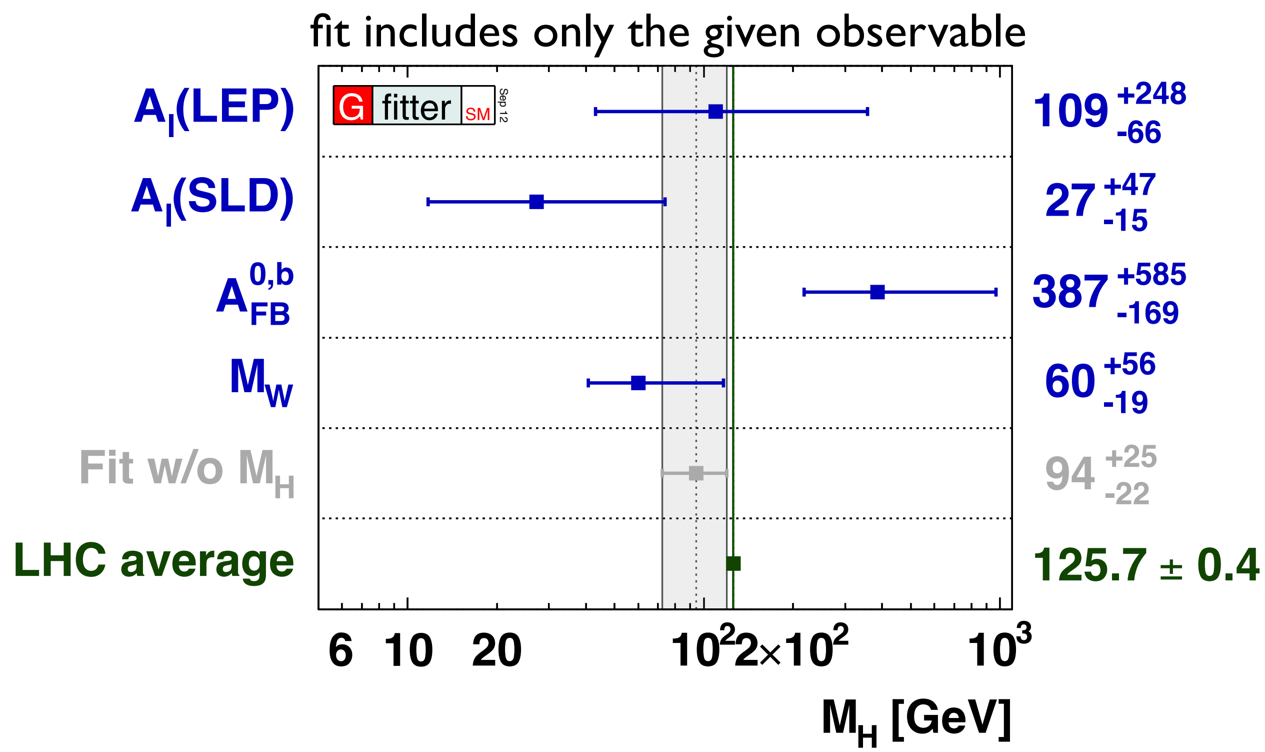
Toy analysis with 20000 toy experiments

- p-value: probability for getting $\chi^2_{min, toy}$ larger than χ^2_{min} from data
- p-value: probability for wrongly rejecting the SM: 0.07 ± 0.01 (theo)



Global Fit: Results





Scan of the $\Delta\chi^2$ profile versus M_H

- blue line: full SM fit
- grey band: fit without M_H measurement
- fit without M_H input gives $M_H = 94^{+25}_{-22}$ GeV
- consistent within 1.3σ with measurement

