Heavy Quark Threshold Physics

Maximilian Stahlhofen

Outline

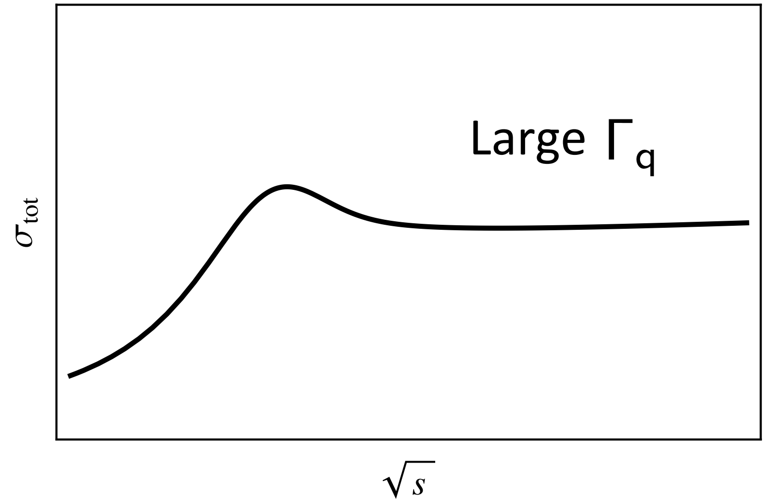
- Introduction
- Theory
  - Coulomb singularities
  - Large logarithms
  - vNRQCD
  - New results
- Applications
  - Top-antitop threshold @ ILC
  - Bottom mass determination
- Summary

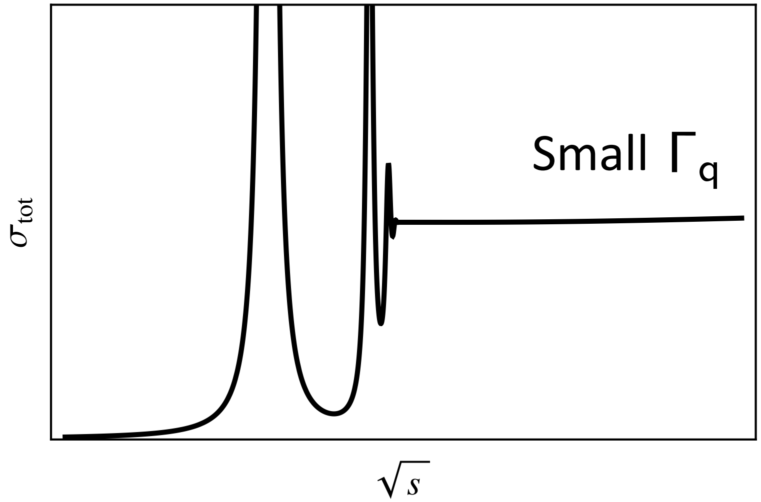
Introduction

Heavy quark pair production @ lepton colliders:

$e^+e^- \to t\,\bar{t}$ $e^+e^- \to b\,\bar{b}$

σ_{tot} in the threshold region (qualitative):





Introduction

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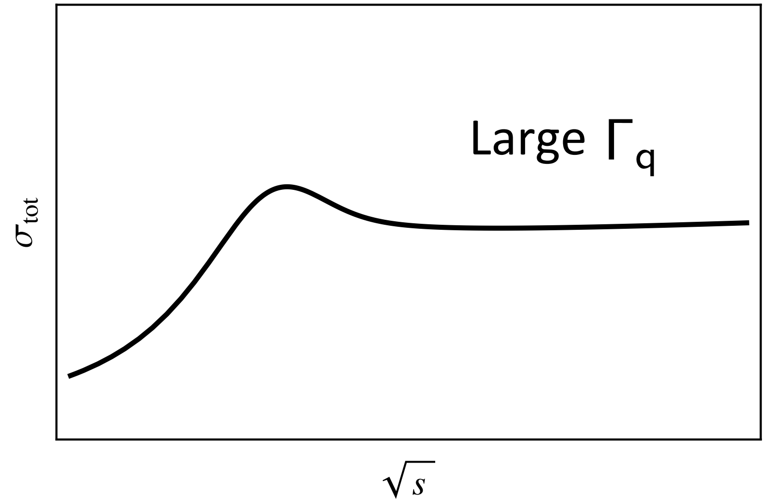

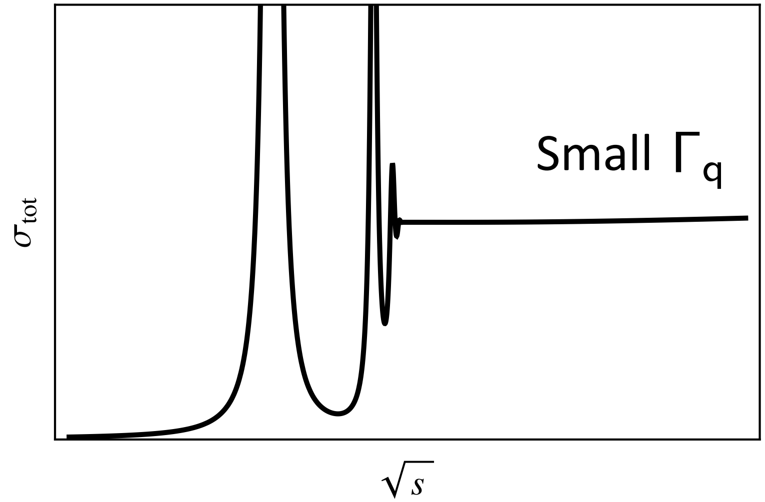

heavy quark velocity (CMS): $v = \sqrt{\frac{\sqrt{s} - 2m}{m}} \sim \alpha_s \ll 1$

(neglect nonperturbative effects for the moment)

Introduction

Motivation to study heavy quark threshold/resonances:

- $e^+e^- \longrightarrow t\,\bar{t}$ (threshold production @ ILC) $\longrightarrow$ $m_t, y_t, \alpha_s, \Gamma_t$
- $e^+e^- \longrightarrow b\,\bar{b}$ (Υ sum rules) $\longrightarrow$ m_b
- $e^+e^- \longrightarrow t\,\bar{t}H$ (ILC phase 1) $\longrightarrow$ y_t
- $e^+e^- \longrightarrow \tilde{t}\,\bar{\tilde{t}}$ (BSM @ ILC ?) $\longrightarrow$?
- Heavy Quarkonium: spectrum, decays

Introduction

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- Heavy Quarkonium: spectrum, decays

Theory

Heavy quark threshold: $v \sim \alpha_s \ll 1$ "nonrelativistic bound state"

multiscale problem:

$$m \gg \vec{p} \sim mv \gg E_{kin} \sim mv^2 \quad (\gg \Lambda_{QCD})$$

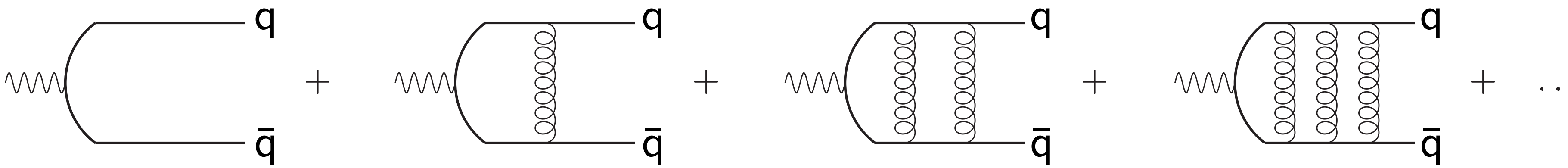
hard soft ultrasoft

- "Coulomb singularities" $\sim (\alpha_s/v)^n$
- Large logarithms $\sim [\alpha_s \ln(v)]^n$

⇒ Resummation using Effective Field Theory

Theory

Problem of Coulomb singularities:

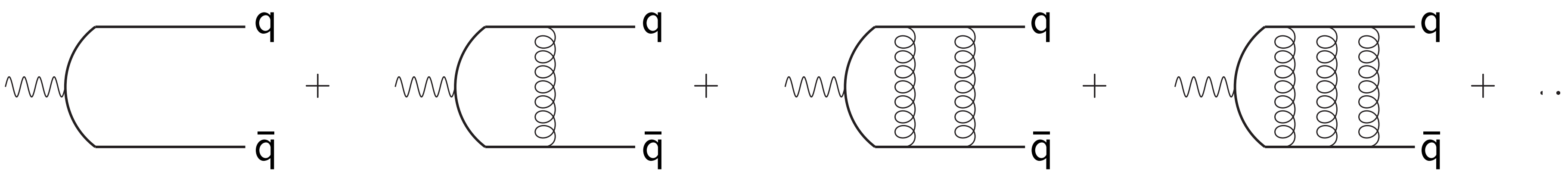


~ 1 $\quad\sim \alpha_s/v$ $\quad\sim (\alpha_s/v)^2$ $\quad\sim (\alpha_s/v)^3$

@Threshold: $\boxed{v \sim \alpha_s \ll 1}$ $\Rightarrow$ breakdown of perturbation theory

Theory

Problem of Coulomb singularities:



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@Threshold: $v \sim \alpha_s \ll 1$ $\Rightarrow$ breakdown of perturbation theory

Solution:

