Precise determination of the Higgs production cross-section at the LHC

Marco Bonvini

DESY Hamburg

DESY, November 12, 2012

Work in collaboration with:

Stefano Forte, Giovanni Ridolfi + Richard Ball, Simone Marzani

Outline

High precision measurements and predictions

Higgs boson: small cross-section, huge background.

Need for accurate prediction from the theory side.

Are theoretical predictions accurate enough?

- EW: coupling small, perturbative corrections to LO processes known to be small
- QCD: completely another story...
  1. protons are not described directly in the field theory: PDFs and their uncertainty
  2. α_s not that small, even at high energies: $\alpha_s(m_Z^2) \sim 0.1$
  3. in some kinematical regimes the perturbativity of QCD may be spoiled

What can we do?

1. better predictions and more accurate data may improve PDFs
2. higher order computations
   (not so easy actually: NNLO is already very complicated)
3. *resummation*

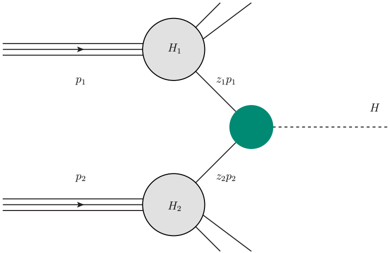
Factorization theorem in QCD

$$\sigma(p_1,p_2) = \sum_{\substack{i,j \\ \text{partons}}} \int dx_1 \int dx_2 \; f_i^{(1)}(x_1,\mu^2)\, f_j^{(2)}(x_2,\mu^2)\; \hat\sigma_{ij}(x_1p_1,\, x_2p_2, \mu^2)$$

$x_{1,2}$: momentum fraction carried by the parton

$\hat\sigma_{ij}(\hat p_1, \hat p_2)$: **partonic cross-section** $(\hat p_i = x_i p_i)$

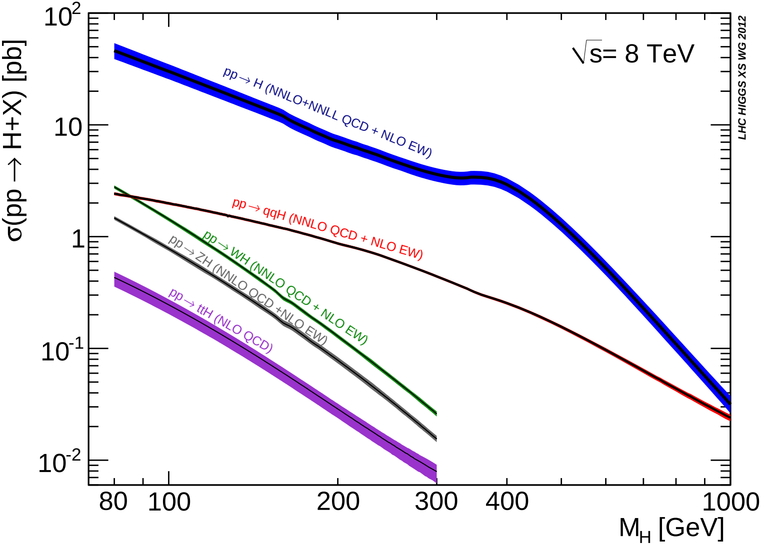
$f_i^{(1,2)}(x_{1,2})$: **PDFs:** parton distribution functions, universal



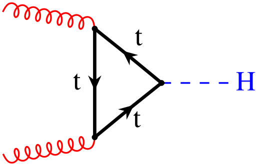
PDFs $f_i(x,\mu^2)$ are non-perturbative objects, cannot be computed in pQCD

$\rightarrow$ they must be extracted ("measured") from data

Higgs production at LHC



Several production modes, but the dominant channel is **gluon fusion**



The partonic cross-section starts at order α_s^2:

$$\hat{\sigma}_{\mathrm{LO}}(z, \alpha_s) = \sigma_0 \, \alpha_s^2 \, \delta(1 - z)$$

where I have defined

$$z = \frac{m_H^2}{\hat{s}} = \frac{m_H^2}{x_1 x_2 s}, \qquad \begin{aligned} \sqrt{\hat{s}} &= \text{partonic c.m. energy} \\ \sqrt{s} &= \text{hadronic c.m. energy} \end{aligned}$$

At LO, $z = 1$ fixed by the kinematics.