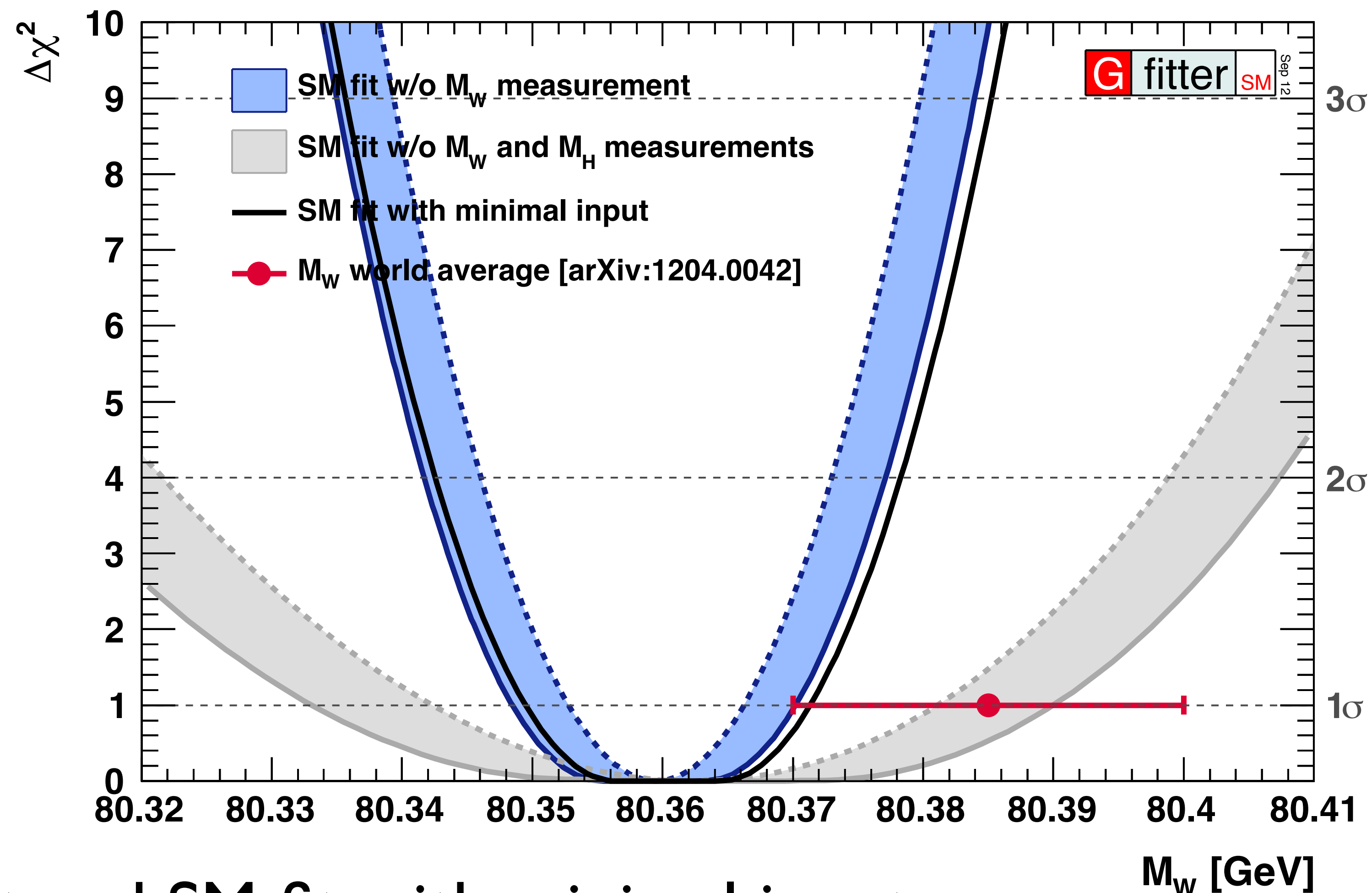
Determination of M_H removing all sensitive observables except the given one:

Tension (2.5σ) between $A^{0,b}_{FB}$, A_{lep}(SLD) and M_W visible

Indirect Determination: W Mass

Scan of the $\Delta\chi^2$ profile versus M_W

- M_H measurement allows for precise constraint of M_W
- also shown: SM fit with minimal input: M_Z, G_F, $\Delta\alpha_{had}^{(5)}(M_Z)$, $\alpha_s(M_Z)$, M_H, m_c, m_b, m_t



- Consistency between total fit and SM fit with minimal input
- Fit result for the indirect determination of M_W:

