Testing Lorentz Invariance of Dark Matter

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CERN

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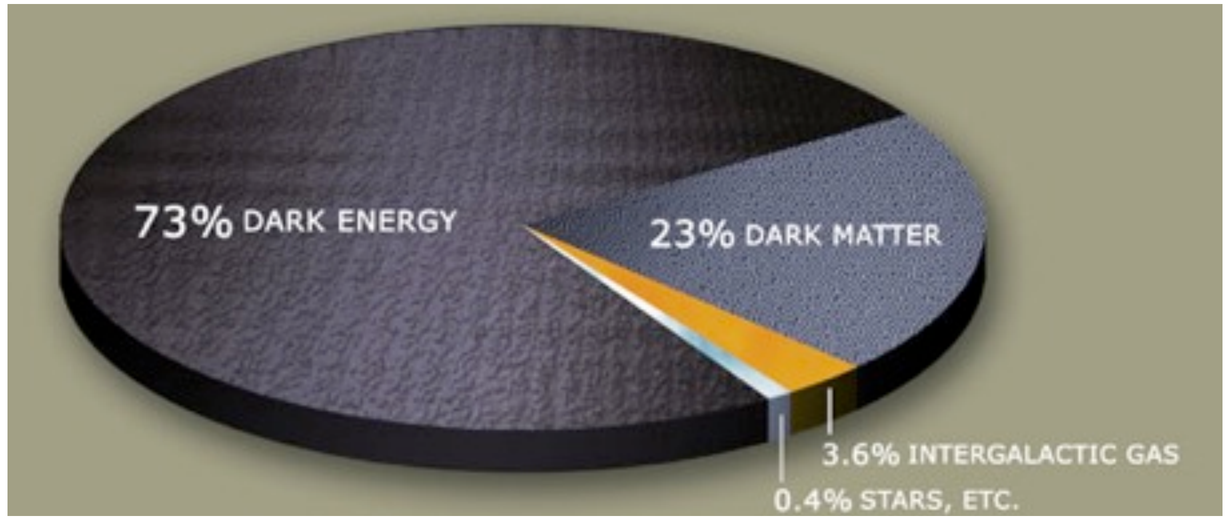
arXiv:1211.xxxx

Why testing Lorentz Invariance (LI)?



Lorentz invariance (LI) is assumed in
Standard Model, Gravity, Dark Matter

Bounds: $\sim 10^{-20}$ $\sim 10^{-7}$ $??$

Faces of breaking LI:

- properties of matter (and DM)
- better for quantum gravity (Hořava gravity)
- alternative to GR/ΛCDM

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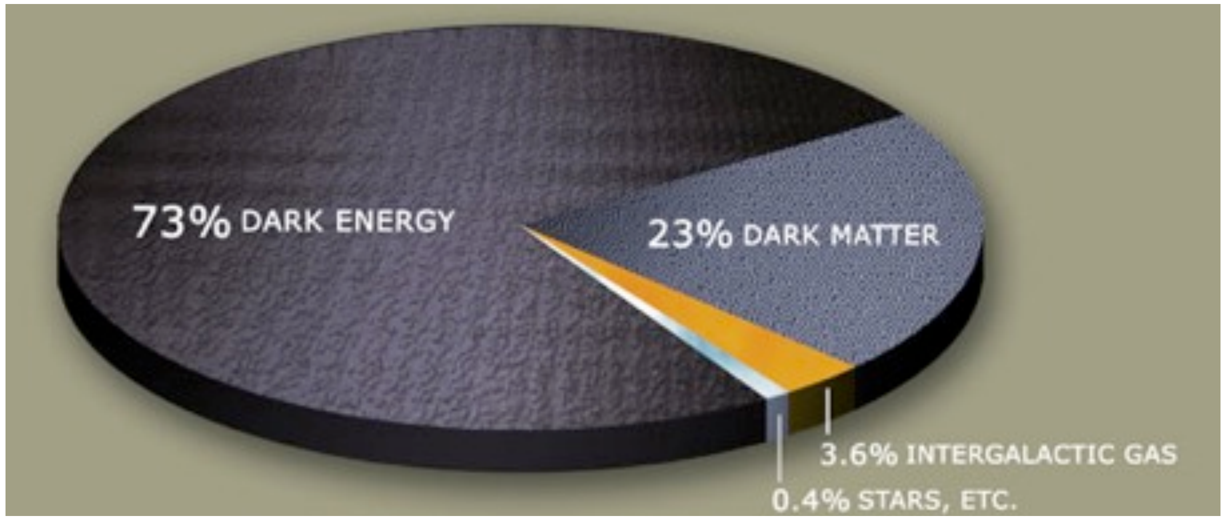

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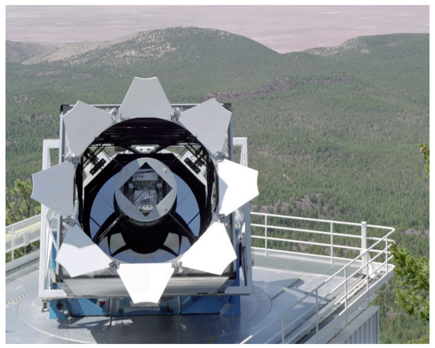
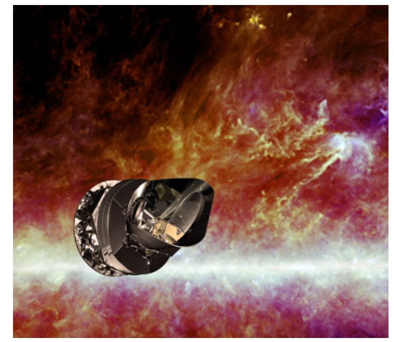
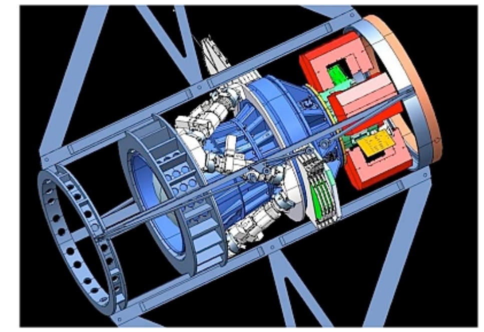
Dark sector missions