Nonrelativistic EFT: **vNRQCD**

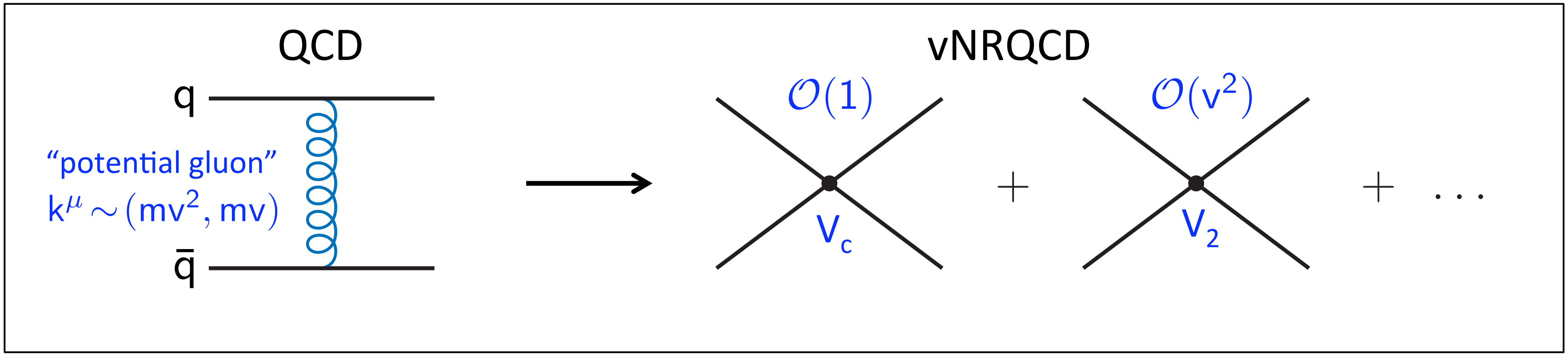
→ Use Schrödinger Equation to resum $(\alpha_s/v)^n$ terms !

Theory

Constructing vNRQCD:

"Expansion in $v \ll 1$"

- Integrate out nonresonant degrees of freedom, e.g.:



$$\Rightarrow \mathcal{L}_{NR} = \mathcal{L}_{kin} + \left[\frac{\mathcal{V}_c}{\mathbf{k}^2} + \frac{\mathcal{V}_k \pi^2}{m\mathbf{k}} + \frac{\mathcal{V}_r(\mathbf{p}^2+\mathbf{p}'^2)}{2m^2\mathbf{k}^2} + \frac{\mathcal{V}_2}{m^2} + \frac{\mathcal{V}_s}{m^2}\mathbf{S}^2 + \ldots \right] \psi^\dagger\psi\,\chi^\dagger\chi + \ldots$$

Theory

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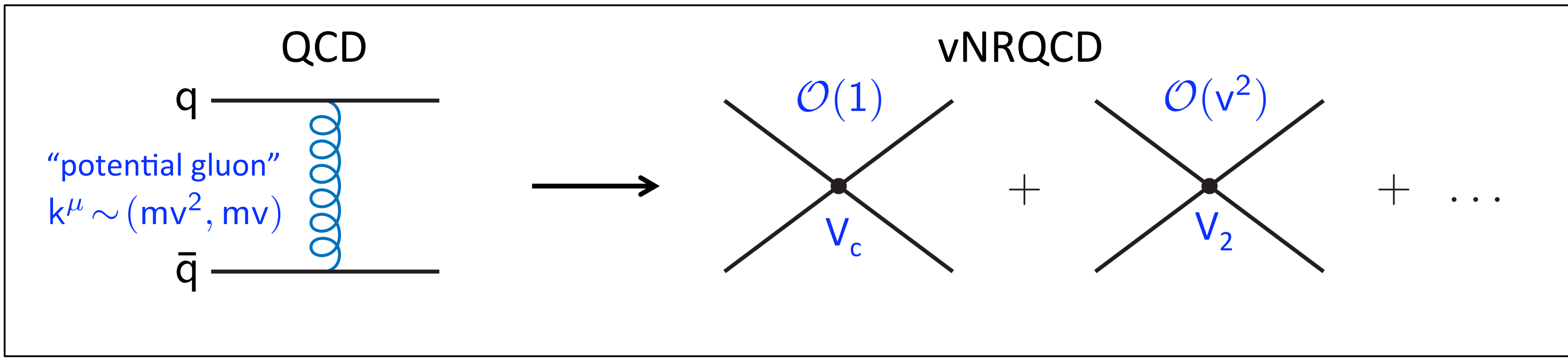

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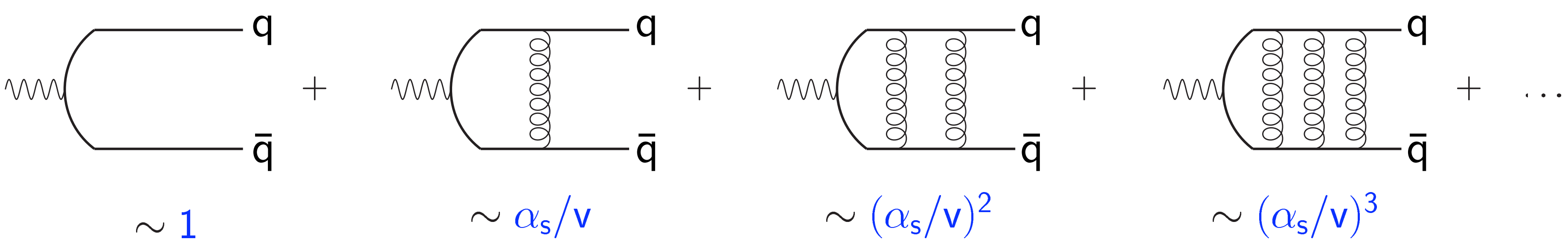
- Separate center-of-mass motion

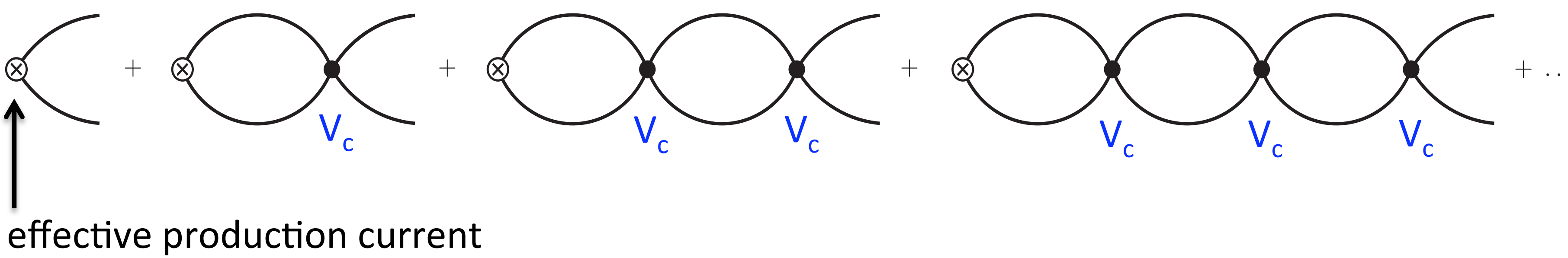
$\Rightarrow$ Schrödinger equation: $$E\,\Psi = \Big[\frac{\mathbf{k}^2}{m} + V + \ldots \Big] \Psi$$

Theory

QCD

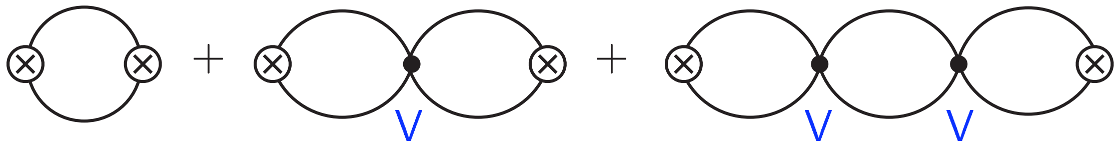


vNRQCD (leading order)

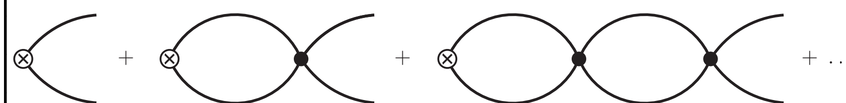


Theory

Green function: $\left[-\frac{\nabla_{\vec{r}}^2}{m} + V(r) - E \right] G(\vec{r}, \vec{r}\,', E) = \delta^{(3)}(\vec{r} - \vec{r}\,')$

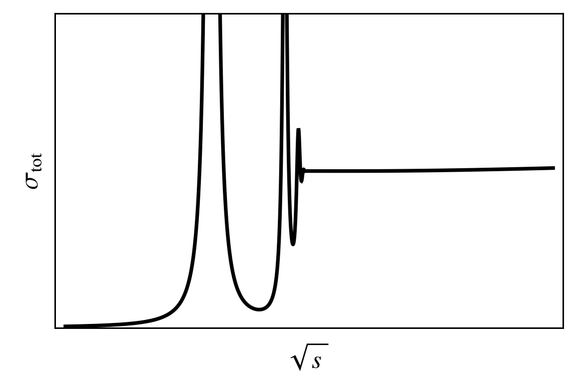
$G(0,0,E) \sim$  $+ \dots$

Cross section:

$\sigma_{tot} \sim \int dPS \left| \right.$  $\left. \right|^2 \sim \mathrm{Im}\left[G(0,0,E)\right]$

Unstable quark:

$$G(0,0,E+i\Gamma_q) \sim \sum_n \frac{|\Psi_n(0)|^2}{E_n - E - i\Gamma_q} + \text{continuum}$$



