Physics beyond the Standard Model

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Scope

The lectures focus on the phenomenology of physics beyond the Standard Model associated with new particles in the 100 GeV– 10 TeV range. I will discuss indirect searches exploiting precision measurements.

The concept is bottom-up, starting from the Standard Model and its problems, with emphasis on the *standard way beyond the Standard Model*: Grand unification and supersymmetry.

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Not covered:
Search strategies for new particles at colliders, theories of gravitation, large extra dimensions, strongly interacting Higgs sectors, . . .

Contents of Lecture I

The theorist's toolbox

A theory's particles and their interactions are encoded in the Lagrangian $\mathcal{L}$. To construct a Lorentz-invariant theory specify:

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- the internal symmetries of $\mathcal{L}(\phi_j, \psi_k, A_l^\mu)$: Transforming

$$\phi_j \to U_{jj'}\phi_{j'}, \quad \psi_k \to U'_{kk'}\psi_{k'}, \quad A_l^\mu \to U''_{ll'}A_{l'}^\mu$$

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The set G of matrices U form a *group*, meaning that $\mathbb{1} \in G$, with $U^{(1)}, U^{(2)} \in G$ also $U^{(1)}U^{(2)} \in G$, and for each $U \in G$ there is an inverse $U^{-1} \in G$. The corresponding matrices U', U'' fulfill the same multiplication law, e.g. $U^{(1)}U^{(2)} = U^{(3)} \Rightarrow U^{(1)\prime}U^{(2)\prime} = U^{(3)\prime}$. That is, the sets $\{U\}$, $\{U'\}$, and $\{U''\}$ are all *representations* of the symmetry group G.

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  leaves $\mathcal{L}$ invariant, $\mathcal{L} \to \mathcal{L}$. The set G of matrices U form a *group*.
- the representations of G according to which the fields $\phi_j(x), \psi_k(x), A^\mu_l(x)$ transform.
- whether $\mathcal{L}$ shall be *renormalisable by power-counting* or not.

More on symmetries

Example:

$$\mathrm{SU}(2)=\{U \in \mathbb{C}^{2\times 2} : U^\dagger U = \mathbb{1} \text{ and } \det U = 1\}$$

$U(\vec{\phi}) = \exp[i\phi_j\sigma^j/2]$ is an SU(2) rotation with angle $\phi \equiv |\vec{\phi}|$ around the axis $\vec{\phi}/\phi$. Pauli matrices:

$$\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \qquad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

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Application: Weak isospin: The left-handed (left-chiral) fermion fields of the Standard Model,

$$L^1=\begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix}, \; L^2=\begin{pmatrix} \nu_{\mu L} \\ \mu_L \end{pmatrix}, \; L^3=\begin{pmatrix} \nu_{\tau L} \\ \tau_L \end{pmatrix}, \quad Q^1=\begin{pmatrix} u_L \\ d_L \end{pmatrix}, \; Q^2=\begin{pmatrix} c_L \\ s_L \end{pmatrix}, \; Q^3=\begin{pmatrix} t_L \\ b_L \end{pmatrix}$$

transform under the weak SU(2) group according to the defining representation, e.g. $L^j \to \exp[i\phi_k \sigma^k/2] L^j$, they are doublets.

The right-handed fermion fields $e_R, \mu_R, \tau_R, u_R, d_R, s_R, c_R, b_R, t_R$ are singlets of SU(2), they live in the trivial representation: e.g. $e_R \to e_R$.

SU(2)=$\{\exp[i\phi_j\sigma^j/2] : \phi_{1,2,3} \in \mathbb{R}\}$

The Pauli matrices are the generators of SU(2), they satisfy the commutation relations

$$\left[\frac{\sigma^k}{2}, \frac{\sigma^l}{2}\right] = i\epsilon_{klm}\frac{\sigma^m}{2}$$

with the Levi-Civita tensor defined by
$\epsilon_{123} = \epsilon_{231} = \epsilon_{312} = 1, \;\; \epsilon_{lkm} = -\epsilon_{klm}$.