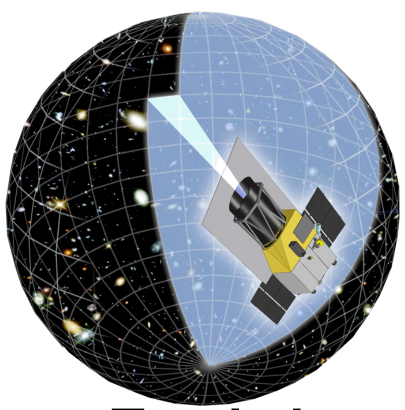
SDSS



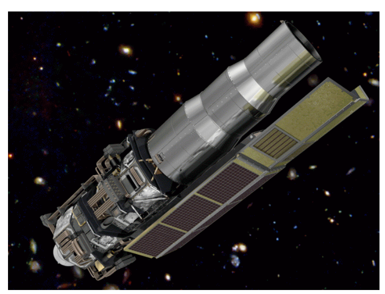
Planck



DES



Euclid

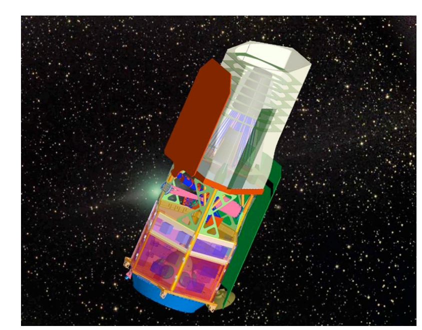


JEDI

Large Synoptic Survey Telescope

WIGGLEZ

SNAP



WFIRST

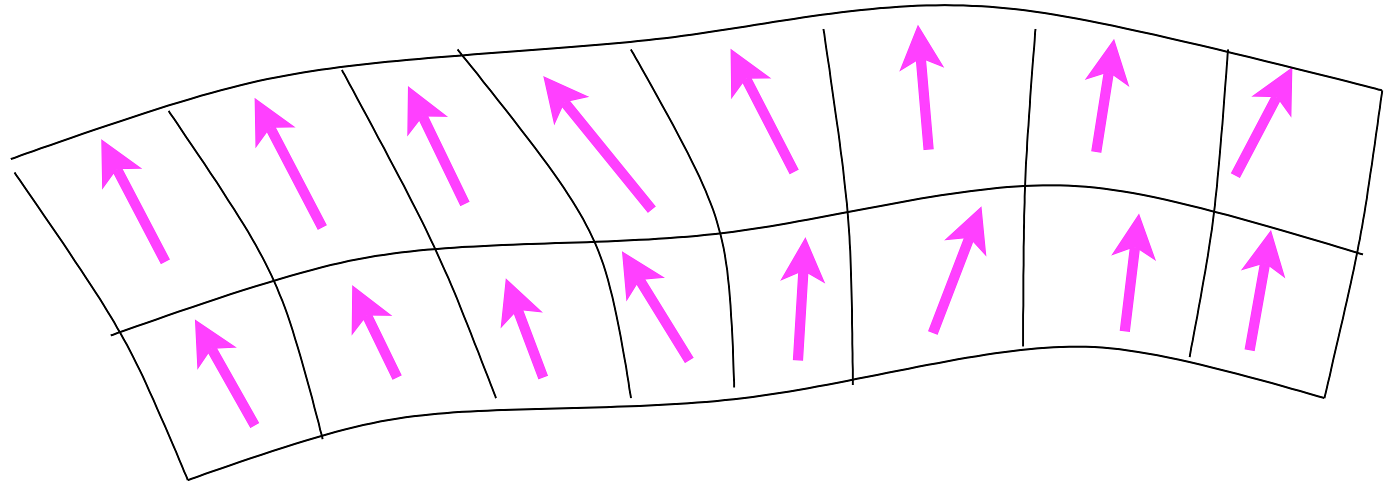
....

Precision data to constrain any compelling model!

Breaking Lorentz Invariance

Space-time filled by a preferred **time** direction

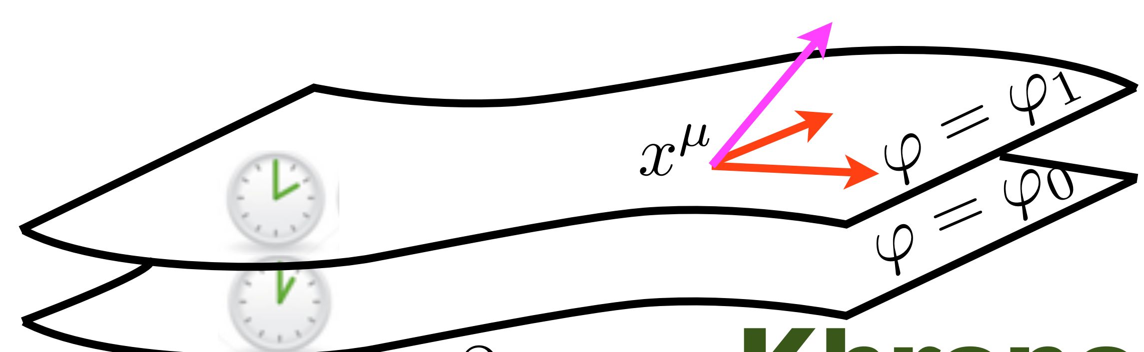
Associated to a time-like unit vector u_μ



Generic: **Einstein-aether**	Hypersurface orthogonal: **Khronometric**

$$u_\mu u^\mu = 1$$

Scalar-vector



Khronon

$$u_\mu \equiv \frac{\partial_\mu \varphi}{\sqrt{\partial_\alpha \varphi \partial^\alpha \varphi}}$$

Gravitational Lagrangian

Ingredients: $u_\mu \;,\; g_{\mu\nu}$

Khronometric case $\qquad u_\mu \equiv \dfrac{\partial_\mu \varphi}{\sqrt{\partial_\alpha \varphi \partial^\alpha \varphi}}$

$$\mathcal{L}_{\chi GR} = \mathcal{L}_{EH} + \sqrt{-g}\left(\lambda \left(\nabla^\mu u_\mu\right)^2 + \alpha \left(u^\nu \nabla_\nu u_\mu\right)^2 + \beta \nabla_\mu u_\nu \nabla^\nu u^\mu\right)$$

- ★ Massless Spin 2 graviton $\quad \omega^2 = c_t^2\, \mathrm{k}^2$
- ★ Extra massless scalar (new force!) $\quad \omega^2 = c_\chi^2\, \mathrm{k}^2$ $\quad \varphi = t + \chi \leftarrow$ **Khronon**

$$c_t^2 = \frac{1}{1-\beta} \qquad c_\chi^2 = \frac{\beta + \lambda}{\alpha}$$

Einstein-aether (generic u_μ): extra term

- ★ Extra vector polarizations $\quad u_\mu = \bar{u}_\mu + \delta u_\mu$

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EFT with cut-off

$\Lambda_{IR} \sim \sqrt{\alpha}\, M_P$

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Known UV completion: Hořava gravity $+O(1/\Lambda_{IR})$

EFT with cut-off $\Lambda_{IR} \sim \sqrt{\alpha} M_P$

Matter Lagrangian

Ingredients: $u_\mu \; , \; g_{\mu\nu}$ + SM Fields + DM

$$\mathcal{L}_m = \mathcal{L}_{LI}(\text{SM}, \text{DM}, g_{\mu\nu}) + \kappa_1 \; \mathcal{L}_{LB}(\text{SM}, g_{\mu\nu}, u_\mu) + \kappa_2 \; \mathcal{L}_{LB}(\text{DM}, g_{\mu\nu}, u_\mu)$$

e.g. $\bar{\psi}\, u^\mu u^\nu \gamma_\mu \partial_\nu \psi$

$\bar{\psi}\, \gamma_0 \partial_0 \psi$

SM: $\kappa_1 \lesssim 10^{-20}$

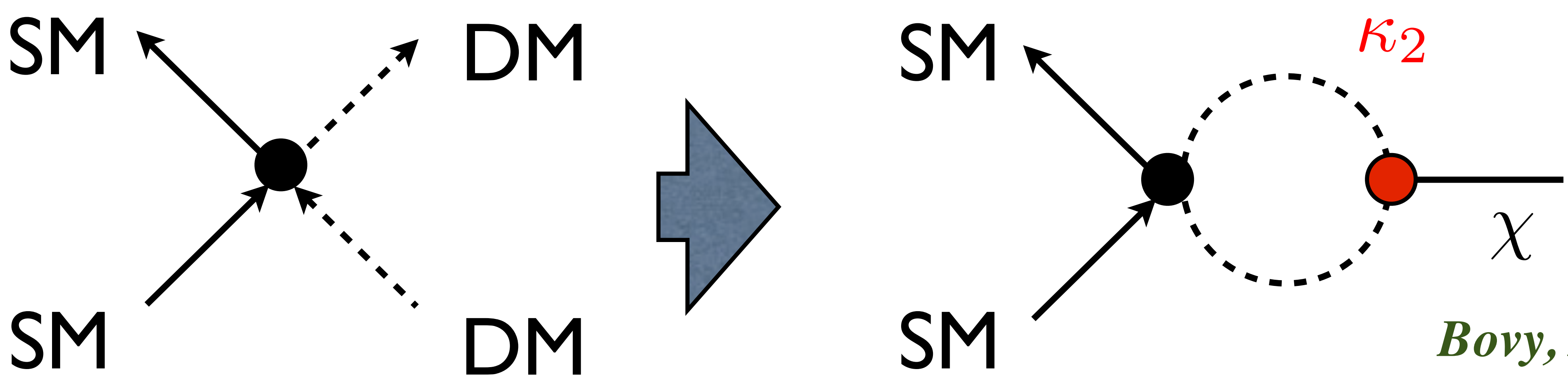
Dynamical explanation?