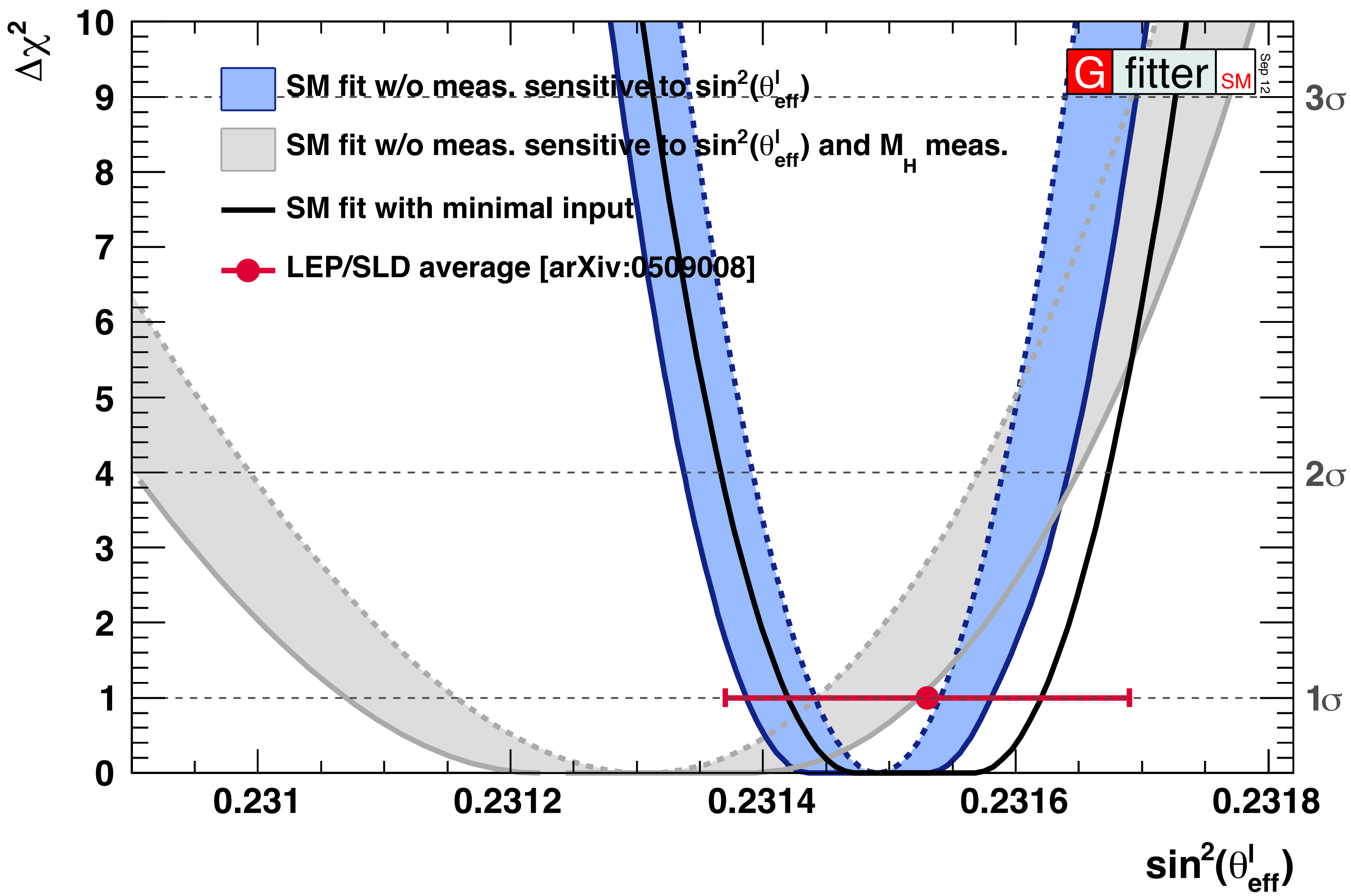
$$
\begin{aligned}
M_W &= 80.3593 \pm 0.0056_{m_t} \pm 0.0026_{M_Z} \pm 0.0018_{\Delta\alpha_{\rm had}} \\
&\qquad \pm 0.0017_{\alpha_S} \pm 0.0002_{M_H} \pm 0.0040_{\rm theo}\,, \\
&= 80.359 \pm 0.011_{\rm tot}\,,
\end{aligned}
$$

More precise than the direct measurements

The Effective Weak Mixing

Scan of the $\Delta\chi^2$ profile versus $\sin^2\theta^l_{eff}$

- all observables sensitive to $\sin^2\theta^l_{eff}$ removed from fit
- M_H measurement allows for precise constraint of $\sin^2\theta^l_{eff}$
- also shown: SM fit with minimal input