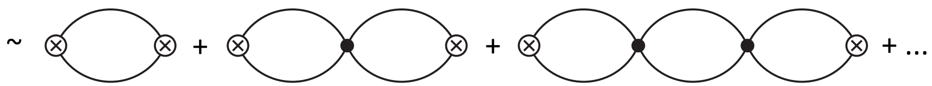
Theory

Leading order:

$$\left[-\frac{\nabla^2_{\vec{r}}}{m} + V_c(r) - E\right] G^{LO}(\vec{r}, \vec{r}\,', E) = \delta^{(3)}(\vec{r} - \vec{r}\,')$$

Analytic solution:

$$G^{LL}(0,0,E,\nu) = \frac{m^2}{4\pi}\left\{iv - C_F\alpha_s\left[\ln\left(\frac{-iv}{\nu}\right) - \frac{1}{2} + \ln 2 + \gamma_E + \Psi\left(1 - \frac{iC_F\alpha_s}{2v}\right)\right]\right\} + \frac{m^2 C_F \alpha_s}{16\pi\epsilon}$$

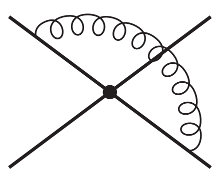


Higher orders:

$$\delta G^{NLO}(0,0,E) = -\int d^3\vec{r}\; G^{LO}(0,\vec{r},E)\, \delta V^{NLO}(\vec{r})\, G^{LO}(\vec{r},0,E)$$

↑ $\mathcal{O}(\alpha_s\,, v)$

Renormalization:



$$\Rightarrow\; V \equiv V(\mu) \;\Rightarrow\; G(0,0,E) \equiv G(0,0,E,\mu)$$

⇒ G^{NNLL} known ✓ [Hoang, Manohar, Stewart, Teubner; 2002] [Pineda, Signer; 2006]

G^{N^3LO} known ✓ [Beneke, Kiyo, Schuller; 2007]

Theory

Problem of large logarithms:

$$\mathrm{m} \gg \vec{\mathrm{p}} \sim \mathrm{mv} \gg \mathrm{E_{kin}} \sim \mathrm{mv}^2$$

hard soft ultrasoft

$$\Rightarrow \quad \alpha_s \ln(\mathrm{E}^2/\mathrm{m}^2),\ \alpha_s \ln(\mathbf{p}^2/\mathrm{m}^2),\ \alpha_s \ln(\mathrm{E}^2/\mathbf{p}^2) \sim \alpha_s \ln \mathrm{v} \sim 1$$

Theory

Problem of large logarithms:

$$m \gg \vec{p} \sim mv \gg E_{kin} \sim mv^2$$

hard soft ultrasoft

$\Rightarrow$ $\alpha_s \ln(E^2/m^2),\ \alpha_s \ln(\mathbf{p}^2/m^2),\ \alpha_s \ln(E^2/\mathbf{p}^2) \sim \alpha_s \ln v \sim 1$

Solution:

Two renormalization scales: $\mu_s = m\nu,\ \mu_u = m\nu^2$ → "v"NRQCD

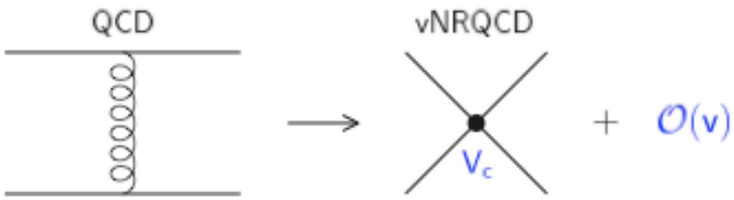
ν "subtraction velocity"

→ RGE's resum $[\alpha_s \ln v]^n$, $\alpha_s [\alpha_s \ln v]^n$, $\alpha_s^2 [\alpha_s \ln v]^n$ ... terms

LL NLL NNLL

Theory

- Nonresonant dof's integrated out, e.g.:



- Resonant dof's $\rightarrow$ fields in the vNRQCD Lagrangian:

nonrel. quark:	$(E, \mathbf{p}) \sim (mv^2, mv)$	$\psi_{\mathbf{p}}(x)$	
soft gluon:	$(q_0, \mathbf{q}) \sim (mv, mv)$	$A_q(x)$	
ultrasoft gluon:	$(q_0, \mathbf{q}) \sim (mv^2, mv^2)$	$A(x)$	

- Systematic expansion in v $\Rightarrow$ consistent power counting in $v \sim \alpha_s$

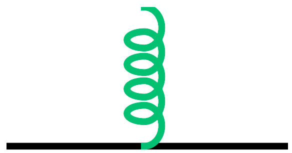
Theory

[Luke, Manohar, Rothstein, '00]

$$\boxed{\mathcal{L}_{\mathrm{vNRQCD}} = \mathcal{L}_{\mathrm{usoft}} + \mathcal{L}_{\mathrm{pot}} + \mathcal{L}_{\mathrm{soft}}}$$

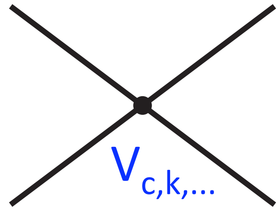
$$D^\mu = \partial^\mu + igA^\mu(x)$$

$$\mathcal{L}_{\mathrm{usoft}} : \quad \psi^\dagger_{\mathbf{p}}(x)\left[iD^0 - \frac{(\mathbf{p}-i\mathbf{D})^2}{2m} + \ldots \right]\psi_{\mathbf{p}}(x) + \ldots$$

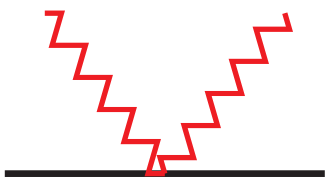


$$\mathcal{L}_{\mathrm{pot}} : \quad -\, V \;\psi^\dagger_{\mathbf{p}'}\psi_{\mathbf{p}}\chi^\dagger_{-\mathbf{p}'}\chi_{-\mathbf{p}} + \ldots$$

$$V \sim \frac{\mathcal{V}_c}{\mathbf{k}^2} + \frac{\mathcal{V}_k \pi^2}{m\mathbf{k}} + \frac{\mathcal{V}_r(\mathbf{p}^2+\mathbf{p}'^2)}{2m^2\mathbf{k}^2} + \frac{\mathcal{V}_2}{m^2} + \frac{\mathcal{V}_s}{m^2}\mathbf{S}^2 + \ldots$$



$$\mathcal{L}_{\mathrm{soft}} :$$



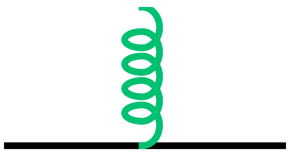
Theory

[Luke, Manohar, Rothstein, '00]

$$\boxed{\mathcal{L}_{\rm vNRQCD} = \mathcal{L}_{\rm usoft} + \mathcal{L}_{\rm pot} + \mathcal{L}_{\rm soft}}$$

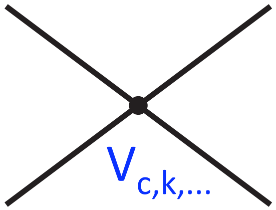
$$D^\mu = \partial^\mu + igA^\mu(x)$$

$$\mathcal{L}_{\rm usoft}: \quad \psi^\dagger_{\mathbf{p}}(x)\left[iD^0 - \frac{(\mathbf{p}-i\mathbf{D})^2}{2m} + \ldots \right]\psi_{\mathbf{p}}(x) + \ldots$$

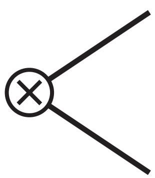


$$\mathcal{L}_{\rm pot}: \quad -V \;\psi^\dagger_{\mathbf{p}'}\psi_{\mathbf{p}}\chi^\dagger_{-\mathbf{p}'}\chi_{-\mathbf{p}} + \ldots$$

$$V \sim \frac{\mathcal{V}_c}{\mathbf{k}^2} + \frac{\mathcal{V}_k \pi^2}{m\mathbf{k}} + \frac{\mathcal{V}_r(\mathbf{p}^2+\mathbf{p}'^2)}{2m^2\mathbf{k}^2} + \frac{\mathcal{V}_2}{m^2} + \frac{\mathcal{V}_s}{m^2}\mathbf{S}^2 + \ldots$$



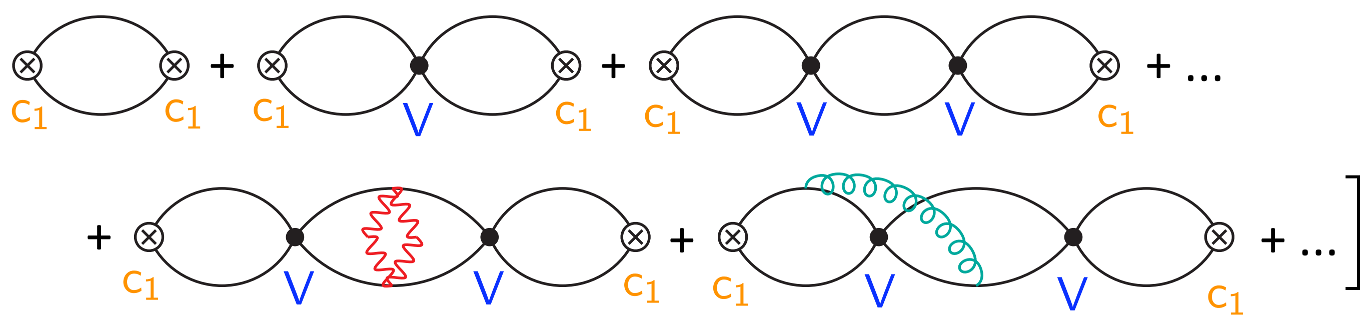
Production/annihilation current ($^3S_1$):



$$\sim \quad c_1(\nu)\cdot \underbrace{\vec{j}_1^{\,\rm eff}(x)}_{\psi^\dagger_{\mathbf{p}}\vec{\sigma}(i\sigma_2)\chi^*_{-\mathbf{p}}} + \ldots \qquad \text{(CMS)}$$

