

A simple, yet subtle, invariance of the two-body decay kinematics

Roberto Franceschini (University of Maryland)

arXiv:1209.0772

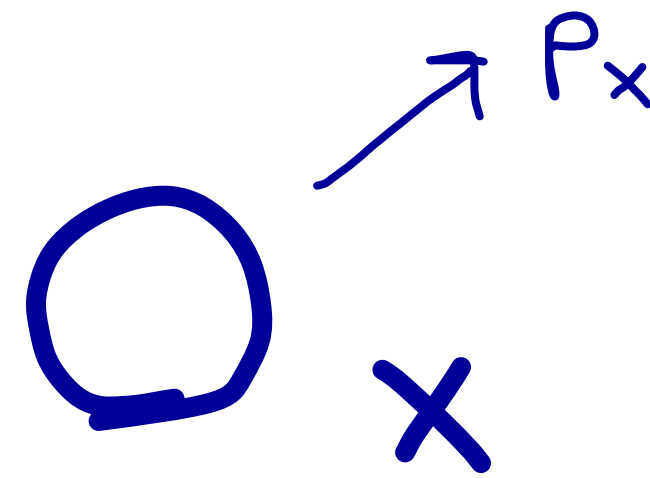
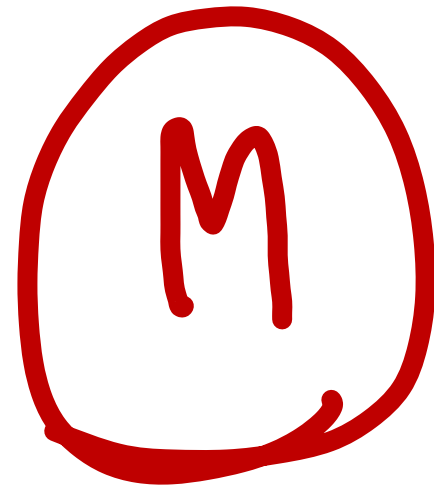
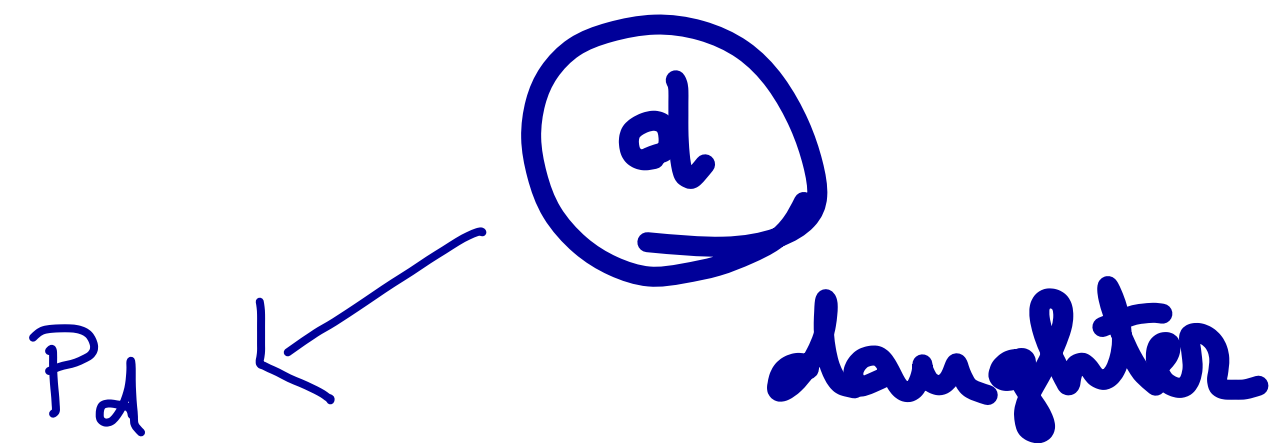
with K.Agashe and D.Kim

TWO-BODY DECAY : $M \rightarrow d X$



$$P_M^\mu = P_d^\mu + P_X^\mu$$

KINEMATICS FULLY FIXED BY THE MASSES



$$E_d =: E_d^* = \frac{m_M^2 + m_d^2 - m_x^2}{2m_M}$$

$$\vec{P}_d + \vec{P}_x = 0$$

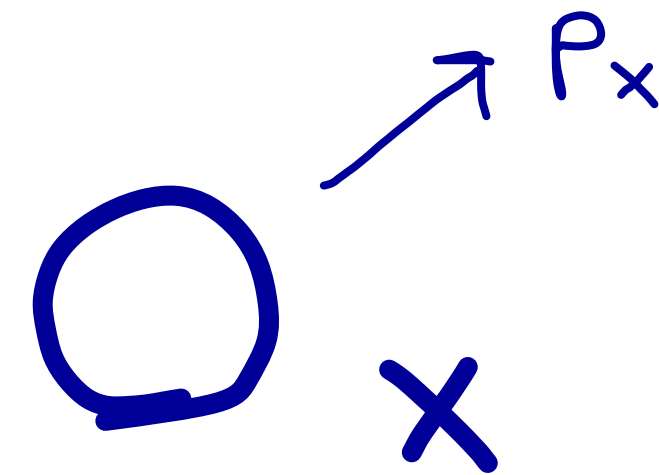
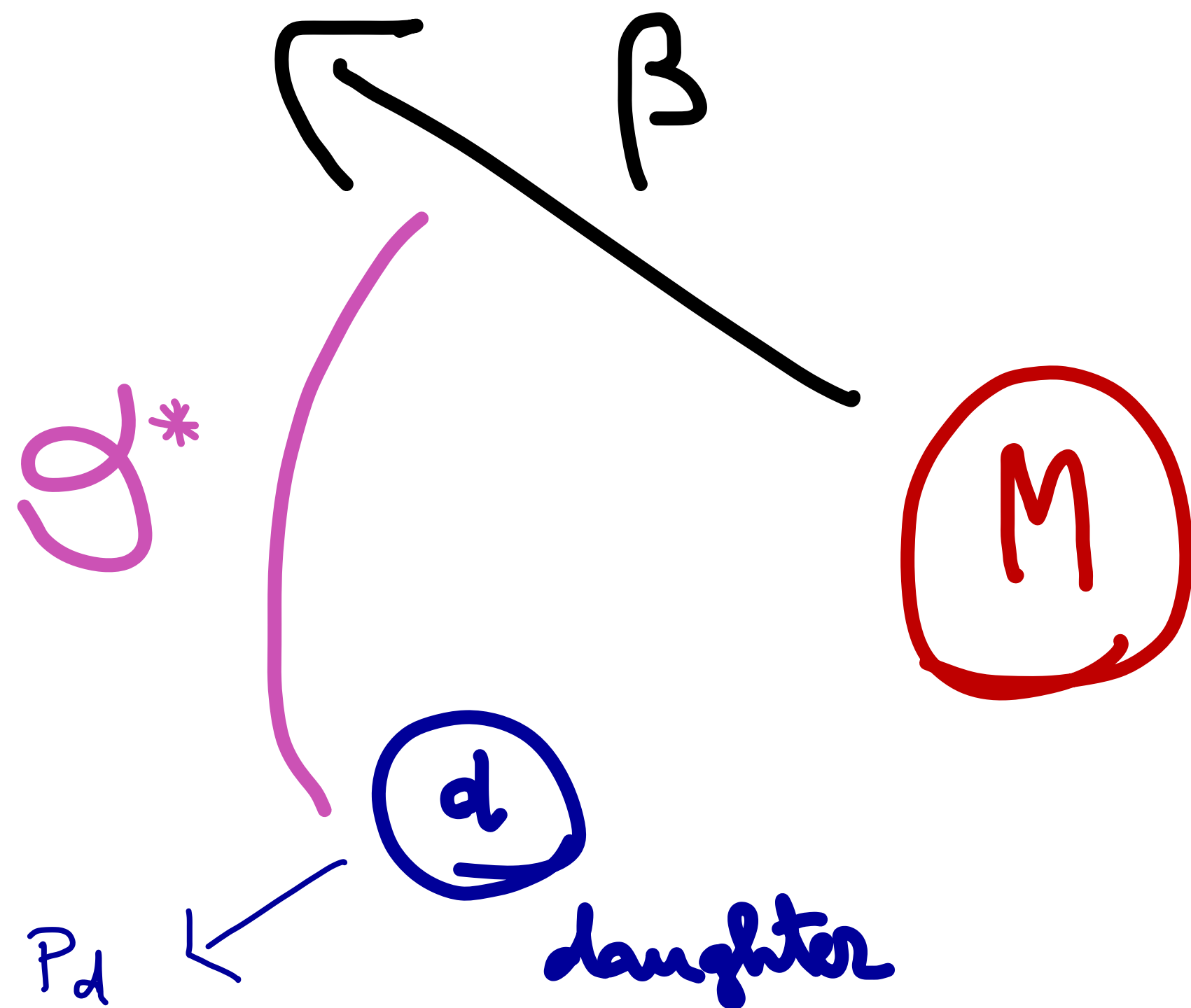
IN THE REST FRAME OF M

WHAT DOES IT LOOK LIKE IN ANOTHER FRAME?

IN GENERAL WE KNOW THE ANSWER

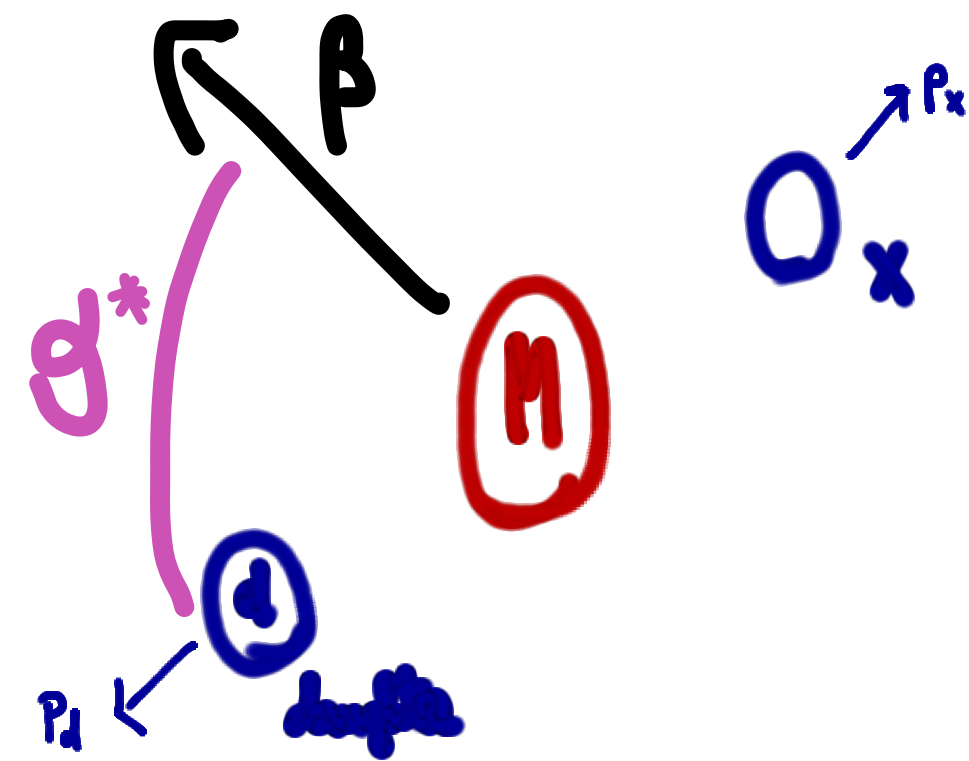
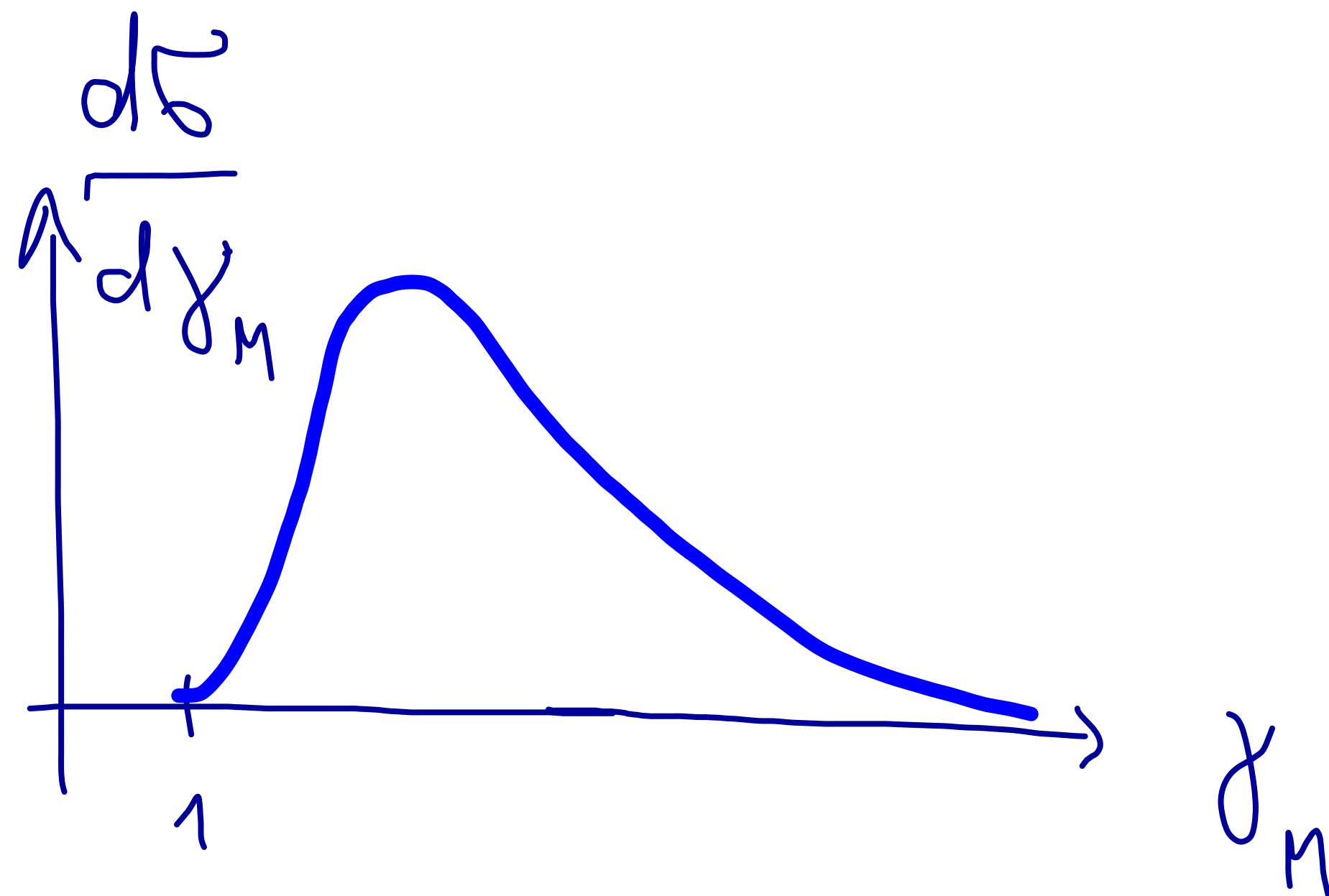
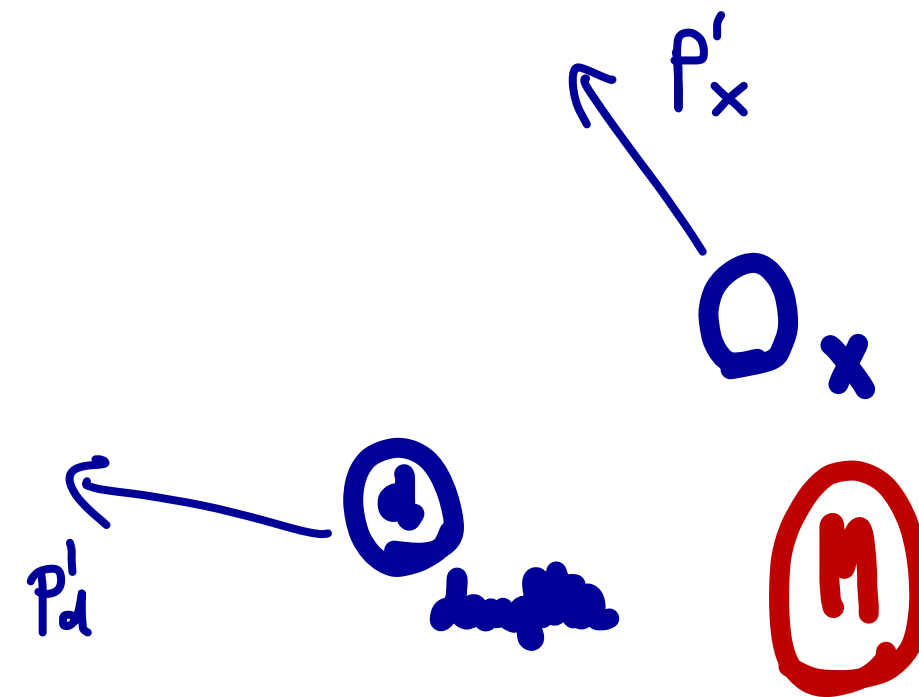
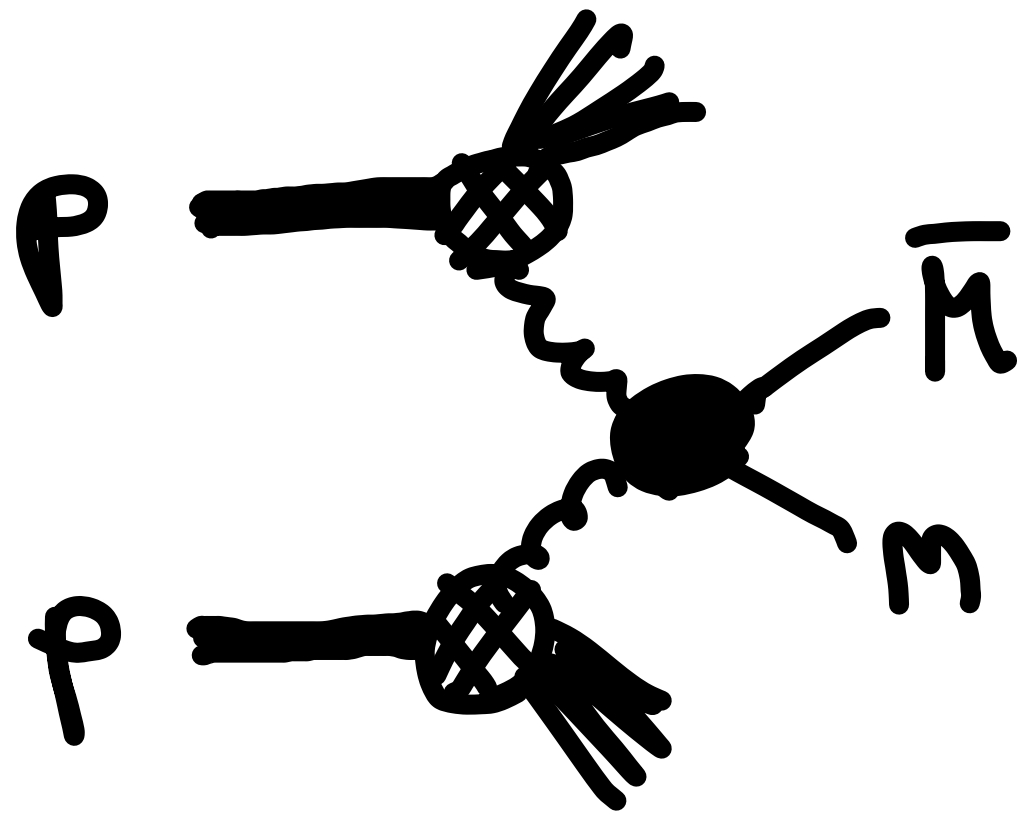
IF THE FRAME OF THE OBSERVER AND THAT OF REST OF
THE MOTHER ARE CONNECTED BY A BOOST β

$$E'_d = E_d^* \gamma + p_d^* \gamma \beta \cos \vartheta^*$$



... BUT

BUT IN MOST CASES WE DO NOT KNOW THE BOOST OF THE MOTHER



SOLUTION TO OVERCOME THE UNKNOWN BOOST

USE BOOST INVARIANT QUANTITIES

- CONSERVED EVENT BY EVENT
- SIMPLE TO UNDERSTAND
- UNIVERSAL (SPECIAL RELATIVITY IS THE SAME FOR ALL PARTICLES)
- DEMANDING
 - GENERICALLY THEY ARE FUNCTION OF SEVERAL QUANTITIES
TO MAKE AN INVARIANT MASS YOU NEED TWO FOUR-VECTORS
WITH BOTH ENERGY AND ANGLES

IN THIS TALK :

LORENTZ VARIANT QUANTITIES
WITH SOME KIND OF "PHENOMENOLOGICAL INVARIANCE"
TO ACCESS INVARIANTS OF THE DECAY

FOR INSTANCE:

THE OBSERVED ENERGY DEPENDS ON THE FRAME

THE ENERGY DISTRIBUTION IN PHENOMENOLOGICALLY RELEVANT
SITUATIONS HAS SOME INVARIANCE

- DAUGHTER d IS MASSLESS (for now)

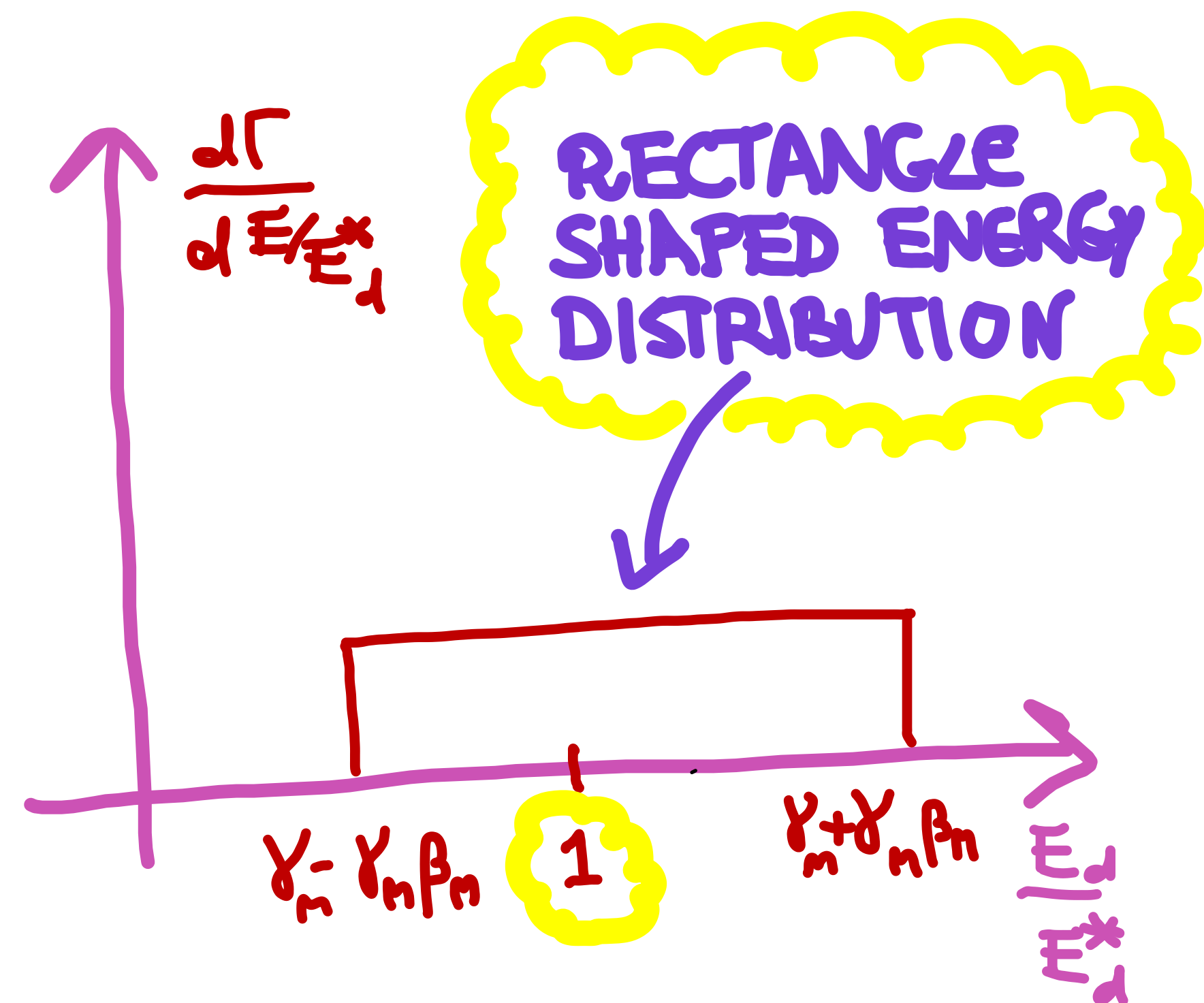
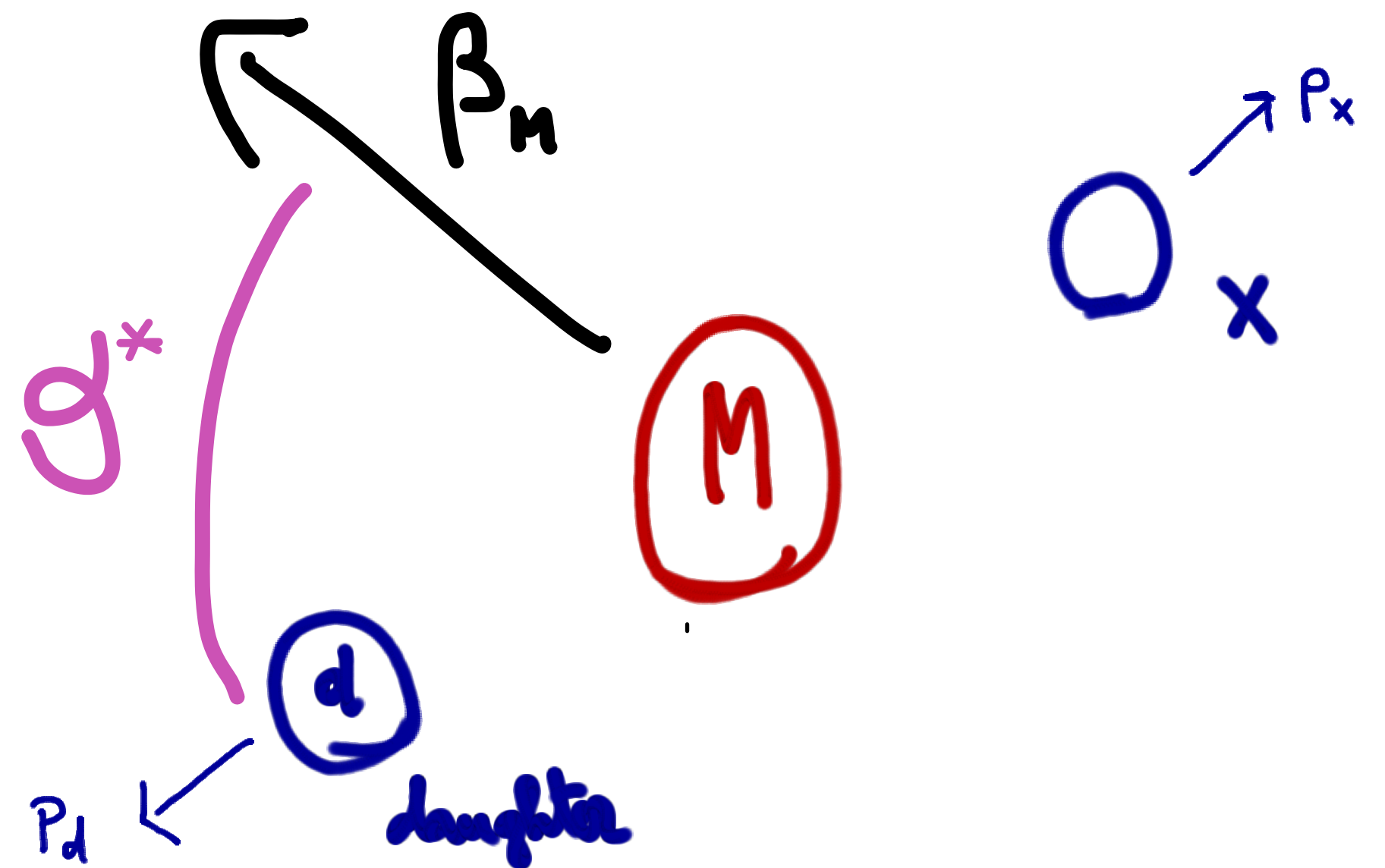
- IMAGINE THE MOTHER HAS A BOOST β_M IN THE LAB FRAME