Perturbative corrections to $gg \to H$

The partonic cross-section can be computed in perturbation theory

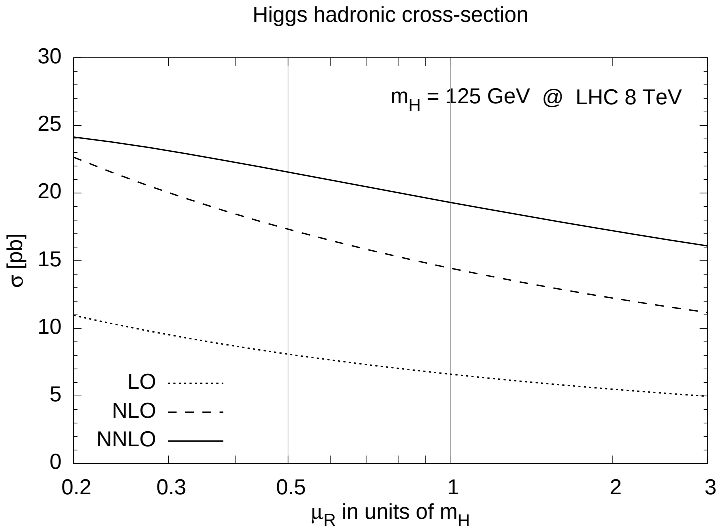
$$\hat\sigma(z,\alpha_s) = z\,\sigma_0\,\alpha_s^2 \underbrace{\left[\delta(1-z) + \alpha_s C^{(1)}(z) + \alpha_s^2 C^{(2)}(z) + \alpha_s^3 C^{(3)}(z) + \ldots\right]}_{\text{coefficient function } C(z,\alpha_s)}$$

State of the art:

- NLO $C^{(1)}(z)$:
  - large m_t approximation [Dawson 1991; Djouadi, Spira, Zerwas 1991]
  - full m_t dependence [Spira, Djouadi, Graudenz, Zerwas 1995]
- NNLO $C^{(2)}(z)$:
  - large m_t approximation [Harlander, Kilgore 2002; Anastasiou, Melnikov 2002]
  - finite m_t as an expansion in $\frac{m_H}{2m_t}$ and in $(1-z)$ [Harlander, Ozeren 2009]
- NNNLO $C^{(3)}(z)$:
  - ONGOING: large m_t as an expansion in $(1-z)$ [Anastasiou et al.]
  - soft and other approximations: more details later...

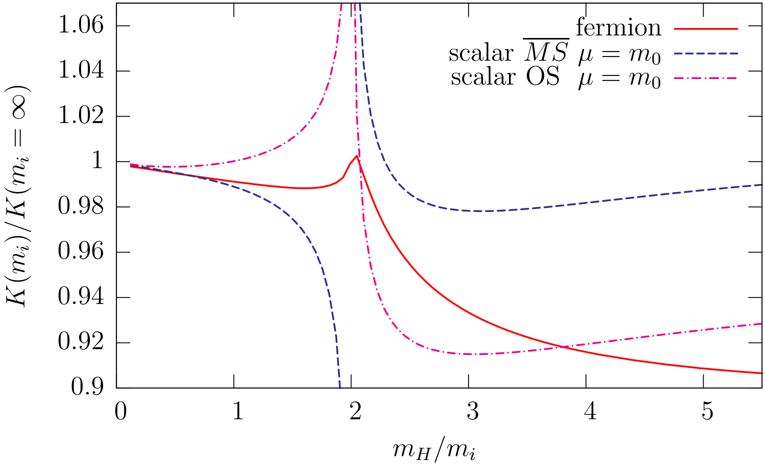
Problem

Perturbative corrections are large!!



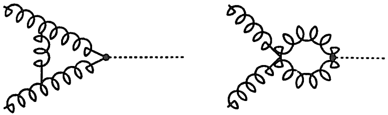
Observed fact

The large-m_t approximation works *surprisingly well* even for large m_H.



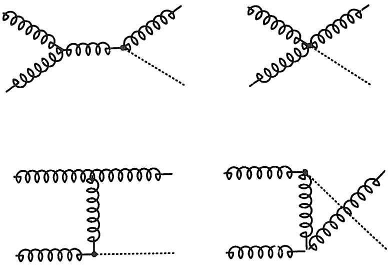
NLO, LHC @ 14 TeV [Bonciani, Degrassi, Vicini 2007]

NLO virtual diagrams



$\propto \delta(1 - z)$

NLO real emission diagrams



non-trivial z dependence

Soft logs

Infrared divergences in both virtual and real emission diagrams.