Theory

$\sigma_{\text{tot}} \sim \mathrm{Im}\Big[$



$\Big]$

$$\sim \mathrm{Im}\Big[c_1(\nu)^2 \cdot G(0,0,E,\nu) \Big] + \ldots$$

↓

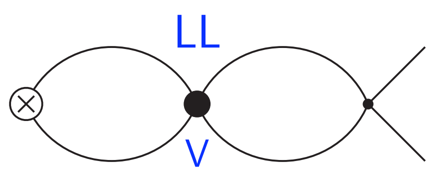
G^{NNLL} known ✓ [Hoang, Manohar, Stewart, Teubner, '02]
[Pineda, Signer, '06]

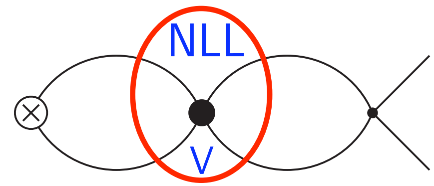
G^{NNNLO} known ✓ [Beneke, Kiyo, Schuller, '07]

Theory

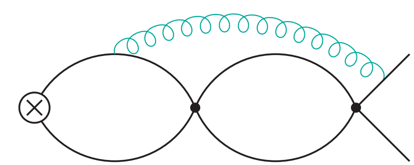
$$\sigma_{\rm tot} \sim \mathrm{Im}\Big[\, c_1(\nu)^2 \cdot G(0,0,E,\nu) \,\Big]$$

current renormalization





$$\ln\left[\frac{c_1(\nu)}{c_1(1)}\right] = \underbrace{\xi^{\rm LL}}_{0} + \xi^{\rm NLL} + \xi^{\rm NNLL}_{\rm mix} + \xi^{\rm NNLL}_{\rm nonmix}$$



[Luke, Manohar, Rothstein, '00]
[Pineda, '02]
[Hoang, Stewart, '03]

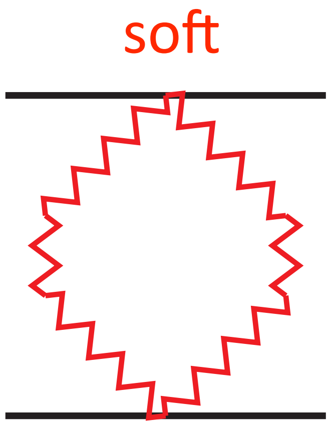
[Hoang, '03]

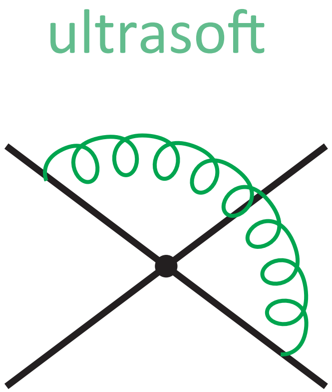
NEW

[Hoang, MS, '11]

Theory

NLL: $$\nu \frac{\partial}{\partial \nu} \ln[c_1(\nu)] = -\frac{\mathcal{V}_c(\nu)}{16\pi^2}\left[\frac{\mathcal{V}_c(\nu)}{4} + \mathcal{V}_2(\nu) + \mathcal{V}_r(\nu) + \mathbf{S}^2\, \mathcal{V}_s(\nu)\right] + \frac{1}{2}\mathcal{V}_k(\nu)$$





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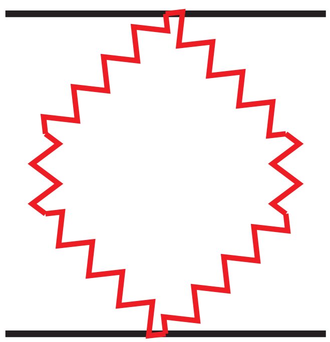
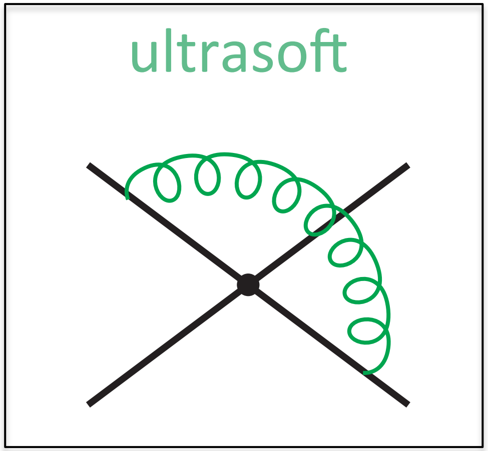
corrections

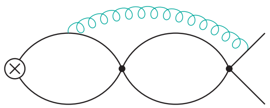
soft



- known soft corrections small



- dominant: $\alpha_s(mv^2) > \alpha_s(mv)$
- large contribution to ξ^{NNLL}_{nonmix}

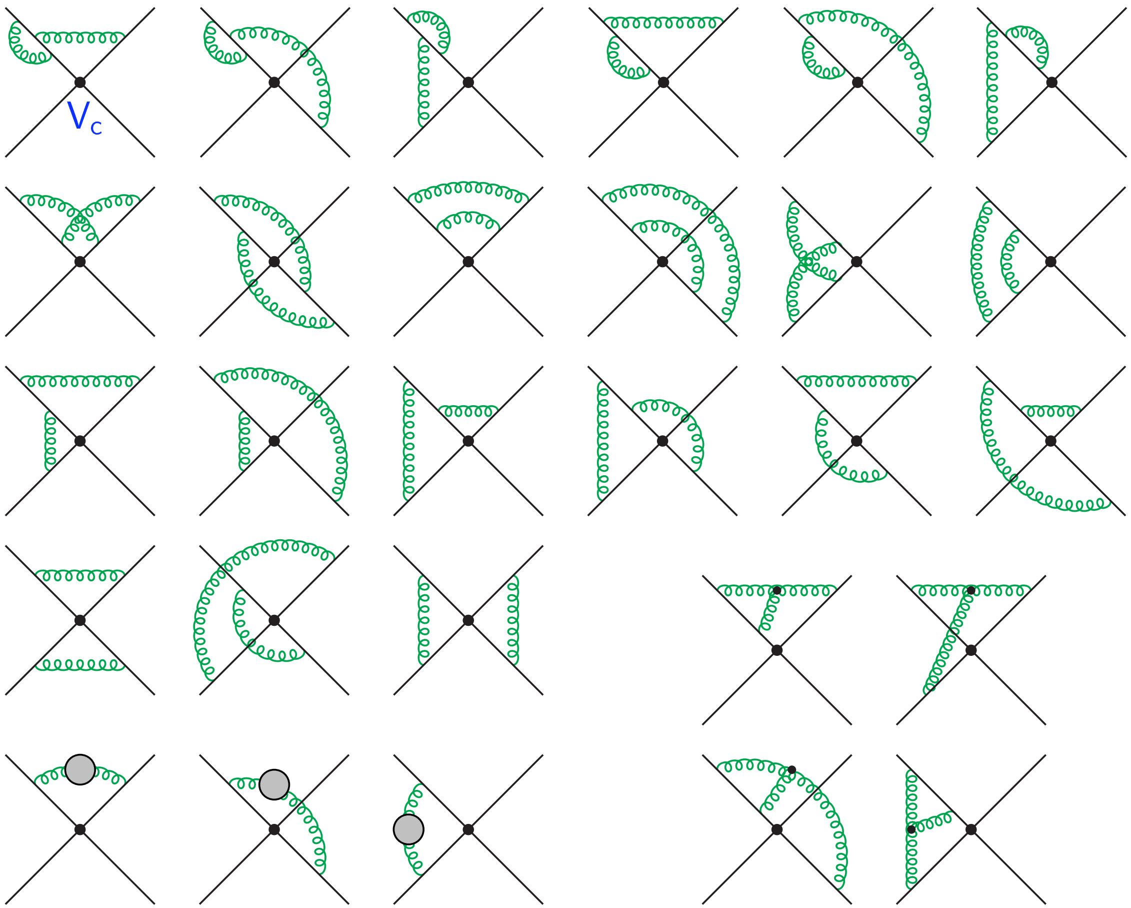


[Hoang, '03]