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$\epsilon_{123}=\epsilon_{231}=\epsilon_{312}=1,\ \ \epsilon_{lkm}=-\epsilon_{klm}$.
The SU(2) matrices $U(\vec{\phi})$ are continuous functions of the parameters ϕ_j, making SU(2) a *Lie group* of matrices.
The generators of a Lie group span a vector space, the *Lie algebra*. The Lie algebra su(2) of SU(2) is spanned by $\frac{\sigma^1}{2},\frac{\sigma^2}{2},\frac{\sigma^3}{2}$ and therefore consists of all hermitian 2×2 matrices with trace zero. (su(2) is a *real* Lie algebra, meaning that only linear combinations with real coefficents are allowed.)

Any set of matrices T^1, T^2, T^3 satisfying the commutation relations

$$\left[T^k, T^l\right] = i\epsilon_{klm} T^m$$

form a *representation* of su(2). The $3^3 = 27$ numbers ϵ_{klm} are the *structure constants* of su(2).

Important: adjoint representation

$$\left[T^k_{\mathrm{su}(2)}\right]_{lm} := -i\epsilon_{klm}$$

$$T^1_{\mathrm{su}(2)} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \quad T^2_{\mathrm{su}(2)} = \begin{pmatrix} 0 & 0 & i \\ 0 & 0 & 0 \\ -i & 0 & 0 \end{pmatrix}, \quad T^3_{\mathrm{su}(2)} = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

The generators are related to quantum numbers (charges) of the fields:
Doublets: Take L^1 as an example:

$$\frac{(\sigma^1)^2 + (\sigma^2)^2 + (\sigma^3)^2}{4} L^1 = \frac{1}{2}\left(\frac{1}{2}+1\right) L^1, \quad \frac{\sigma^3}{2} L^1 = \frac{1}{2}\begin{pmatrix} \nu_{eL} \\ -e_L \end{pmatrix}.$$

That is, the weak isospin quantum number I_W of the doublets is found as $I_W = 1/2$, the third component of the weak isospin is $I_W^3 = \pm 1/2$ for neutrino and electron, respectively.

Here I_W and I_W^3 are defined in analogy to the spin quantum numbers s and s_3 in quantum mechanics.

The adjoint representation has $I_W = 1$:

$$\left(T^1_{\mathrm{su}(2)}\right)^2 + \left(T^2_{\mathrm{su}(2)}\right)^2 + \left(T^3_{\mathrm{su}(2)}\right)^2 = 1\,(1+1)\,\mathbb{1}$$

Eigenvectors of $T^3_{\mathrm{su}(2)}$:

$$T^3_{\mathrm{su}(2)} \begin{pmatrix} 1 \\ i \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ i \\ 0 \end{pmatrix}, \quad T^3_{\mathrm{su}(2)} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = 0, \quad T^3_{\mathrm{su}(2)} \begin{pmatrix} 1 \\ -i \\ 0 \end{pmatrix} = - \begin{pmatrix} 1 \\ -i \\ 0 \end{pmatrix}.$$

We find eigenvalues $I^3_W = 1, 0, -1$, i.e. the adjoint representation is a triplet.

Gauge principle

If the group parameters (ϕ_j in the case of SU(2)) depend on the space-time coordinate $x = (t, \vec{x})$, one calls the symmetry *local* or *gauged*.

A lagrangian which is bilinear in the fields describes a free particle.

Example: Electron field e, a 4-component Dirac spinor field, with mass m:

$$\mathcal{L}_{\text{free}} = \overline{e}\left[i\partial\!\!\!/ - m\right] e$$

Here $\partial\!\!\!/ = \gamma^\mu \partial_\mu$ with the Dirac matrices γ^μ and $\overline{e} = e^\dagger \gamma^0$.

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Here $\partial\!\!\!/ = \gamma^\mu \partial_\mu$ with the Dirac matrices γ^μ and $\overline{e} = e^\dagger \gamma^0$.
$\mathcal{L}_{\text{free}}$ is invariant under *global* (i.e. x-independent) phase transformations

$$e \to e^{i\phi} e.$$

$$\mathcal{L}_{\text{free}} = \bar{e}\left[i\not{\partial} - m\right] e$$

is not invariant under local phase transformation with $\exp\left[-i\phi(x)\right]$. Spoiler:
$\partial_\mu \exp\left[-i\phi(x)\right] = -i(\partial_\mu\phi(x)) \exp\left[-i\phi(x)\right]$

The group of phase transformation is called U(1), for "unitary 1×1 matrices". The structure constants of U(1) vanish, thus different U(1) transformations always commute and U(1) is called *Abelian*.