In the following $\kappa_1 = 0$

DM: $\kappa_2 ?$

Digression: Direct Detection

Very strong bounds can be placed if **DM** is **directly detected** soon



Bovy, Farrar 08

$$\mathcal{L} \supset A \; \bar{q} q \bar{\psi}_{DM} \psi_{DM}$$

$$\sigma \sim A^2 \; m_q^2$$

$$\kappa_1^{eff} = A \, \Lambda^2 \kappa_2 \lesssim 10^{-12} \frac{\mathrm{GeV}}{M_P}$$

A κ_2 relevant for cosmology requires $\sigma \lesssim 10^{-46} \ \mathrm{cm}^{-2}$

$$\sigma_{exp} \lesssim 10^{-43} \ \mathrm{cm}^{-2}$$

Gravitational Constraints

Theoretical

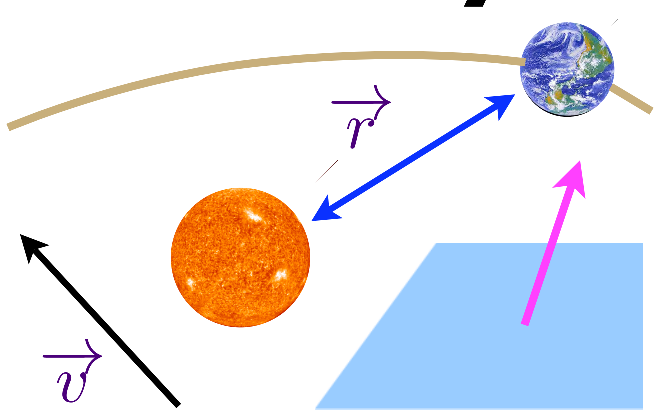
no singular limits, stability, no Cerenkov

$$c_\chi^2 > 0, \quad c_t^2 > 0$$

$$0 < \alpha < 2 \qquad c_t^2 \geq 1, \; c_\chi^2 \geq 1$$

cut-off $\Lambda_{IR} \sim \sqrt{\lambda}\, M_P$

Solar system



$\vec{v} \sim 10^{-2}$

$$h_{00} = -2G_N \frac{M}{r}\left(1 - \frac{(\alpha_1^{PPN} - \alpha_2^{PPN})v^2}{2} - \frac{\alpha_2^{PPN}}{2}\frac{(x^i v^i)^2}{r^2}\right)$$

$$h_{0i} = \frac{\alpha_1^{PPN}}{2} G_N \frac{m}{r} v^i$$

$$\alpha_1^{PPN} = -4(\alpha - 2\beta) \qquad \lesssim 10^{-4}$$

$$G_N = \frac{1}{8\pi M_P^2 (1 - \alpha/2)}$$

$$\alpha_2^{PPN} = \frac{(\alpha - 2\beta)(\alpha - \lambda - 3\beta)}{2(\lambda + \beta)} \qquad \lesssim 10^{-7}$$

Gravitational Waves

Bounds $O(.01)$

Summary Local Tests

The models without LI are phenomenologically viable (and maybe UV complete) provided

$\kappa_1 = 0$ cut-off $\Lambda_{IR} \sim \sqrt{\lambda}\, M_P$

i) $\alpha = 2\beta$ $\alpha \sim \lambda \lesssim 10^{-2}$ (GW)

ii) $\beta = 0$ $\alpha = \lambda \lesssim 10^{-4}$ α_1^{PPN}

iii) Generic $\alpha \sim \lambda \sim \beta \lesssim 10^{-7}$ α_2^{PPN}

Breaking LI and Dark Matter

Lorentz Breaking (LB) of Dark Matter

$$\mathcal{L}_m = \mathcal{L}_{LI}(\mathrm{SM}, \mathrm{DM}, g_{\mu\nu}) + \cancel{\kappa_1\, \mathcal{L}_{LB}(\mathrm{SM}, g_{\mu\nu}, u_\mu)}$$
$$+\kappa_2\, \mathcal{L}_{LB}(\mathrm{DM}, g_{\mu\nu}, u_\mu)$$

but... CDM is non-relativistic, can we probe κ_2?

$$E = \sqrt{f(p^2) + M^2} = M + O(v)$$

LB effects from the coupling to $u_\mu = \bar{u}_\mu + \delta u_\mu$:

(i) the background $\bar{u}_\mu$ modifies the inertial mass

(ii) new interaction from δu_μ

DM gravitates differently:

no equivalence principle and enhanced collapse

LB Dark Matter: Point particles

$$S_{pp} = -m \int \mathrm{d}s \qquad \Rightarrow \qquad S_{pp} = -m \int \mathrm{d}s\, f(u_\mu v^\mu)$$

$$\mathrm{d}s = \sqrt{g_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu} \qquad v^\mu = \frac{\mathrm{d}x^\mu}{\mathrm{d}s}\,, \quad u_\mu = \bar{u}_\mu + \delta u_\mu$$

Newtonian limit: $v^i,\ \delta u^i \ll 1 \qquad g_{00} = 1 + 2\phi$

$$S = M_P^2 \int \mathrm{d}^4x \left[\phi\Delta\phi + \frac{\alpha}{2}\delta u^i \Delta \delta u^i\right] + \int \mathrm{d}^4x\, \rho \left[\frac{(v^i)^2}{2} - \phi - Y(\delta u^i - v^i)^2\right]$$

Extra force — DM density — Extra force

$$\rho \equiv \sum m_i \delta^{(3)}(x - x_i(t)) \qquad Y \equiv f'(1)$$

$$a_i \equiv \frac{\mathrm{d}v_i}{\mathrm{d}t} = -\frac{\partial_i \phi}{1 - Y} + F_i^{LB}(\delta u, v)\,, \qquad 2M_P^2\,\Delta\phi = \rho$$

Modified inertial mass: no equivalence principle

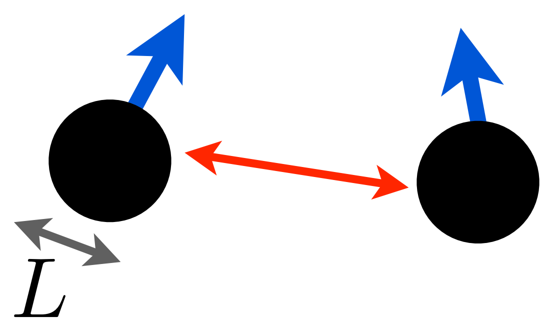
Newtonian Cosmology: Jeans Instability

$$S = M_P^2 \int \mathrm{d}^4x \left[\phi\Delta\phi + \frac{\alpha}{2}\delta u^i \Delta \delta u^i\right] + \int \mathrm{d}^4x\, \rho \left[\frac{(v^i)^2}{2} - \phi - Y(\delta u^i - v^i)^2\right]$$

$$\rho(x,t) \equiv \rho(t)(1+\delta(x,t))$$

Potential for DM and aether: $\rho\, Y$

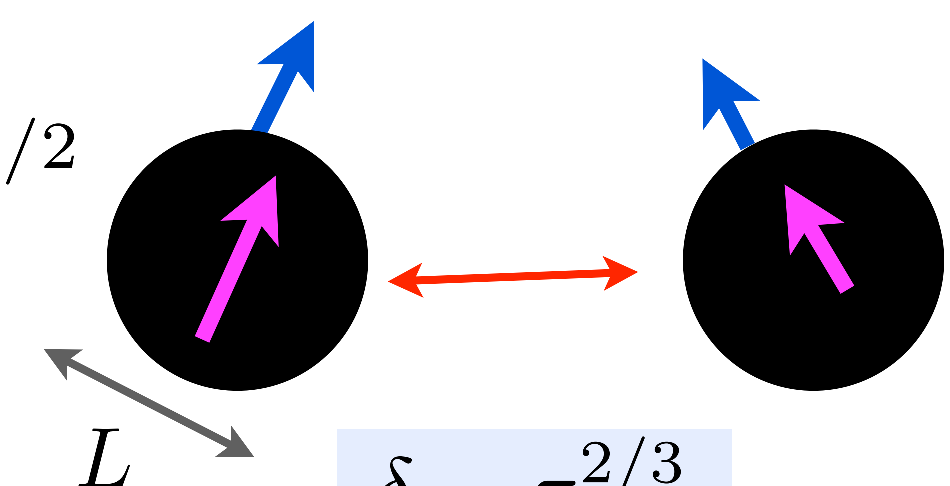
(i) $L \ll \left(\frac{\alpha M_P^2}{\rho\, Y}\right)^{1/2}$



$$F = \frac{F_N}{1-Y} \qquad Y > 0$$

Faster Jeans instability: $\delta \sim \tau^{\gamma}$, $\gamma = \frac{1}{6}\left[-1 + \sqrt{\frac{25-Y}{1-Y}}\right]$