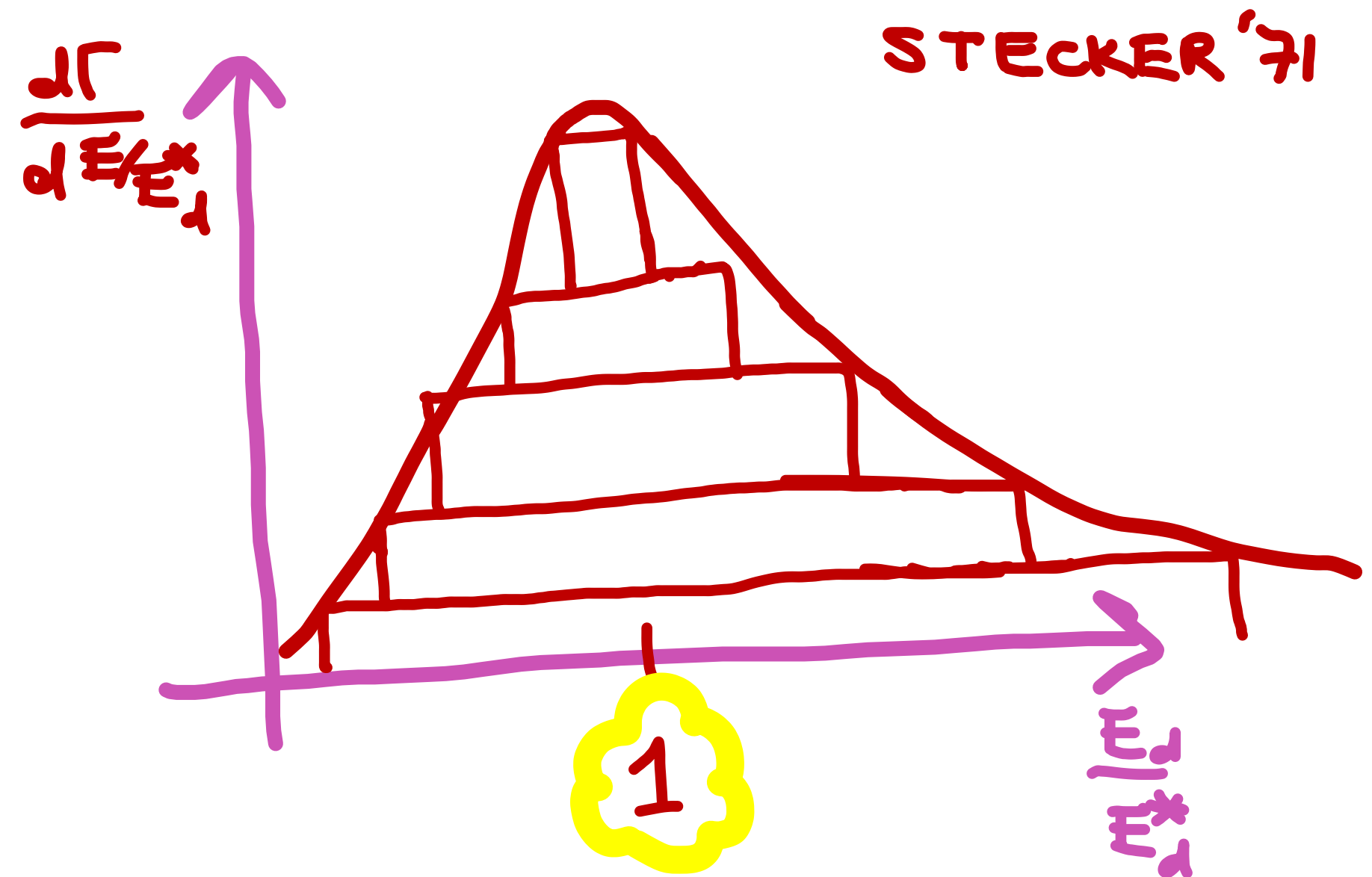
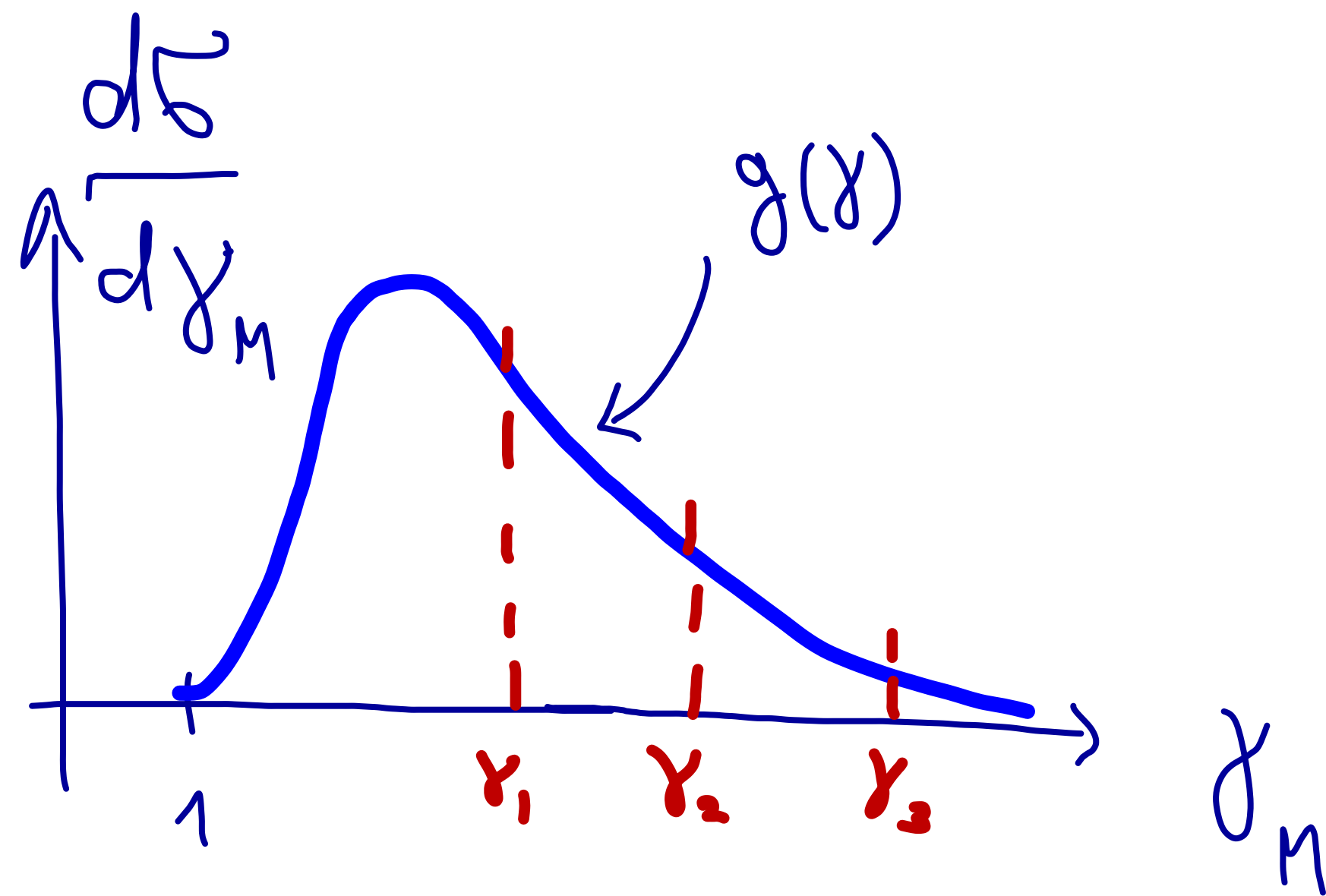
- THE DAUGHTER MOMENTUM IS AT AN ANGLE ϑ W.R.T. β_M

IN THE LAB

$$E_d = E_d^* (\gamma_M + \cos \vartheta^* \beta_M \gamma_M)$$

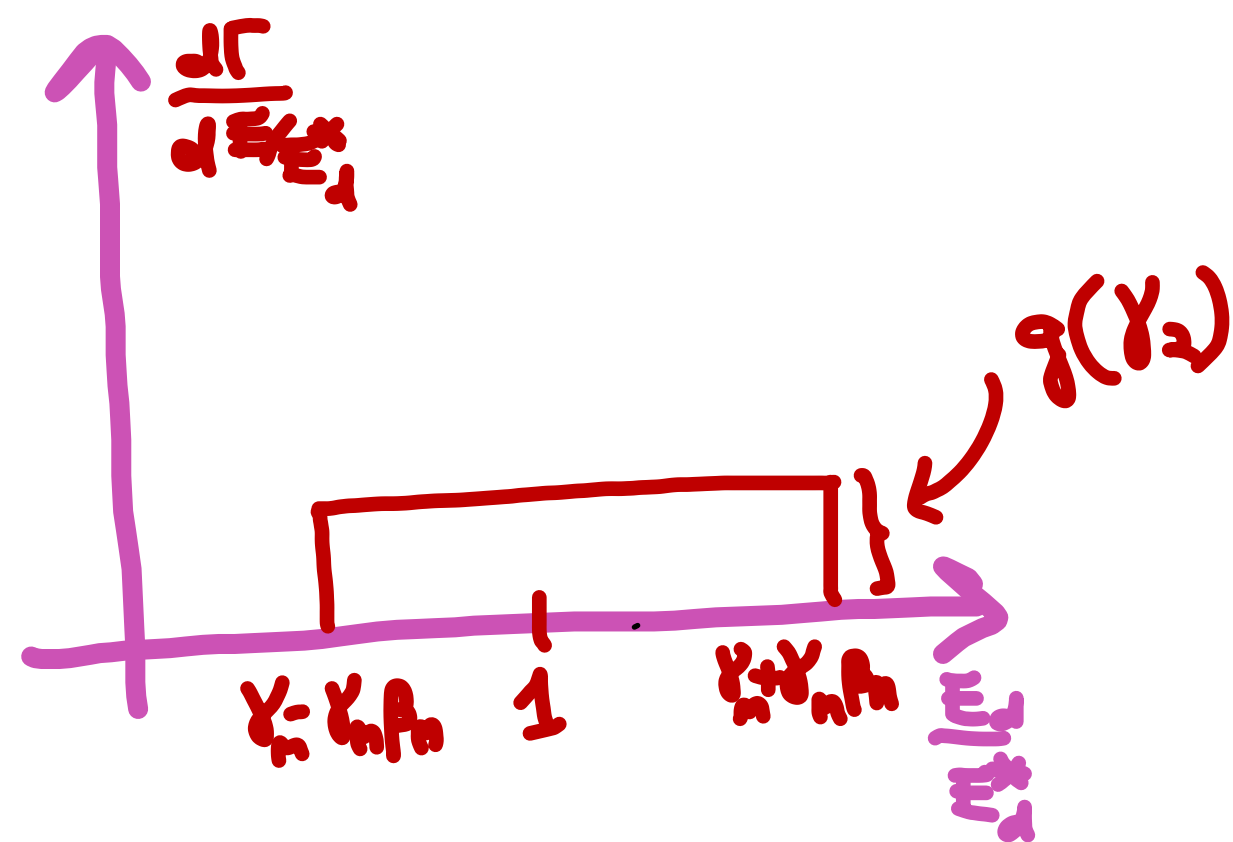
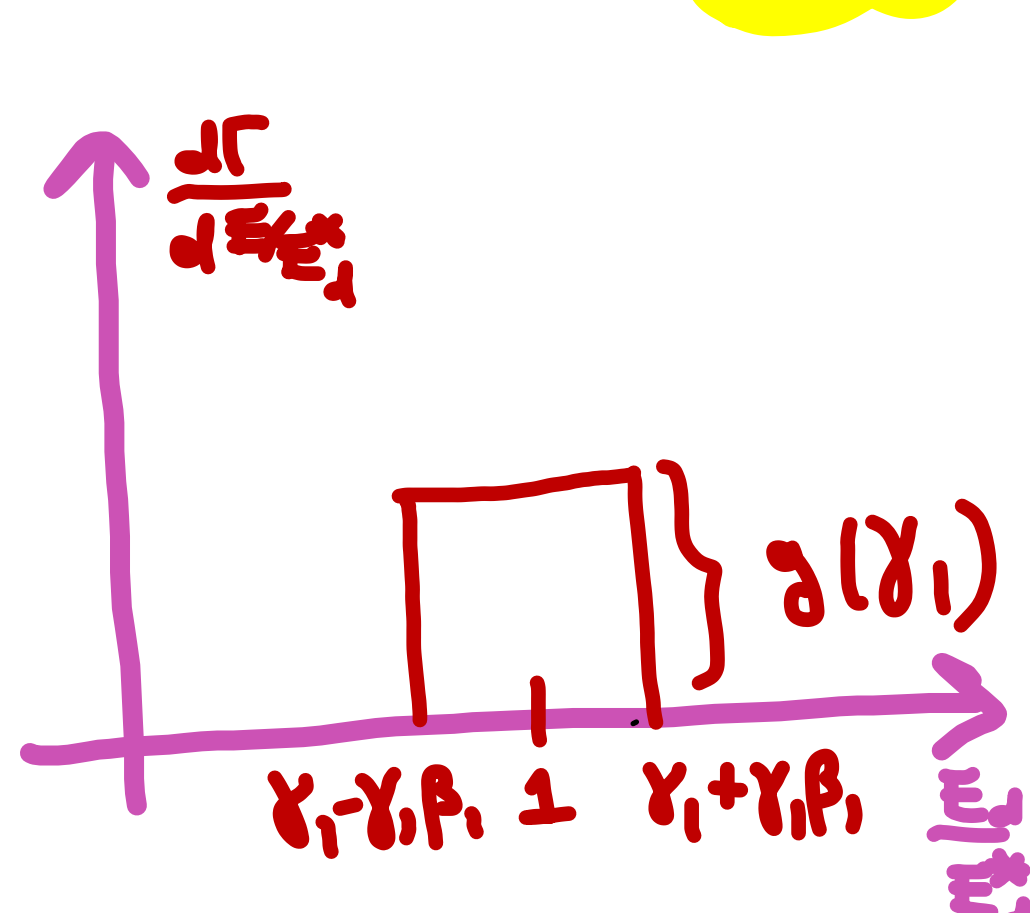
IF THE MOTHER IS A SCALAR
 ϑ IS FLAT FROM -1 TO 1





STECKER '71

THE ENERGY DISTRIBUTION IN THE LAB
IS THE SUM OF ALL THE RECTANGLES



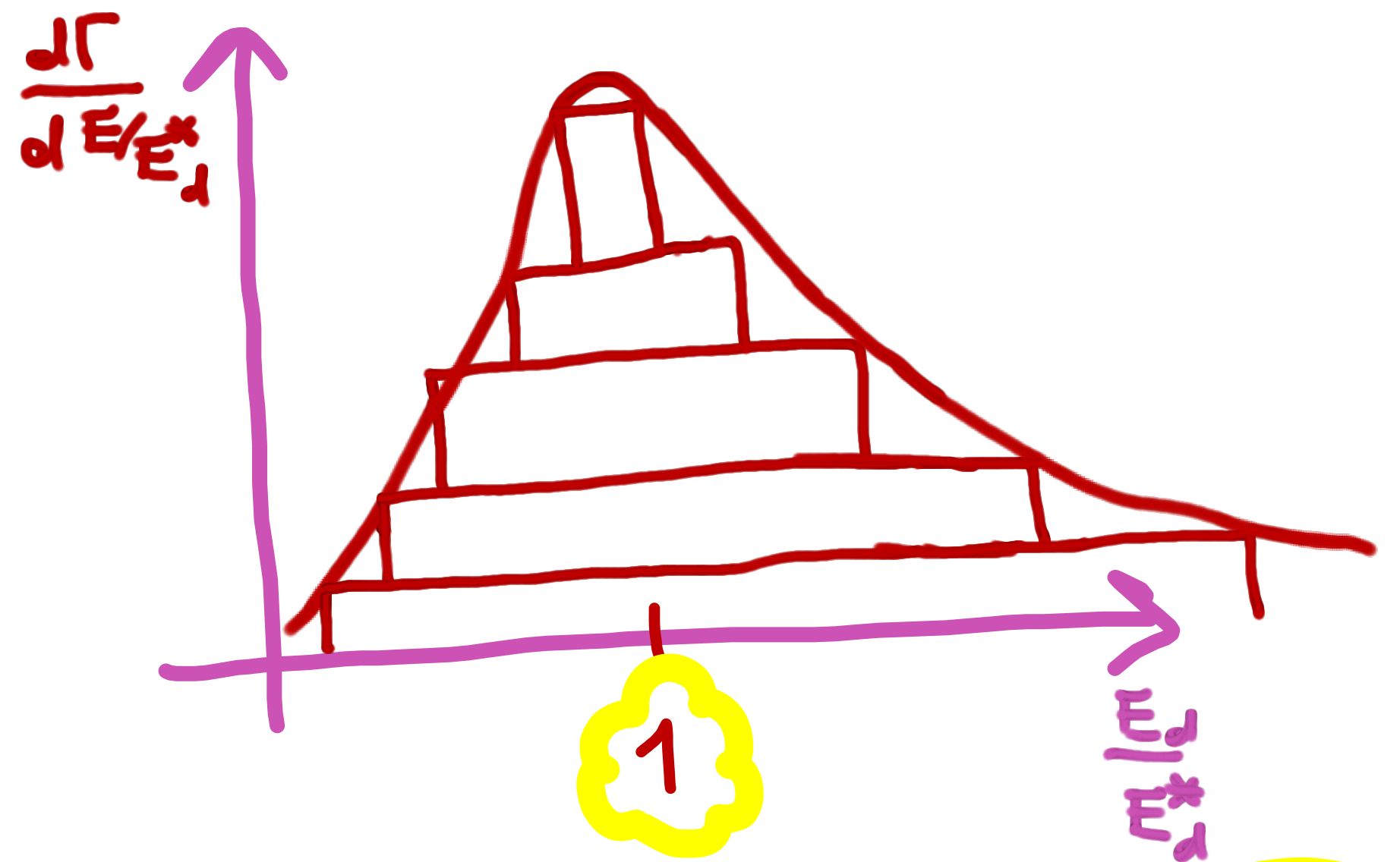
GENERALIZATIONS:

- INSTEAD OF A SCALAR MOTHER ONE CAN TAKE AN UNPOLARIZED ENSEMBLE OF PARTICLES WITH SPIN

- THE DAUGHTER CAN BE MASSIVE IF $g(\gamma) = 0$ for $\gamma \geq 2\gamma^* - 1$

WHERE $\gamma^* = \frac{E_d^*}{m_d}$

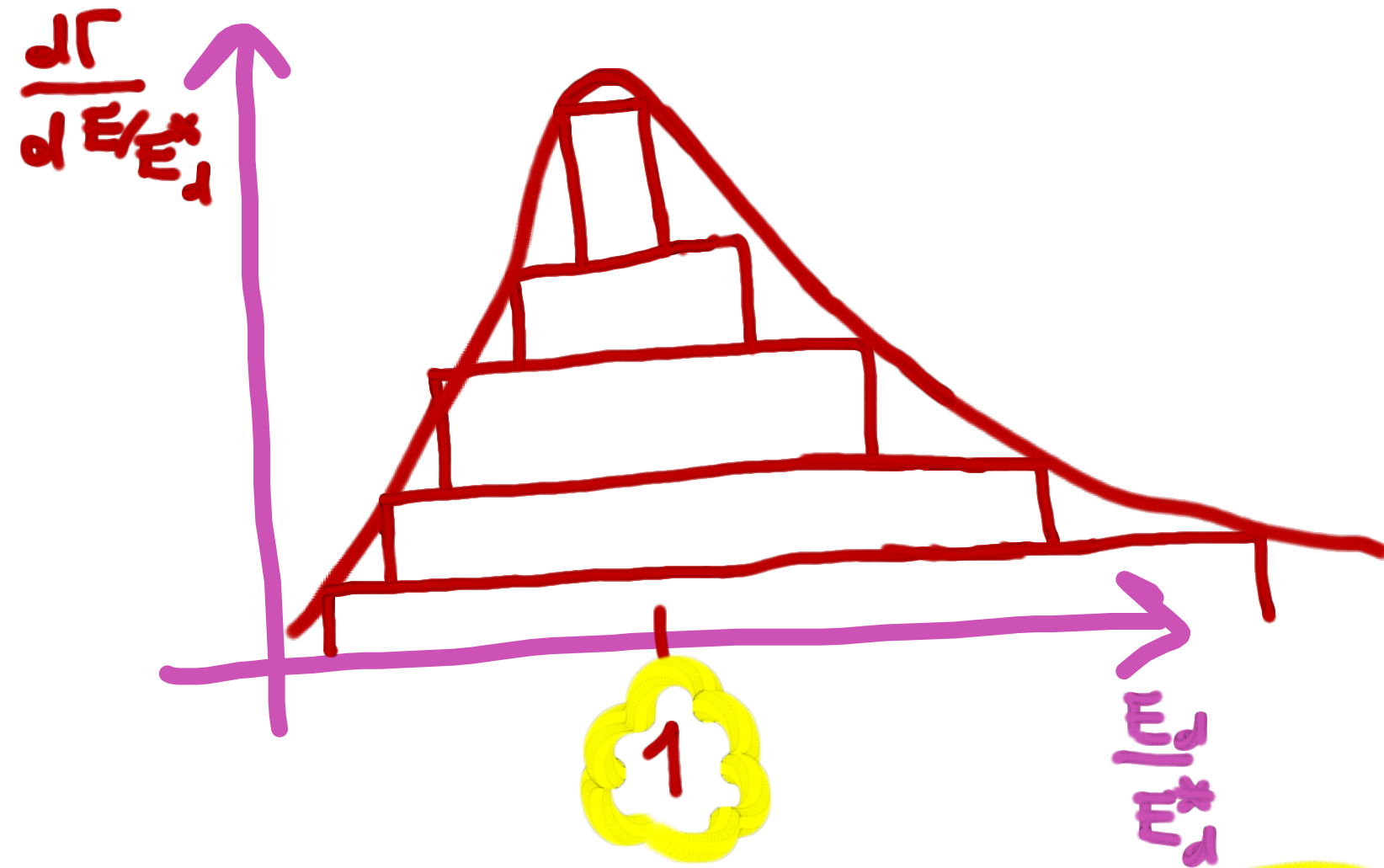
$$E_d^* = \frac{m_m^2 + m_d^2 - m_x^2}{2m_m}$$



$E = E_d^*$ IS THE PEAK

THE FRAME-DEPENDENT ENERGY DISTRIBUTION ENCODES THE INVARIANT E_d^* IN A VERY SIMPLE WAY

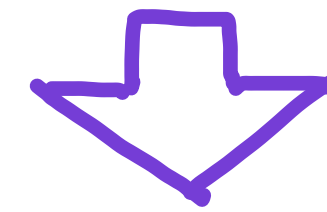
ADVANTAGES (GENERAL: ALMOST ONLY KINEMATICS)



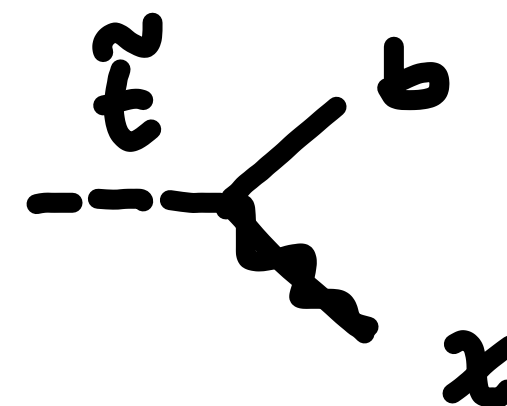
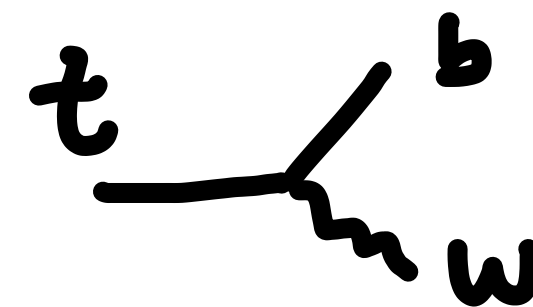
$E = E_d^*$ IS THE PEAK

THE FRAME-DEPENDENT
ENERGY DISTRIBUTION ENCODES
THE INVARIANT E_d^* IN A
VERY SIMPLE WAY

THE ONLY DYNAMICAL ASSUMPTION
WAS THE MOTHER TO BE NOT POLARIZED

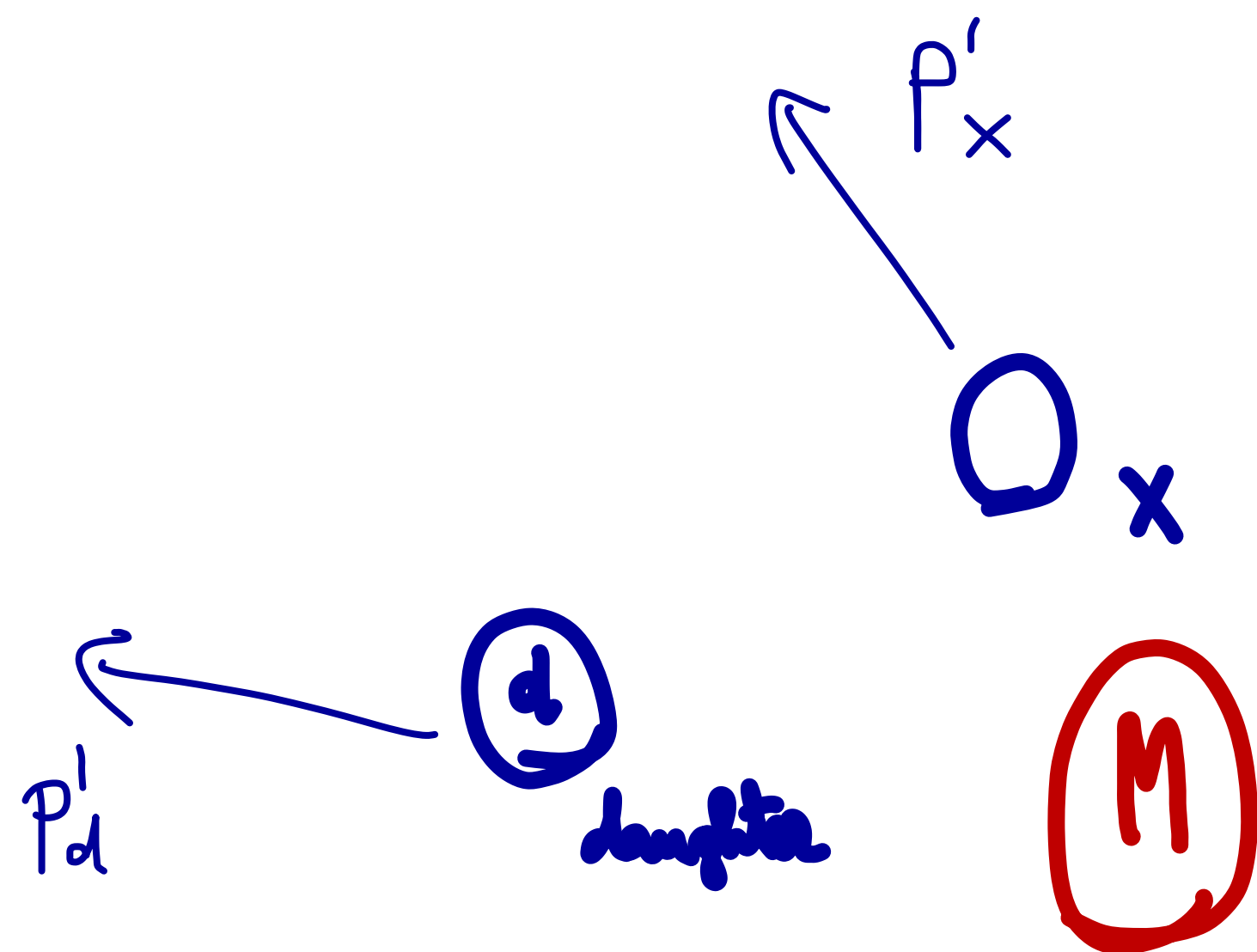


THE RESULT APPLIES FOR BOTH
KNOW PARTICLES OF THE SM
AND FOR NEW PHYSICS

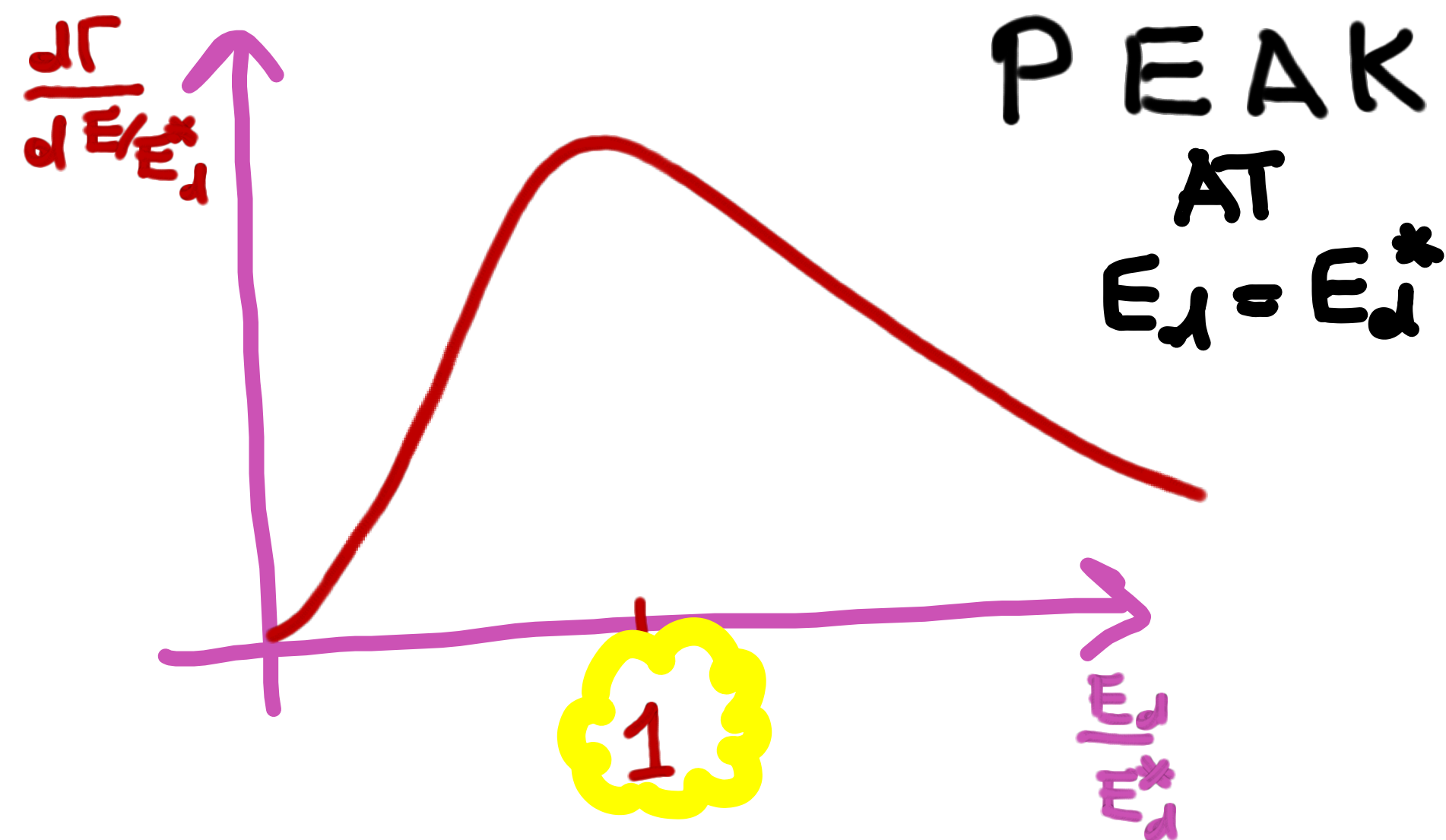


ADVANTAGES

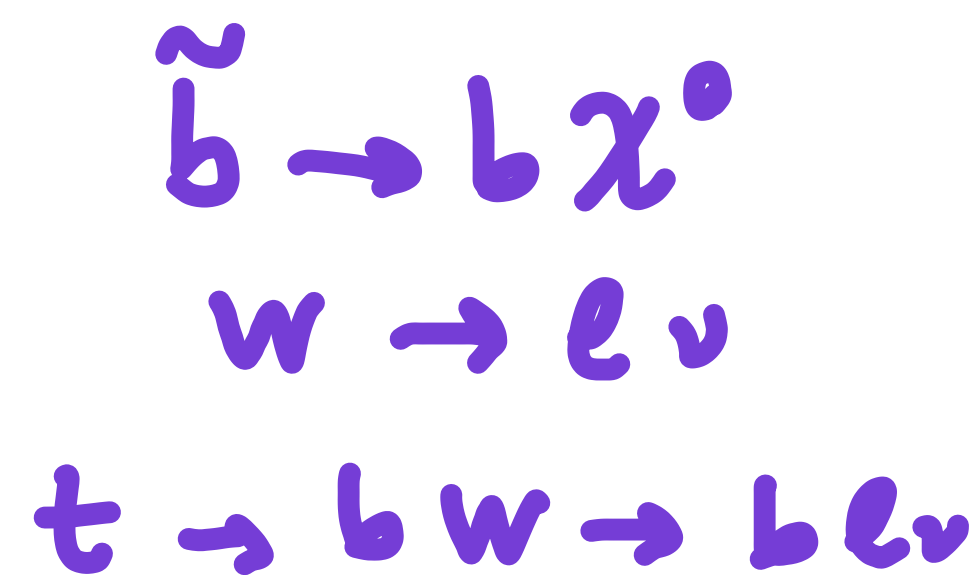
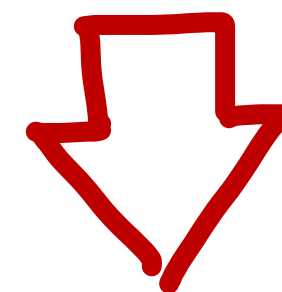
(OVER INVARIANT MASSES FOR INSTANCE)