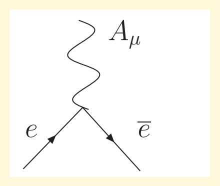
Remedy: *Gauge field A_μ* and *covariant derivative*

$$D_\mu = \partial_\mu + i g_e A_\mu$$

with $A_\mu \to A_\mu + \partial_\mu \phi(x)/g_e$. Need also kinetic term (i.e. bilinear term) for A^μ, conveniently expressed in terms of the field strength tensor $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$. Find *quantum electrodynamics*:

$$\mathcal{L}_{\text{QED}} = \overline{e}\left[i\not{D} - m\right] e - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

Interpretation: A_μ is the *photon field*. It mediates the electromagnetic force between electrons. $g_e \approx 0.30$ accompanying A_μ is the electromagnetic coupling constant.



Next SU(2): Consider lepton doublet $L^1 = \begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix}$: The covariant derivative involves three gauge fields $W_\mu^1, W_\mu^2, W_\mu^3$:

$$D_\mu = \partial_\mu - igW_\mu^a \frac{\sigma^a}{2}$$

and

$$\mathcal{L} = \bar{L} i \not{D} L - \frac{1}{4} F_{\mu\nu}^a F^{\mu\nu\, a}$$

Novel feature of *non-Abelian* gauge theory:

$$F_{\mu\nu}^a = \partial_\mu W_\nu^a - \partial_\nu W_\mu^a + g \epsilon_{abc} W_\mu^b W_\nu^c$$

Thanks to the third term in $F_{\mu\nu}^a$ the lagrangian contains *self-interactions* of the gauge bosons.

Gauge bosons live in the adjoint representation $\left[T^k_{\mathrm{su}(2)}\right]_{lm} := -i\epsilon_{klm}$:

Three parameters needed to describe an SU(2) rotation multiply three Pauli matrices.

$\Rightarrow$ Adjoint representation consists of 3×3 matrices.

$\Rightarrow$ three W-bosons!

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One parameter needed to describe a U(1) phase rotation $\exp\left[-i\phi(x)\right]$. The structure constant is zero, because U(1) is Abelian.

$\Rightarrow$ Adjoint representation consists of the 1×1 matrix 0.

$\Rightarrow$ one gauge boson A_μ!

Recall the eigenvectors of of $T^3_{\mathrm{su}(2)}$:

$$\begin{pmatrix} W^1 \\ W^2 \\ W^3 \end{pmatrix} = \frac{W^+}{\sqrt{2}} \begin{pmatrix} 1 \\ i \\ 0 \end{pmatrix} + \frac{W^-}{\sqrt{2}} \begin{pmatrix} 1 \\ -i \\ 0 \end{pmatrix} + W^3 \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

with $W^\pm = \dfrac{W^1 \mp iW^2}{\sqrt{2}}$.

Hence

$$T^3_{\mathrm{su}(2)} \begin{pmatrix} W^1 \\ W^2 \\ W^3 \end{pmatrix} = \frac{W^+}{\sqrt{2}} \begin{pmatrix} 1 \\ i \\ 0 \end{pmatrix} - \frac{W^-}{\sqrt{2}} \begin{pmatrix} 1 \\ -i \\ 0 \end{pmatrix}$$

and we realise that W^+, W^-, W^3 have the I^3_W quantum numbers $1, -1, 0,$ respectively.

Everything learned from U(1) and SU(2) generalises to other Lie groups as well:

We'll encounter SU(N) gauge theories with $N = 2, 3, 4, 5$:
The Lie algebra su(N) is spanned by $N^2 - 1$ traceless hermitian $N \times N$ matrices, therefore there are $N^2 - 1$ gauge bosons and the adjoint representation consists of $(N^2 - 1) \times (N^2 - 1)$ matrices.