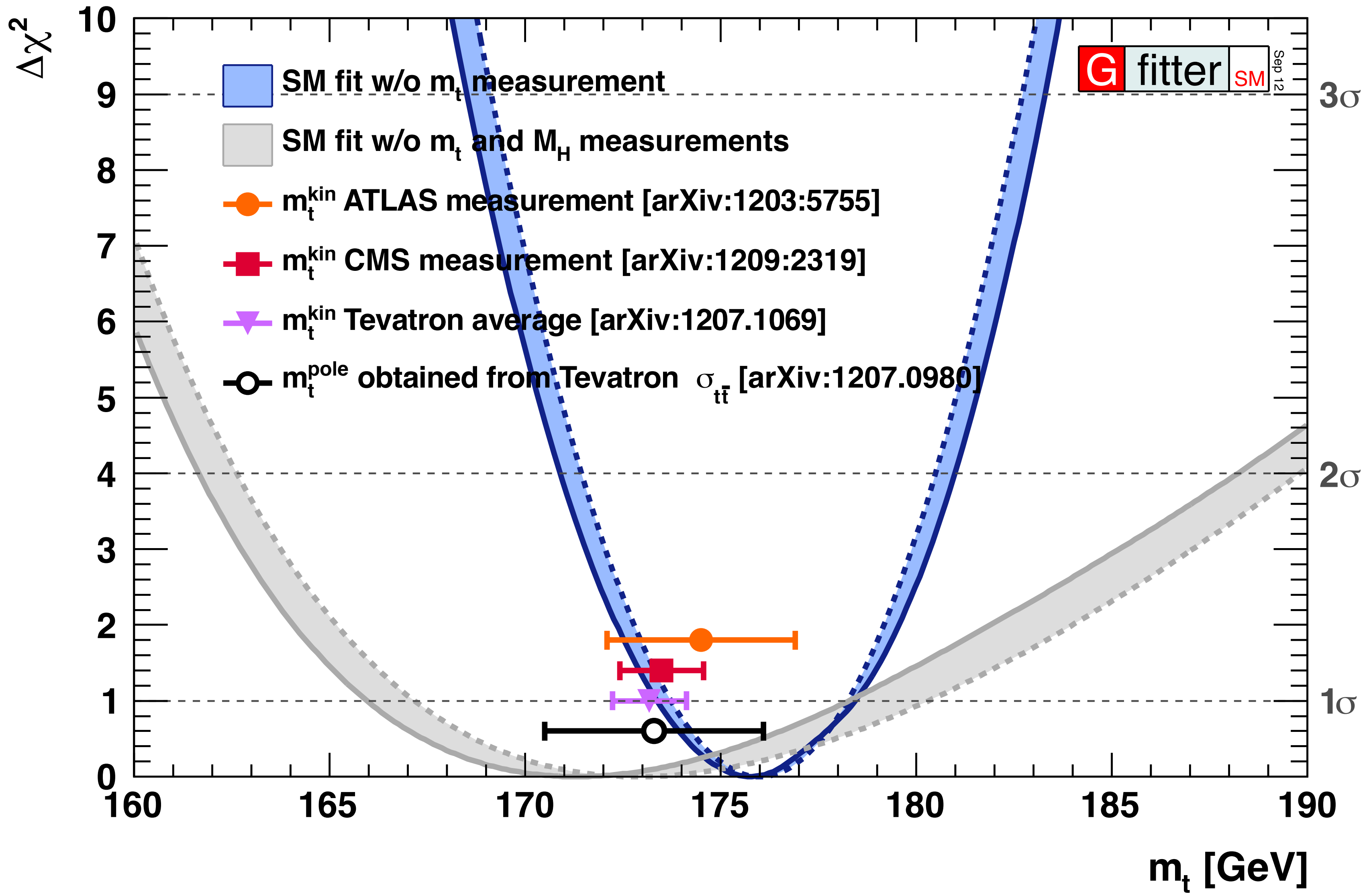


$$\sin^2\theta^\ell_{\rm eff} = 0.231496 \pm 0.000030_{m_t} \pm 0.000015_{M_Z} \pm 0.000035_{\Delta\alpha_{\rm had}} \pm 0.000010_{\alpha_S} \pm 0.000002_{M_H} \pm 0.000047_{\rm theo}\,,$$