Theory

NLL: $$\nu \frac{\partial}{\partial \nu} \ln[c_1(\nu)] = -\frac{\mathcal{V}_c(\nu)}{16\pi^2}\left[\frac{\mathcal{V}_c(\nu)}{4} + \mathcal{V}_2(\nu) + \mathcal{V}_r(\nu) + \mathbf{S}^2\,\mathcal{V}_s(\nu)\right] + \frac{1}{2}\mathcal{V}_k(\nu)$$

renormalize



- Feynman gauge
- $\overline{\mathrm{MS}}$, dim. reg.
- $O(10^3)$ diagrams

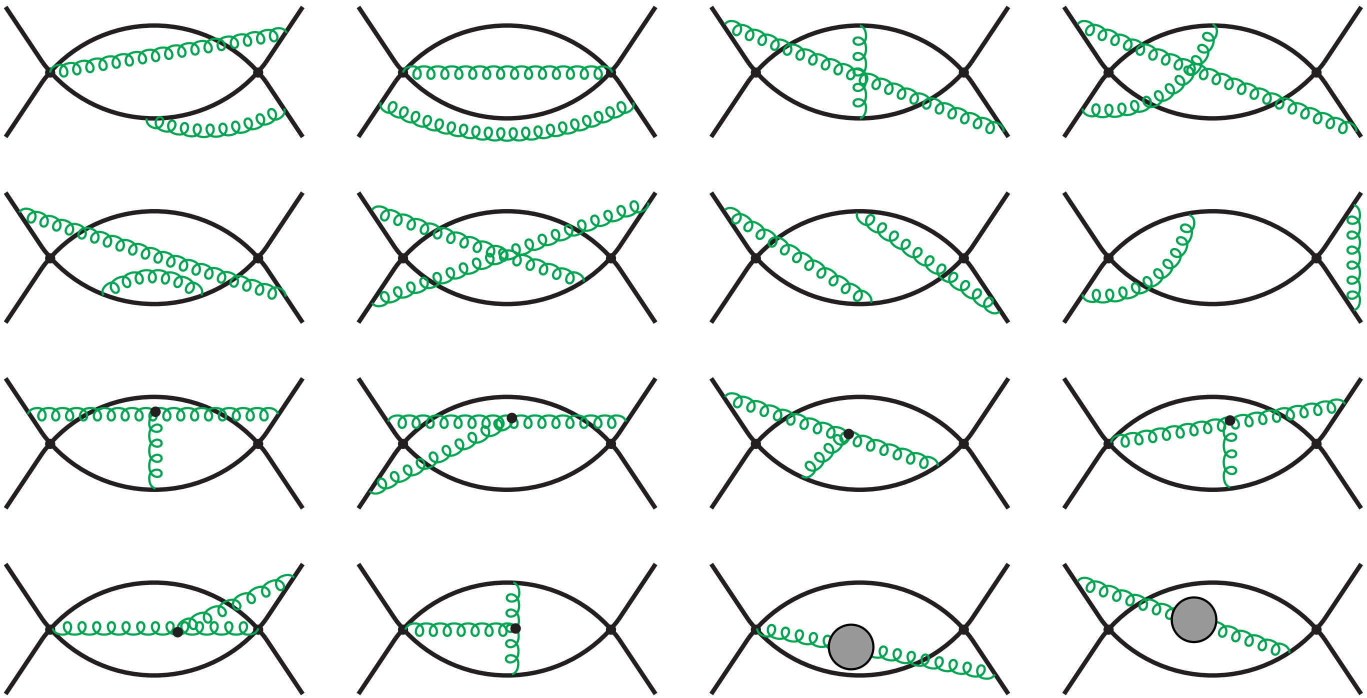
$$\Rightarrow \quad \delta\mathcal{V}_{r,2}^{2\,\text{loop}} \xrightarrow{\text{RGE}} \mathcal{V}_{r,2}^{\text{NLL}}(\nu)$$

[Hoang, MS, '06]
[Pineda, '11]

Theory

NLL: $$\nu \frac{\partial}{\partial \nu} \ln[c_1(\nu)] = -\frac{\mathcal{V}_c(\nu)}{16\pi^2}\left[\frac{\mathcal{V}_c(\nu)}{4} + \mathcal{V}_2(\nu) + \mathcal{V}_r(\nu) + \mathbf{S}^2\, \mathcal{V}_s(\nu)\right] + \frac{1}{2}\mathcal{V}_k(\nu)$$

renormalize



- 3 loops:
  2 x usoft
  1 x potential (finite)
- Feynman gauge
- $\overline{\mathrm{MS}}$, dim. reg.
- $\Leftrightarrow$ $O(10^4)$ 2-loop diagrams
- Generation:
  own **Mathematica** code
- Integrals:
  IBP & partial frac.

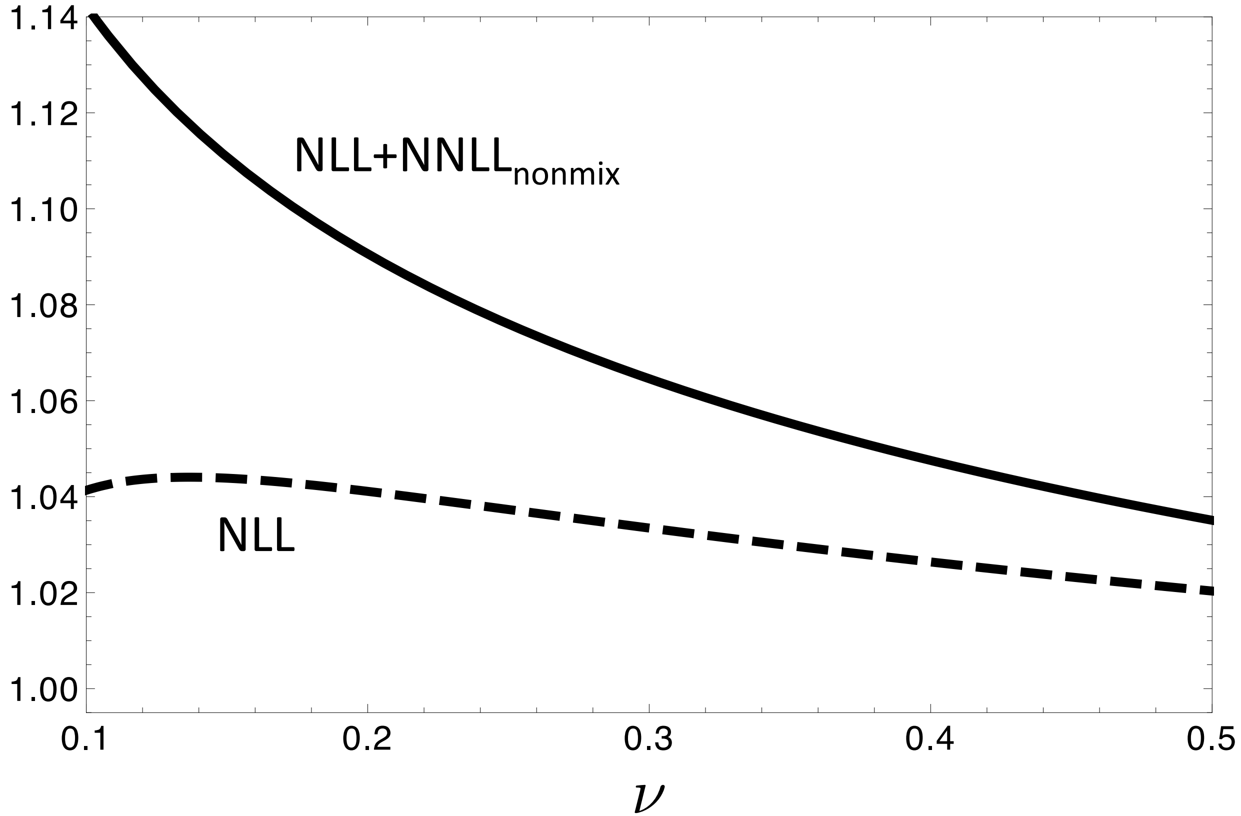
$$\Rightarrow \quad \boxed{\delta \mathcal{V}_k^{2\,\text{loop}} \xrightarrow{\text{RGE}} \mathcal{V}_k^{\text{NLL}}(\nu)}$$

[Hoang, MS, ‘11]
[Pineda, ‘11]