(ii) $L \gg \left(\frac{\alpha M_P^2}{\rho\, Y}\right)^{1/2}$



$\delta \sim \tau^{2/3}$

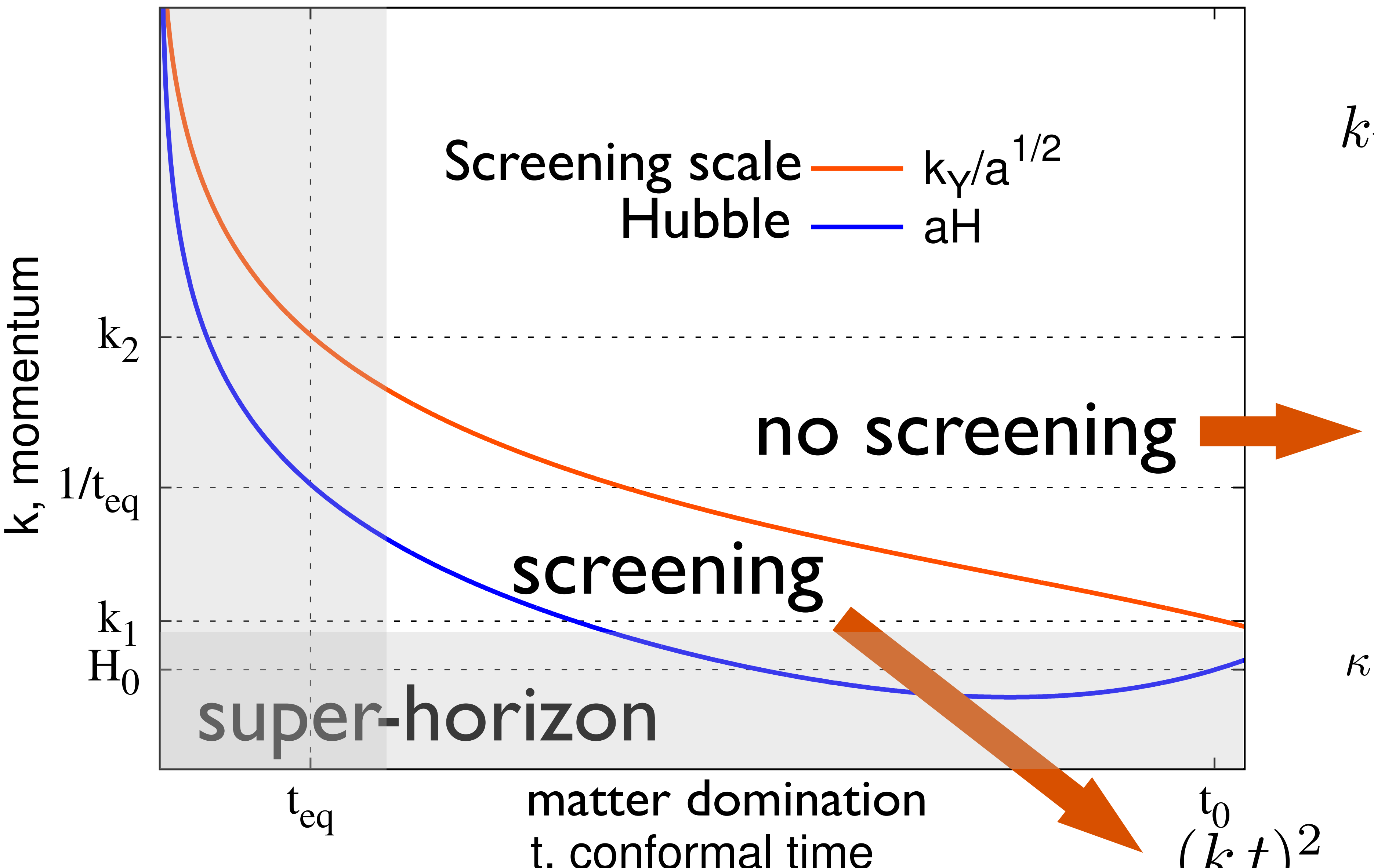
$$F = F_N$$

Screening

Cosmological perturbations

$$\rho(x,t) \equiv \rho(t)(1+\delta(x,t))$$

Scalars: All effects summarized in $Y \equiv \tilde{f}'(1)$



$$k_Y^2 \equiv \frac{3H_0^2\Omega_{dm}Y}{(\beta+\lambda)(1-Y)}$$

$$\delta \sim \frac{(k\,t)^2}{6(1-Y)}t^\kappa$$

$$\kappa = \sqrt{25+\frac{24\Omega_{dm}Y}{\Omega_{cm}(1-Y)}}-5$$

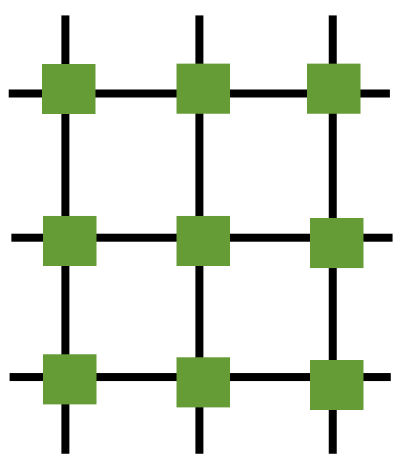
$$\delta \sim \frac{(k\,t)^2}{6}$$

Relativistic fluid without LI

At cosmological distances matter behaves like a relativistic (almost) pressureless perfect **fluid:**

$\rho(x,t)$ $\quad$ $v^\mu(x,t)$ $\quad$ equation of state

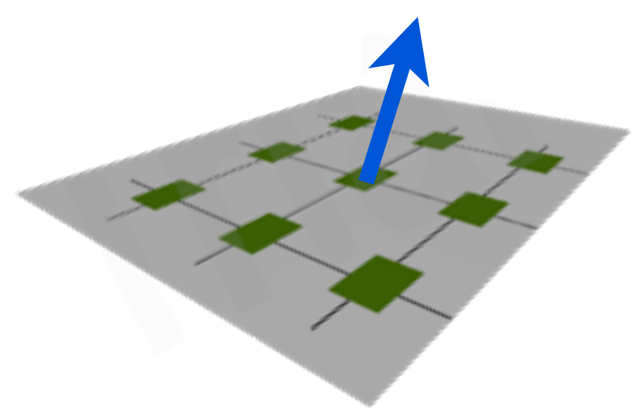
Pull-back formalism of **fluids**



$\varphi^I(x,t)$

$I = \{1,2,3\}$

Internal space of fluid elements



Space-time

 $\equiv v^\mu$, $\quad v^\mu v_\mu = 1$

$$v^\mu \partial_\mu \varphi^I = 0$$

$I = \{1,2,3\}$

$$B \equiv -\det\left[g^{\mu\nu}\partial_\mu\varphi^I\partial_\nu\varphi^J\right]$$

$$\nabla^\mu(\sqrt{B}v^\mu) = 0 \quad \Longleftrightarrow \quad n \equiv \sqrt{B}$$

Relativistic fluid without LI

Perfect fluids → Only compression modes

Physics invariant under transformations preserving the volume

$$\varphi \mapsto \tilde{\varphi}(\varphi)$$
$$\det \frac{\partial \tilde{\varphi}^I}{\partial \varphi^J} = 1$$

LI:

$$\underset{fluid}{S} = \int \mathrm{d}^4x \sqrt{-g} f(B)$$

$$B \equiv -\det\left[g^{\mu\nu}\partial_\mu\varphi^I\partial_\nu\varphi^J\right]$$
$$\overset{fluid}{T_{\mu\nu}} = (\rho + p) v_\mu v_\nu - p\, g_{\mu\nu}$$
$$p = 2f'(B)B - f(B)$$
$$\rho = f(B)$$

~~LI~~:

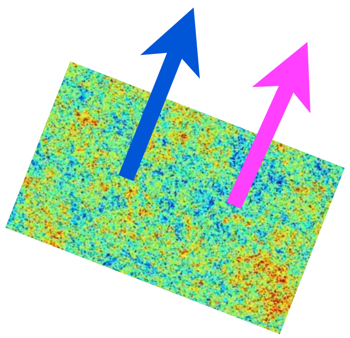
$$\underset{fluid}{S} = \int \mathrm{d}^4x \sqrt{-g} f(B, u^\mu v_\mu) = -m \int \mathrm{d}^4x \sqrt{B}\, \tilde{f}(u^\mu v_\mu)$$

Plausible assumption $\underset{fluid}{S} \propto \; n = \sqrt{B}$

Relativistic Cosmology

$$G_{\mu\nu} = \frac{1}{M_P^2} T^m_{\mu\nu} + \frac{1}{M_P^2} T^{fluid}_{\mu\nu} + \frac{1}{M_P^2} T^{aether}_{\mu\nu} + \Lambda \, g_{\mu\nu}$$

Background: Homogeneous and isotropic (preferred foliation aligned with CMB frame)



$$\mathrm{d}s^2 = g_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu = \mathrm{d}t^2 - a(t)^2 \mathrm{d}x^i \mathrm{d}x^i$$

$$u_\mu = (u_0(t), 0, 0, 0) = v_\mu \quad , \quad \rho(t)$$

Friedmann equations almost not modified!