$$= 0.23150 \pm 0.00010_{\rm tot}\,,$$

More precise than the direct determination from LEP/SLD measurements

Indirect Determination: Top Mass

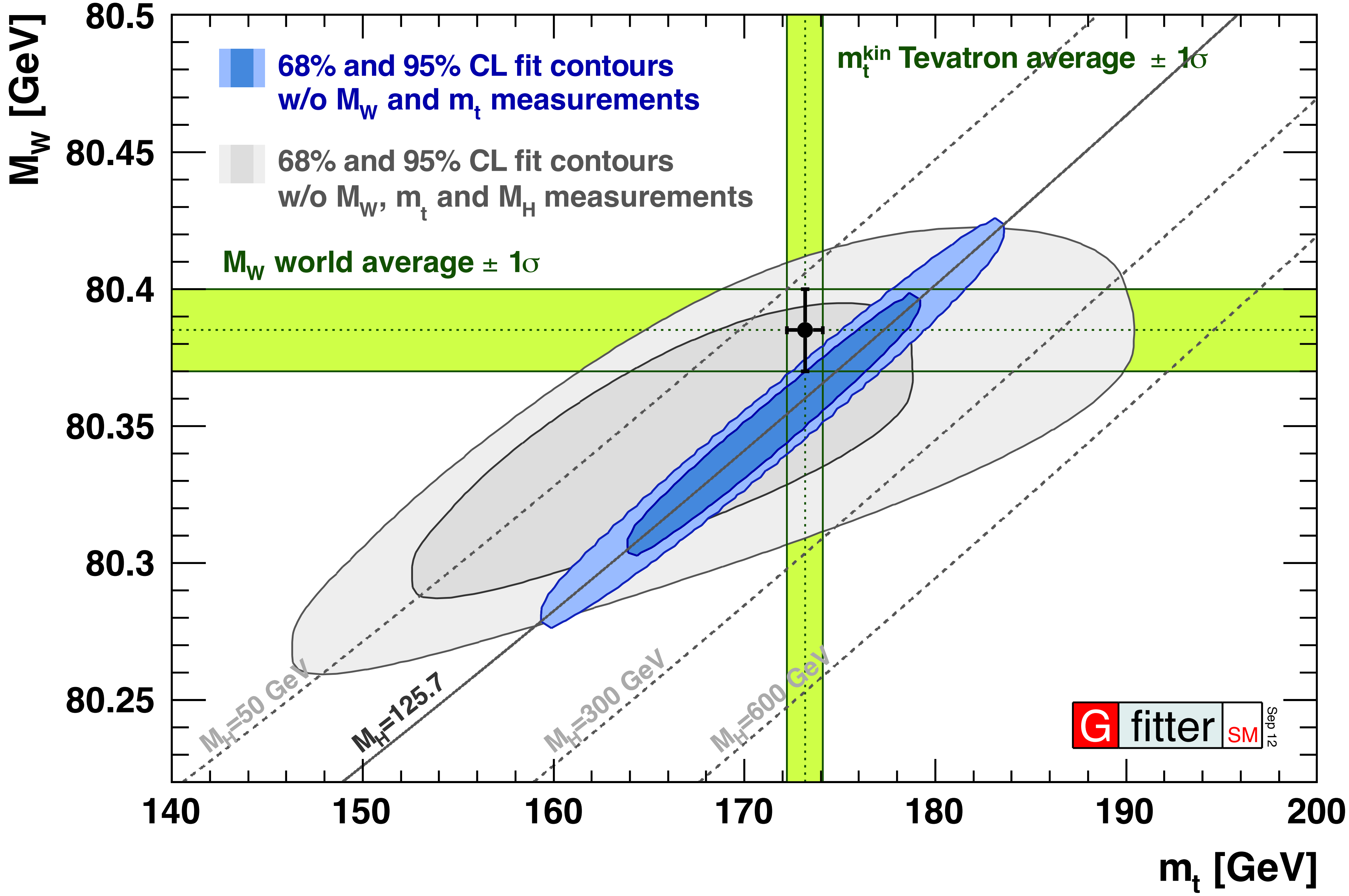


Scan of the $\Delta\chi^2$ profile versus m_t

- consistency with direct measurements
- M_H measurement allows for better constraint of m_t

$m_t = 175.8\,^{+2.7}_{-2.4}\,\mathrm{GeV}$ (Tevatron average: m_t = 173.2 ± 0.9 GeV)

W and Top Mass



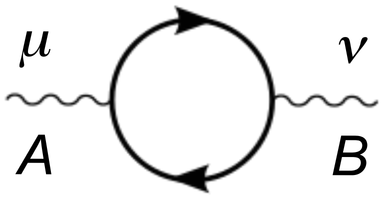
68% and 95% CL contours of fit without using M_W, m_t (and M_H)

- Impressive consistency of SM model

Beyond the SM

At low energies, BSM physics appears dominantly through vacuum polarisation

- Aka, *oblique corrections*



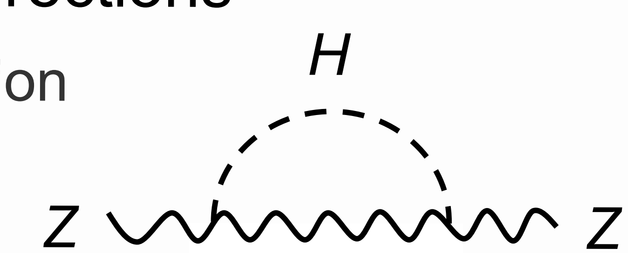
$$= i\Pi^{\mu\nu}_{AB=\{W,Z,\gamma\}}(q)$$

- Direct corrections (vertex, box, brems-strahlung) generally suppressed by m_f / Λ

Oblique corrections reabsorbed into electroweak parameters $\Delta\rho$, $\Delta\kappa$, Δr

Electroweak fit sensitive to BSM physics through oblique corrections

- In direct competition with Higgs loop corrections



- Oblique corrections from New Physics described through ***STU* parameters**
  [Peskin-Takeuchi, Phys. Rev. D46, 381 (1992)]

$$\boxed{O_{\text{meas}} = O_{\text{SM,ref}}(M_H, m_t) + c_S S + c_T T + c_U U}$$

S : (S+U) New Physics contributions to neutral (charged) currents

T : Difference between neutral and charged current processes – sensitive to weak isospin violation

U : Constrained by M_W and Γ_W. Usually very small in NP models (often: U=0)

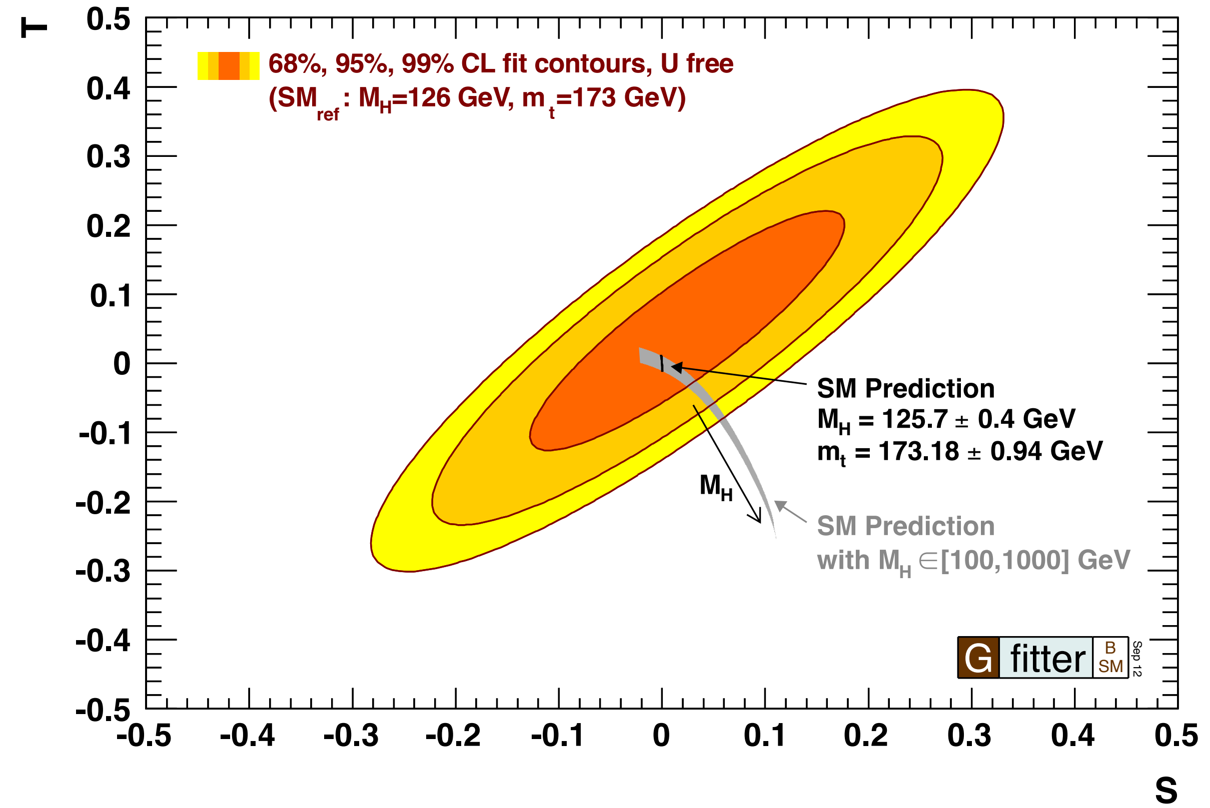
- Also considered: correction to $Z \to bb$ coupling, and extended parameters (VWX)
  [Burgess et al., PLB 326, 276 (1994), PRD 49, 6115 (1994)]