Theory

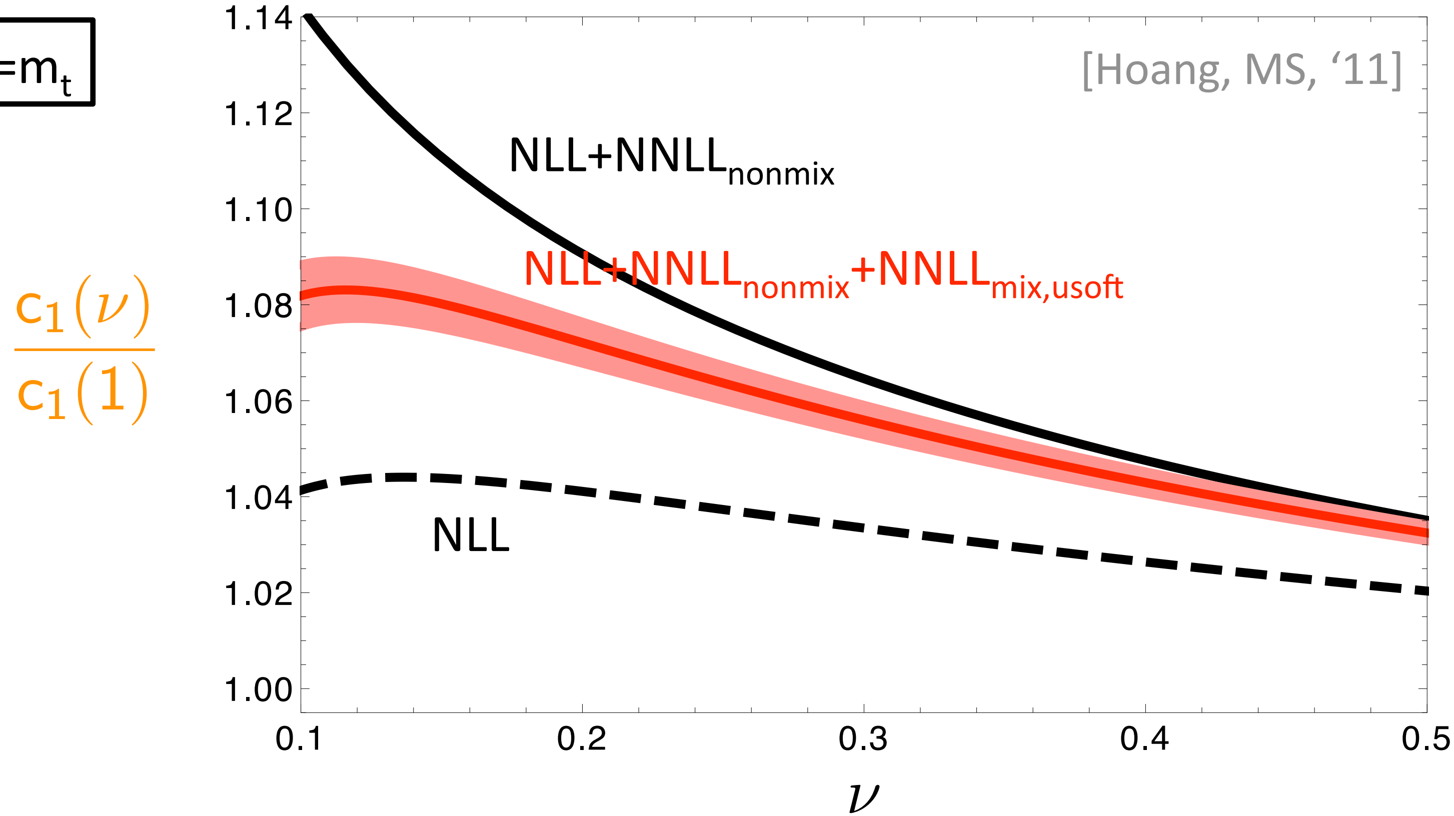
$m = m_t$

$$\frac{c_1(\nu)}{c_1(1)}$$



Theory

$m=m_t$



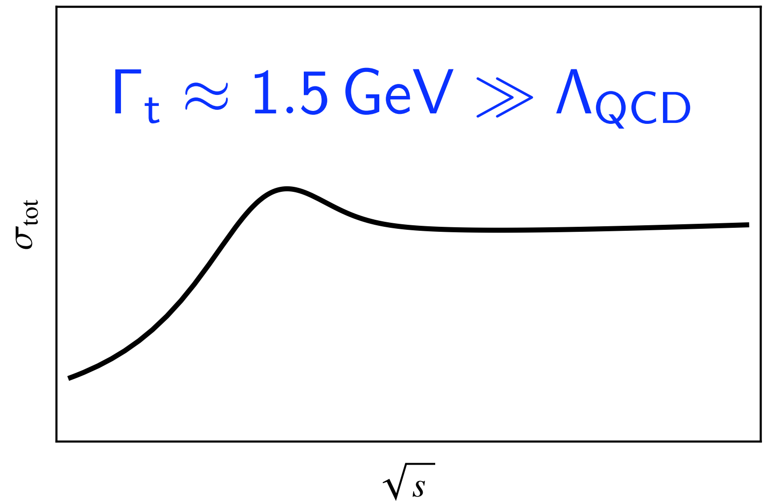
- large ultrasoft NNLL contributions compensate each other
- known soft NNLL contributions are small

$$\boxed{e^+e^- \longrightarrow t\,\bar{t}}$$

Top-antitop threshold @ ILC

Applications

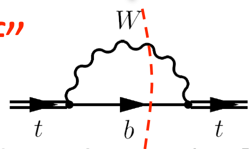
$t\bar{t}$ threshold scan @ ILC:



$$v_{eff} \equiv \sqrt{\frac{\sqrt{s}-2m_t}{m_t}} \to \sqrt{\frac{\sqrt{s}-2m_t+i\Gamma_t}{m_t}}$$

$$|v_{eff}| \gtrsim 0.1$$

"IR cutoff"



[Fadin, Khoze, '87]

⇨ Nonpert. effects suppressed!

- EW effects known up to NNLL

[Hoang, Reisser, Ruiz-Femenia, '10]
[Beneke, Jantzen Ruiz-Femenia, '10]

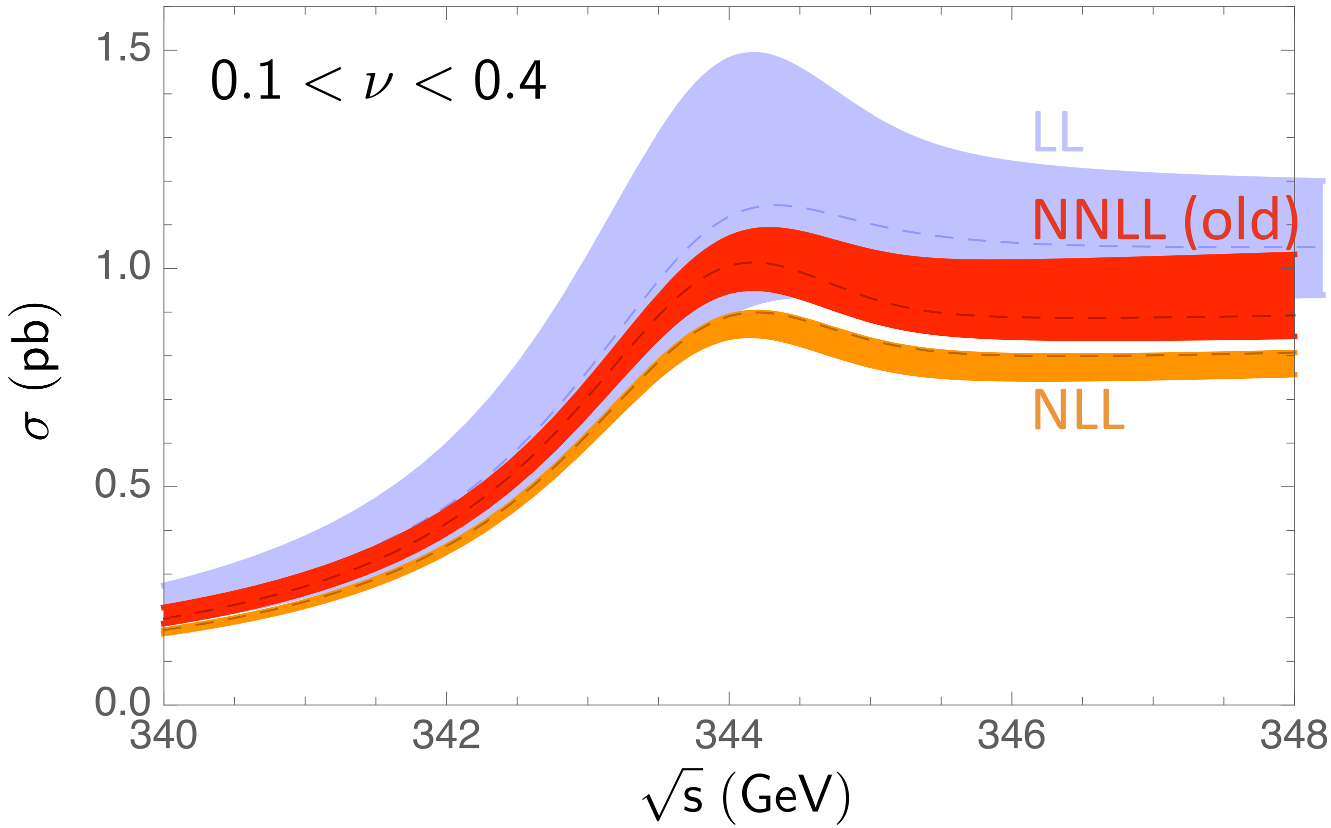
Experiment (simulation):

[Martinez, Miquel, '02]

$$\Delta m_t \sim 20\mathrm{MeV},\ \Delta\Gamma_t \sim 30\mathrm{MeV},\ \Delta\alpha_s \sim 0.0012,\ \Delta y_t/y_t \sim 35\%$$