$$E_d^* = \frac{m_M^2 + m_d^2 - m_x^2}{2m_M}$$

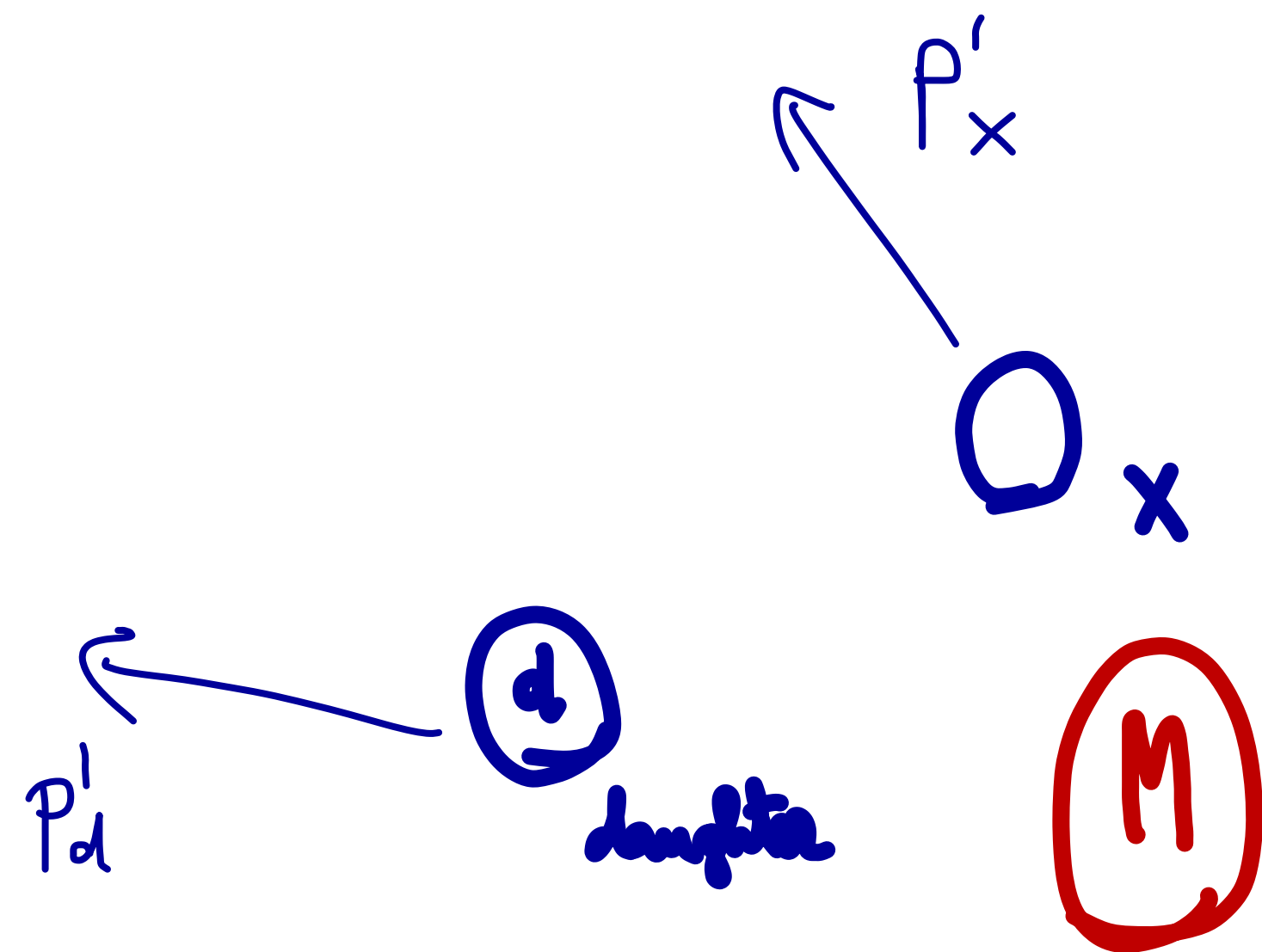


- NO NEED TO MEASURE THE OTHER DECAY PRODUCT

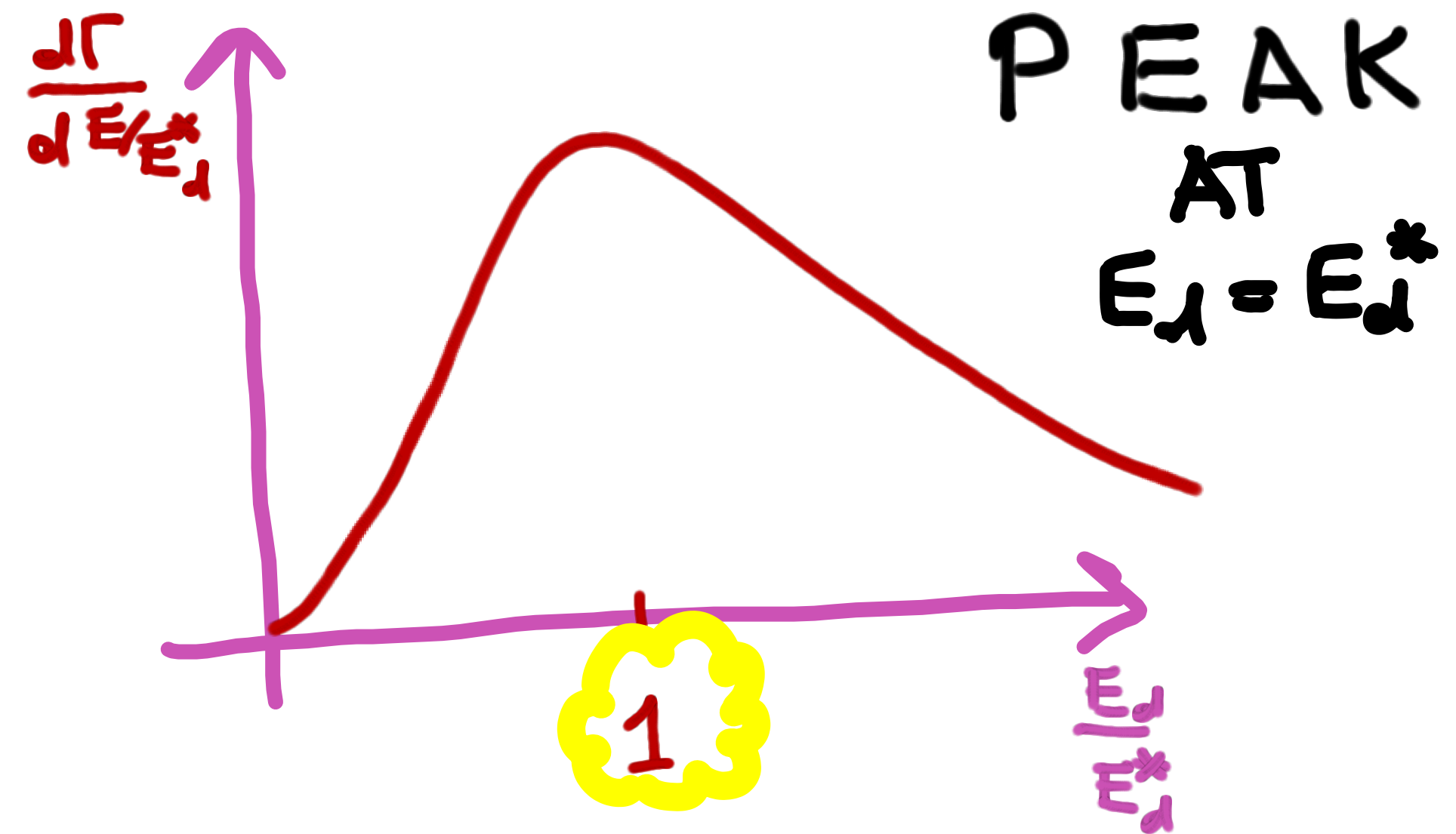


ADVANTAGES

OVER MANY TRANSVERSE KINEMATICAL VARIABLES
IN USE IN COLLIDER PHYSICS (m_{T2}, \dots)

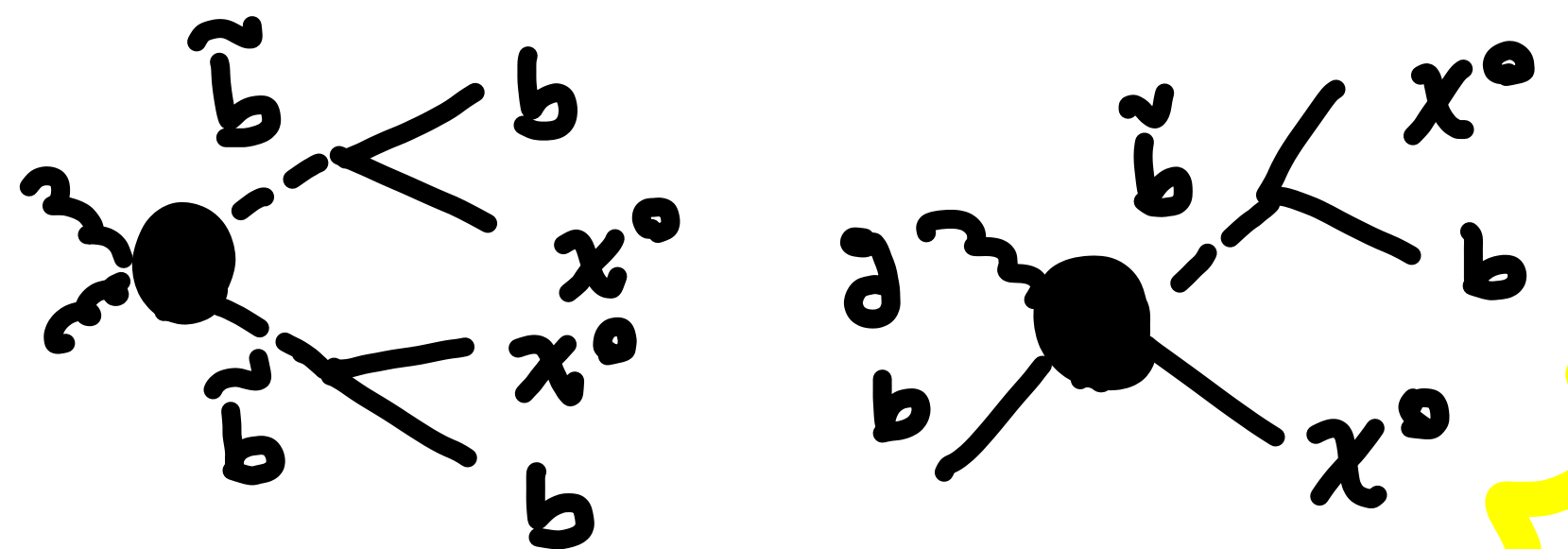


$$E_d^* = \frac{m_M^2 + m_d^2 - m_x^2}{2m_M}$$



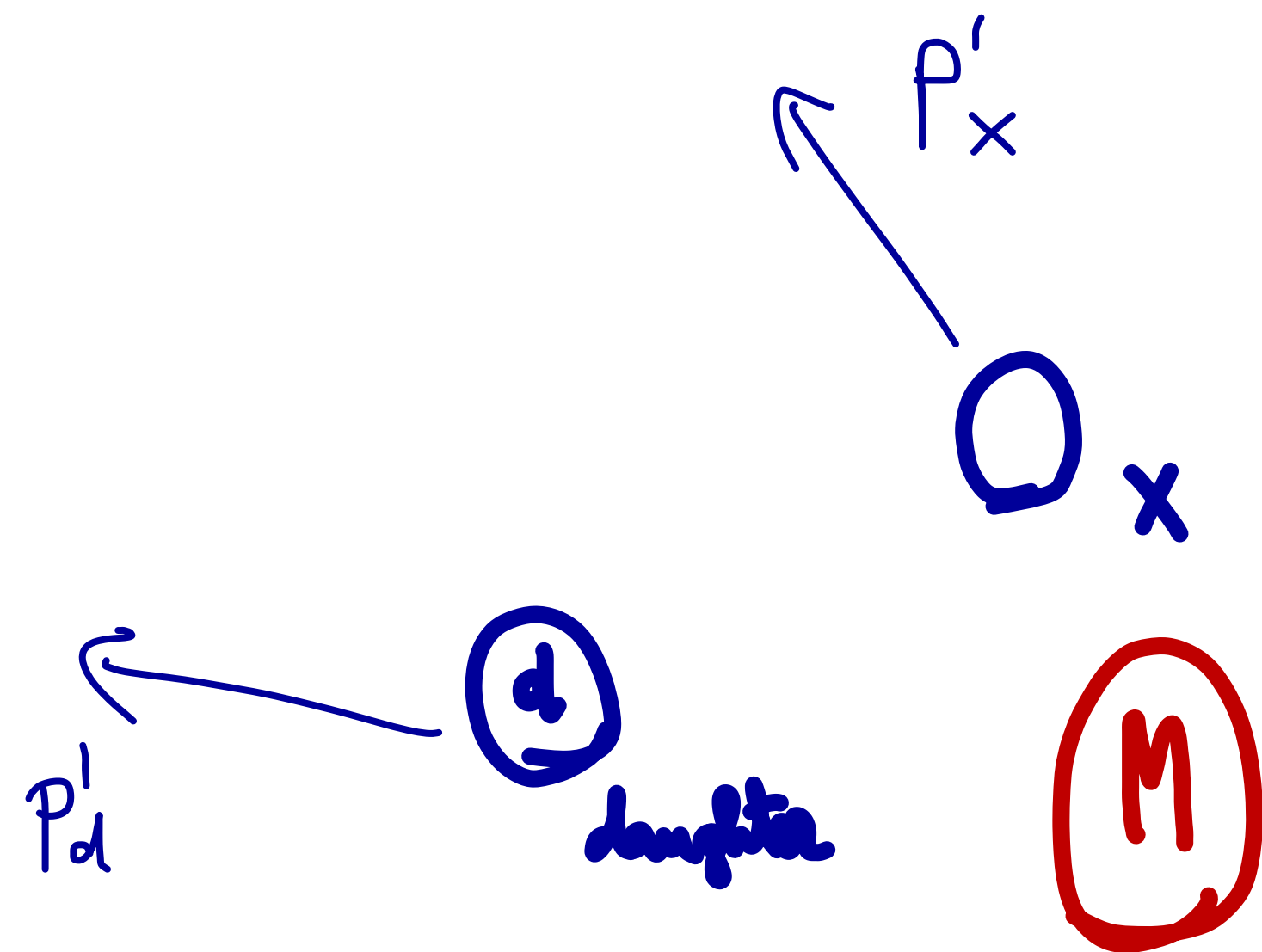
PEAK
AT
 $E_d = E_d^*$

• NO NEED TO KNOW
ANYTHING ABOUT THE REST
OF THE EVENT

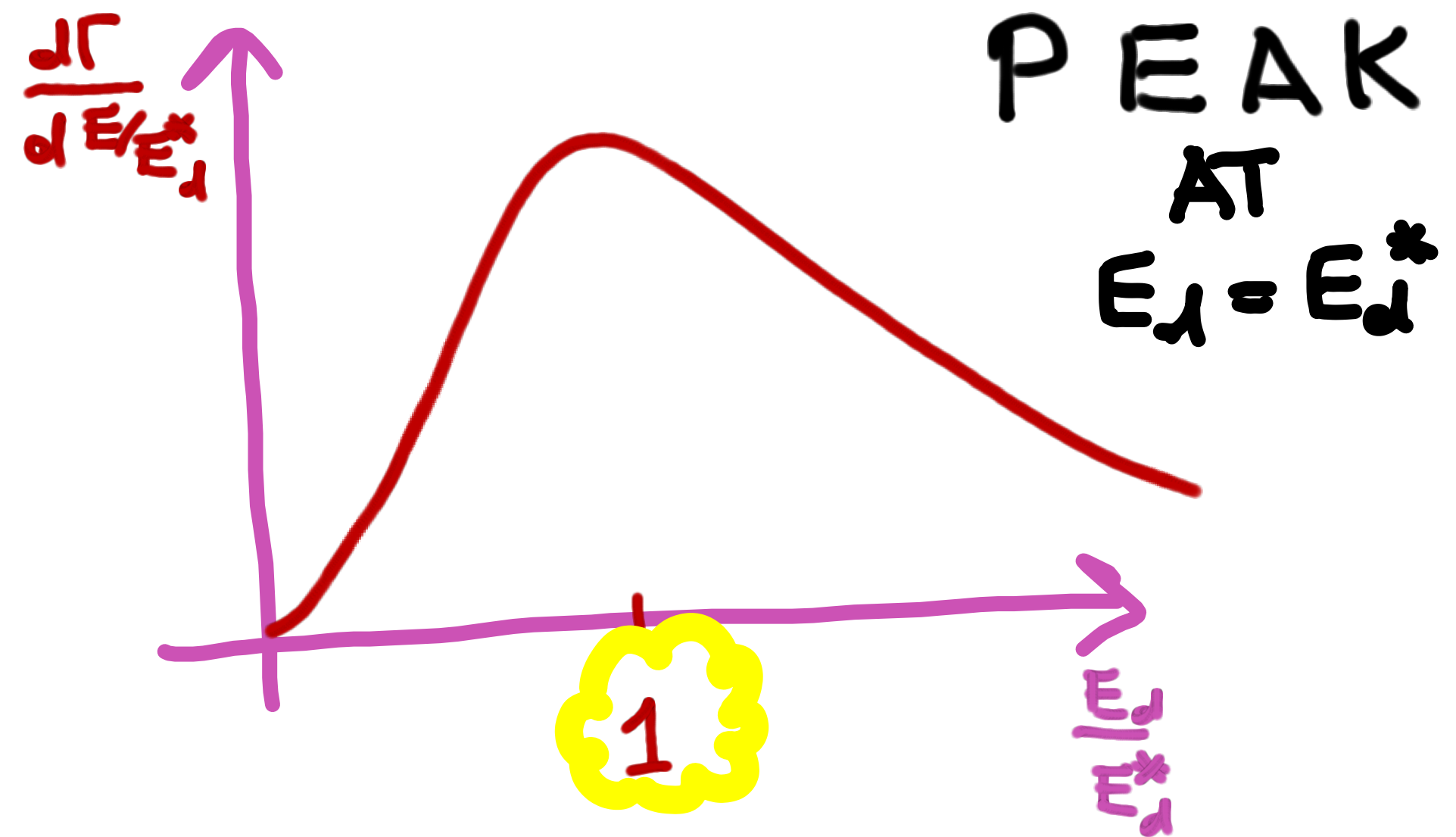


ADVANTAGES

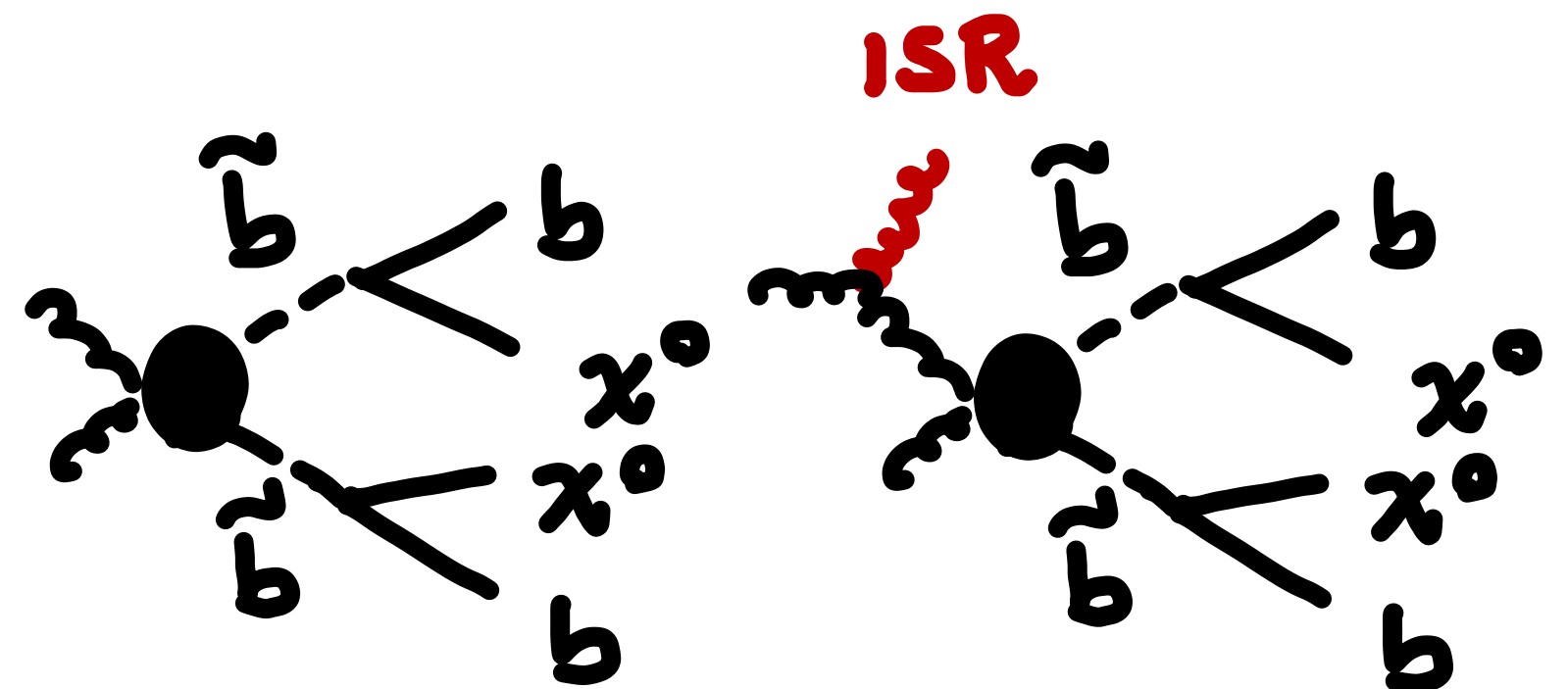
OVER MANY TRANSVERSE KINEMATICAL VARIABLES
IN USE IN COLLIDER PHYSICS (m_{T2}, \dots)



$$E_d^* = \frac{m_M^2 + m_d^2 - m_x^2}{2m_M}$$



• NO NEED TO KNOW
ANYTHING ABOUT THE REST
OF THE EVENT



SOME MORE INSIGHTS BY
GOING THROUGH AN ANALYTIC PROOF :