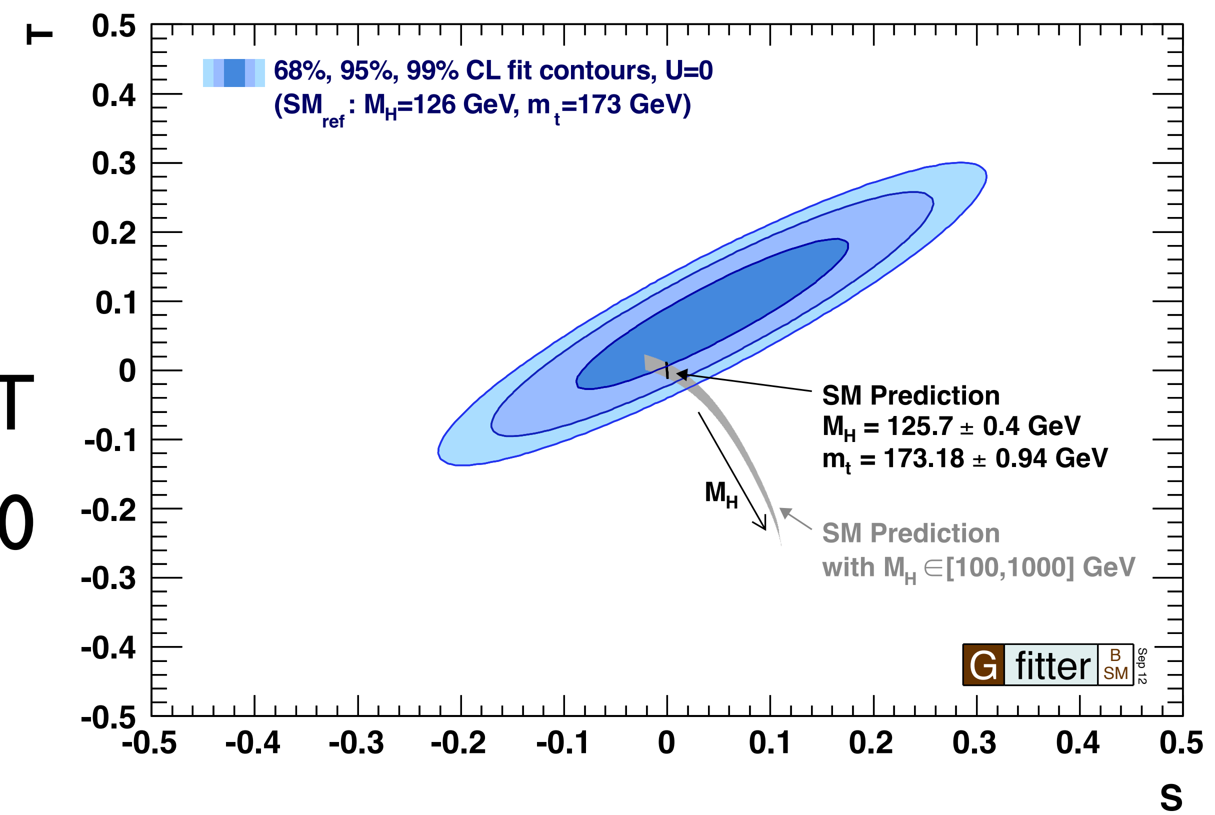
Constraints on S, T and U

S, T, U obtained by fit to EW observables

- SM reference chosen to be
  $M_{H,ref}$ = 126 GeV
  $m_{t,ref}$ = 173 GeV
  - this defines (0, 0, 0)
  - S,T depend logarithmically on M_H
- Fit result:
  S = 0.03 ± 0.10
  T = 0.05 ± 0.12
  U = 0.03 ± 0.10
  with large correlation between S and T
- Stronger constraints from fit with U=0