Soft divergences: the energy is small
(in the case of real emission, the emitted gluon is soft)

Soft divergences *cancel* by the Kinoshita-Lee-Nauenberg theorem.
Residual terms are the so called *soft logarithms*

$$C(z, \alpha_s) \ni \left[\frac{\log^k(1-z)}{1-z}\right]_+$$

The $+$ symbol defines a distribution via

$$\int_0^1 dz\, [f(z)]_+\, g(z) = \int_0^1 dz\, f(z)\, [g(z) - g(1)]$$

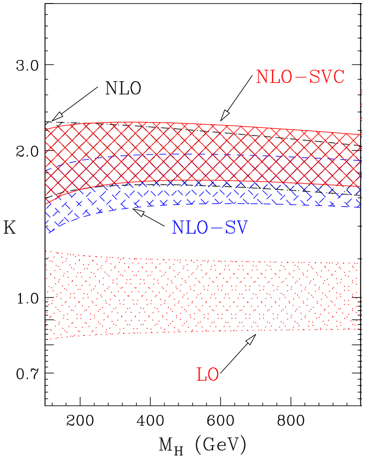
Formally:

$$[f(z)]_+ = f(z) - \delta(1-z) \int_0^1 dx\, f(x)$$

Another observed fact

The soft part approximates well the full result.

$$C^{(1)}(z) = C^{(1)}_{\mathsf{soft}}(z) + C^{(1)}_{\mathsf{non\text{-}soft}}(z)$$



LHC @ 14 TeV [Catani, de Florian, Grazzini 2001]

Connection between these facts

The two facts are not unrelated...

The *dominant* soft terms are m_t-independent,

i.e. they are the same in the full (finite m_t) result
and in the large-m_t approximated result.

Then, the success of the large m_t approximation can thus be justified (at least in part) by the success of the soft approximation.

Soft-gluon resummation

The N^{n}LO term in the expansion of $C(z, \alpha_s)$ contains

$$\alpha_s^n \left[\frac{\log^k(1-z)}{1-z}\right]_+, \qquad 0 \le k \le 2n-1$$

due to n gluon emissions.

As $z \to 1$ (soft limit), the soft logs become large.

When

$$\boxed{\alpha_s \log^2(1-z) \sim \mathcal{O}(1)}$$

all the soft terms in the perturbative series are equally important, and any truncation would be meaningless.

Resummation of soft-gluon logarithms

[Catani, Trentadue, NPB 327 (1989) 323] [Sterman, NPB 281 (1987) 310]

Brief summary

We have discussed large-m_t and soft approximations and soft-gluon resummation.

Three questions:

1. when does the large-m_t approximation work so good?
2. when does the soft part of the cross-section approximate well the full result?
3. when is soft-gluon resummation *actually* relevant/needed?

We are going to answer these questions...

Dominance of the soft limit

Driver: soft approximation, good in the *partonic* soft limit $z \to 1$
In real life, we are interested in the *hadron-level* cross-section.

The connection is **not** straightforward!

$$\sigma(\tau) = \int_\tau^1 \frac{dz}{z} \mathscr{L}\left(\frac{\tau}{z}, \mu^2\right) C(z, \alpha_s(\mu^2)) \qquad \tau = \frac{m_H^2}{s} \qquad \left(z = \frac{m_H^2}{\hat{s}}\right)$$

where I have defined the *gluon luminosity*

$$\mathscr{L}(x, \mu^2) = \int_x^1 \frac{dy}{y}\, g\left(\frac{x}{y}, \mu^2\right) g(y, \mu^2)$$

The region $z \sim 1$ is ALWAYS included in the integration region!
When is that region dominant?

$\tau \sim 1$ (hadronic threshold limit): $z \in [\tau, 1]$ always in the threshold region
$\Rightarrow$ **The soft terms dominate for sure**

$\tau \ll 1$ (at LHC 8 TeV and $m_H = 125$ GeV: $\tau \simeq 10^{-4}$): let's see...