$$x := \frac{E_d}{E_d^*}$$

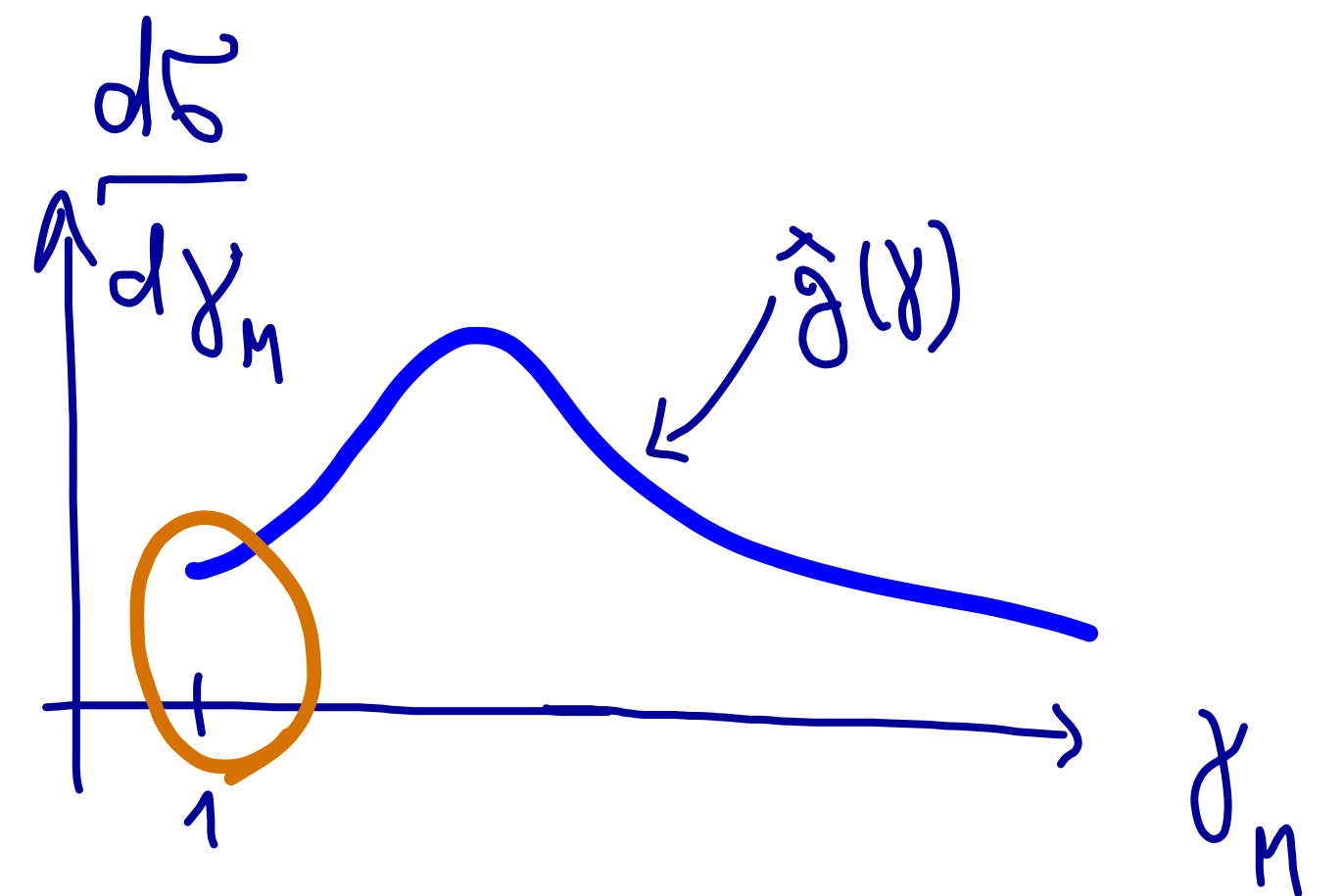
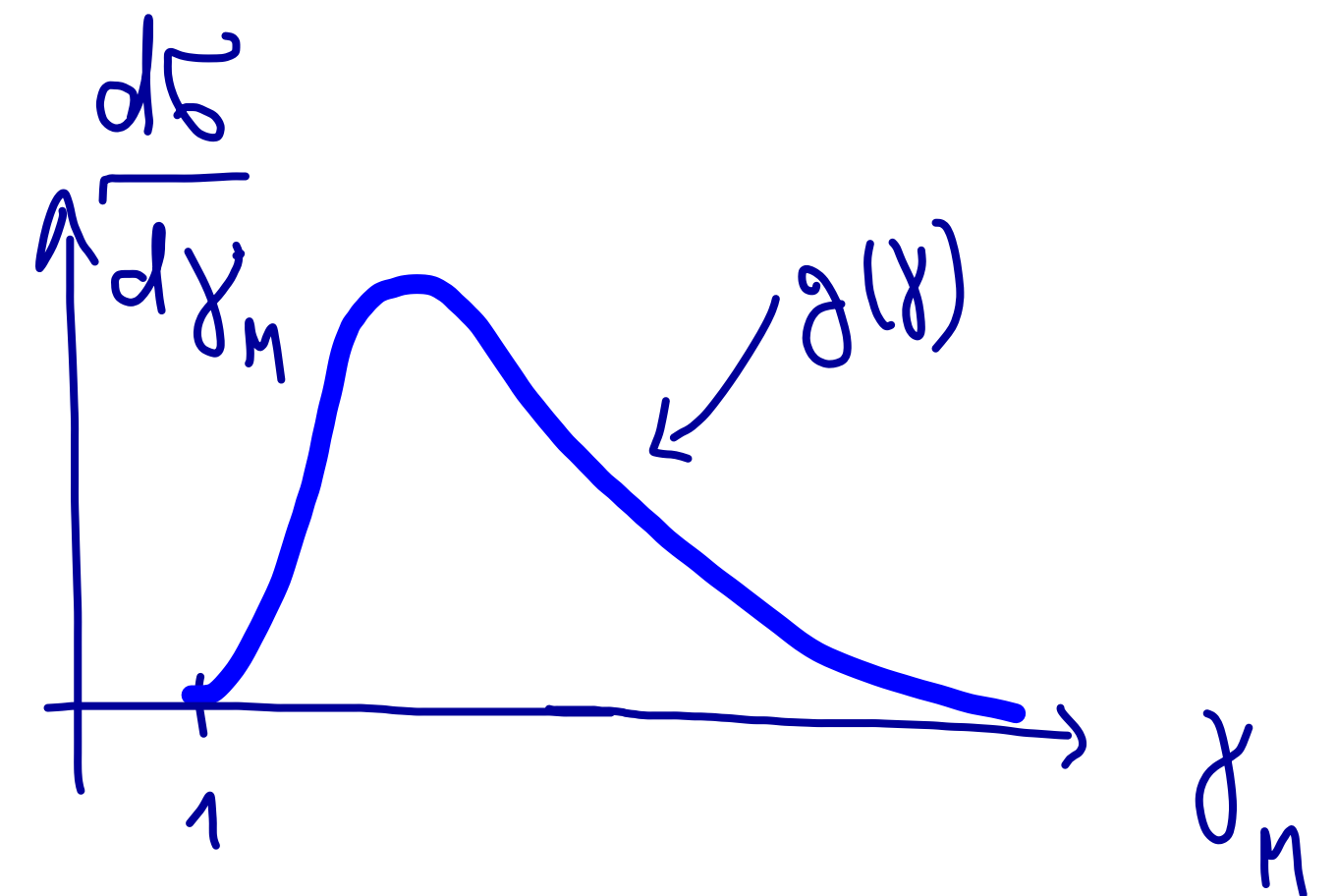
(MASSLESS DAUGHTER)

$$f(x) = \frac{1}{\Gamma} \frac{d\Gamma}{dx} = \int_{\frac{1}{2}(x+\frac{1}{x})}^{\infty} d\gamma \frac{g(\gamma)}{2\sqrt{\gamma^2-1}}$$

$$f'(x) = \frac{\text{sign}(1-x)}{2x} g\left(\frac{1}{2}\left(x+\frac{1}{x}\right)\right)$$

$$g(1) = 0$$

$g(1) \neq 0$ the derivative changes sign \Rightarrow



KINK IN THE
OBSERVED
ENERGY DISTRIBUTION

SOME MORE INSIGHTS BY
GOING THROUGH AN ANALYTIC PROOF :

$$x := \frac{E_d}{E_d^*}$$

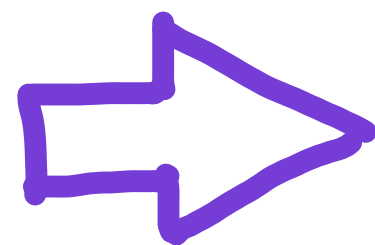
(MASSLESS DAUGHTER)

$$f(x) = \frac{1}{\Gamma} \frac{d\Gamma}{dx} = \int_{\frac{1}{2}(x+\frac{1}{x})}^{\infty} d\gamma \frac{g(\gamma)}{2\sqrt{\gamma^2-1}}$$

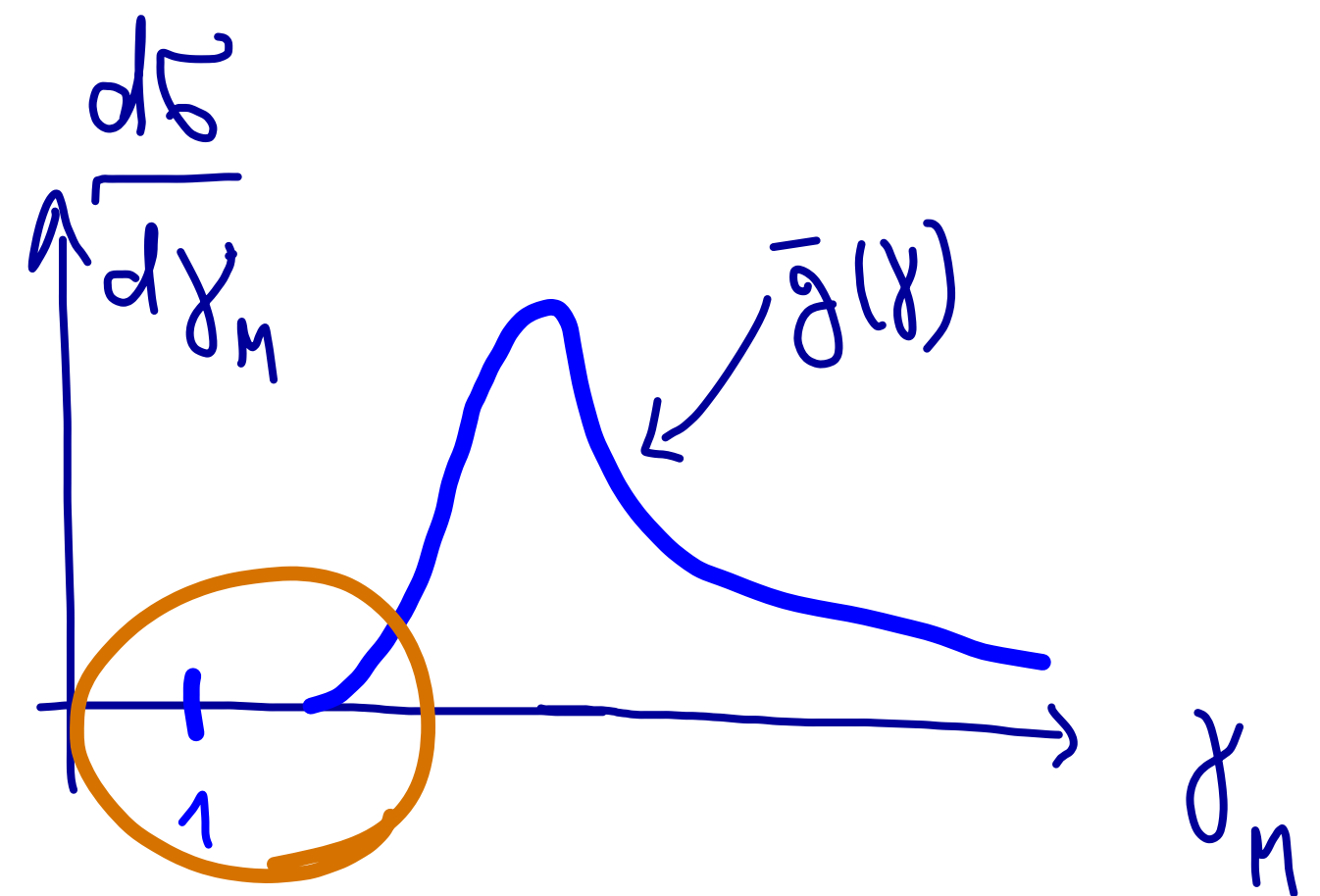
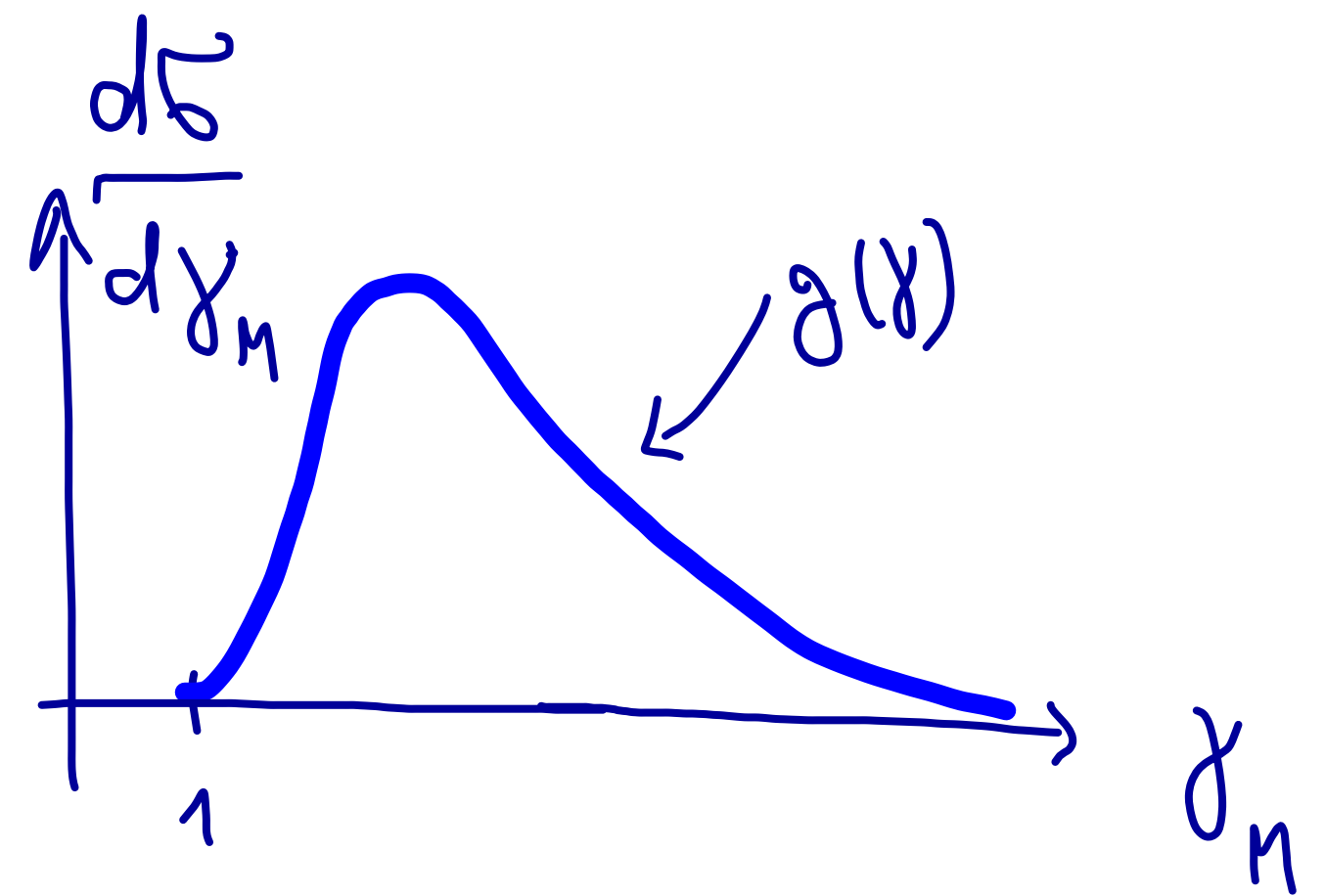
$$f'(x) = \frac{\sinh(1-x)}{2x} g\left(\frac{1}{2}\left(x+\frac{1}{x}\right)\right)$$

$$g(\gamma) = 0$$

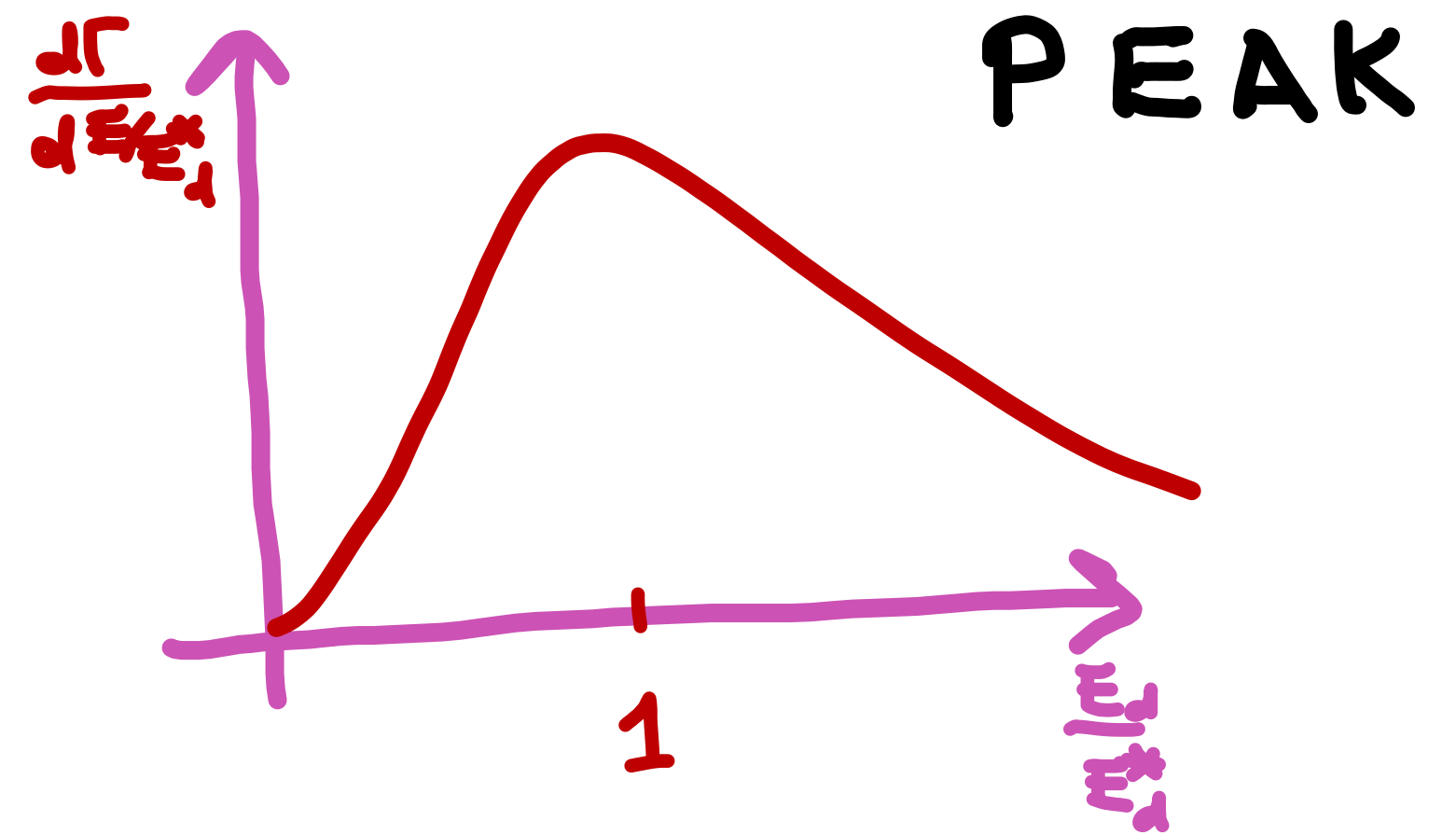
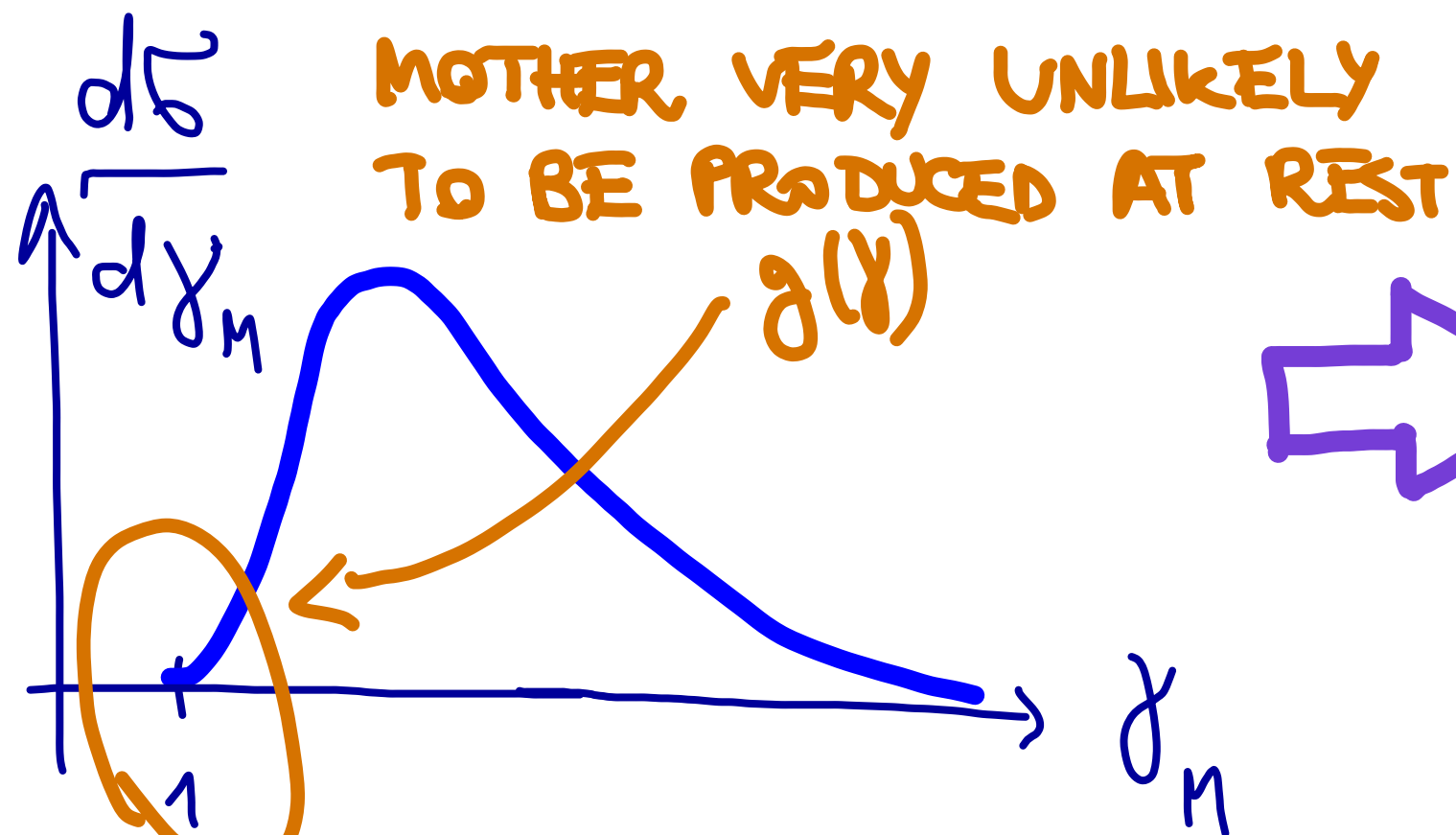
IN A RANGE $[1, \gamma^c]$



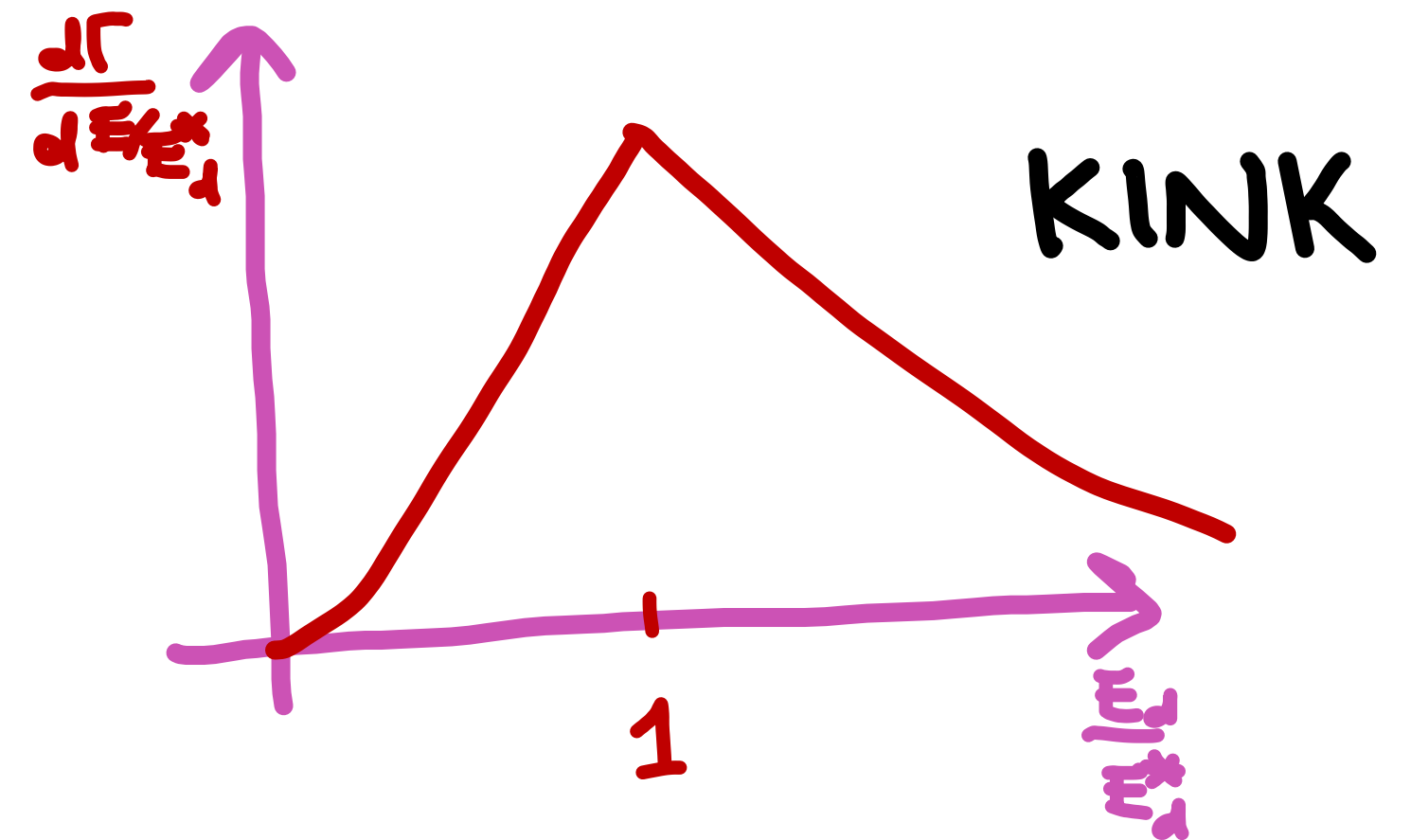
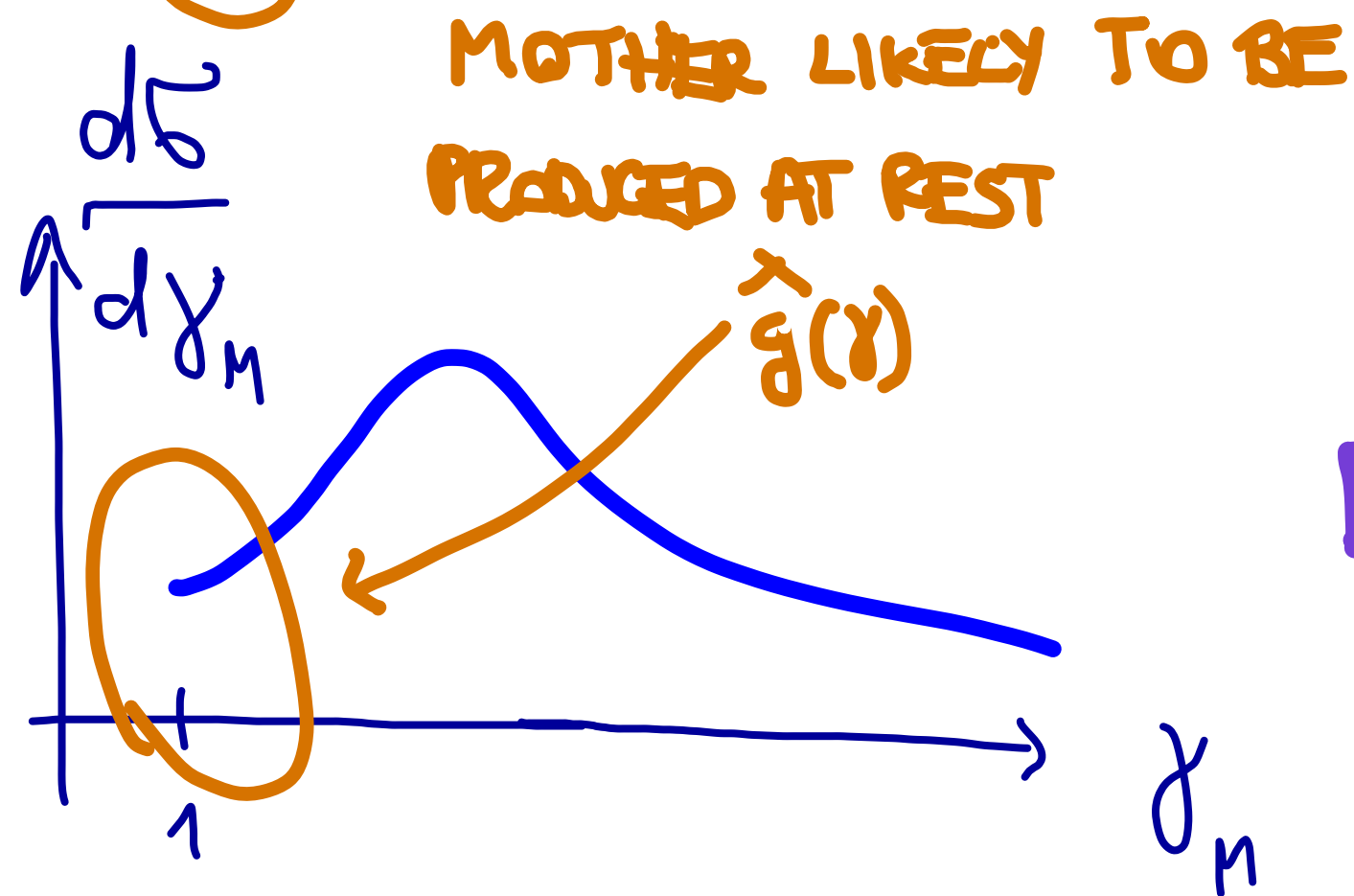
PLATEAU IN
THE OBSERVED
ENERGY DISTRIBUTION