No indication of new physics





Summary

Assuming the newly discovered boson is the SM Higgs

- all fundamental parameters of the SM are known
- possibility to overconstrain the SM at the electroweak scale
- global EW fit has been redone, with a p-value of 0.07
- small p-value comes mostly from R^0_b and $A^{0,b}_{FB}$

Knowledge of M_H allows for precision determinations of

- W mass, top mass, $\sin^2\theta^l_{eff}$
- detailed information in arXiv:1209.2716 and recent updates on www.cern.ch/gfitter

EW Fit allows to constrain many BSM models

- no signs of new physics from oblique parameters
- stay tuned for more results

Additional Material

Parameter	Input value	Free in fit	Fit result incl. M_H	Fit result not incl. M_H	Fit result incl. M_H but not exp. input in row
M_H [GeV]$^{(\circ)}$	125.7 ± 0.4	yes	125.7 ± 0.4	$94\,^{+25}_{-22}$	$94\,^{+25}_{-22}$
M_W [GeV]	80.385 ± 0.015	–	80.367 ± 0.007	80.380 ± 0.012	80.359 ± 0.011
Γ_W [GeV]	2.085 ± 0.042	–	2.091 ± 0.001	2.092 ± 0.001	2.091 ± 0.001
M_Z [GeV]	91.1875 ± 0.0021	yes	91.1878 ± 0.0021	91.1874 ± 0.0021	91.1983 ± 0.0116
Γ_Z [GeV]	2.4952 ± 0.0023	–	2.4954 ± 0.0014	2.4958 ± 0.0015	2.4951 ± 0.0017
$\sigma^0_{\rm had}$ [nb]	41.540 ± 0.037	–	41.479 ± 0.014	41.478 ± 0.014	41.470 ± 0.015
R^0_ℓ	20.767 ± 0.025	–	20.740 ± 0.017	20.743 ± 0.018	20.716 ± 0.026
$A^{0,\ell}_{\rm FB}$	0.0171 ± 0.0010	–	0.01627 ± 0.0002	0.01637 ± 0.0002	0.01624 ± 0.0002
$A_\ell\ ^{(\star)}$	0.1499 ± 0.0018	–	$0.1473\,^{+0.0006}_{-0.0008}$	0.1477 ± 0.0009	$0.1468 \pm 0.0005^{(\dagger)}$
$\sin^2\theta^\ell_{\rm eff}(Q_{\rm FB})$	0.2324 ± 0.0012	–	$0.23148\,^{+0.00011}_{-0.00007}$	$0.23143\,^{+0.00010}_{-0.00012}$	0.23150 ± 0.00009
A_c	0.670 ± 0.027	–	$0.6680\,^{+0.00025}_{-0.00038}$	$0.6682\,^{+0.00042}_{-0.00035}$	0.6680 ± 0.00031
A_b	0.923 ± 0.020	–	$0.93464\,^{+0.00004}_{-0.00007}$	0.93468 ± 0.00008	0.93463 ± 0.00006
$A^{0,c}_{\rm FB}$	0.0707 ± 0.0035	–	$0.0739\,^{+0.0003}_{-0.0005}$	0.0740 ± 0.0005	0.0738 ± 0.0004
$A^{0,b}_{\rm FB}$	0.0992 ± 0.0016	–	$0.1032\,^{+0.0004}_{-0.0006}$	0.1036 ± 0.0007	0.1034 ± 0.0004
R^0_c	0.1721 ± 0.0030	–	0.17223 ± 0.00006	0.17223 ± 0.00006	0.17223 ± 0.00006
R^0_b	0.21629 ± 0.00066	–	0.21474 ± 0.00003	0.21475 ± 0.00003	0.21473 ± 0.00003
$\overline{m}_c$ [GeV]	$1.27\,^{+0.07}_{-0.11}$	yes	$1.27\,^{+0.07}_{-0.11}$	$1.27\,^{+0.07}_{-0.11}$	–
$\overline{m}_b$ [GeV]	$4.20\,^{+0.17}_{-0.07}$	yes	$4.20\,^{+0.17}_{-0.07}$	$4.20\,^{+0.17}_{-0.07}$	–
m_t [GeV]	173.18 ± 0.94	yes	173.52 ± 0.88	173.14 ± 0.93	$175.8\,^{+2.7}_{-2.4}$
$\Delta\alpha^{(5)}_{\rm had}(M_Z^2)\ ^{(\triangle\bigtriangledown)}$	2757 ± 10	yes	2755 ± 11	2757 ± 11	$2716\,^{+49}_{-43}$
$\alpha_s(M_Z^2)$	–	yes	0.1191 ± 0.0028	0.1192 ± 0.0028	0.1191 ± 0.0028
$\delta_{\rm th} M_W$ [MeV]	$[-4, 4]_{\rm theo}$	yes	4	4	–
$\delta_{\rm th} \sin^2\theta^\ell_{\rm eff}\ ^{(\triangle)}$	$[-4.7, 4.7]_{\rm theo}$	yes	-1.4	4.7	–