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G_c}{3} \rho_m$$

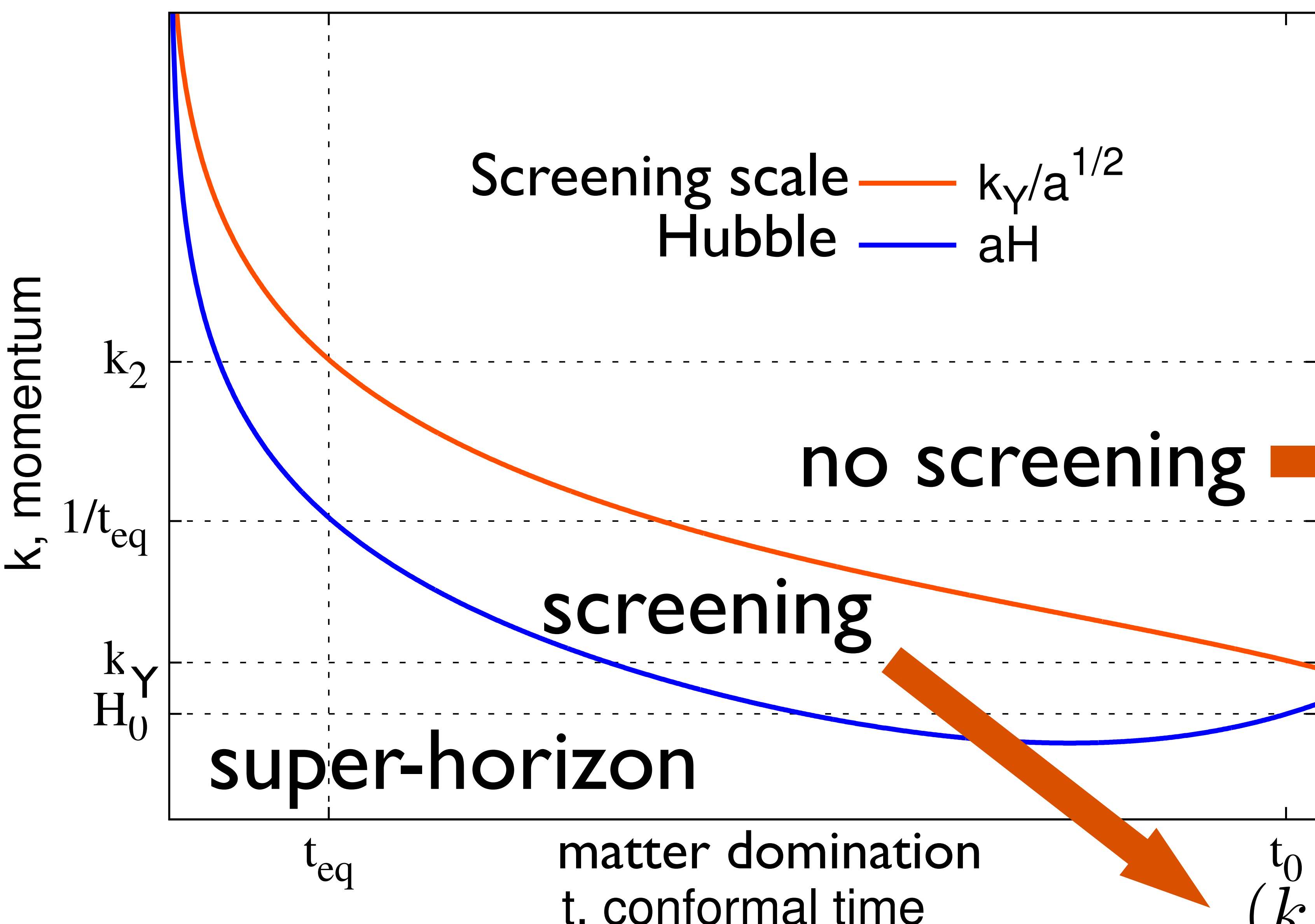
$$G_c = \frac{1}{8\pi M_P^2 [1 + 3\lambda/2 + \beta/2]}$$

From BBN ($^4\mathrm{He}$ abundance) $G_c = G_N + O(.01)$

Cosmological perturbations

$$\rho(x,t) \equiv \rho(t)(1+\delta(x,t))$$

Scalars: All effects summarized in $\color{red}{Y} \equiv \tilde{f}'(1)$



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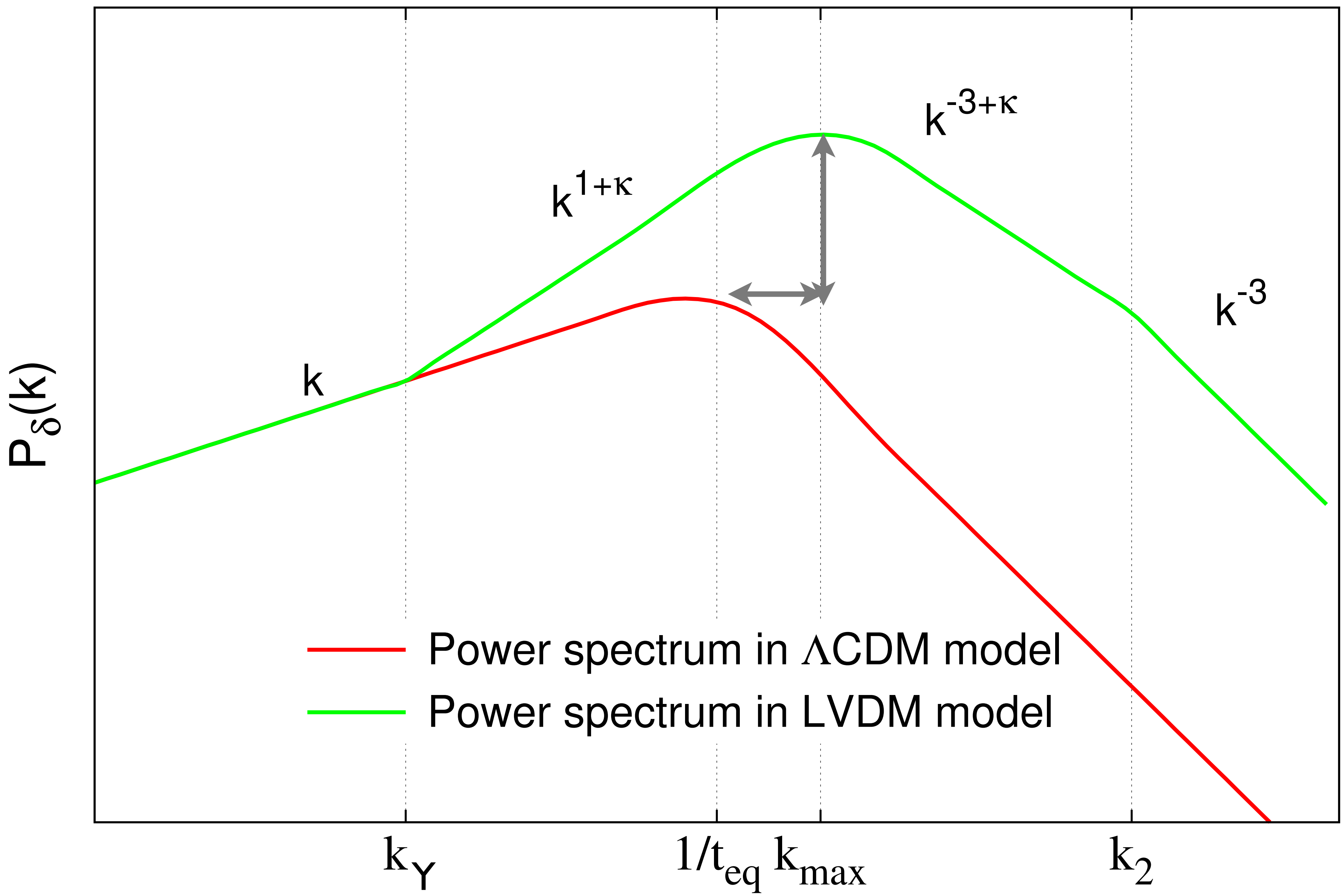
$$\kappa = \sqrt{25 + \frac{24\Omega_{dm}Y}{\Omega_{cm}(1-Y)}} - 5$$