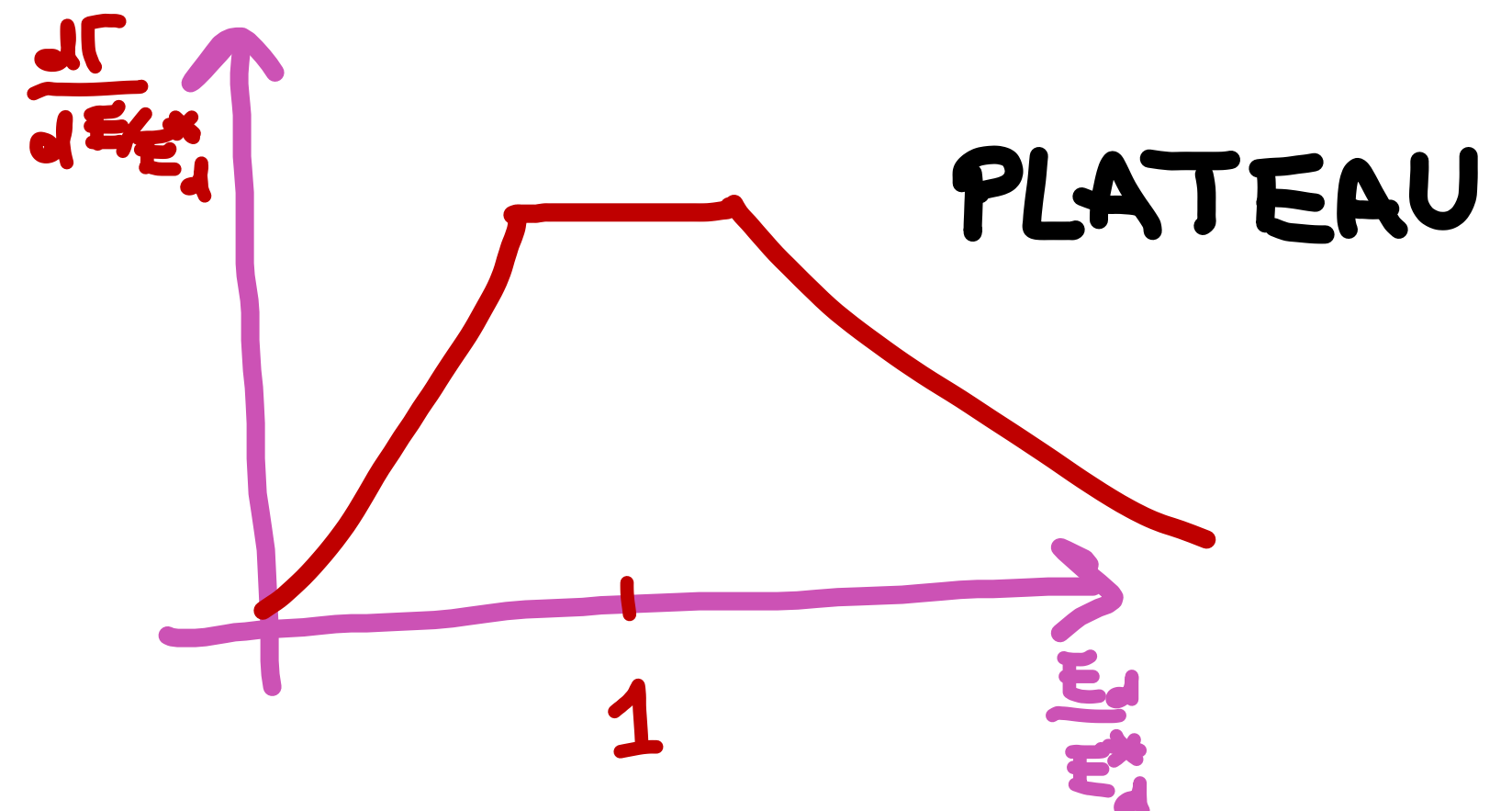
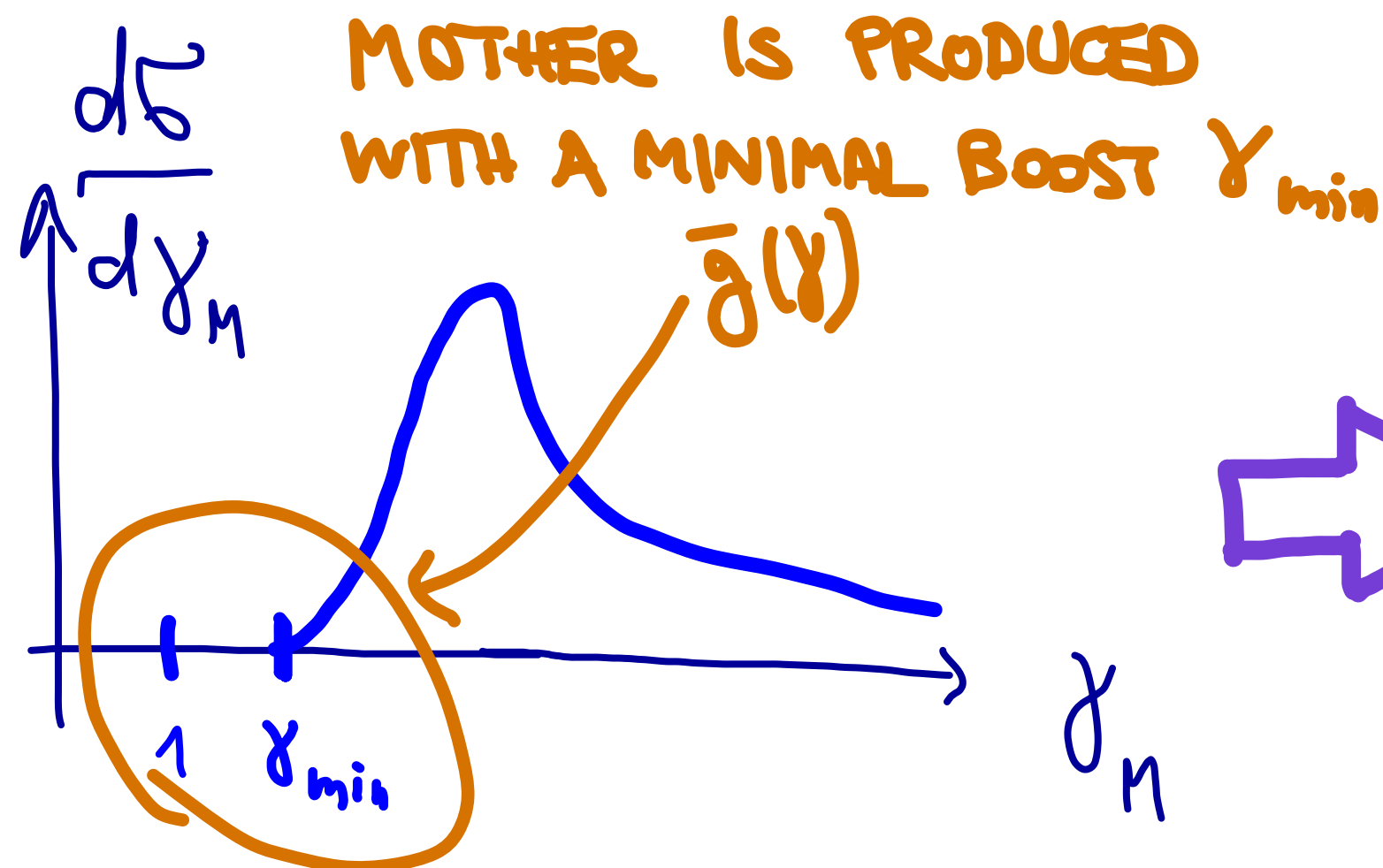
RISE-AND-FALL
BOOST
DISTRIBUTION
OF THE MOTHER



$\hat{g}(1) \neq 0$



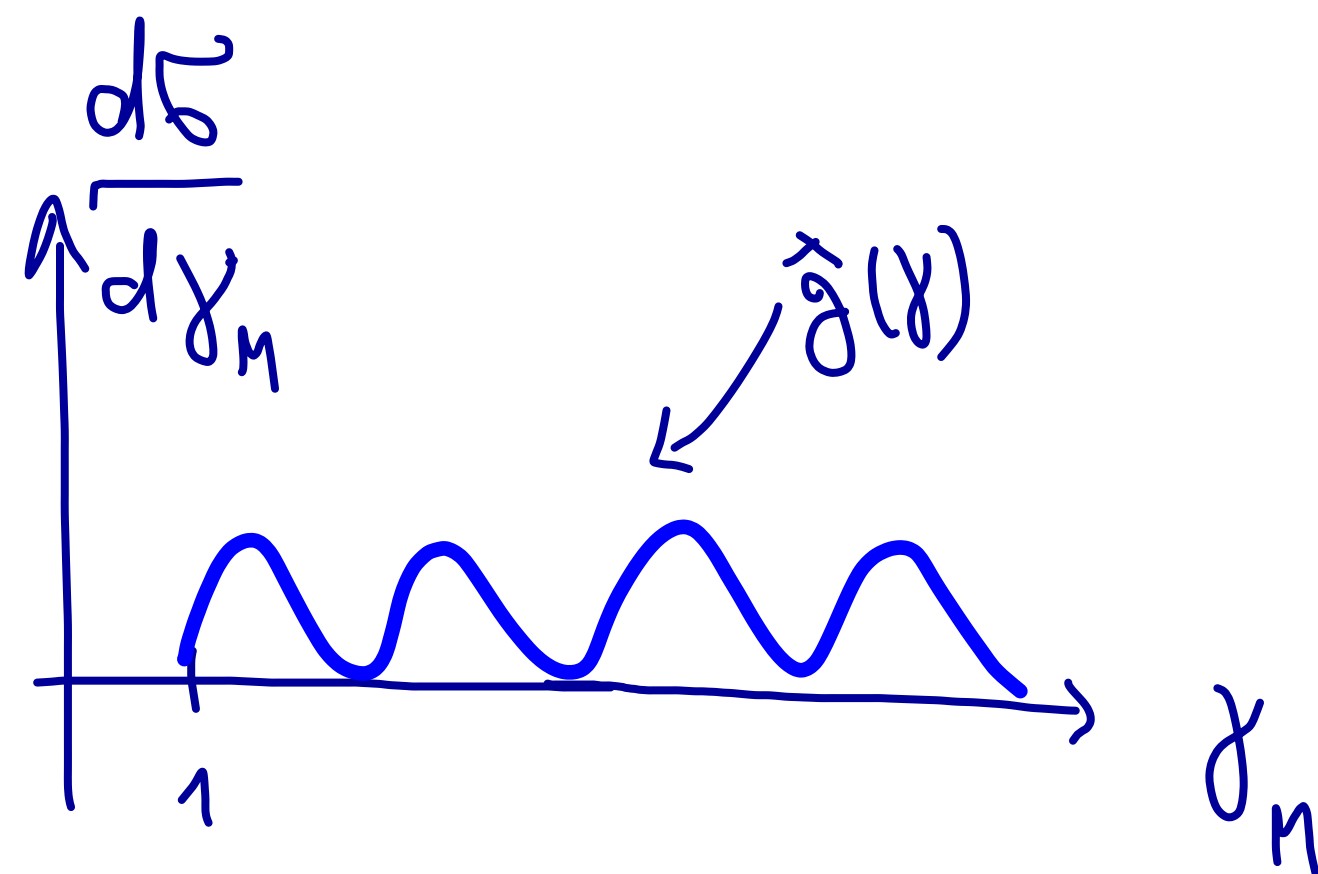
$g(\gamma) = 0$
in a range
 $\gamma \in [1, \gamma^c]$



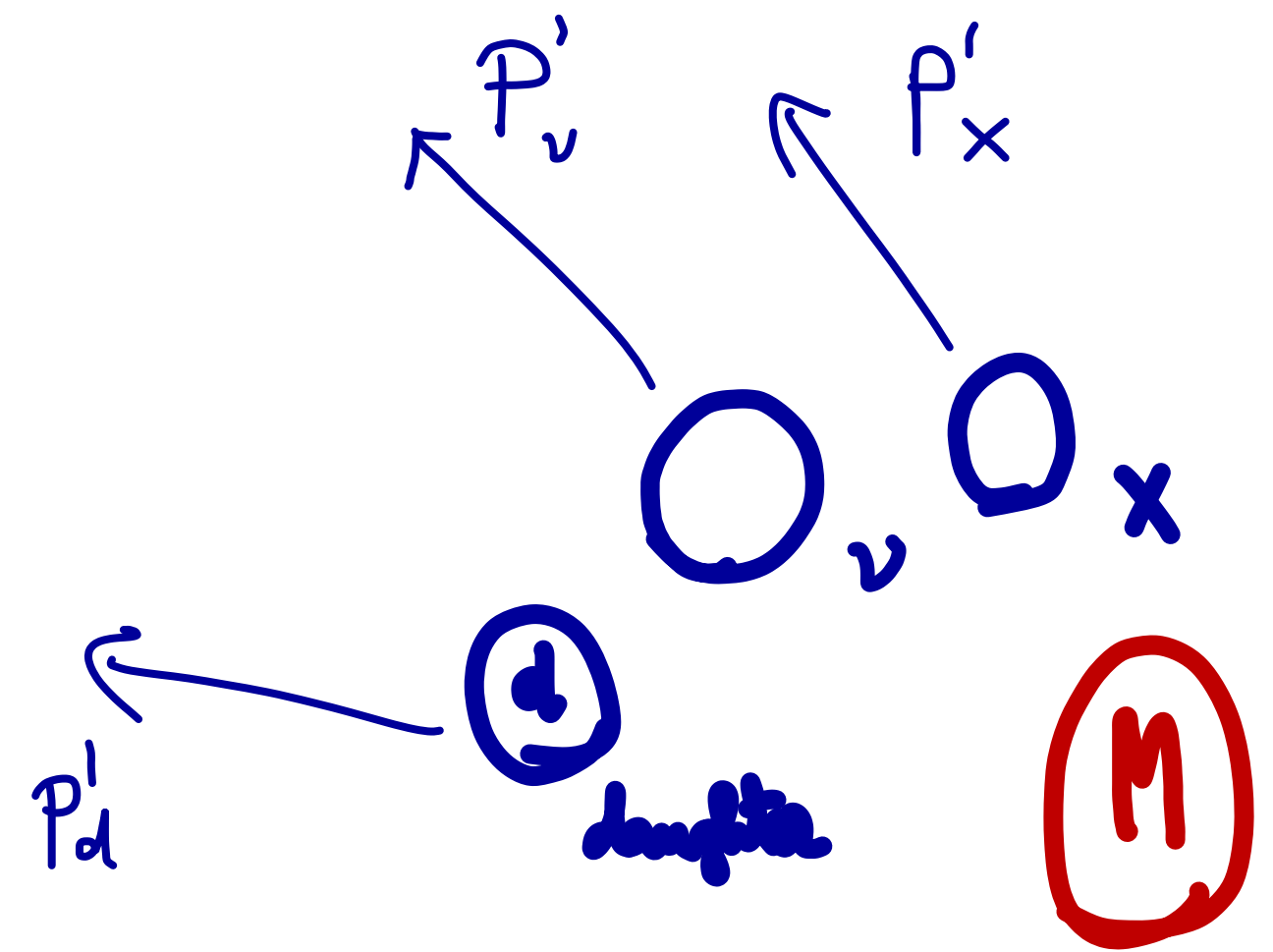
WHEN AND WHY THIS BREAKS DOWN ?

① BOOST DISTRIBUTION
OF THE MOTHER WITH
SPECIAL FEATURES

(MANY MINIMA, LARGE FLAT PORTIONS,...)



② THE DECAY WAS NOT TWO-BODY
EXTRA INVISIBLE/UNDETECTED
PARTICLES IN THE DECAY



WHEN AND WHY THIS BREAKS DOWN ?

CAVEAT:

$$M \rightarrow dX$$

IS TWO BODY ONLY UP TO EXTRA RADIATION

IF THE FINAL STATES ARE COLORED $M \rightarrow dX + \text{gluons}$

JET CLUSTERING SOLVES THIS ISSUE TO SOME EXTENT

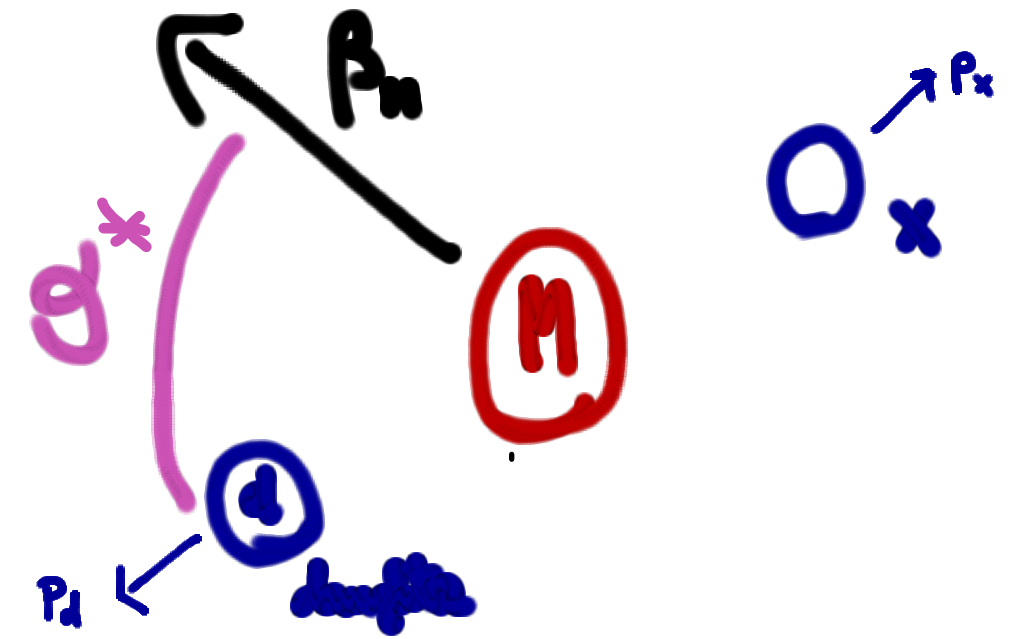
HARD RADIATION MAY BE RESOLVABLE AND EFFECTIVELY
GIVE RISE TO A THREE-BODY DECAY

RESOLVABLE RADIATION CAN BE VETOED

WHEN AND WHY THIS BREAKS DOWN?

③ THE DAUGHTER'S MASS

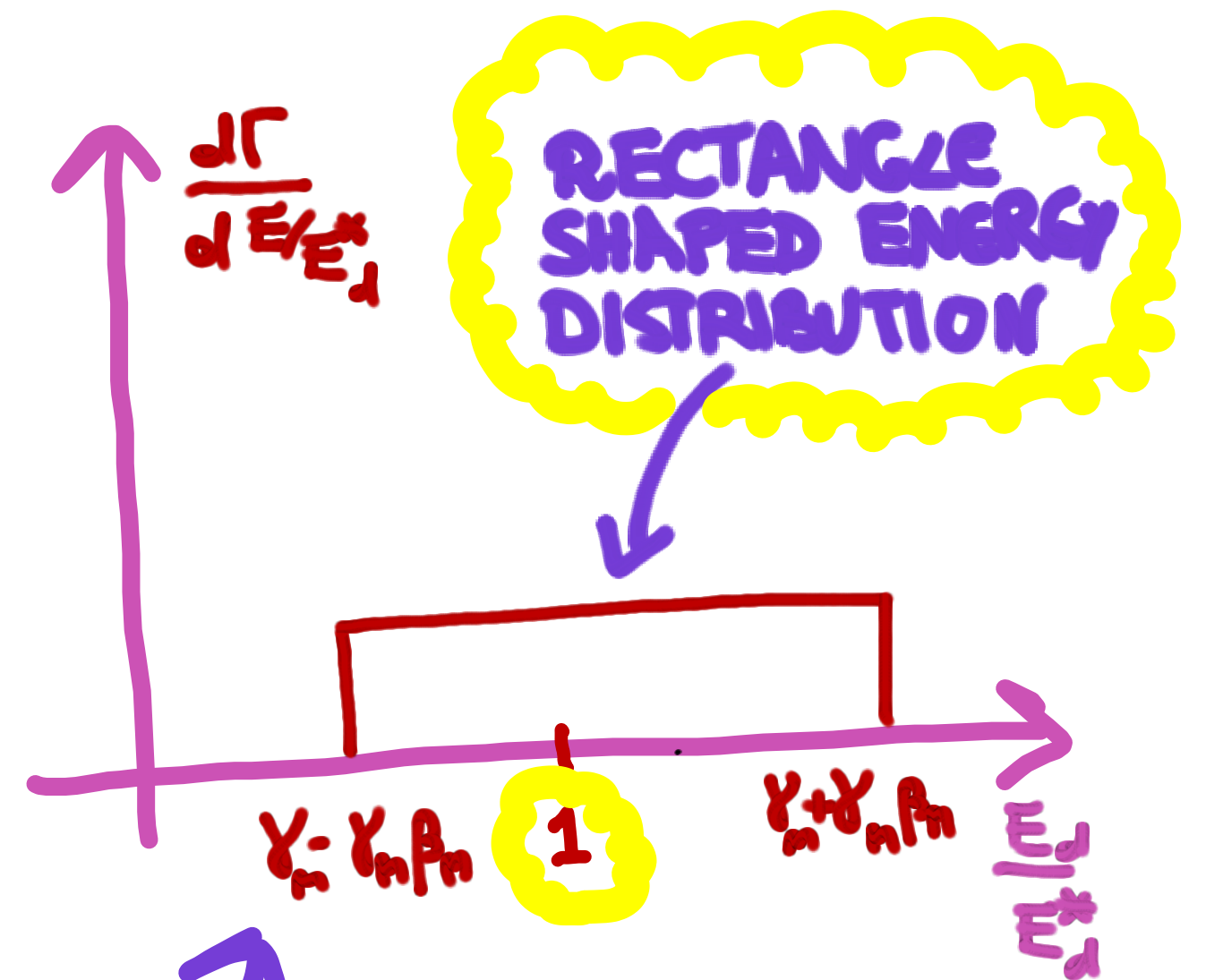
$$E_d' = E_d^* \gamma_n + \cos \theta^* \gamma_n \beta_n p_d^*$$



THE MINIMUM OF THIS QUANTITY AT $\theta^* = \pi$
(BOOST ANTI-PARALLEL TO THE MOMENTUM OF d)

IF $p_d^* = E_d^*$ (MASSLESS DAUGHTER)

$$E_{d,min}' = E_d^* (\gamma_n - \sqrt{\gamma_n^2 - 1}) < E_d^*$$



LOWER EDGE OF EACH
RECTANGLE BELOW E^*

WHEN AND WHY THIS BREAKS DOWN?

• THE DAUGHTER'S MASS

$$E'_d = E_d^* \gamma_n + \cos \theta^* \gamma_n \beta_n p_d^*$$

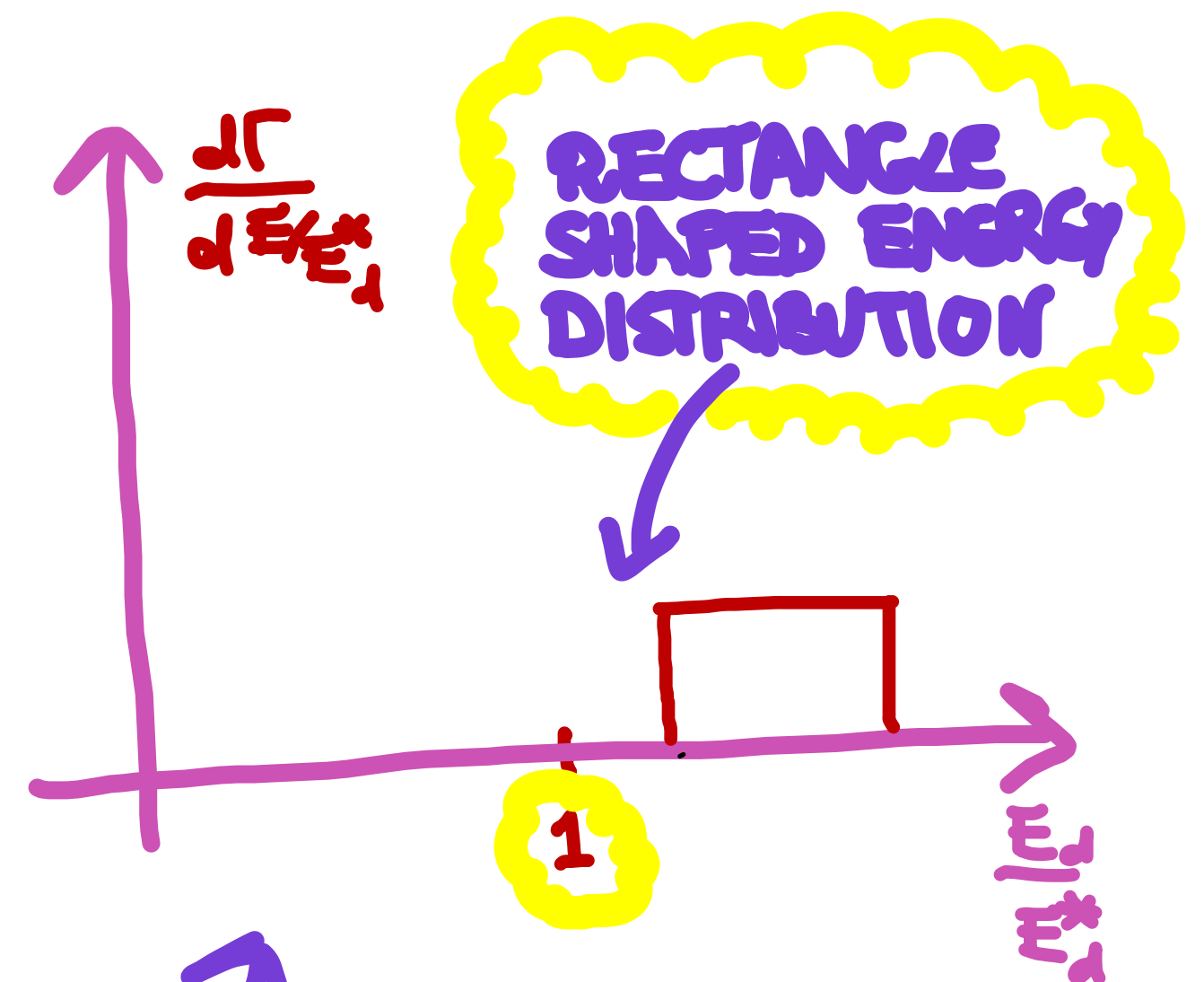
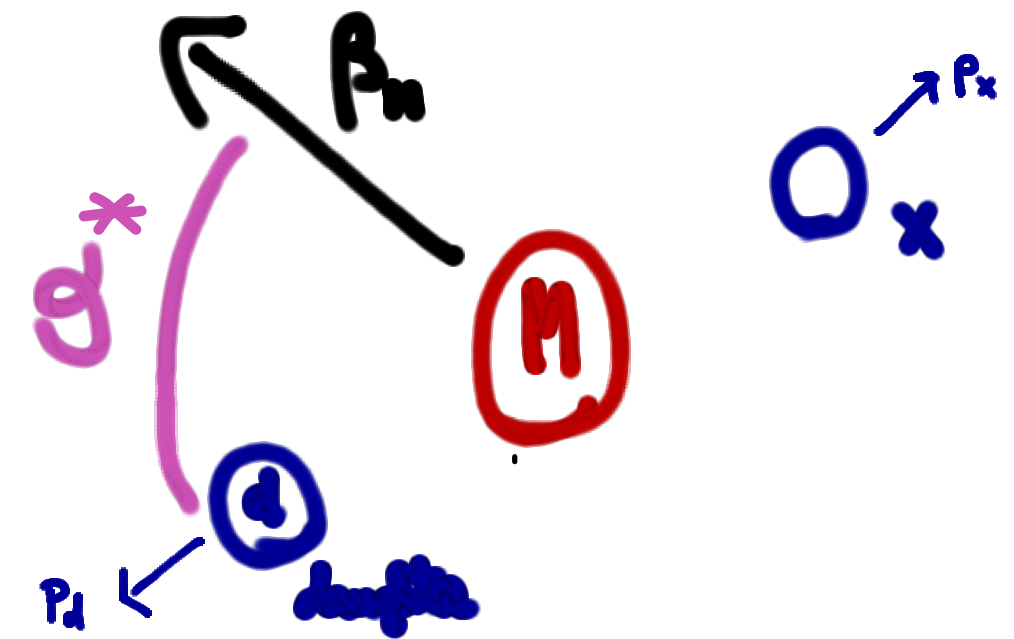
THE MINIMUM OF THIS QUANTITY AT $\theta^* = \pi$
(BOOST ANTI-PARALLEL TO THE MOMENTUM OF d)

IF $p_d^* \leq E_d^*$ (MASSIVE DAUGHTER)

$$p_d^* \rightarrow 0 \quad E_d^* \rightarrow m_d$$

$$E'_d = m_d \gamma_n + \dots$$

FOR γ_n LARGE GIVES
RECTANGLES $E_{d,\min} > E_d^*$



LOWER EDGE OF EACH
RECTANGLE ABOVE E^*

WHEN AND WHY THIS BREAKS DOWN?