$\alpha_s(M_Z)$ from Z→hadrons

- Fit of electroweak precision observables
- Input mostly from LEP data from the Z-peak
- Determination of α_s: most sensitivity through total hadronic cross section at the Z-pole and the partial leptonic width

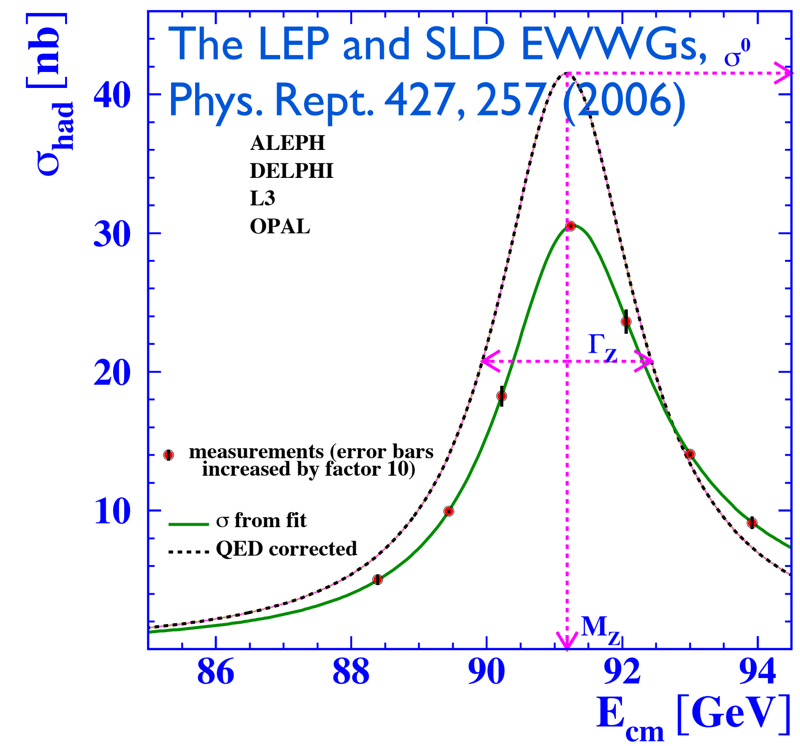
$$\sigma^0_{\rm had} \equiv \frac{12\pi}{m_Z^2}\frac{\Gamma_{\rm ee}\Gamma_{\rm had}}{\Gamma_Z^2} \qquad R^0_\ell \equiv \Gamma_{\rm had}/\Gamma_{\ell\ell}$$

obtained from the four LEP experiments, 17 million Z decays

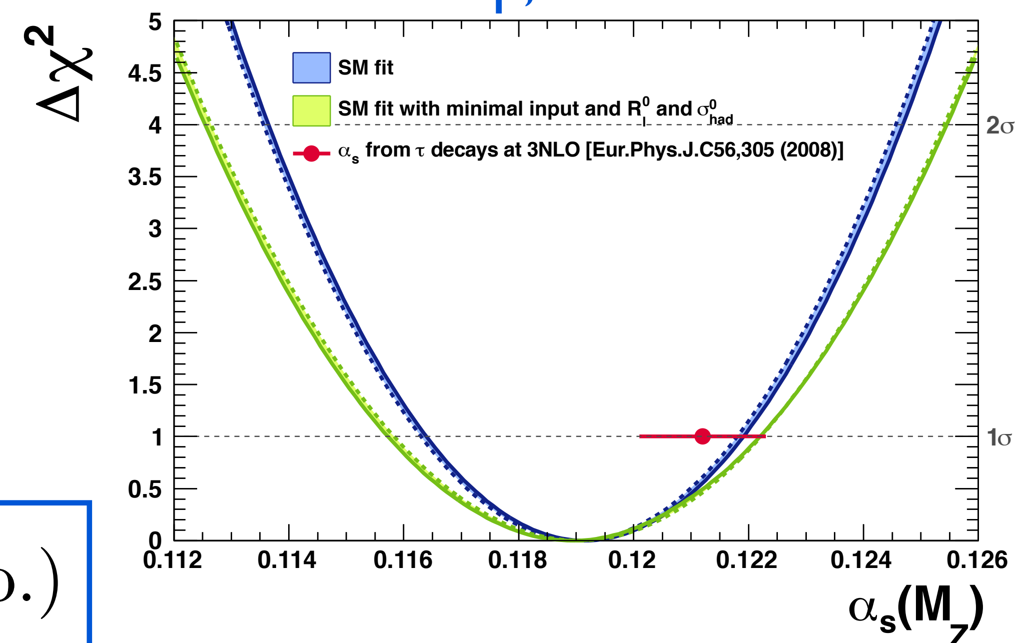
Complete $O(\alpha_s^4)$ calculation available:

[P. Baikov et al., Phys. Rev. Lett. 108, 222003 (2012)]

$$\boxed{\alpha_s(M_Z) = 0.1191 \pm 0.0028\,(\mathrm{exp.}) \pm 0.0001\,(\mathrm{theo.})}$$



Gfitter Group, arXiv:1209.2716



Improvement in precision only with ILC/GigaZ expected