Saddle point argument

$$\sigma(\tau) = \int_\tau^1 \frac{dz}{z}\, \mathscr{L}\left(\frac{\tau}{z}\right) C(z,\alpha_s)$$

Mellin transform:

$$\sigma(N) = \int_0^1 d\tau\, \tau^{N-1}\, \sigma(\tau) = \mathscr{L}(N)\, C(N,\alpha_s)$$

Inverse Mellin transform

$$\sigma(\tau) = \int_{c-i\infty}^{c+i\infty} \frac{dN}{2\pi i}\, \tau^{-N} \mathscr{L}(N)\, C(N,\alpha_s)$$

The integral is dominated by the values of N in the proximity of the saddle point $N = N_0$:

$$\log\frac{1}{\tau} = -\frac{d}{dN}\log \mathscr{L}(N)\bigg|_{N_0} - \frac{d}{dN}\log C(N,\alpha_s)\bigg|_{N_0}$$

[MB, Forte, Ridolfi, NPB 847 (2011) 93-159]

[MB, Forte, Ridolfi, PRL 109 (2012) 102002]

Then we rephrase the questions:

1. when does the large-m_t approximation work so good?
2. when does the soft part of the cross-section approximate well the full result?
3. when is soft-gluon resummation *actually* relevant/needed?

$$\Downarrow$$

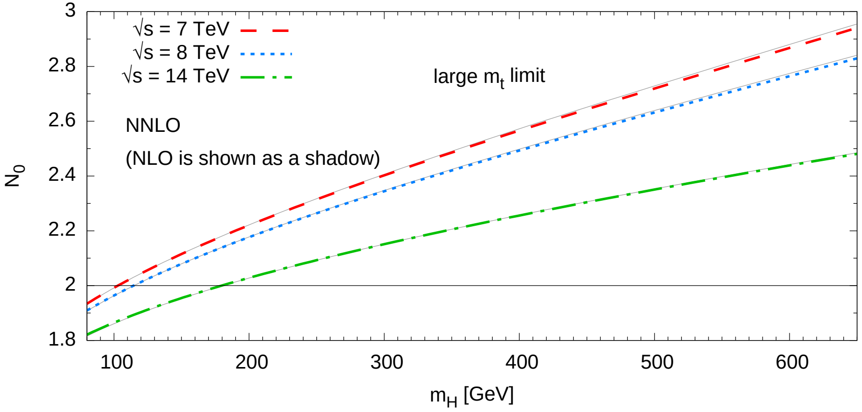
1. Is the large-m_t approximation good at the saddle point N_0?
2. Is the soft approximation good at the saddle point N_0?
3. Is soft-gluon resummation needed at the saddle point N_0?

$$\sigma(N) = \mathscr{L}(N)\, C(N, \alpha_s)$$

Hadron level kinematics $m_H, \sqrt{s}$ (or τ) $\quad\Rightarrow\quad$ saddle point N_0

Position of the saddle point

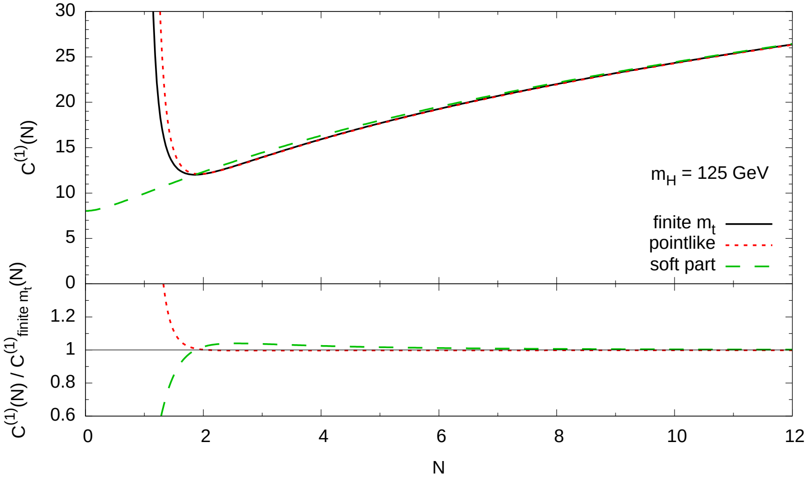
A *unique real* saddle N_0 always exists.



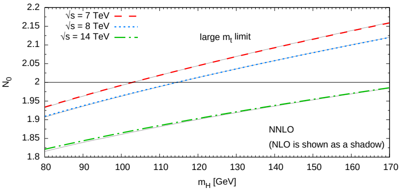
Note that these curves **do not depend on the perturbative order.**
This is due to the fact that they are mainly determined by the PDFs.