• THE DAUGHTER'S MASS

$$E'_d = E_d^* \gamma_n + \cos \theta^* \gamma_n \beta_n p_d^*$$

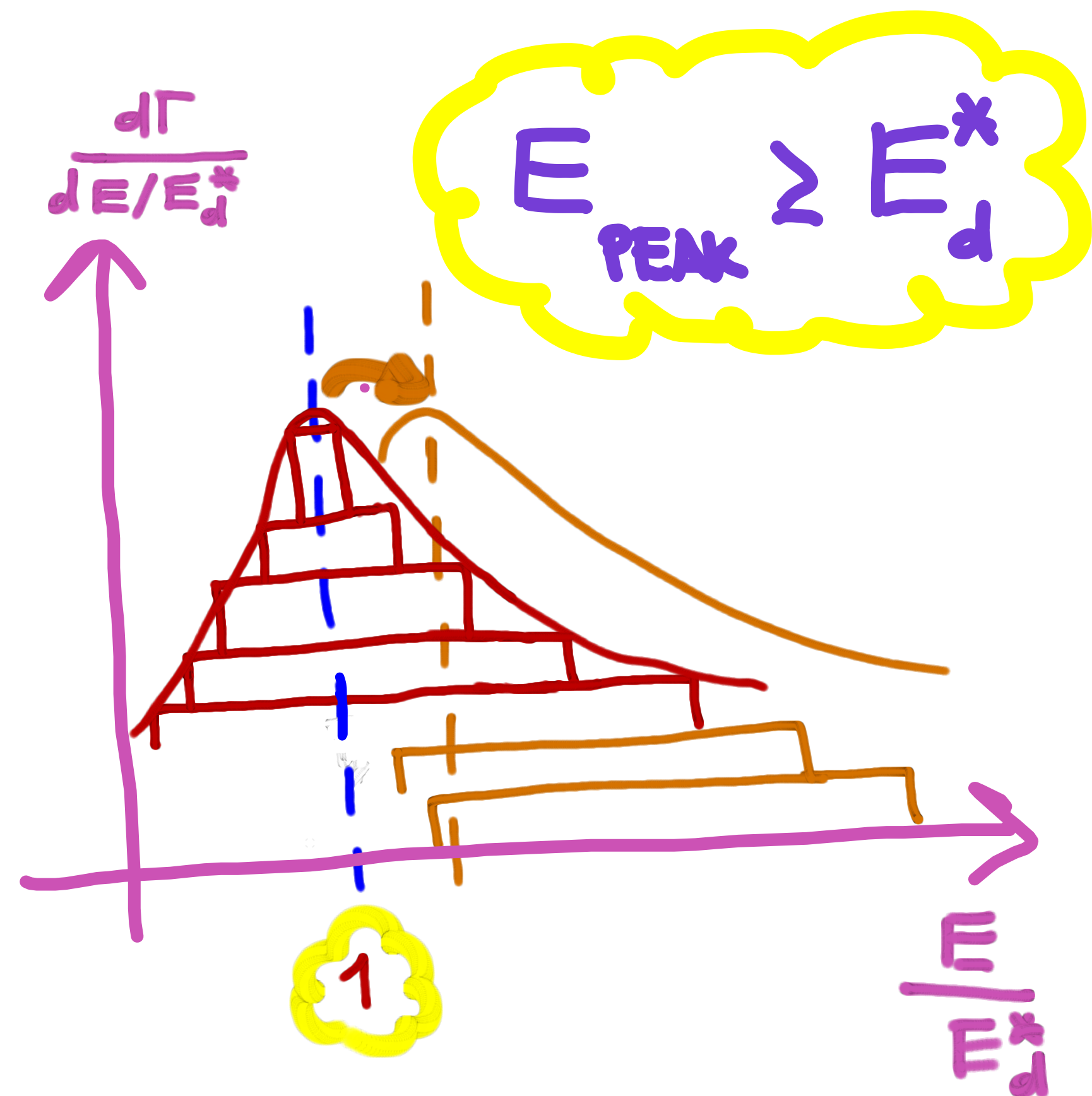
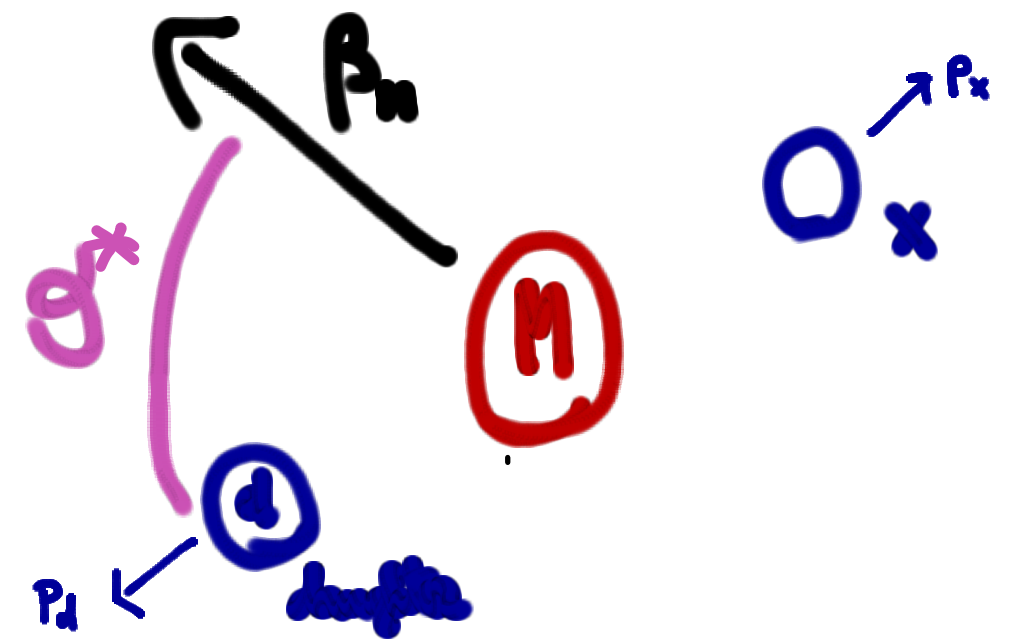
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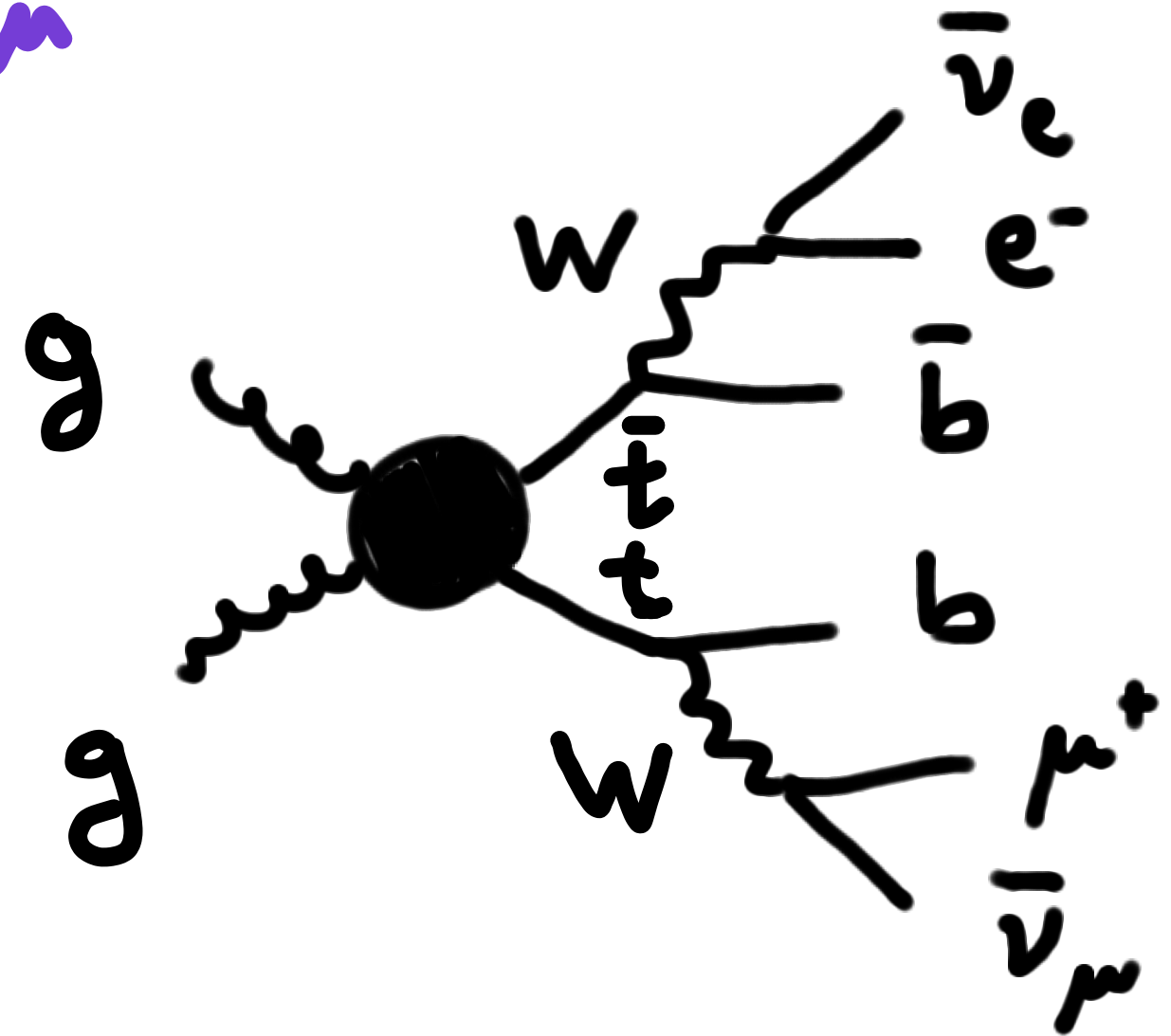
FOR γ_n LARGE GIVES
RECTANGLES $E_{d,\min} \geq E_d^*$



APPLICATIONS:

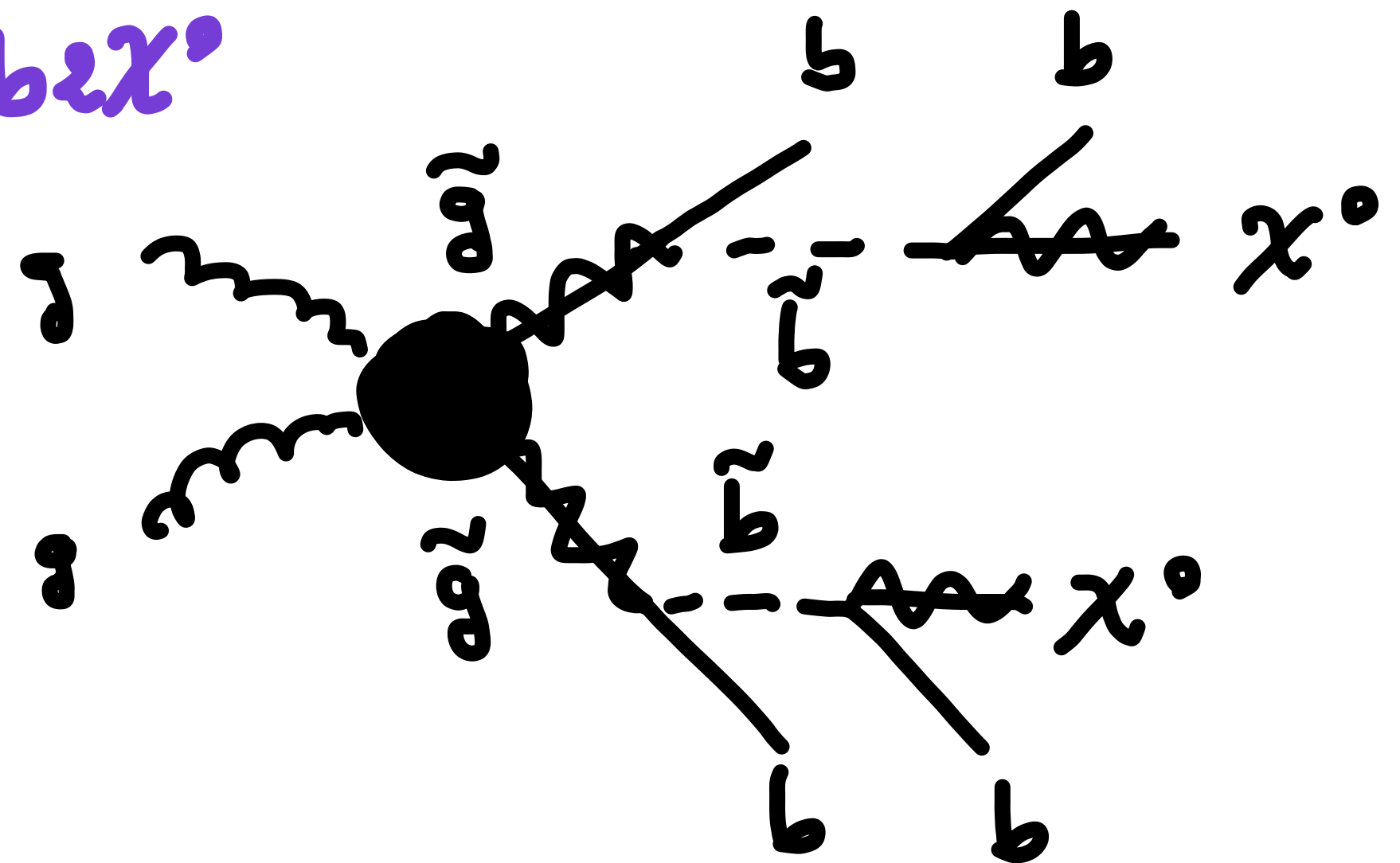
1) $pp \rightarrow t\bar{t} \rightarrow b\bar{b} \mu^+ e^- \bar{\nu}_e \nu_\mu$

m_{top}
AS PROOF OF
PRINCIPLE



2) $pp \rightarrow \tilde{g}\tilde{g} \rightarrow b\tilde{b} b\tilde{b} \rightarrow 4b2\chi^0$

$m_{\tilde{g}}, m_{\tilde{b}}, m_\chi$

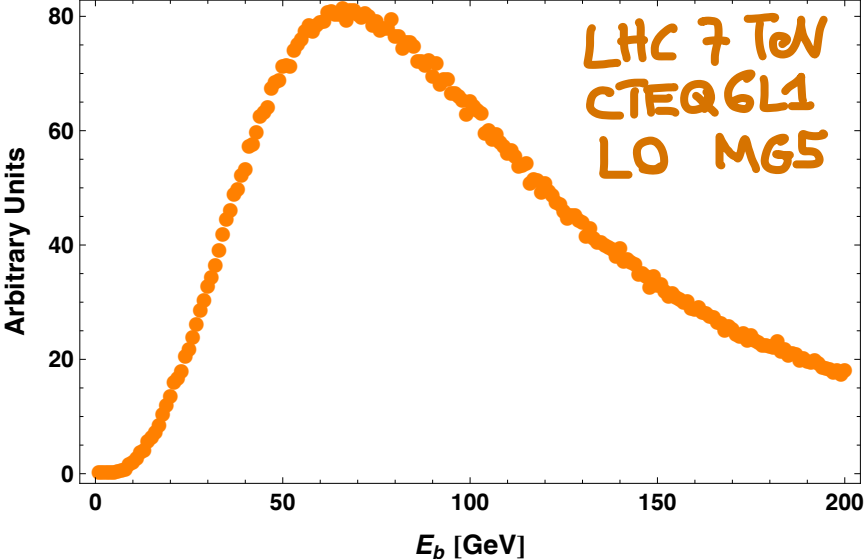


$$pp \rightarrow t\bar{t} \rightarrow b\bar{b} e\bar{\mu} \cancel{\nu_\tau}$$

- QCD PAIR PRODUCTION OF $t\bar{t}$ ENSURES THAT THE OVERALL SAMPLE OF TOP DECAYS IS UNPOLARIZED

$$E_b^* = \frac{m_t^2 - m_W^2 + m_b^2}{2m_t} \cong 67 \text{ GeV} \quad E_b^* \gg m_b$$

THE b QUARK CAN BE TAKEN AS MASSLESS



FINDING THE PEAK :

- LOOK BY EYE

- FIND A TEMPLATE AND USE IT TO FIT DATA

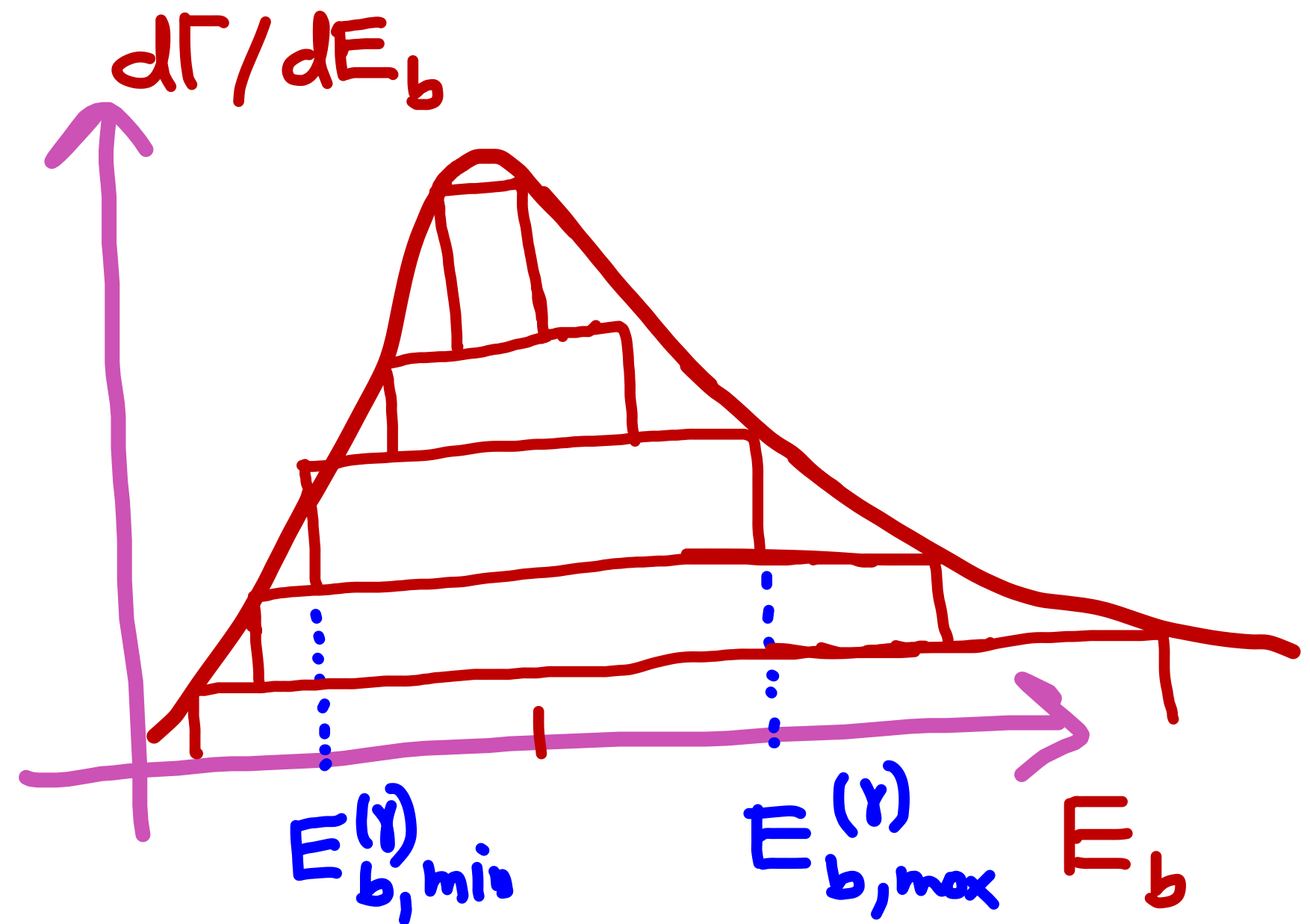
- A TEMPLATE MOTIVATED FROM PRIME PRINCIPLES SEEMS UNATTAINABLE BECAUSE IT DEPENDS ON PARTON DISTRIBUTION FUNCTIONS AND ON THE MATRIX ELEMENT FOR THE PRODUCTION PROCESS

FIND A TEMPLATE AND USE IT TO FIT DATA

$$E_b = E_b^* (\gamma_n + \cos \vartheta^* \sqrt{\gamma_n^2 - 1})$$

$$E_{b,\min}^{(\gamma)} = E_b^* (\gamma_n - \sqrt{\gamma_n^2 - 1})$$

$$\sqrt{E_{b,\min} E_{b,\max}} = E_b^*$$



$d\Gamma/dE_b$
MUST BE A FUNCTION OF

$$\frac{E_b}{E_b^*} + \frac{E_b^*}{E_b}$$

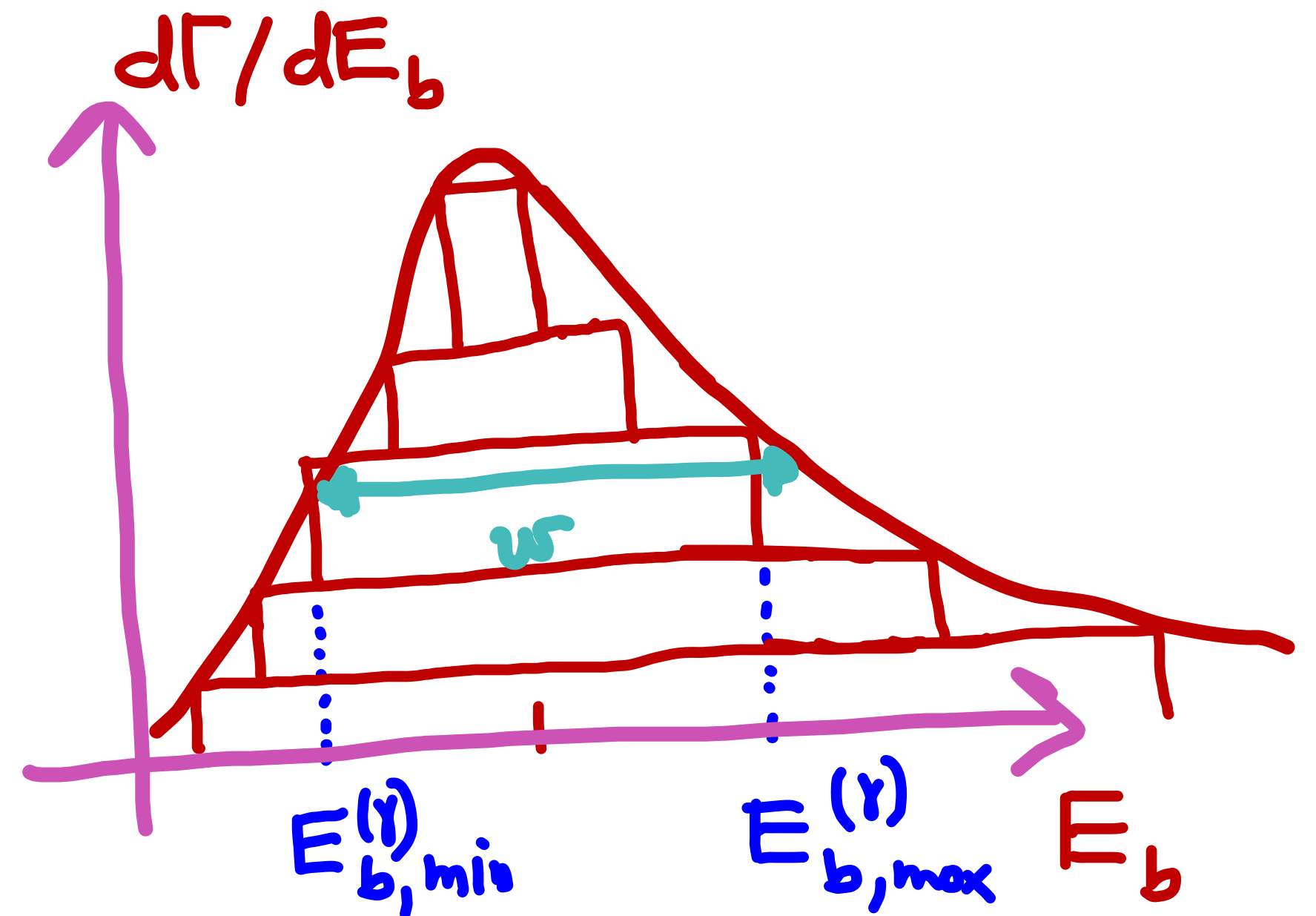
FIND A TEMPLATE AND USE IT TO FIT DATA

- $d\Gamma/dE_b \xrightarrow{E_b \rightarrow 0, \infty} 0$ (at least)

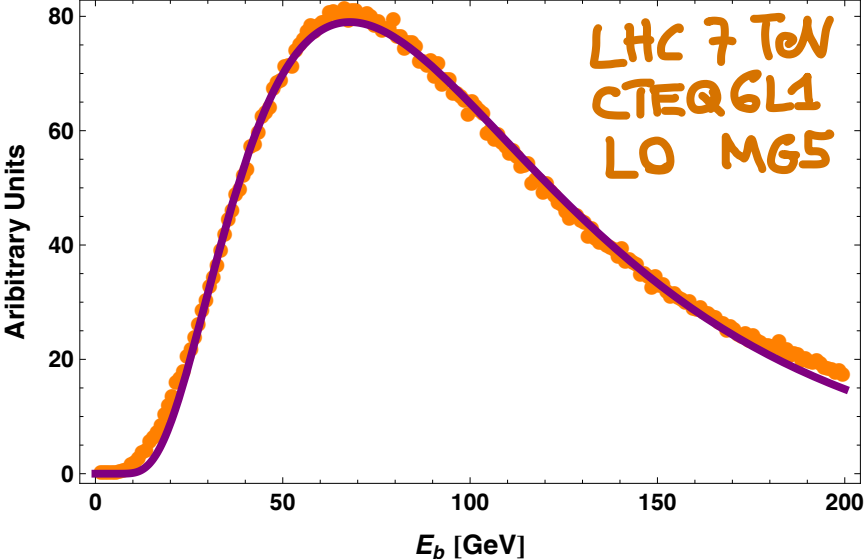
- $d\Gamma/dE_b$ max at $E_b = E_b^*$

- IN SOME LIMIT SHOULD BE A δ -FUNCTION (MOTHER AT REST)

- $d\Gamma/dE_b$
MUST BE A FUNCTION OF
$$\frac{E_b}{E_b^*} + \frac{E_b^*}{E_b}$$



$$d\Gamma/dE \sim \exp\left(-w\left(\frac{E}{E^*} + \frac{E^*}{E}\right)\right)$$

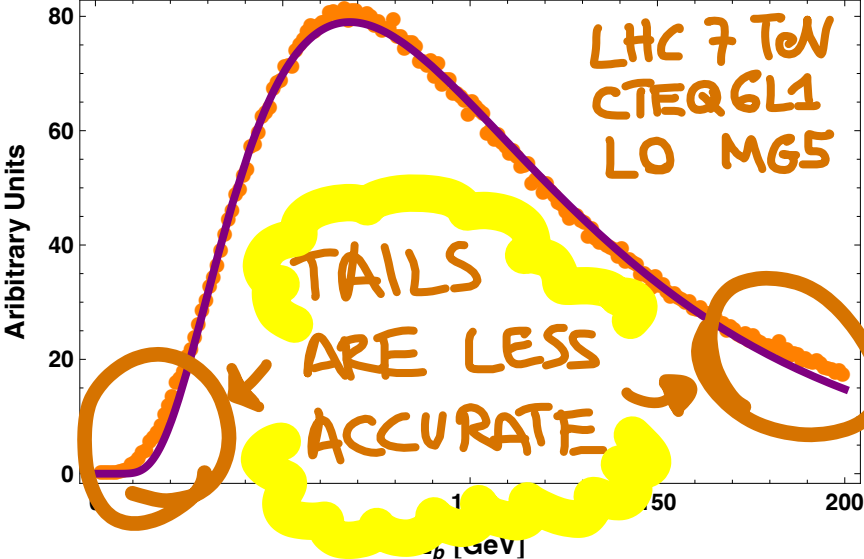


Arbitrary Units

LHC 7 TeV
CTEQ6L1
LO MG5

TAILS
ARE LESS
ACCURATE

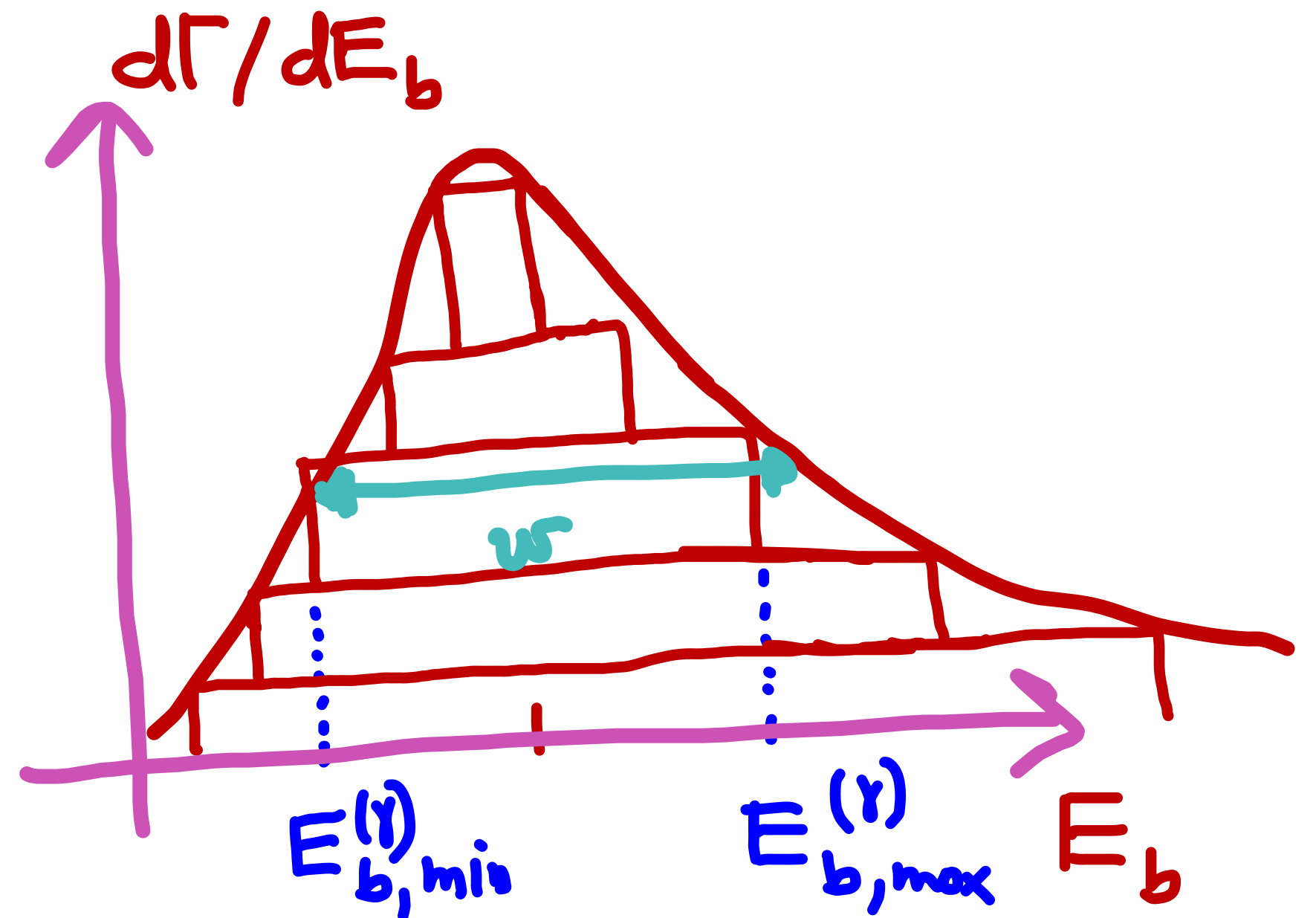
p_T [GeV]



FIND A TEMPLATE AND USE IT TO FIT DATA

- $d\Gamma/dE_b \xrightarrow{E_b \rightarrow 0, \infty} 0$ ☒ ☐
- $d\Gamma/dE_b$ max at $E_b = E_b^*$ ☒
- IN SOME LIMIT SHOULD BE A δ -FUNCTION (MOTHER AT REST) ☒
- $d\Gamma/dE_b$ ☒
MUST BE A FUNCTION OF

$$\frac{E_b}{E_b^*} + \frac{E_b^*}{E_b}$$



$$d\Gamma/dE \sim \exp\left(-w\left(\frac{E}{E^*} + \frac{E^*}{E}\right)\right)$$

CAN WE MEASURE PARTICLE MASSES ?