$$\delta \sim \frac{(k\,t)^2}{6}$$

Cosmological perturbations

$$\langle \delta(k)\delta(k')\rangle \equiv \delta^{(3)}(k+k')P(k)k^3$$

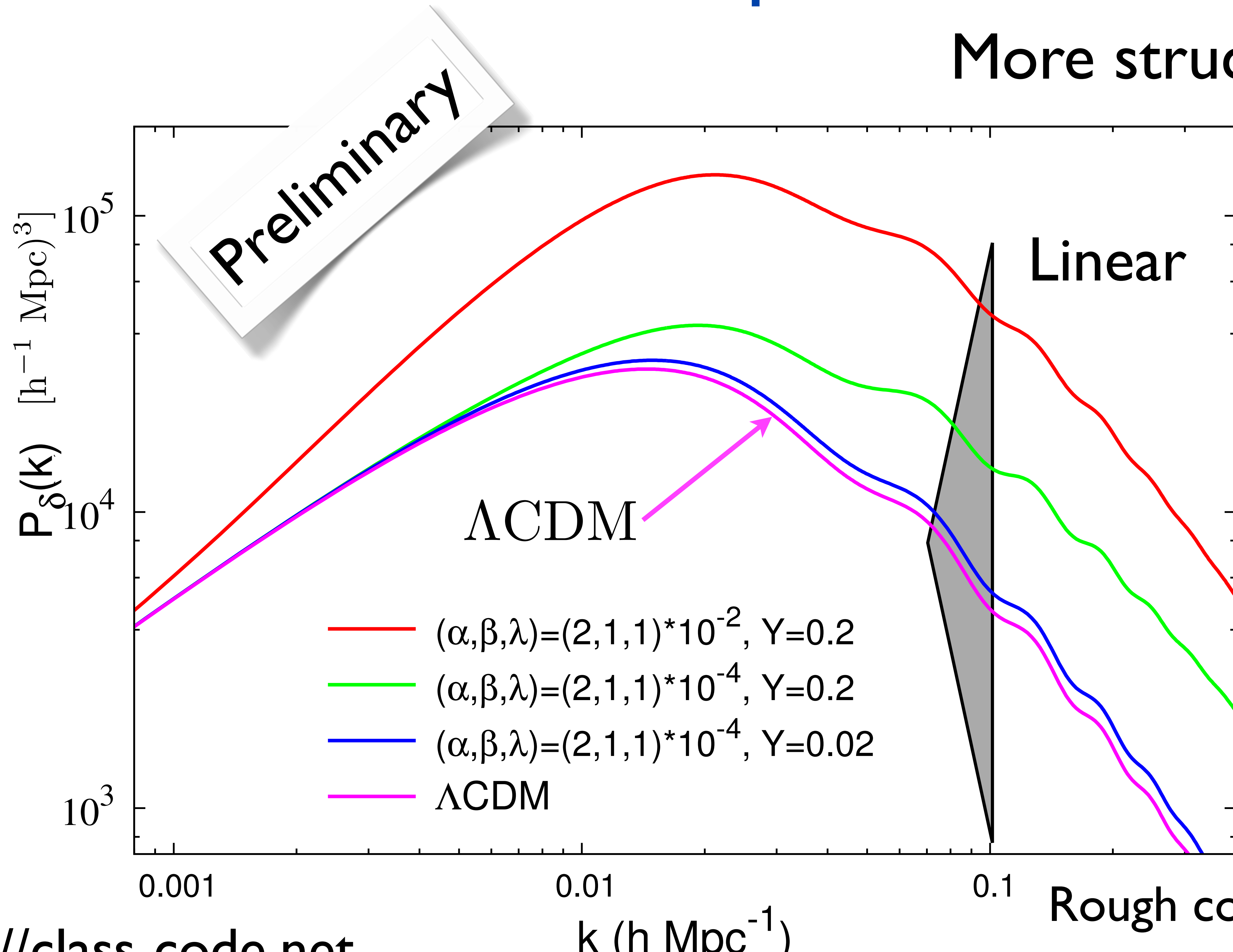


$$k_Y^2 \equiv \frac{3H_0^2\Omega_{dm}Y}{(\beta+\lambda)(1-Y)}$$

$$k_2 = k_Y\sqrt{\frac{\Omega_{dm}+\Omega_b}{\Omega_\gamma}}$$

Matter Power Spectrum

More structure!