Partonic comparison

$$C(N,\alpha_s) = 1 + \alpha_s\, C^{(1)}(N) + \alpha_s^2\, C^{(2)}(N) + \alpha_s^3\, C^{(3)}(N) + \dots$$

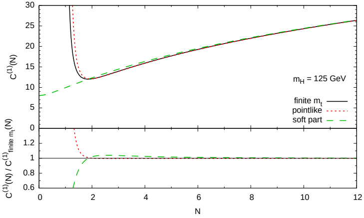


Example of conclusions one may draw



Assuming $m_H = 125$ GeV

- $N_0 \simeq 2.06$ for LHC 7 TeV
- $N_0 \simeq 2.03$ for LHC 8 TeV
- $N_0 \simeq 1.92$ for LHC 14 TeV



LHC	large-m_t	soft
7 TeV	very good ($\sim 1\%$)	quite good ($4 - 9\%$)
8 TeV	very good ($\sim 1\%$)	quite good ($4 - 9\%$)
14 TeV	good ($\sim 1 - 2\%$)	not so good ($5 - 10\%$)

The larger $\sqrt{s}$, the worse the soft and large-m_t approximations.

Is soft-gluon resummation really *needed* ?

Actually no.

In N space, the loss-of-perturbativity condition becomes

$$\alpha_s \log^2(1-z) \gtrsim 1 \qquad \Rightarrow \qquad \alpha_s \log^2 N \gtrsim 1$$

which gives, roughly,

$$N \gtrsim 10$$

But the saddle point is never so large!!!

Soft-gluon resummation can just be used as a (useful!) tool to predict higher order (soft) terms and refine the current NNLO results.

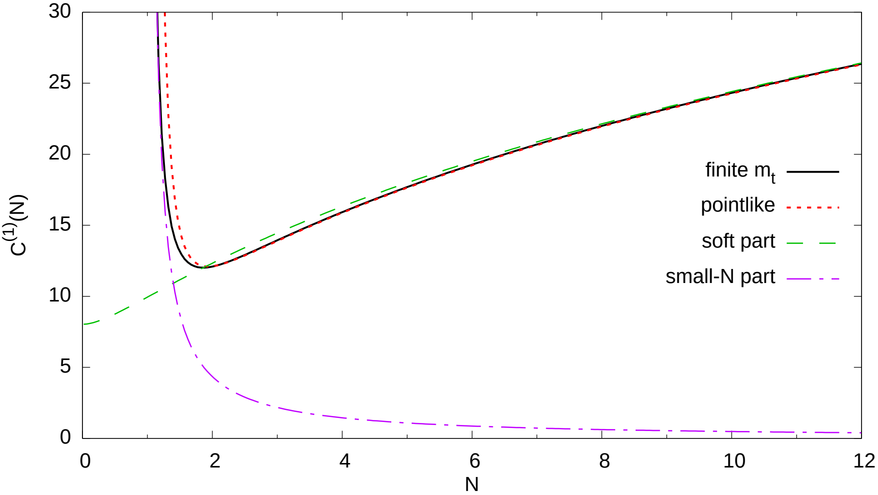
Saddle point method: summary

- Approximations: parton level
- Connection parton- to hadron-level: $\sigma(N) = \mathscr{L}(N) C(N, \alpha_s)$
- The saddle point N_0 determines the region which gives the dominant contribution to the inverse Mellin transform
- The saddle point is independent on the perturbative order and on other details: property of the considered subprocess, determined by PDFs
- Based on all-order considerations, we can argue that soft terms dominate in the region $N \gtrsim 2$
- Soft-gluon resummation never needed, but possibly useful
- The larger the collider energy $\sqrt{s}$, the smaller the saddle point N_0, the worse the soft approximation

What can we do when $N_0 \lesssim 2$?

Singularity structure in N space

$$C(N, \alpha_s) = 1 + \alpha_s C^{(1)}(N) + \dots$$



Singularity structure in N space: idea

Once we know the dominant singularities in N space ($N = 1$, $N \to \infty$) we have a good control of the physical region $1 < N < \infty$.