⇨ **Theory goal: $\Delta\sigma_{tot}/\sigma_{tot} \lesssim 3\%$**

Applications

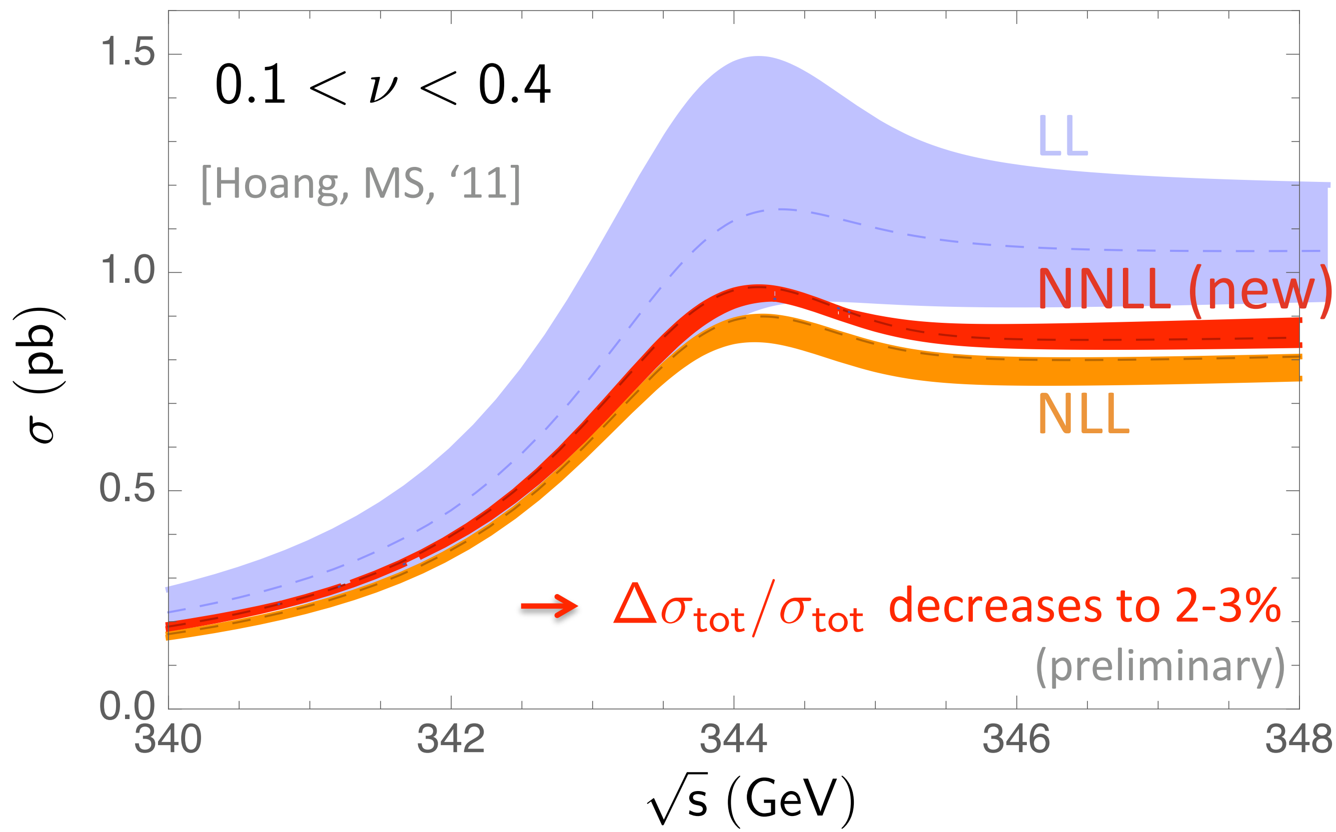


Theory status until 2011:

$\Delta m_t \sim 100\text{MeV}$ ✓

$\Delta\sigma_{tot}/\sigma_{tot} \sim 6\%$ < 3% needed for precise $\Gamma_t,\ y_t,\ \alpha_s$

Applications



- LO EW effects included by $G^{LL}(0, 0, E, \nu) \longrightarrow G^{LL}(0, 0, E + i\Gamma, \nu)$ [Fadin, Khoze, '87]
- Combination with NNLL EW effects and detailed error analysis → W.I.P

Applications

$$\boxed{e^+e^- \longrightarrow b\,\bar{b}}$$

Bottom mass from nonrelativistic sum rules

Applications

m_b from $b\bar{b}$ production near threshold:

R-ratio: $$R_{b\bar{b}}(s) = \frac{\sigma(e^+e^- \to \gamma^* \to \text{“}b\bar{b}\text{”})}{\sigma_{pt}}$$ $\sigma_{pt} = 4\pi\alpha_{em}^2/(3s)$

Moments: $$P_n = \int_0^\infty \frac{ds}{s^{n+1}} R_{b\bar{b}}(s)$$

Sum rule: (hadronic) $$P_n^{exp} = P_n^{th}$$ (partonic) [Novikov, Okun, Shifman, Vainshtein, Voloshin, Zakharov; 1977]

large n (nonrelativistic): $n \gtrsim 4$

[Pineda, Signer, '06] ← NNLO + NLL

[Beneke, Signer, '99]
[Melnikov,Yelkhovsky, '98]
[Hoang, '98/'99]
} NNLO

small n (relativistic): $n \lesssim 3$

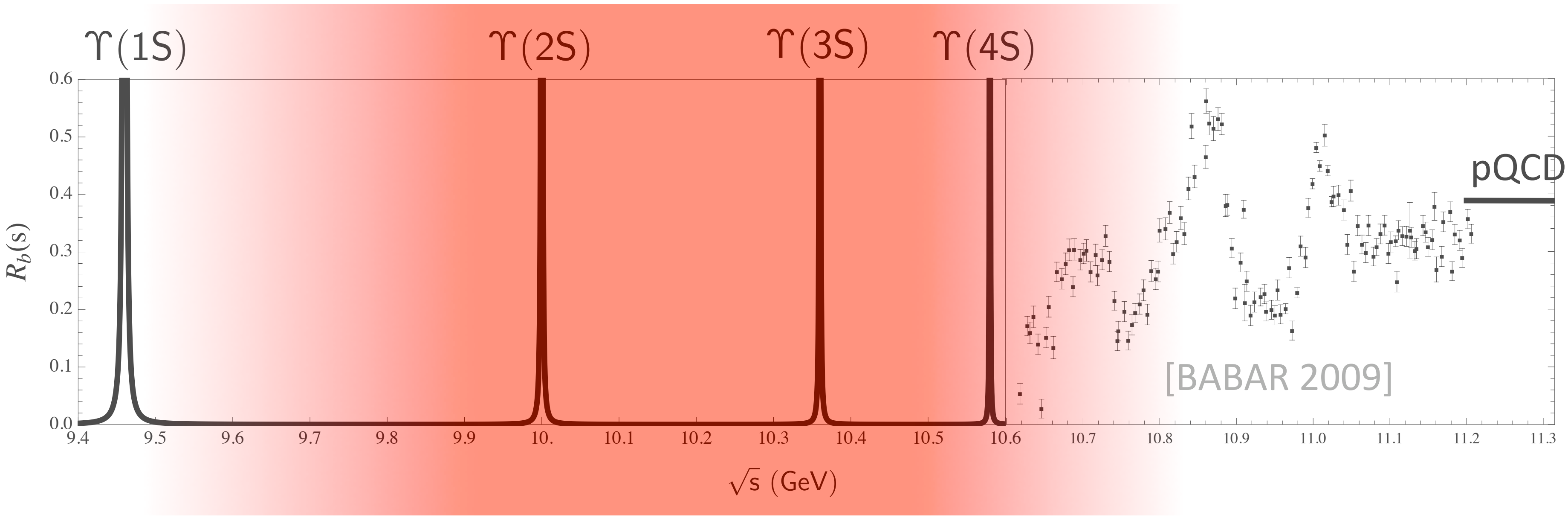
[Bodenstein et al., '11]

[Chetyrkin et al., '09/'10]

[Kühn et al., '07]

Applications

R-Ratio in the $b\bar{b}$ threshold region:

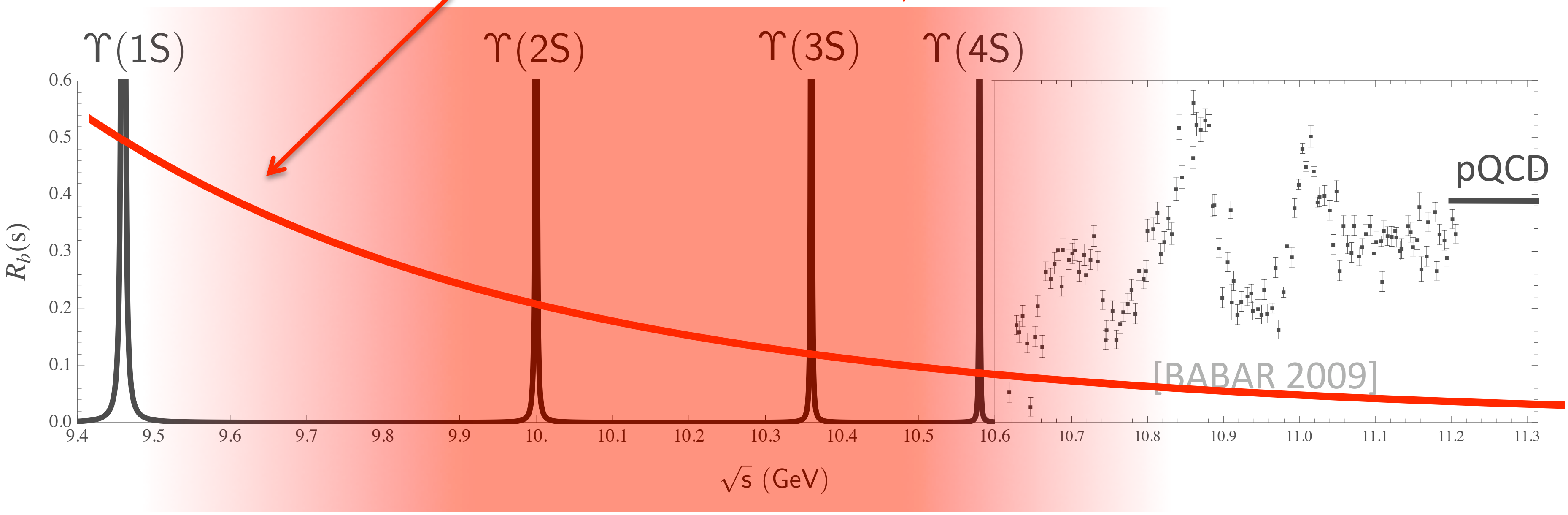


NONPERTURBATIVE

Applications

Large n sum rule: $$P_n = \int_0^\infty \frac{ds}{s^{n+1}} R_{b\bar{b}}(s)$$ $4 \leq n \leq 10$