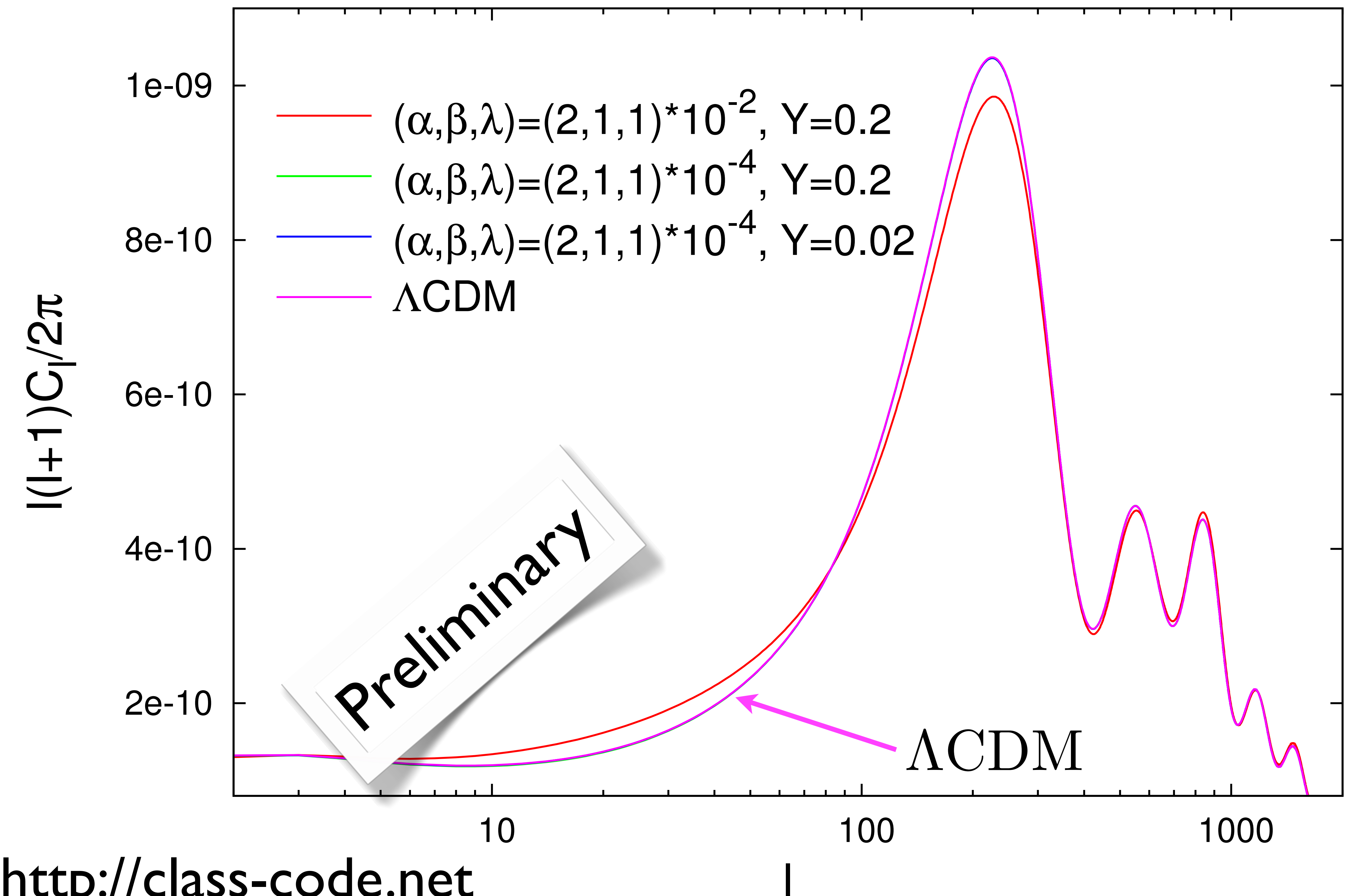


Rough constraint

$Y < 10^{-2}$

http://class-code.net

Cosmic Microwave Background

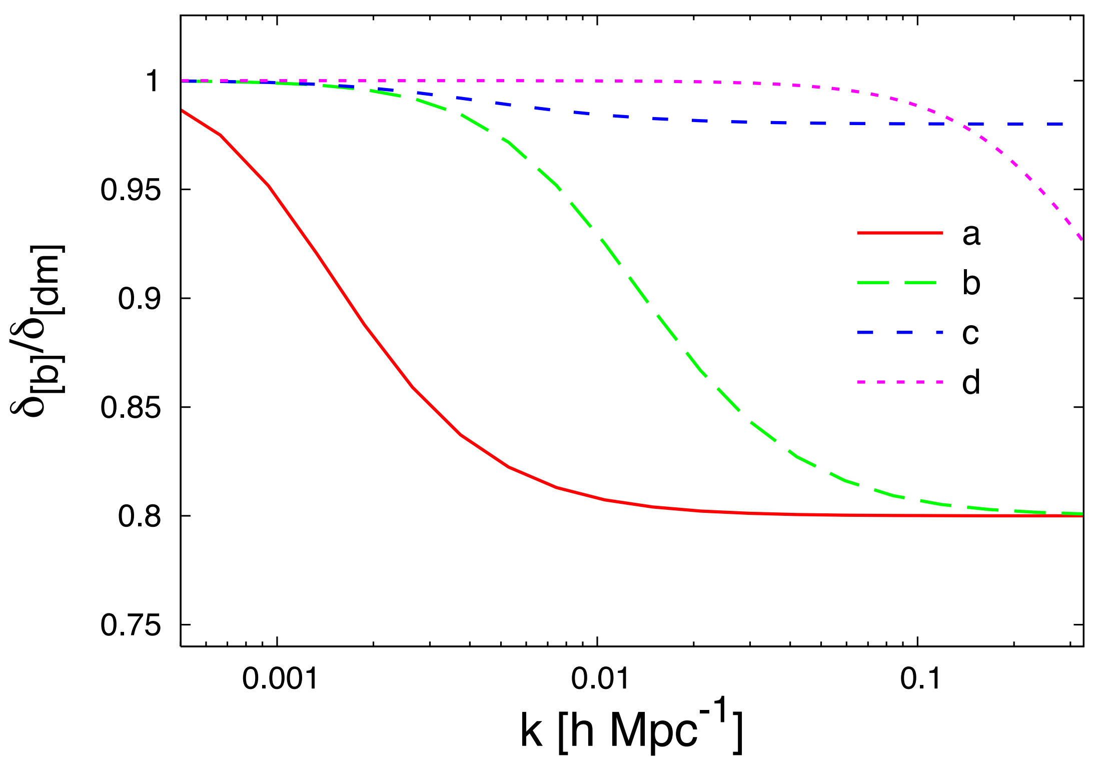


http://class-code.net

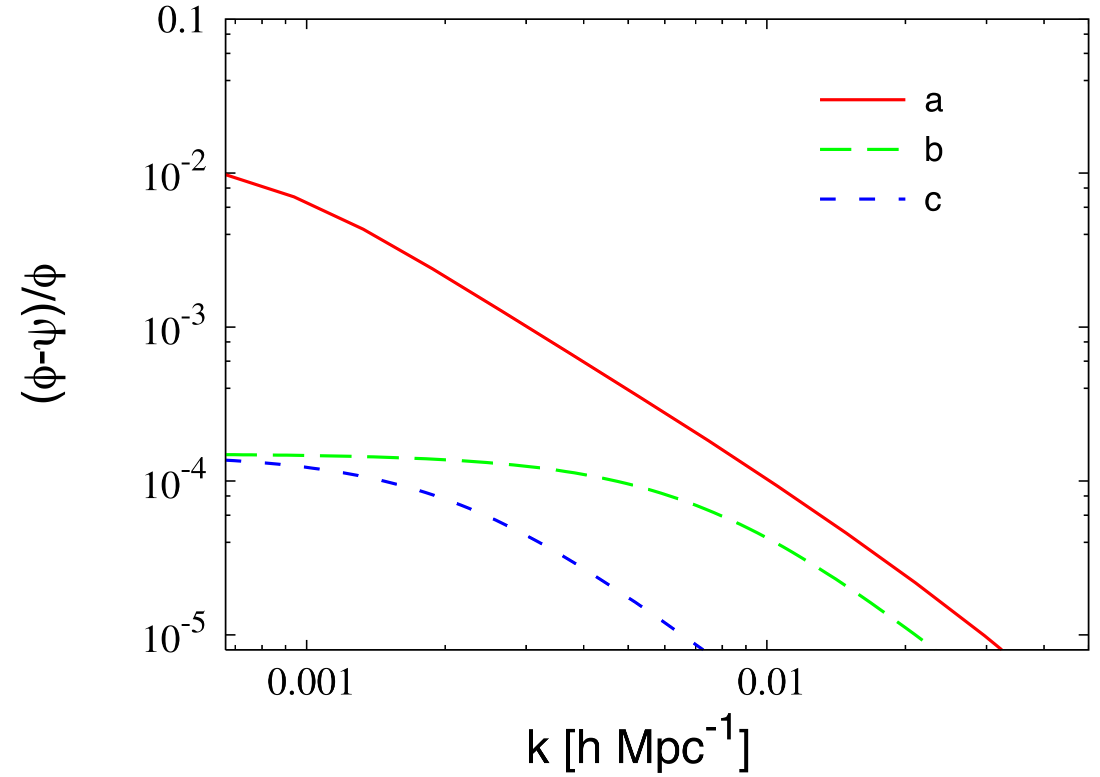
$Y < 10^{-2}$

Other effects

Baryons bias



Anisotropic stress



	α	β	λ	Y	$k_{Y,0}$ (h Mpc^{-1})	$k_{Y,eq}$ (h Mpc^{-1})
a	$2 \cdot 10^{-2}$	10^{-2}	10^{-2}	0.2	$9.2 \cdot 10^{-4}$	$6.5 \cdot 10^{-2}$
b	$2 \cdot 10^{-4}$	10^{-4}	10^{-4}	0.2	$9.1 \cdot 10^{-3}$	0.65
c	$2 \cdot 10^{-4}$	10^{-4}	10^{-4}	0.02	$2.6 \cdot 10^{-3}$	0.18
d	10^{-7}	0	10^{-7}	0.2	0.41	29

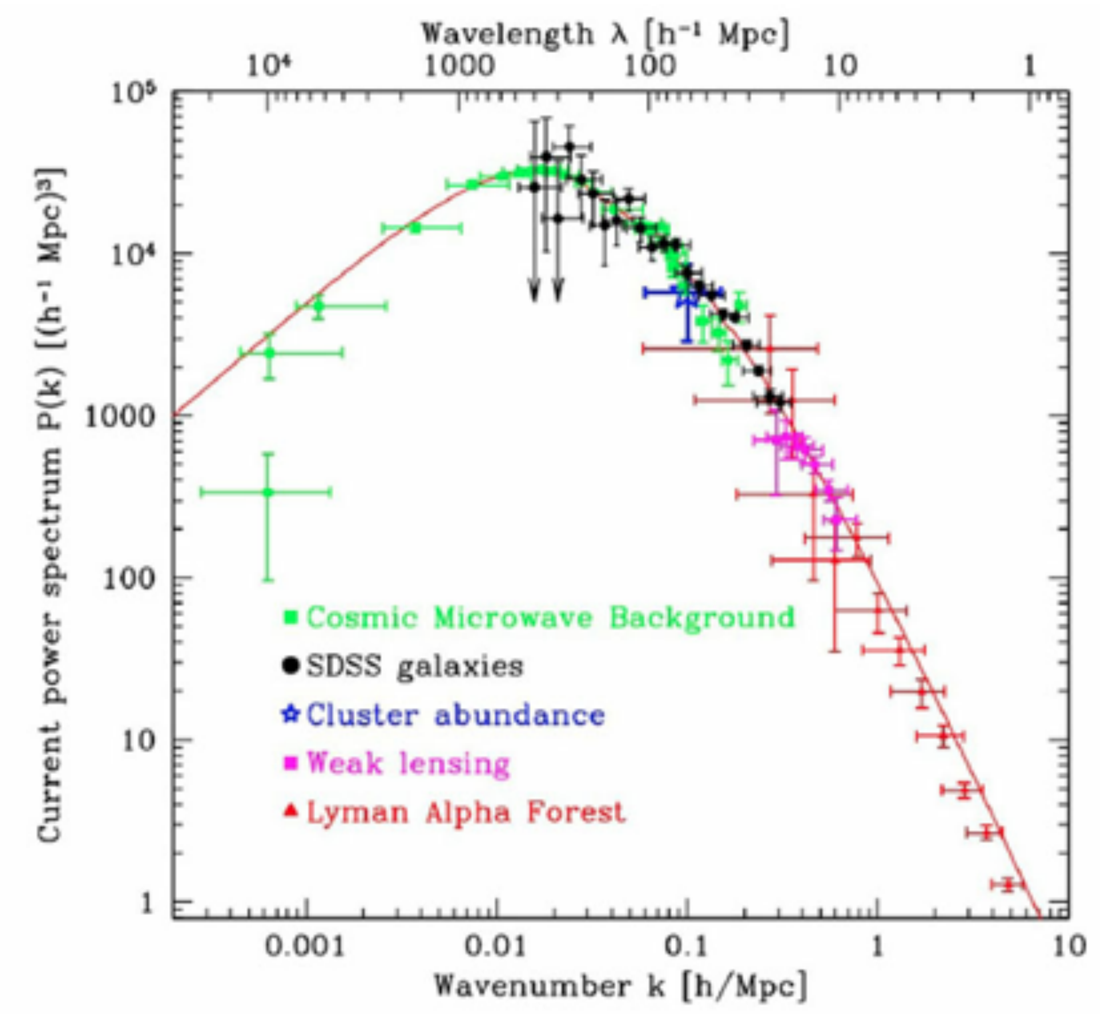
Conclusions

- Breaking Lorentz invariance in the dark sector to better understand the fundamental properties of our universe (may be useful for quantum gravity).
- Cosmological observations allow to constrain deviations from Lorentz invariance. For **dark matter:** same background evolution, but distinct signals for (linear) perturbations.
- Breaking of equivalence principle for dark matter: enhanced (scale dependent) growth of structure at large scales. Lorentz breaking summarized in a parameter $Y < 10^{-2}$