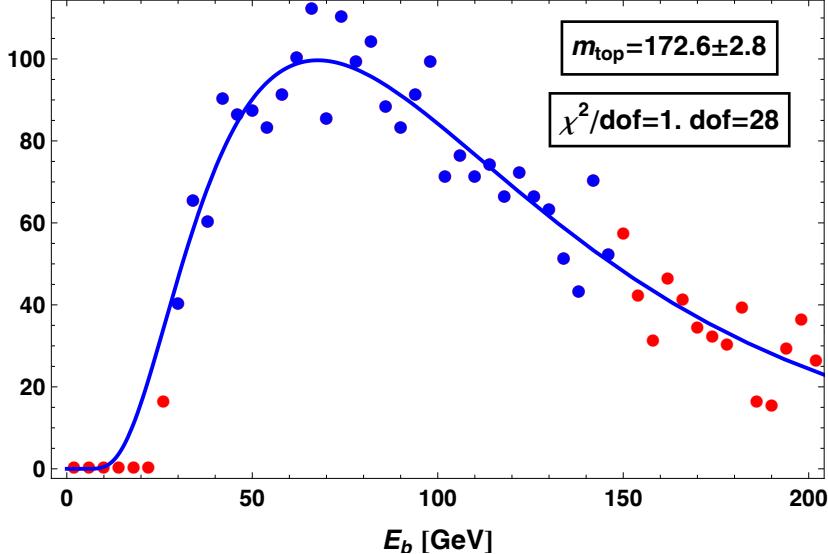
$$\bullet E_b^* = \frac{m_t^2 - m_W^2 + m_b^2}{2m_t} \cong 67 \text{ GeV}$$

FROM THE RESULT OF THE FIT TO THE
LEADING ORDER MATRIX ELEMENT WE HAVE AT
LEAST A CHANCE

NEED TO EVALUATE :

- DETECTOR EFFECTS \longrightarrow DELPHES 1.9
- EXTRA QCD RADIATION \longrightarrow SOFT QCD PYTHIA 6.4
- BIAS FROM EVENT SELECTION \longrightarrow ATLAS-CONF-2012-017

Events/4 GeV



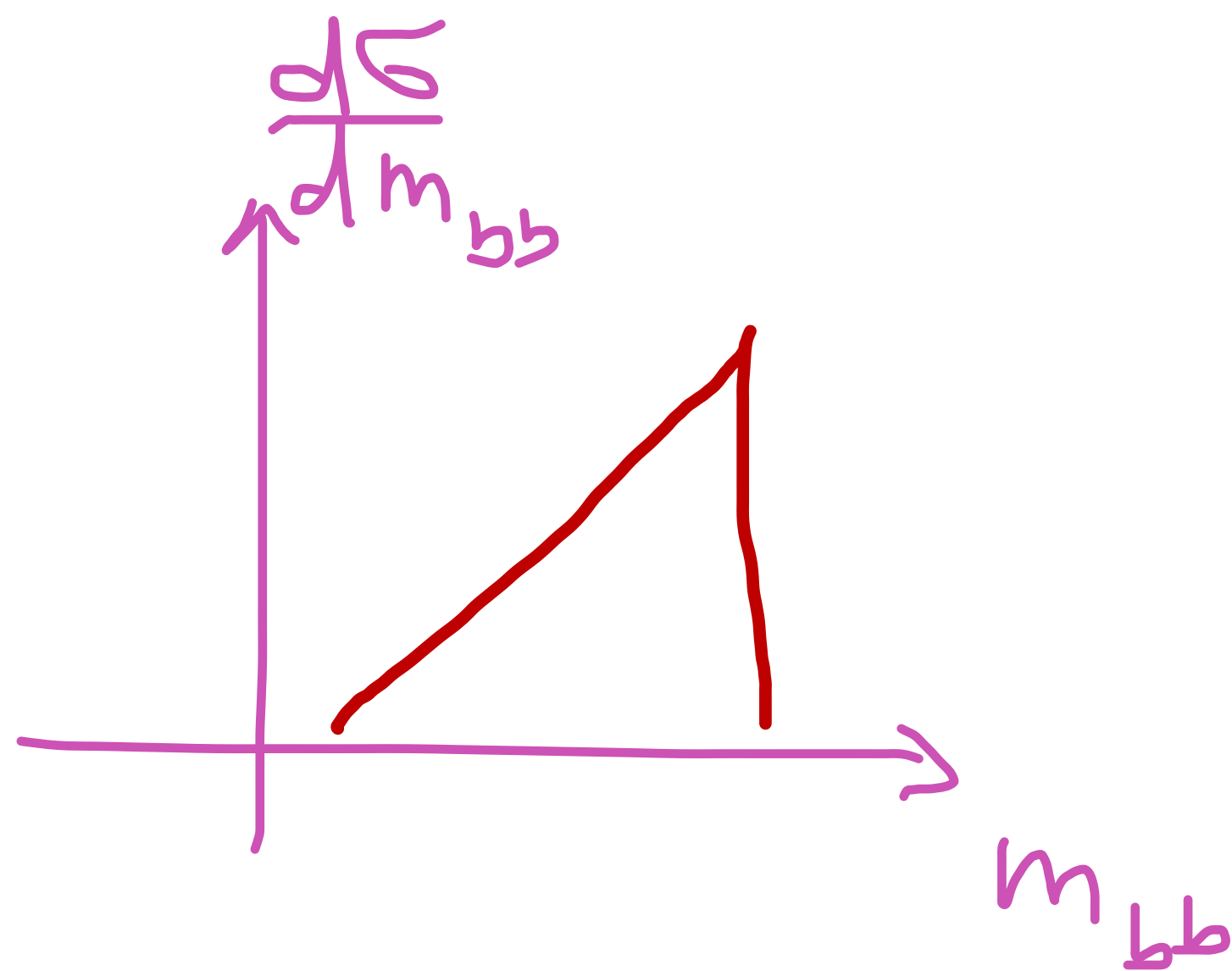
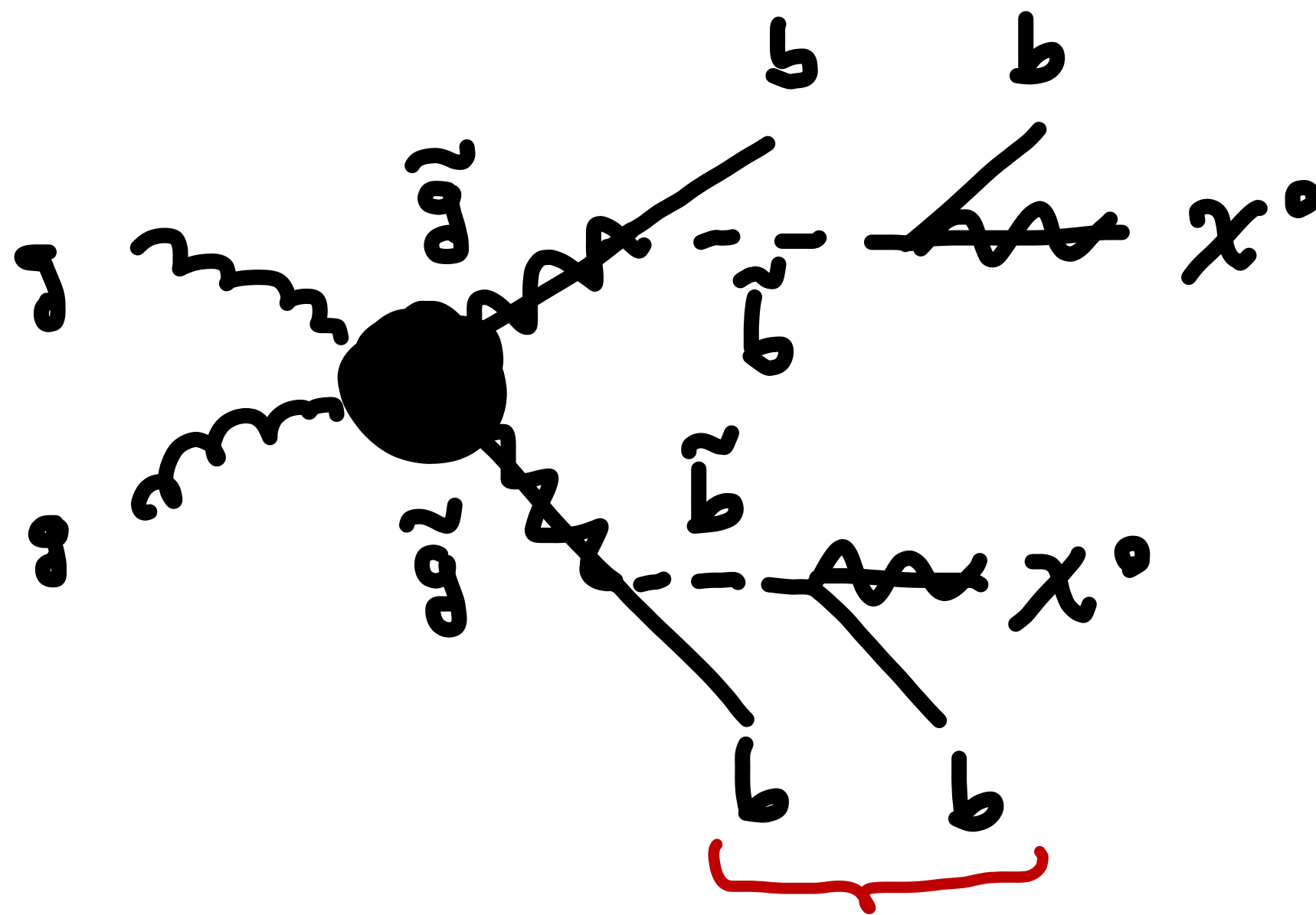
CAN WE MEASURE PARTICLE MASSES ?

FROM 100 PSEUDO EXPERIMENTS
FOR LHC $\sqrt{s} = 7$ TeV AND $\mathcal{L} = 5/\text{fb}$
WE GET

$$m_{\text{top}} = 173.1 \pm 2.5 \text{ GeV}$$

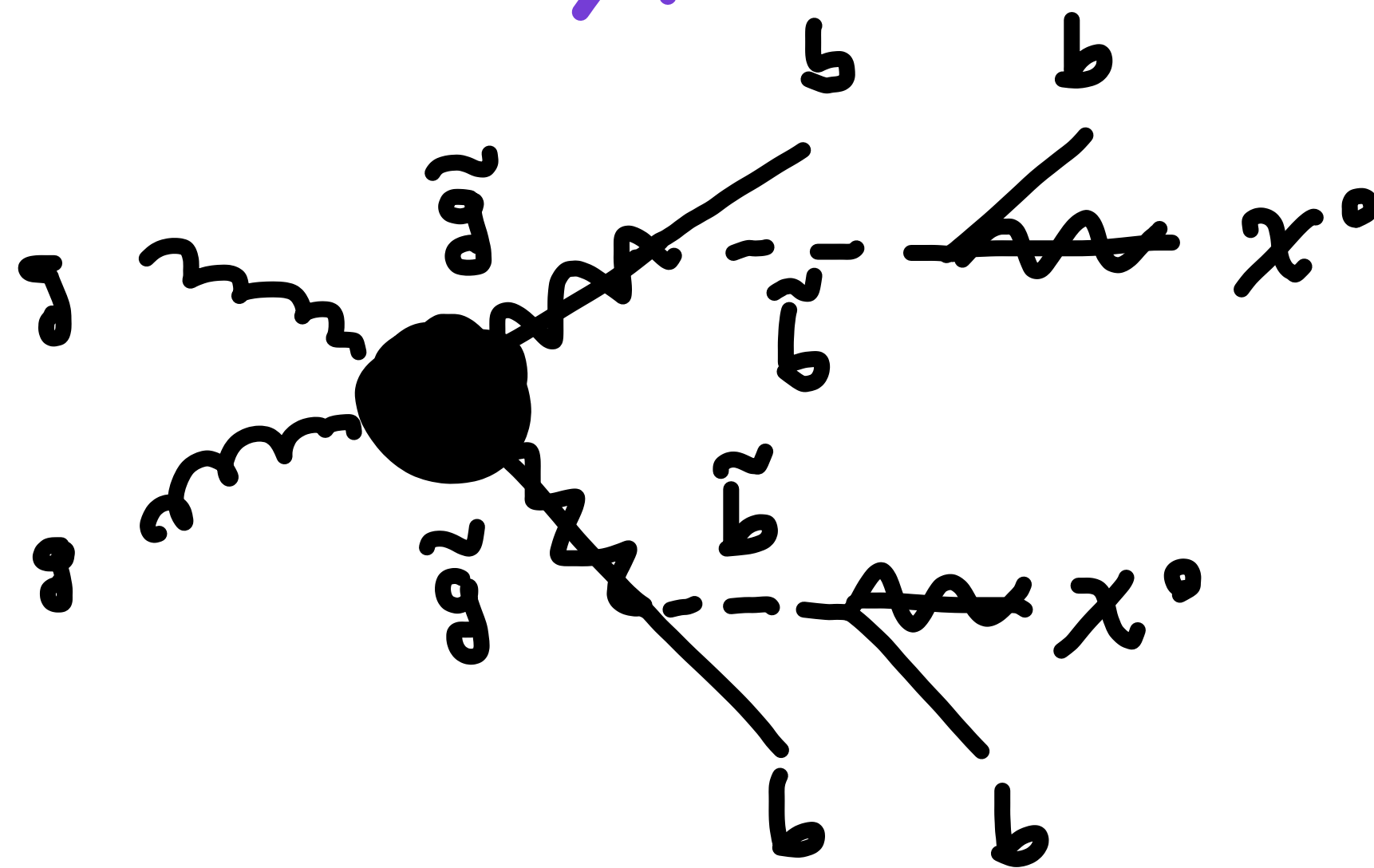
- ALL THE EFFECTS AT LEADING ORDER ARE WELL UNDER CONTROL
- HIGHER ORDER QCD WAS NOT INCLUDED ($\leq 10\%$)

$$pp \rightarrow \tilde{g}\tilde{g} \rightarrow b\tilde{b}b\tilde{b} \rightarrow 4b \text{ E.T.}$$

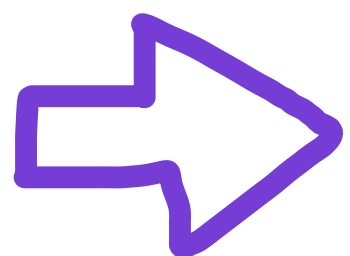


$$m_{bb}^{\max} = \sqrt{\frac{m_g^2 - m_{\tilde{b}}^2}{m_{\tilde{b}}}} \cdot \frac{m_{\tilde{b}}^2 - m_{\chi}^2}{m_{\tilde{b}}}$$

$$pp \rightarrow \tilde{g}\tilde{g} \rightarrow b\tilde{b}b\tilde{b} \rightarrow 4b \cancel{E_T}$$

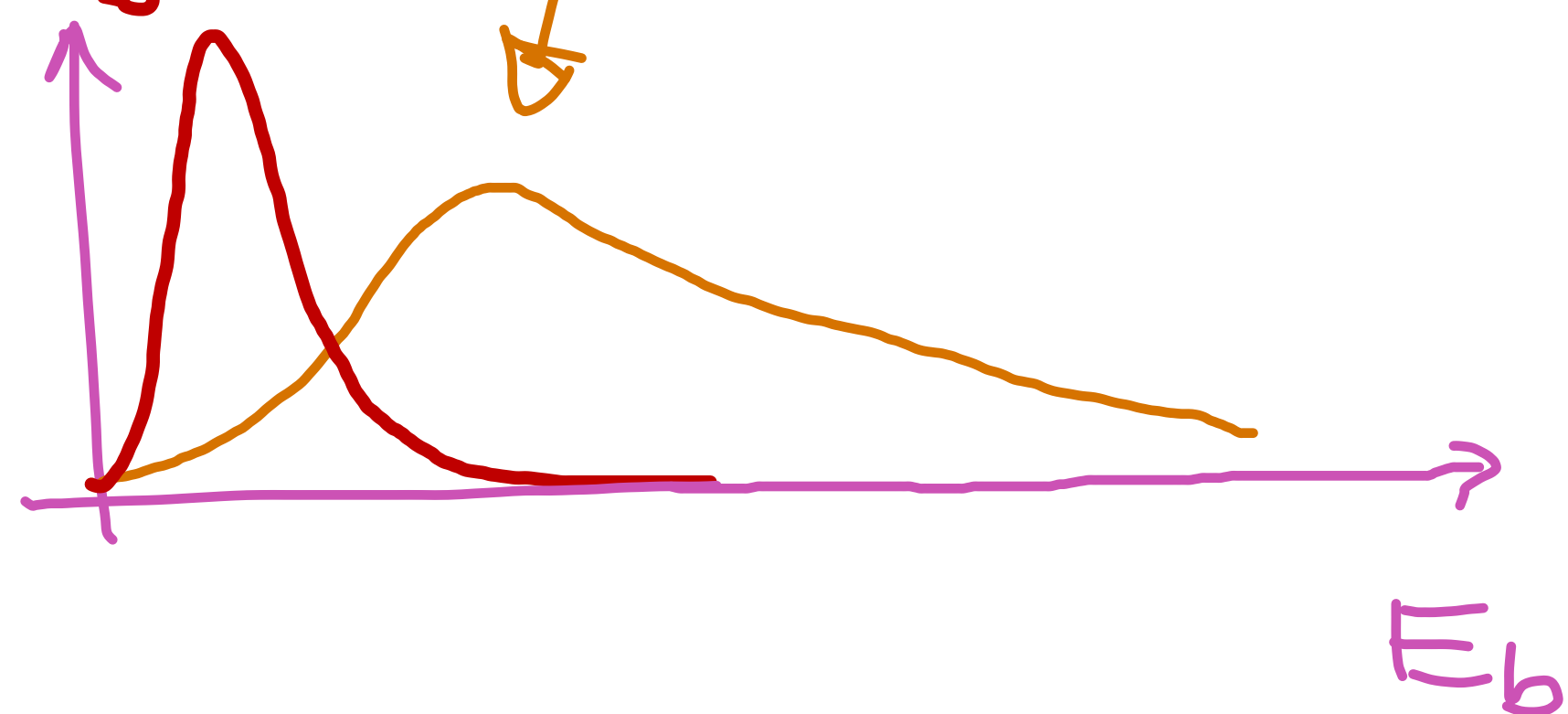


TWO-STEP
DECAY



$$E_b^{\text{peak}} = \frac{m_{\tilde{g}}^2 - m_{\tilde{b}}^2}{2m_{\tilde{g}}}$$

$$E_b^{\text{peak}} = \frac{m_{\tilde{b}}^2 - m_{\chi}^2}{2m_{\tilde{b}}}$$



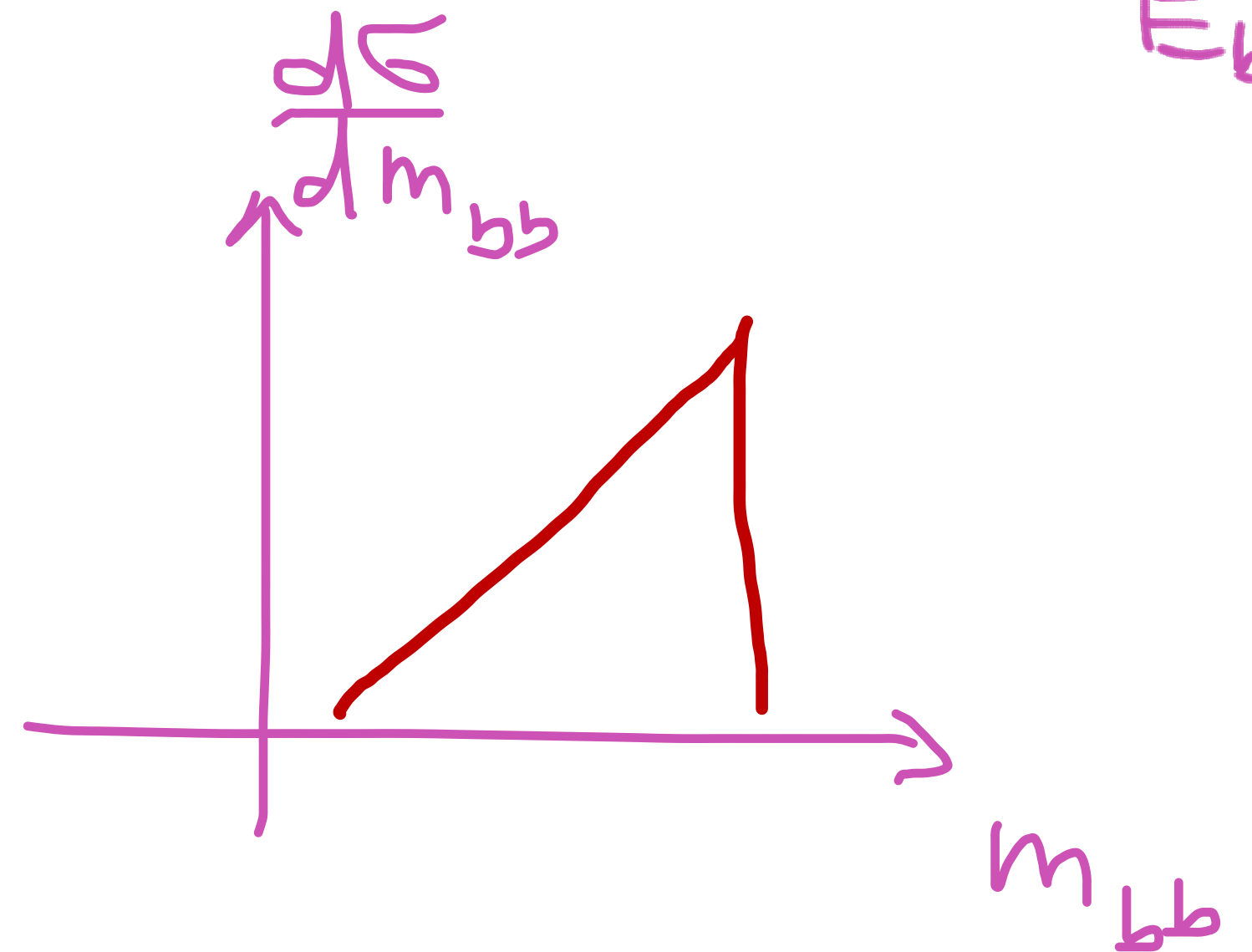
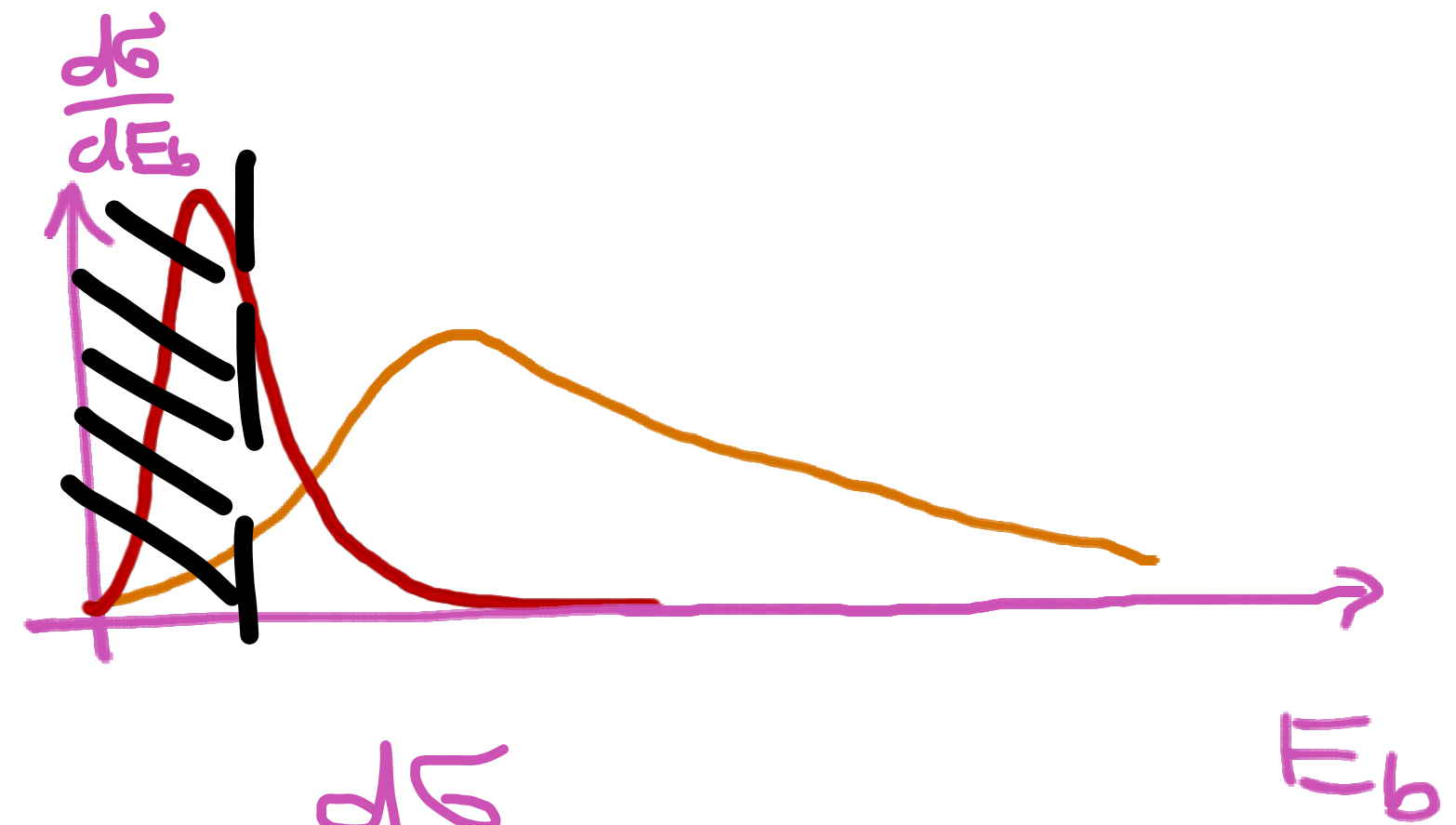
NO COMBINATORICAL ISSUES
JUST LOOK AT $\frac{d\sigma}{dE_b}$

$$pp \rightarrow \tilde{g}\tilde{g} \rightarrow b\tilde{b}b\tilde{b} \rightarrow 4b \cancel{E_T}$$

$$E_{\tilde{b}_H} = \frac{m_{\tilde{b}}^2 - m_x^2}{2m_{\tilde{b}}}$$

$$E_{\tilde{b}_L} = \frac{m_{\tilde{g}}^2 - m_{\tilde{b}}^2}{2m_{\tilde{g}}}$$

$$m_{bb}^{\max} = \sqrt{4 \frac{m_x}{m_{\tilde{b}}} E_{\tilde{b}_H} E_{\tilde{b}_L}}$$