Observations:

- the limit $N \to 1$ doesn't commute with the limit $m_t \to \infty$
- Singularity in $N = 1$ controlled by BFKL resummation
- BFKL resummation affects PDF evolution

GLAP evolution equations

Let's consider only gluons:

$$\mu^2 \frac{d}{d\mu^2} g(x,\mu^2) = \int_x^1 \frac{dz}{z} P\left(\frac{x}{z}, \alpha_s(\mu^2)\right) g(z,\mu^2)$$

or, in Mellin space,

$$\mu^2 \frac{d}{d\mu^2} g(N,\mu^2) = \gamma(N, \alpha_s(\mu^2))\, g(N,\mu^2)$$

P and γ are the Altarelli-Parisi splitting-function and anomalous-dimensions, respectively, related by

$$\gamma(N, \alpha_s(\mu^2)) = \int_0^1 dx\, x^{N-1} P(x, \alpha_s(\mu^2))$$

and describe the splitting of a gluon into two gluons, one with fraction x of the initial gluon momentum.

Small-x logarithms

$P(x, \alpha_s(\mu^2))$ has a perturbative expansion

$$P(x, \alpha_s(\mu^2)) = \alpha_s \left[P^{(0)}(x) + \alpha_s \, P^{(1)}(x) + \alpha_s^2 \, P^{(2)}(x) + \dots \right]$$

At small x, it can be shown that the dominant terms are of the form

$$\alpha_s^{k+1} \frac{\log^j x}{x}, \qquad 0 \le j \le k$$

(*small-x or high-energy or BFKL logarithms*) or, in Mellin space,

$$\alpha_s^{k+1} \frac{1}{(N-1)^j}, \qquad 0 \le j \le k+1$$

When

$$\boxed{\alpha_s \log \frac{1}{x} \sim \mathcal{O}(1) \qquad \Leftrightarrow \qquad \frac{\alpha_s}{N-1} \sim \mathcal{O}(1)}$$

resummation of these terms is needed.

Small-x resummation

The anomalous dimension $\gamma(N, \alpha_s)$ can be resummed by combining GLAP equations with BFKL equations.

[Altarelli, Ball, Forte, NPB 742 (2006) 1-40]

What about coefficient functions?

It can be shown that the resummation of the leading small-N singularity of the coefficient function $C(N, \alpha_s)$ can be obtained from the resummed gluon anomalous dimension $\gamma(N, \alpha_s)$

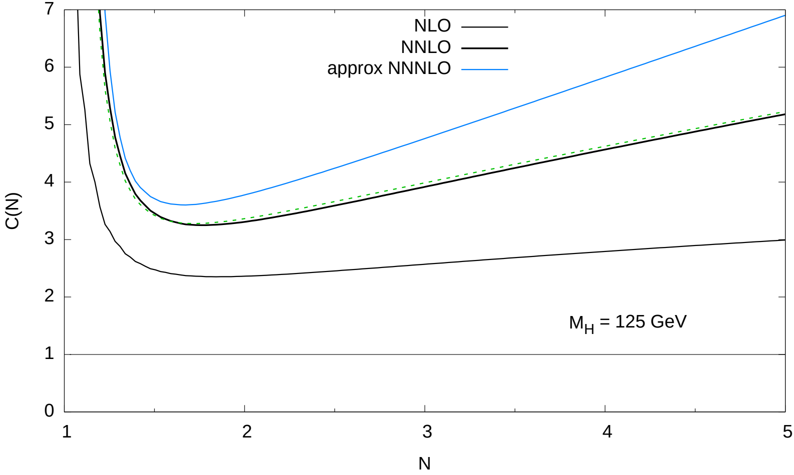
[Catani, Hautmann, NPB 427 (1994) 475-524]

[Altarelli, Ball, Forte, NPB 799 (2008) 199-240]

We have now the tools to build an approximate Higgs coefficient function at higher orders.

Higgs coefficient function at order α_s^3

$$C(N,\alpha_s) = 1 + \alpha_s\, C^{(1)}(N) + \alpha_s^2\, C^{(2)}(N) + \alpha_s^3\, C^{(3)}(N) + \dots$$