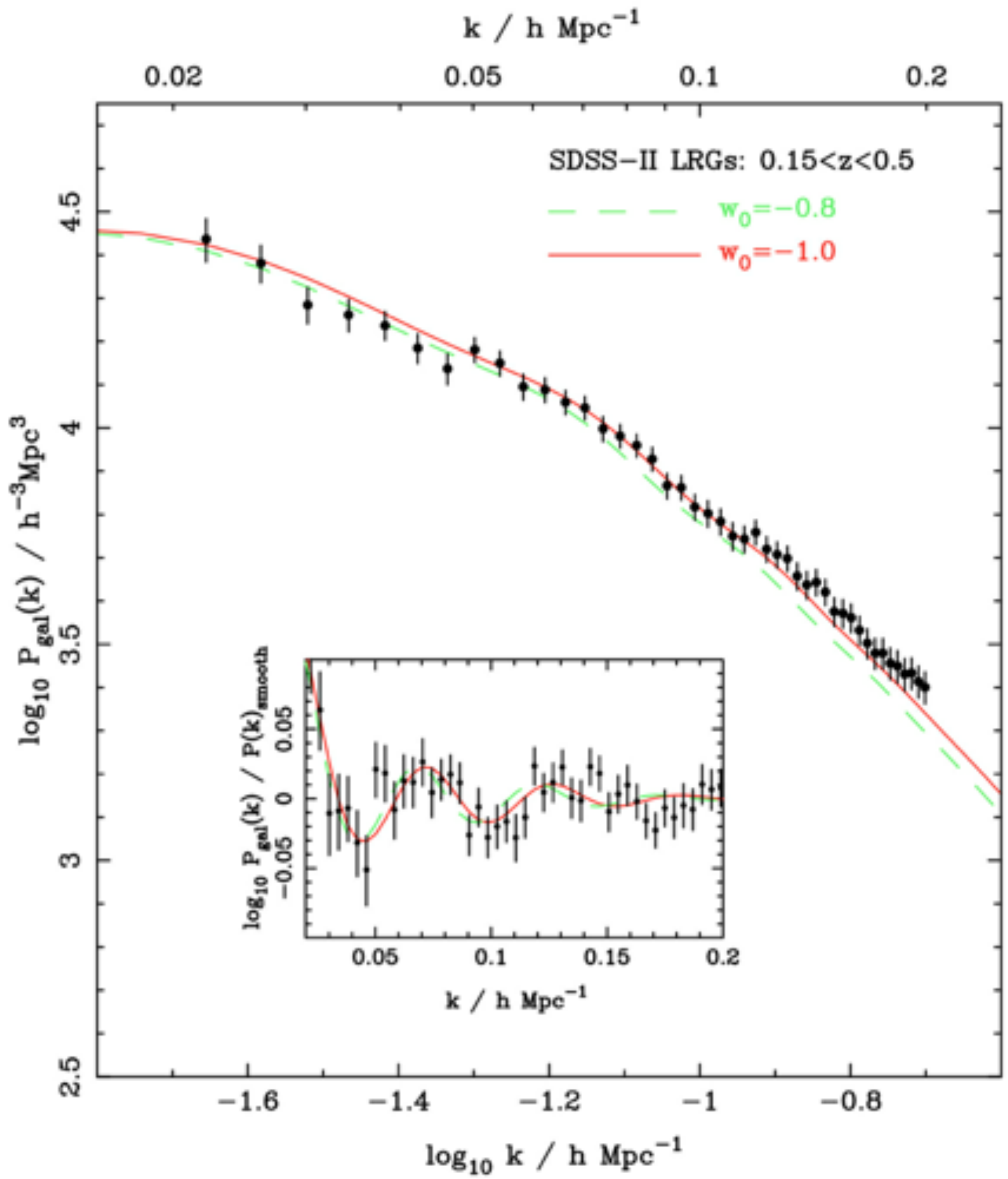
OUTLOOK

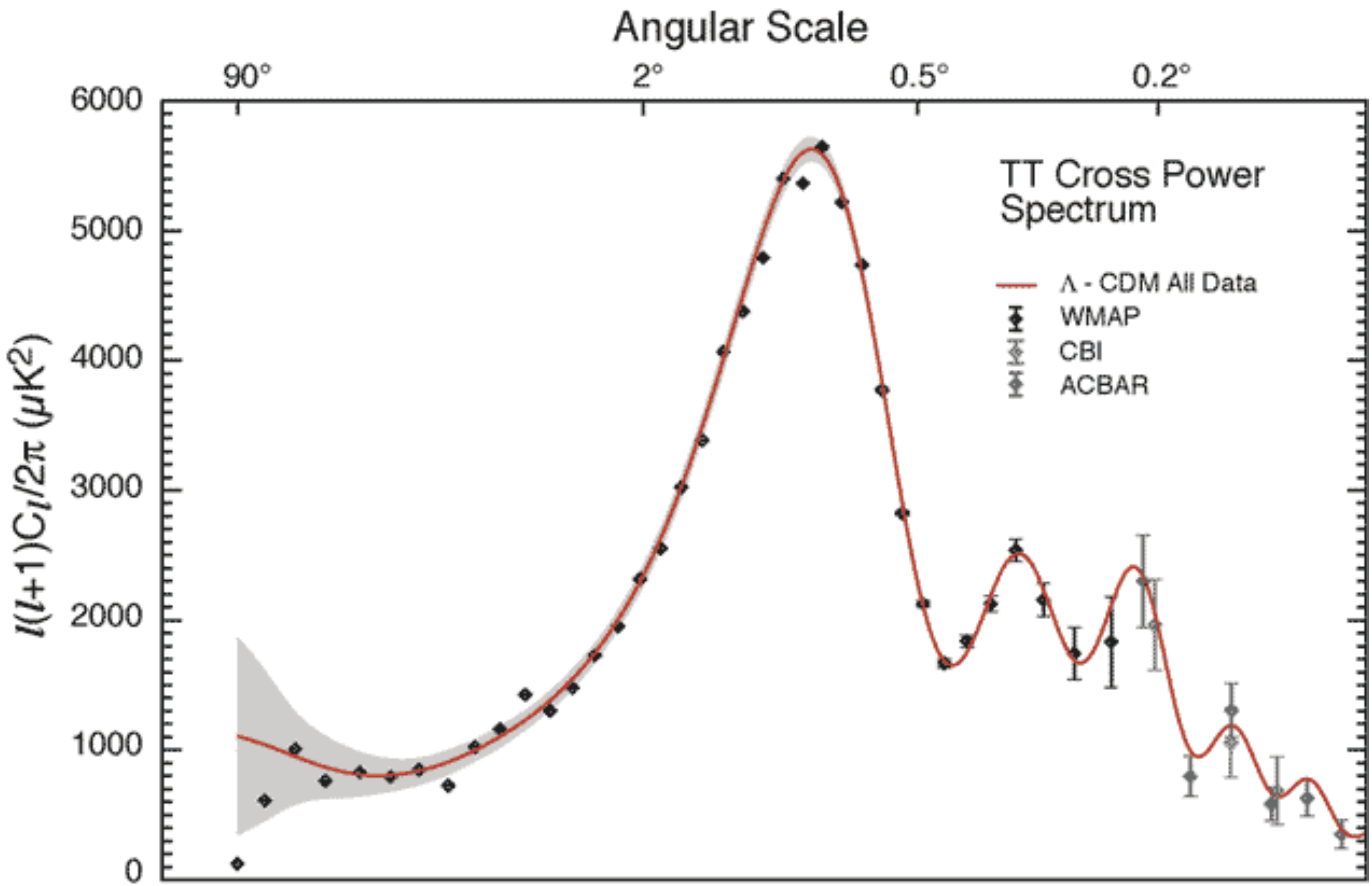
* Detailed study of parameters, comparison with data effect at other scales, DM 'problems'

Tegmark'02



Percival'10





Angular Scale
90°
2°
0.5°
0.2°
6000
5000
4000
3000
2000
1000
0
$l(l+1)C_l/2\pi$ (μK^2)
TT Cross Power Spectrum
Λ - CDM All Data
WMAP
CBI
ACBAR