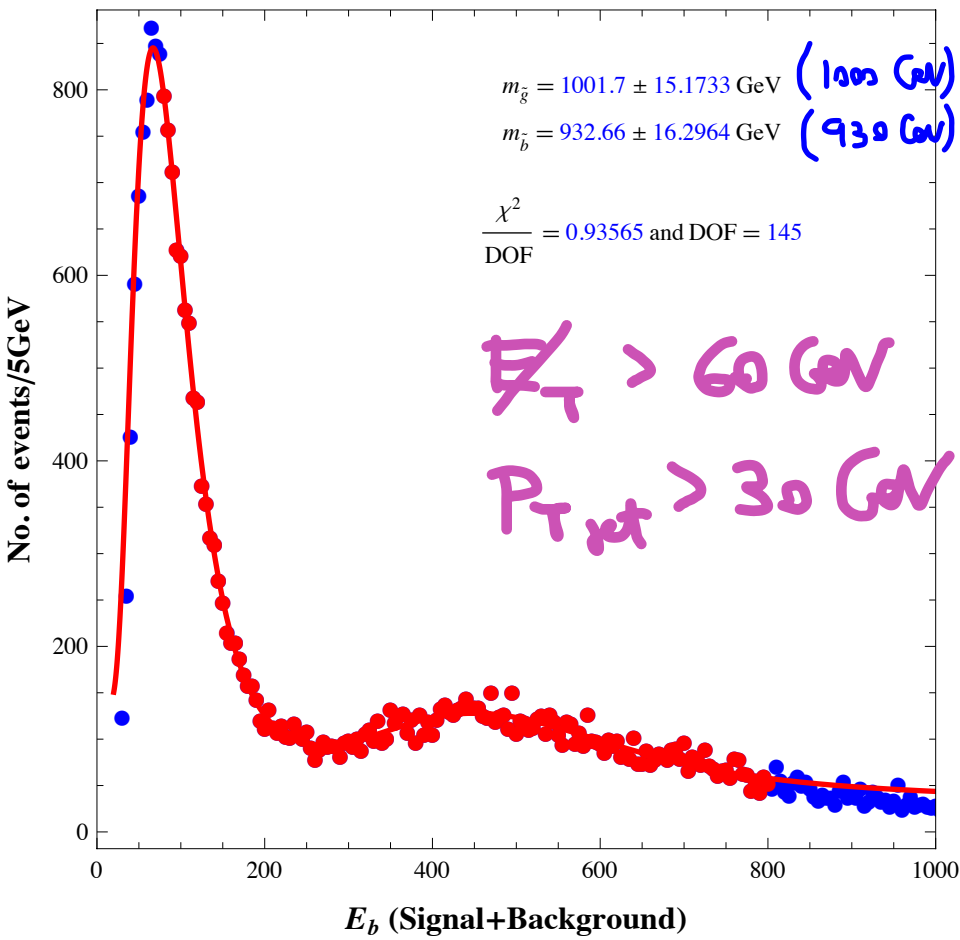


- BACKGROUNDS $Z + \text{jets}$ (mostly $Z b\bar{b}b\bar{b}$) & $t\bar{t}b\bar{b}$ (subdom)
- CUTS MAY AFFECT THE ENERGY DISTRIBUTION

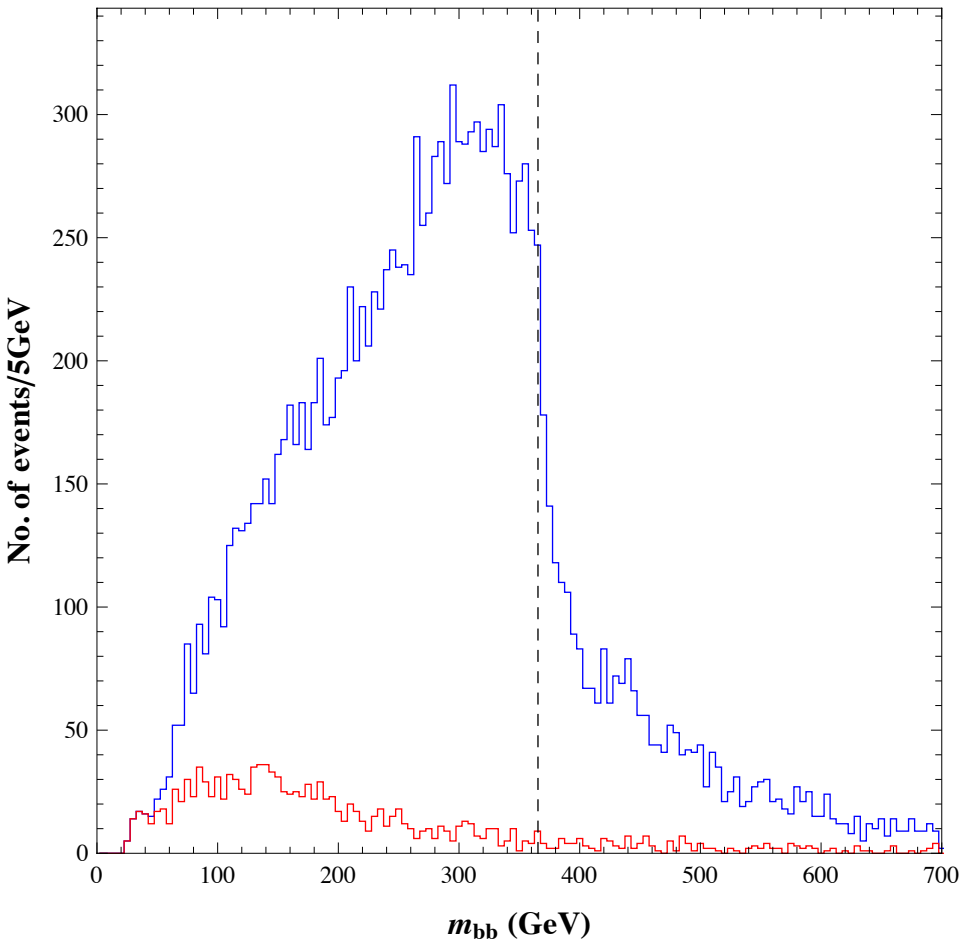
$$p_{T,\text{jet}} > x \Rightarrow E_{\text{jet}} > x$$



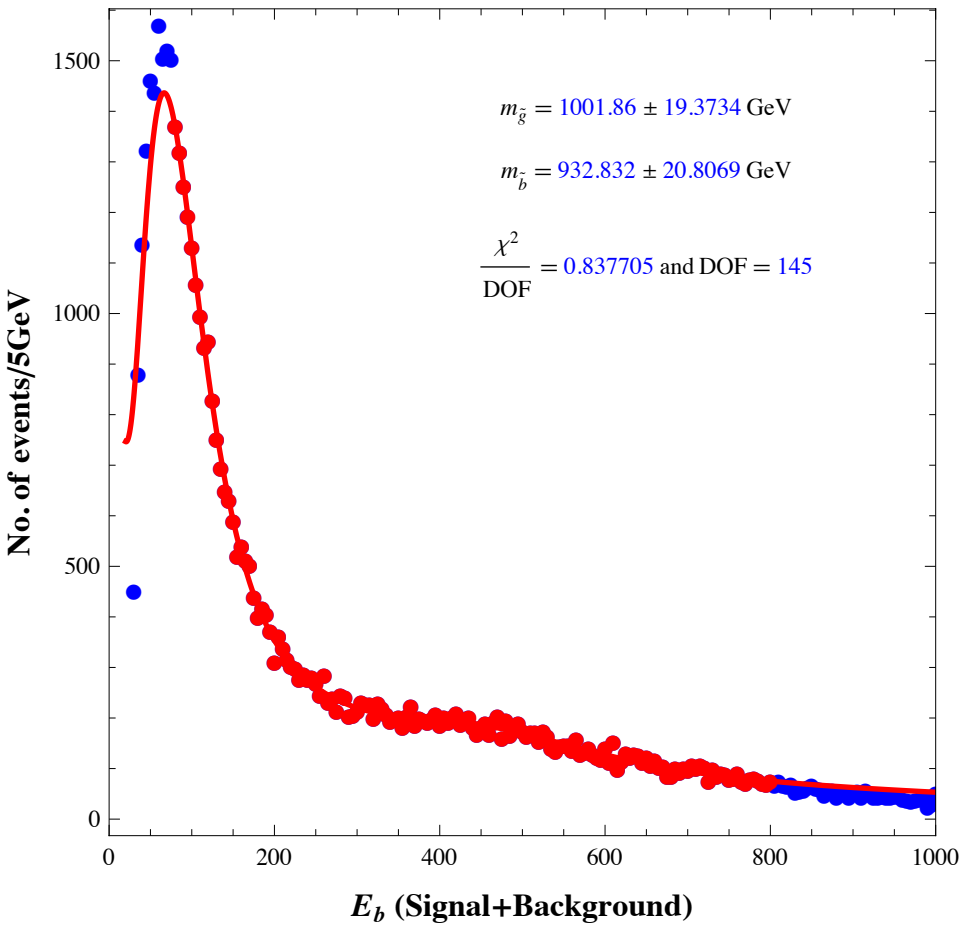
CASE I (S/B=10)



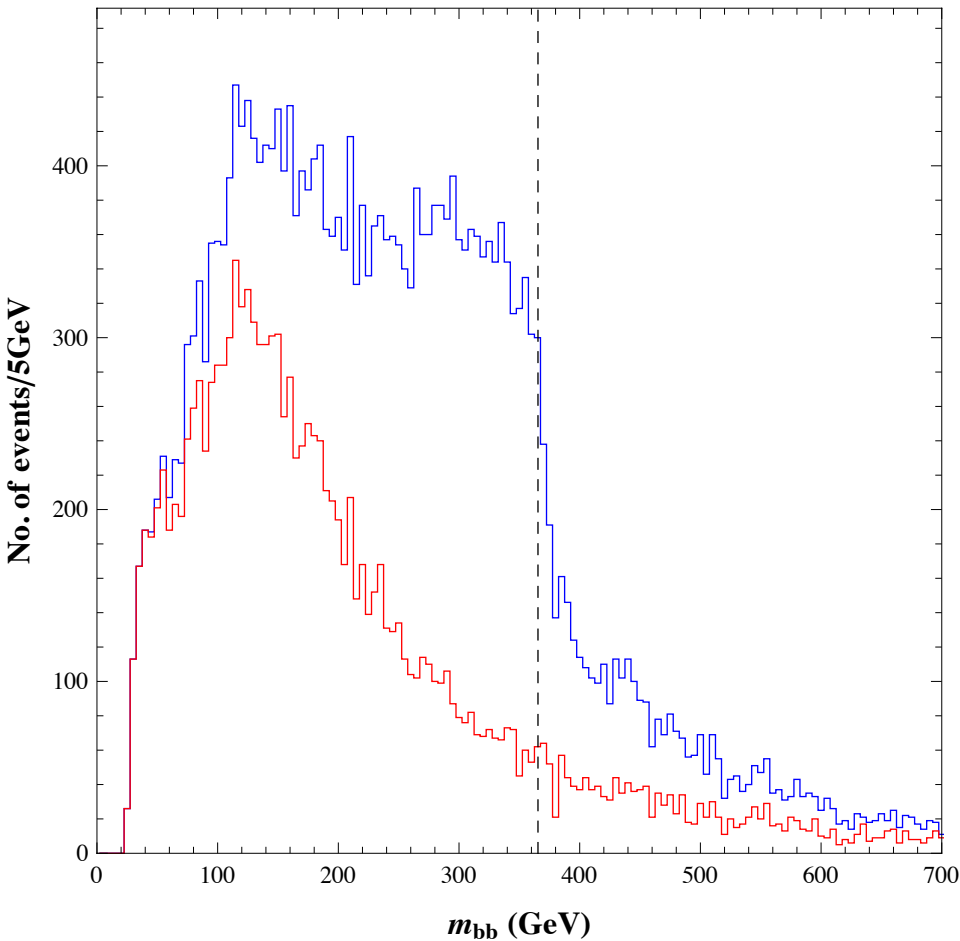
CASE I (S/B=10)



CASE I (S/B=1)



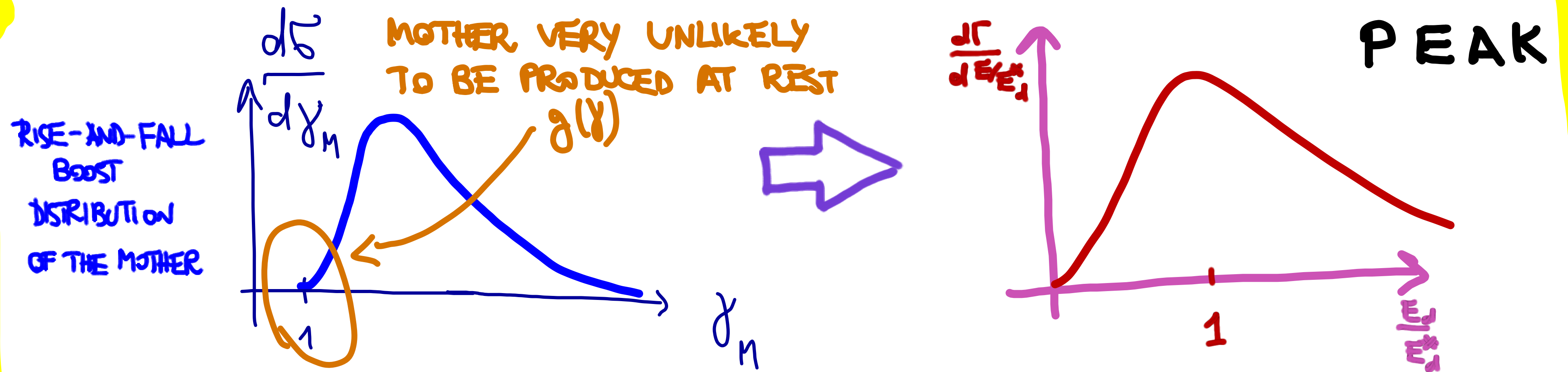
CASE I (S/B=1)



Conclusions

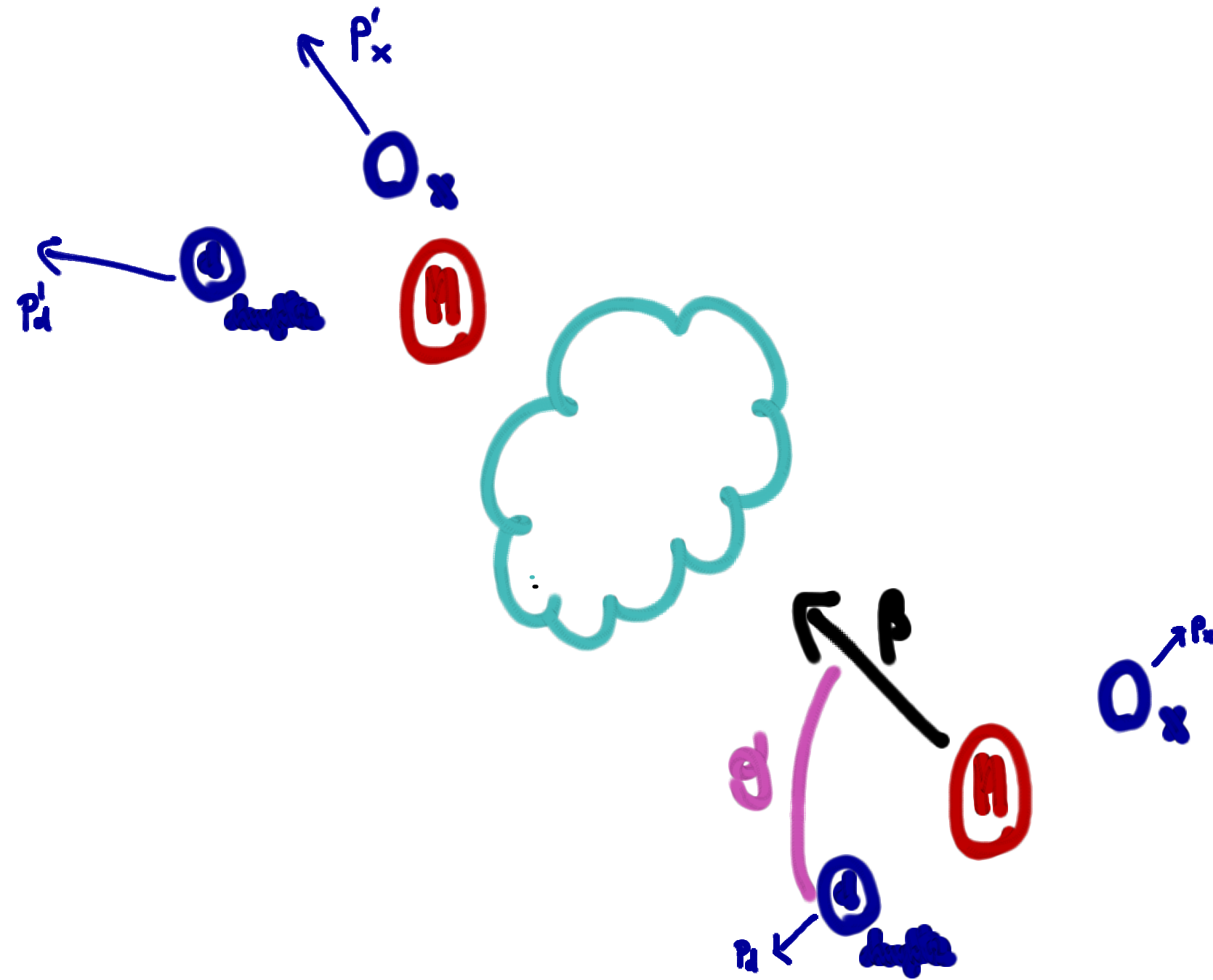
IN PHENOMENOLOGICALLY RELEVANT CASES (HIGH ENERGY COLLIDERS)

THE SPECTRUM OF ENERGY IN TWO BODY DECAYS ENCODES IN A SIMPLE WAY AN INVARIANT OF THE TWO BODY DECAY KINEMATICS



KINKS OR PLATEAUS ARE POSSIBLE AS WELL

Conclusions



THE PEAK OF THE ENERGY DISTRIBUTION IS ROBUST
FOR MASSLESS AND MASSIVE DAUGHTERS

$$E_{\text{peak}} = \frac{m_H^2 - m_x^2}{2m_H}$$

$$E_{\text{peak}} \geq \frac{m_H^2 - m_x^2 + m_d^2}{2m_H}$$

LIMITING FACTORS:

- RADIATIVE CORRECTIONS

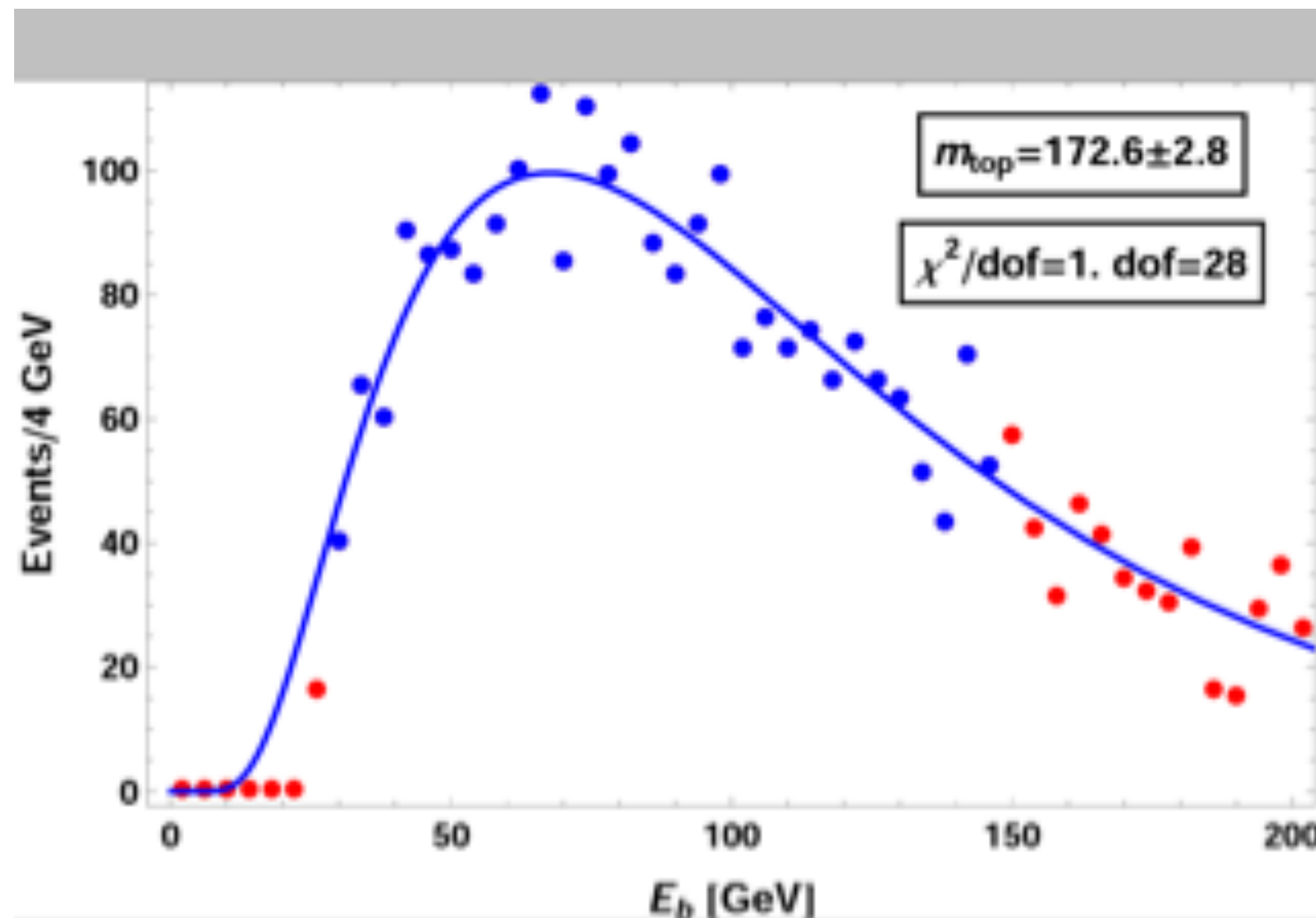
EXTRA RADIATION MAKES THE DECAY 3-BODY

- TOO LARGE MASS OF THE OBSERVED DAUGHTER

- MAY BE SENSITIVE TO SELECTION CUTS

DESPITE THESE LIMITATIONS THE OBSERVATION CAN BE USED TO
MEASURE PARTICLE MASSES WITH 10% ACCURACY OR BETTER

$t \rightarrow b \ell \nu$ in $pp \rightarrow t\bar{t} \Rightarrow \underline{m_{\text{top}}}$ FROM $d\sigma/dE_b$

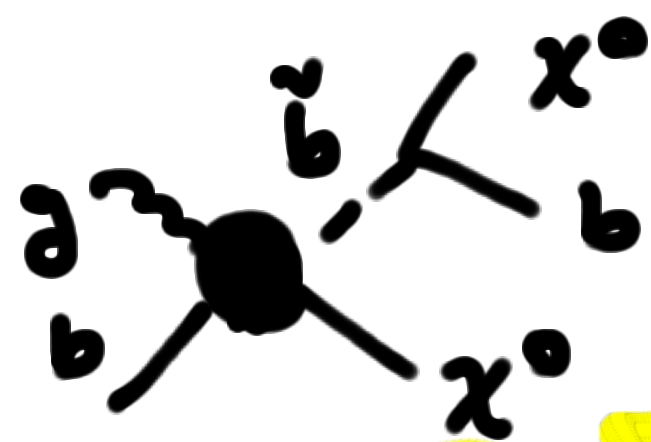
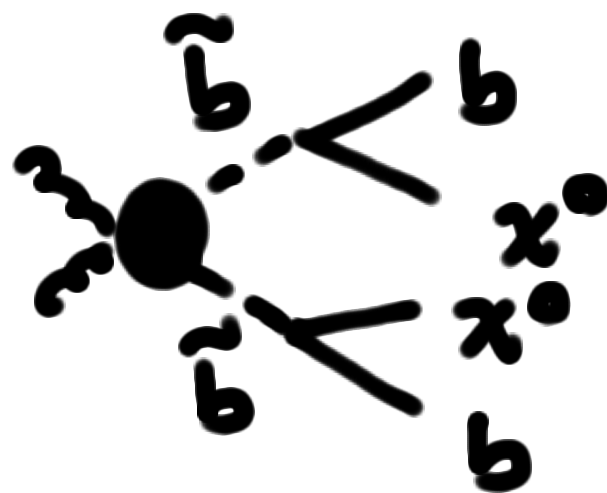


$g \rightarrow b\bar{b} \rightarrow b\bar{b}\chi$
IN PROGRESS

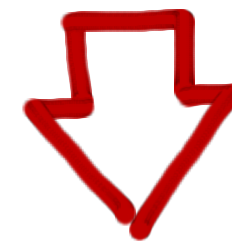
"LOCAL"

SIMPLE

- NO NEED TO KNOW ANYTHING ABOUT THE REST OF THE EVENT

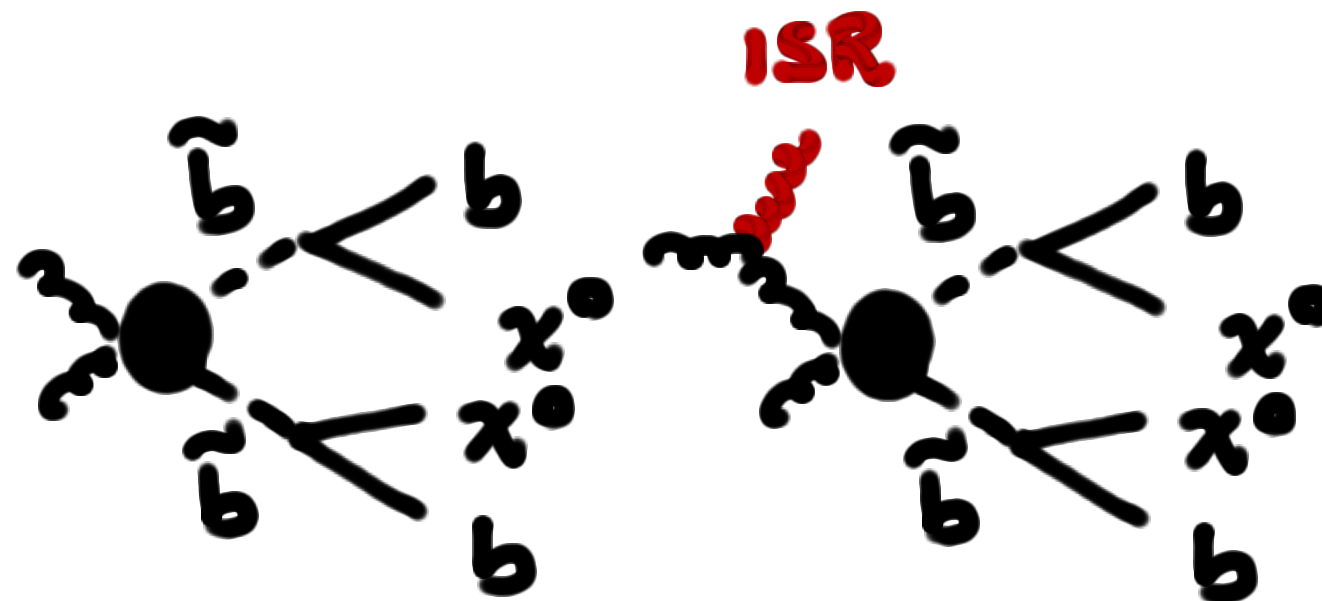


- NO NEED TO MEASURE THE OTHER DECAY PRODUCT



$$\begin{aligned}\tilde{b} &\rightarrow b \chi^0 \\ W &\rightarrow l \nu \\ t &\rightarrow b W \rightarrow b l \nu\end{aligned}$$

- NO NEED TO KNOW ANYTHING ABOUT THE REST OF THE EVENT



ROBUST

Also, since

$$\log E_\gamma = \frac{1}{2}(\log E_{\gamma, \min} + \log E_{\gamma, \max}) = \log \mu \quad (1-225)$$

it follows that, on logarithmic plots of the energy spectra of these γ -rays, the rest-system energy μ will lie halfway between the extremum energies.

We are particularly concerned with decays that are isotropic in the rest system of the decaying particle, such as the π^0 and Σ^0 decays, which we have previously considered. For these decays, we have already shown that the resultant γ -ray energy distribution function is only a function of the momentum of the primary; indeed this function is a constant which is inversely proportional to this momentum for a given primary, within a range proportional to the momentum of the primary, and vanishes outside this range. Thus, for decays of parent particles with a wide range of primary energies, γ -ray spectra are generated which are made up of a superposition of rectangular spectra, as shown in figure 1-11. Higher energy primaries produce the γ -rays at the extremes of the spectrum. We therefore deduce a second important kinematic property, which holds for two-body decays that produce γ -rays isotropically in the rest system of the decaying primary; viz,

The energy spectra of γ -rays produced isotropically in the rest system of the decaying primary will be symmetric on a logarithmic plot with respect to $E_\gamma = \mu$ and will peak at $E_\gamma = \mu$.

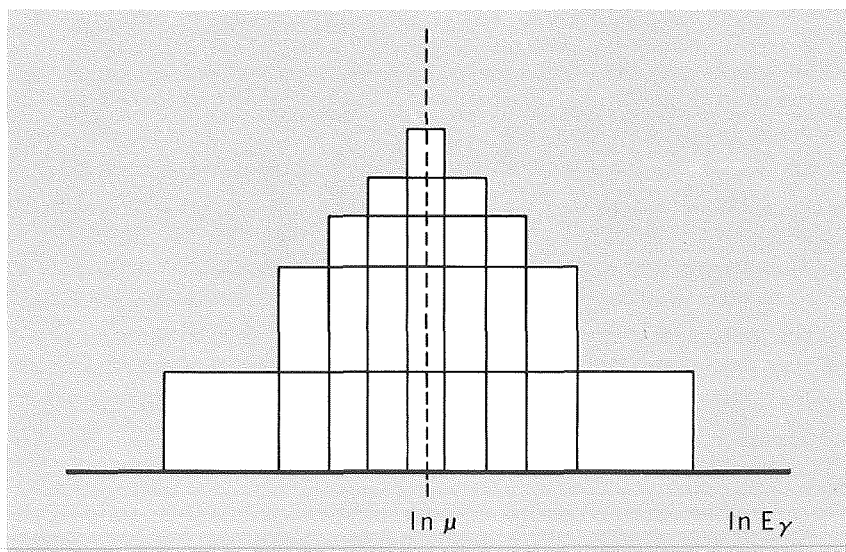


FIGURE 1-11.—Ideal superposition of γ -ray energy spectra from π^0 or Σ^0 particles having discrete values of energy.