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Other popular models of new physics involve SO(N), the group of real orthogonal $N \times N$ matrices with determinant equal to 1.
SO(N) has $N(N - 1)/2$ generators, which are traceless imaginary antisymmetric matrices.
SO(N) matrices describe rotations in $\mathbb{R}^n$.

Simple groups:

subgroup U of group G: $g_1, g_2 \in U \;\Rightarrow\; g_1 g_2 \in U$,
also: $g_2^{-1} g_1 g_2 \in U$

invariant subgroup U of G: $g_1 \in U, g_2 \in G \;\Rightarrow\; g_2^{-1} g_1 g_2 \in U$

A group G is called simple, if it has no invariant Lie subgroups (other than G and 1). The SU(N) and SO(N) groups (except for SO(4)) are simple.
E.g. SO(2) is a subgroup of SO(3), but it is not invariant.

With $g_1, g_1' \in G_1$ and $g_2, g_2' \in G_2$ the pairwise multiplication $(g_1, g_2) \cdot (g_1', g_2') = (g_1 g_1', g_2 g_2')$ defines another group, the direct product $G_1 \times G_2$.
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Physical significance: A direct product of gauge groups describes independent interactions. E.g. the gauge fields of $SU(2)$ (i.e. the W-bosons) carry no color or hypercharge.

Compare this with the electric charge:

The W-bosons carry electric charges!

$U(1)_{\mathrm{em}}$ is a non-invariant subgroup of $SU(2) \times U(1)_Y$.

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- fixes the interaction of gauge bosons, once the group representations of the fermion fields are specified,
- involves only a single coupling constant g for the boson-fermion and all boson-boson couplings.

Electroweak interaction

Gauge group: $SU(2) \times U(1)_Y$

doublets: $Q_L^j = \begin{pmatrix} u_L^j \\ d_L^j \end{pmatrix}$ und $L^j = \begin{pmatrix} \nu_L^j \\ \ell_L^j \end{pmatrix}$

$j = 1, 2, 3$ labels the generation.

Examples: $Q_L^3 = \begin{pmatrix} t_L \\ b_L \end{pmatrix}$, $L^1 = \begin{pmatrix} \nu^{eL} \\ e_L \end{pmatrix}$

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Important: Only left-handed fields couple to the W boson.

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- three gauge interactions
- Yukawa interaction of Higgs with quarks and leptons
- Higgs self-interaction

Spontaneous symmetry breaking

Higgs doublet field $H = \begin{pmatrix} H_1^+ \\ H_2^0 \end{pmatrix}$ with hypercharge quantum number $y = 1/2$.
(Beware of different normalisations of y in the literature!)

The classical Higgs potential is chosen such that it develops minima with $H \neq 0$. To quantise the theory around this minimum identify the value v at which $|H|^2$ is minimal with the vacuum expectation value of the quantised Higgs field.

These minima are related by an $SU(2) \times U(1)_Y$ gauge transformation. Use this to choose $\langle H \rangle = \begin{pmatrix} 0 \\ v \end{pmatrix}$ with $v = 174\,\text{GeV} = \dfrac{246\,\text{GeV}}{\sqrt{2}}$.

Spontaneous symmetry breaking $SU(2) \times U(1)_Y \to U(1)_{\text{em}}$:

$$T^3 \begin{pmatrix} 0 \\ v \end{pmatrix} = \frac{\sigma^3}{2} \begin{pmatrix} 0 \\ v \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 0 \\ -v \end{pmatrix} \neq 0, \qquad Y \begin{pmatrix} 0 \\ v \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 0 \\ v \end{pmatrix} \neq 0$$

but

$$\left[T^3 + Y \mathbb{1} \right] \begin{pmatrix} 0 \\ v \end{pmatrix} = \frac{\sigma^3 + \mathbb{1}}{2} \begin{pmatrix} 0 \\ v \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ v \end{pmatrix} = 0$$

and we recognise the electric-charge operator

$$Q = T^3 + Y \qquad \text{Gell-Mann–Nishijima relation}$$

Note: The mass parameter μ^2 of the Higgs potential is the only dimensionful parameter of the SM Lagrangian. Its value is chosen to give the correct $v = \sqrt{\sqrt{2}/(4G_F)}$, with the Fermi constant G_F determined from muon decay. All masses of elementary particles are proportional to v.