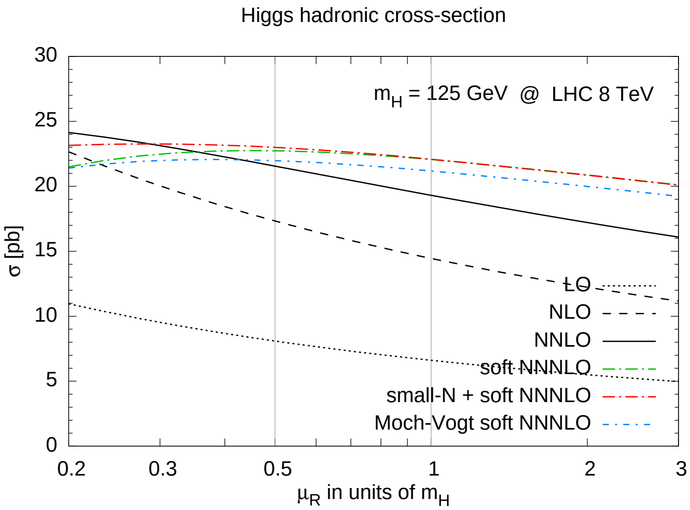


[Ball, MB, Forte, Marzani, Ridolfi (work in progress)]

N^{3}LO prediction for Higgs production

[Moch, Vogt, PLB 631 (2005) 48-57]: soft only

[Ball, MB, Forte, Marzani, Ridolfi (work in progress)]: soft and small-N



Conclusions

Motivations

- precise phenomenology at LHC requires accurate predictions
- Higgs production at the LHC
  - is the large m_t approximation good?
  - is the soft approximation good?
  - is soft-gluon resummation *needed*?

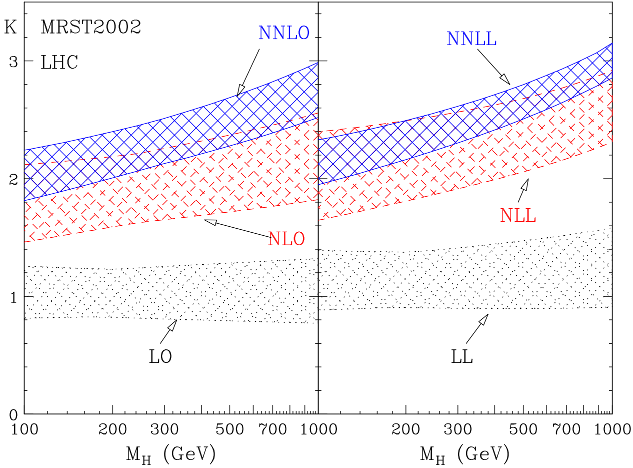
Results

- saddle-point analysis:
  - relates the dominant partonic region to hadron kinematics
  - allows to establish when the approximations are good
  - shows that soft-gluon resummation is never really *needed*
  - hints that small-x (BFKL) resummation may be relevant at LHC
- Approximate prediction for Higgs cross-section at N^{3}LO using both soft and small-z terms

Backup slides

Threshold resummation for Higgs production

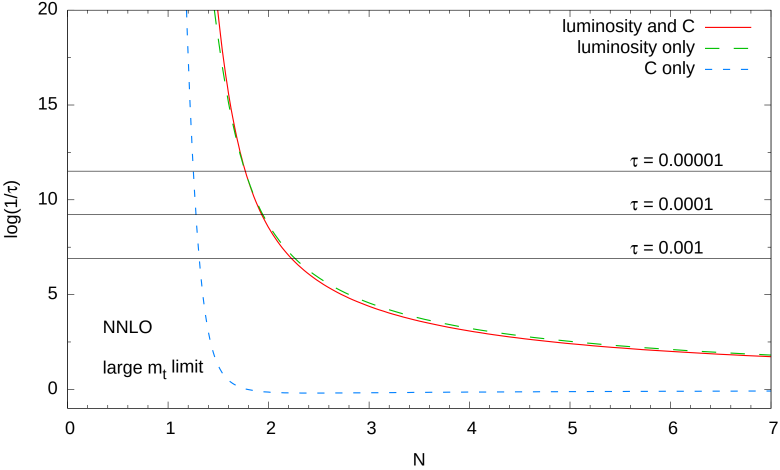
LHC @ 14 TeV



[Catani, De Florain, Grazzini, Nason, 2003]

Saddle point: a closer look

Saddle point: $$\log\frac{1}{\tau} = -\frac{d}{dN}\log\mathscr{L}(N) - \frac{d}{dN}\log C(N,\alpha_s)$$



N_0 depends essentially on $\mathscr{L}(N)$ only, and mostly on its small-N behaviour.

Small-x resummation (1)

Resummation of such logarithms is performed combining GLAP and BFKL evolution equations, which describe the evolution of the gluon PDF $g(x, \mu^2)$ wrt μ^2 and x respectively

$$\mu^2 \frac{d}{d\mu^2} g(x, \mu^2) = \int_x^1 \frac{dz}{z} P\left(\frac{x}{z}, \alpha_s(\mu^2)\right) g(z, \mu^2)$$

$$-x \frac{d}{dx} g(x, \mu^2) = \int_0^\infty \frac{d\nu^2}{\nu^2} K\left(\frac{\mu^2}{\nu^2}, \alpha_s\right) g(x, \nu^2)$$

The BFKL equation is valid at small-x only.
Taking a double Mellin transform we get

$$[M - \gamma(N, \alpha_s)]\, g(N, M) = F_0(N)$$

$$[N - 1 - \chi(M, \alpha_s)]\, g(N, M) = \tilde{F}_0(M)$$

where $\chi(M, \alpha_s)$ is the Mellin transform of $K(\nu^2, \alpha_s)$.

Small-x resummation (2)

$$[M - \gamma(N, \alpha_s)]\, g(N, M) = F_0(N)$$
$$[N - 1 - \chi(M, \alpha_s)]\, g(N, M) = \tilde{F}_0(M)$$

These two solutions must coincide in the region where they are both valid

$$N \sim 1, \qquad M \sim 0$$

This brings to the *duality* relation

$$\chi\left(\gamma(N, \alpha_s), \alpha_s\right) = N - 1 \qquad \leftrightarrow \qquad \gamma\left(\chi(M, \alpha_s), \alpha_s\right) = M$$

At LO, $\chi(M, \alpha_s) = \alpha_s \chi_0(M)$ and then

$$\chi_0\left(\gamma(N, \alpha_s)\right) = \frac{N-1}{\alpha_s} \qquad \to \qquad \gamma(N, \alpha_s) = \chi_0^{-1}\left(\frac{N-1}{\alpha_s}\right)$$

resums all powers in $\dfrac{\alpha_s}{N-1}$!!!

Discussion on what is SOFT (1)

In z-space: $\mathcal{D}_k(z) = \left[\frac{\log^k(1-z)}{1-z}\right]_+$

$2\mathcal{D}_1(N) =$

$= \psi_0^2(N) - \psi_1(N) + 2\gamma\psi_0(N) + \zeta_2 + \gamma^2$	$\left[\frac{\log(1-z)}{1-z}\right]_+$
$\simeq \psi_0^2(N) + 2\gamma\psi_0(N) + \zeta_2 + \gamma^2$	$\left[\frac{\log\frac{1-z}{\sqrt{z}}}{1-z}\right]_+' = \left[\frac{\log(1-z)}{1-z}\right]_+ - \frac{\log\sqrt{z}}{1-z}$
$\simeq \log^2\frac{1}{N-\frac{1}{2}} + 2\gamma\log\frac{1}{N-\frac{1}{2}} + \zeta_2 + \gamma^2$	$\frac{1}{\sqrt{z}}\left[\frac{\log\log\frac{1}{z}}{\log\frac{1}{z}}\right]_+ + (\zeta_2 + \gamma^2)\delta(1-z)$
$\simeq \log^2\frac{1}{N} + 2\gamma\log\frac{1}{N} + \zeta_2 + \gamma^2$	$\left[\frac{\log\log\frac{1}{z}}{\log\frac{1}{z}}\right]_+ + (\zeta_2 + \gamma^2)\delta(1-z)$
$\simeq \psi_0^2(N+1) + 2\gamma\psi_0(N+1) + \zeta_2 + \gamma^2$	$z\left[\frac{\log\frac{1-z}{\sqrt{z}}}{1-z}\right]_+'$

The last option allows to include all the terms $\alpha_s^n\log^{2n-1}(1-z)$ in the soft terms (equivalent to the collinear improvement of [Krämer, Laenen, Spira 1997], [Catani, de Florian, Grazzini 2001], [Catani, de Florian, Grazzini, Nason 2003])

Discussion on what is SOFT (2)

$$\mathcal{D}_k = \left[\frac{\log^k(1-z)}{1-z}\right]_+ \qquad \tilde{\mathcal{D}}_k = \left[\frac{\log^k \frac{1-z}{\sqrt{z}}}{1-z}\right]_+' \qquad \mathcal{D}_k^{\log} = \left[\frac{\log^k \log\frac{1}{z}}{\log\frac{1}{z}}\right]_+ + c_k\delta(1-z)$$