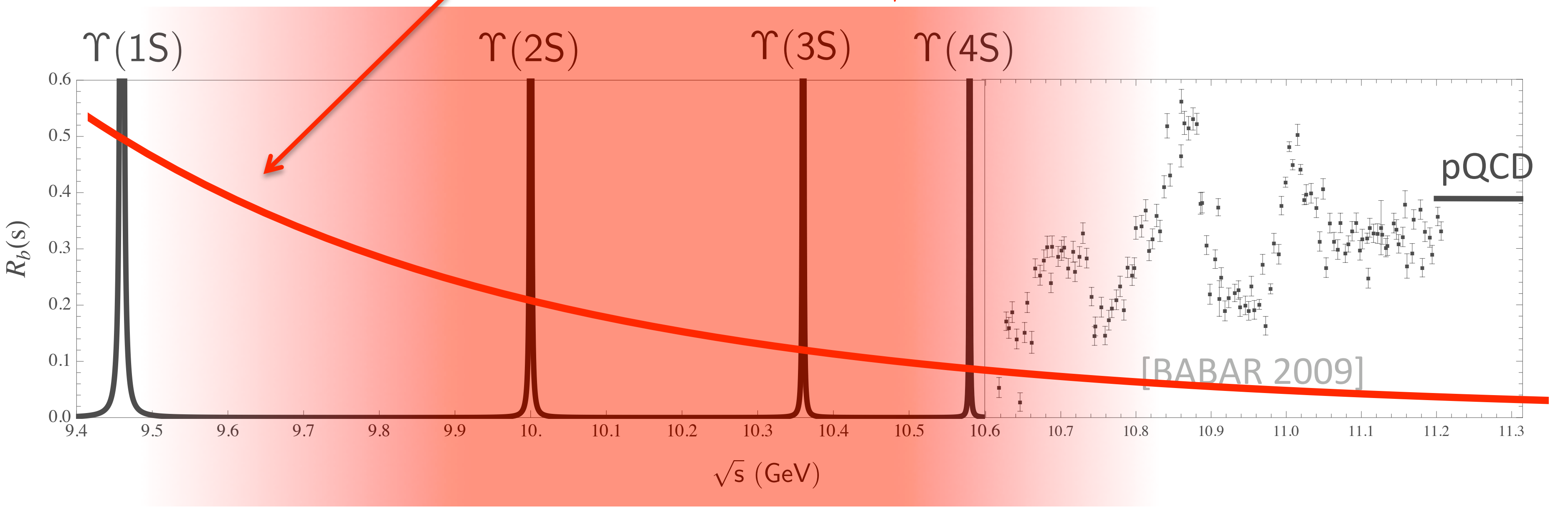
"smearing" over $\Delta E \sim m/n$



Applications

Large n sum rule: $P_n = \int_0^\infty \frac{ds}{s^{n+1}} R_{b\bar{b}}(s)$ $\quad 4 \leq n \leq 10$

"smearing" over $\Delta E \sim m/n$



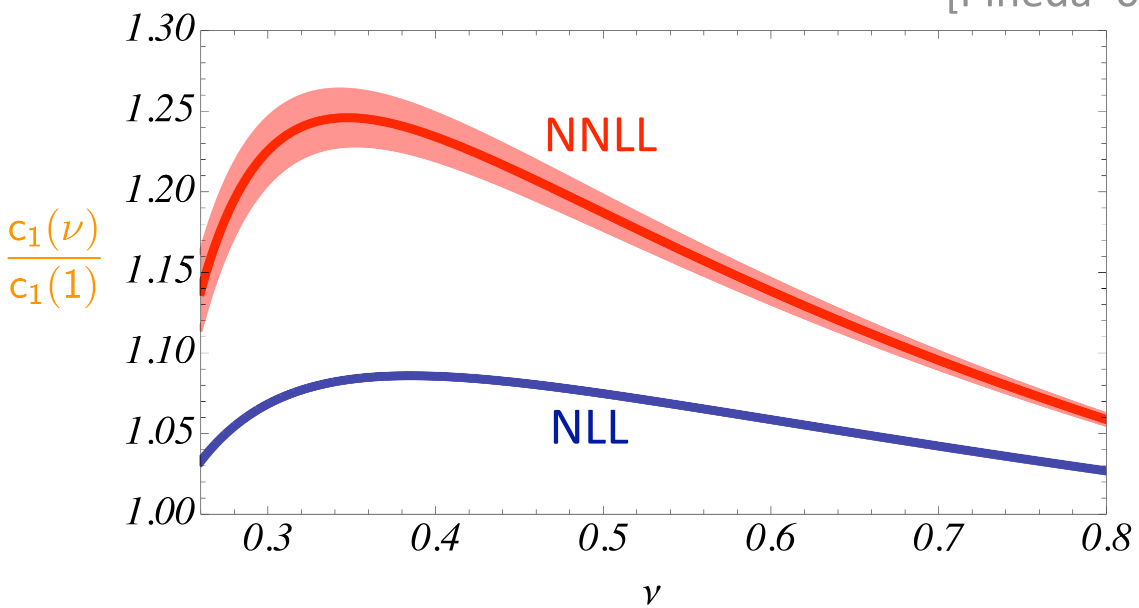
- Nonperturbative effects suppressed for $\Delta E \sim mv^2 \sim m/n \gg \Lambda_{QCD}$
- Average (relative) b-quark velocity $v \sim 1/\sqrt{n} \ll 1$ $\longrightarrow$ vNRQCD

Applications

$$R_{b\bar{b}}(s) \sim \mathrm{Im}\left[c_1(\nu)^2 \cdot G(0,0,E,\nu) \right] + \ldots$$

G^{NNLL} known ✓ [Hoang, Manohar, Stewart, Teubner, '02] [Pineda, Signer, '06]

c_1^{NLL} known ✓ [Luke, Manohar, Rothstein '00] [Pineda '02] [Hoang, Stewart, '03]



c_1^{NNLL} NEW ✓ [Hoang, MS, '11]

Applications

Integration over s ⇨ $P_n = \int_0^\infty \frac{ds}{s^{n+1}} R_{b\bar{b}}(s)$

[Voloshin, '95]

$$P_n^{th} = \frac{3\,N_c\,Q_b^2\,\sqrt{\pi}}{4^{n+1}(M_b^{pole})^{2n}\,n^{3/2}}\,c_1(\nu)^2\left(1 + 2\sqrt{\pi}\,\phi + 4\sqrt{\pi}\sum_{p=2}^{\infty}\phi^p\,\frac{\zeta_p}{\Gamma(\frac{p-1}{2})} + \ldots\right) + \ldots$$

$\phi := C_F/2\,\alpha_s(\nu)\sqrt{n}$

Coulomb resummation: $\sim \sum_m (\alpha_s/v)^m$

- Higher order logarithms $\sim \ln(\nu\sqrt{n})$

⇨ Renormalization parameter: $\nu \sim v \sim 1/\sqrt{n}$ resums nonrel. logs!

- Short distance mass M_b^{1S} to avoid renormalon ambiguity:

$$M_b^{pole} = M_b^{1S}\{1 + \Delta^{LL} + \Delta^{NLL} + [(\Delta^{LL})^2 + \Delta_c^{NNLL} + \Delta_m^{NNLL}]\}$$

Results

Fit for n=10: $P_{10}^{\text{exp}} = P_{10}^{\text{th}}(M_b^{1S}) \Rightarrow M_b^{1S}$

matching scale:

$$\mu_{\text{hard}} = hm$$

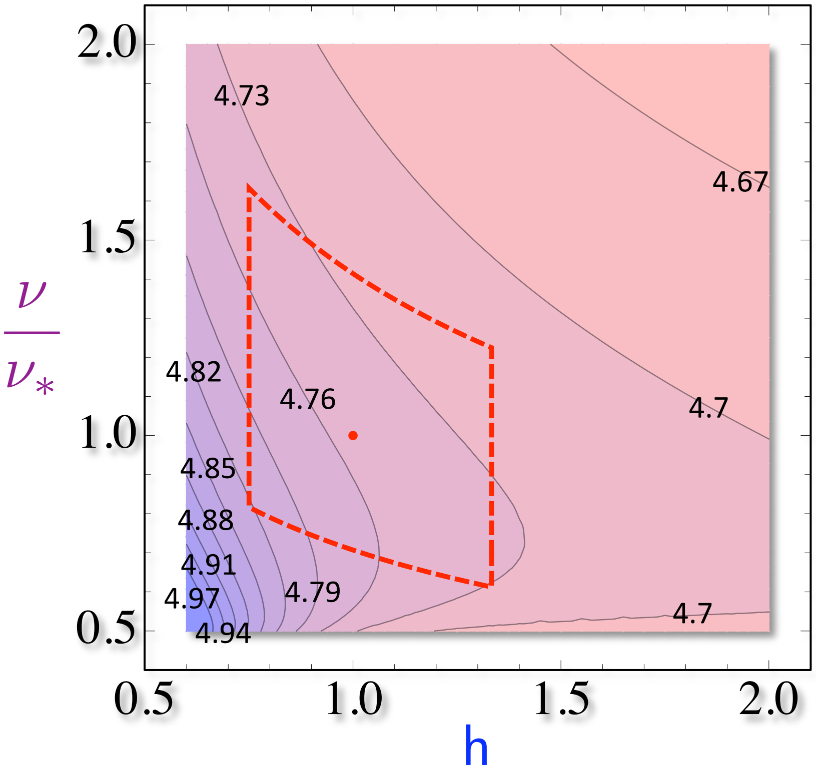
renormalization scales:

$$\mu_{\text{soft}} = hm\nu$$

$$\mu_{\text{usoft}} = hm\nu^2$$

default choice:

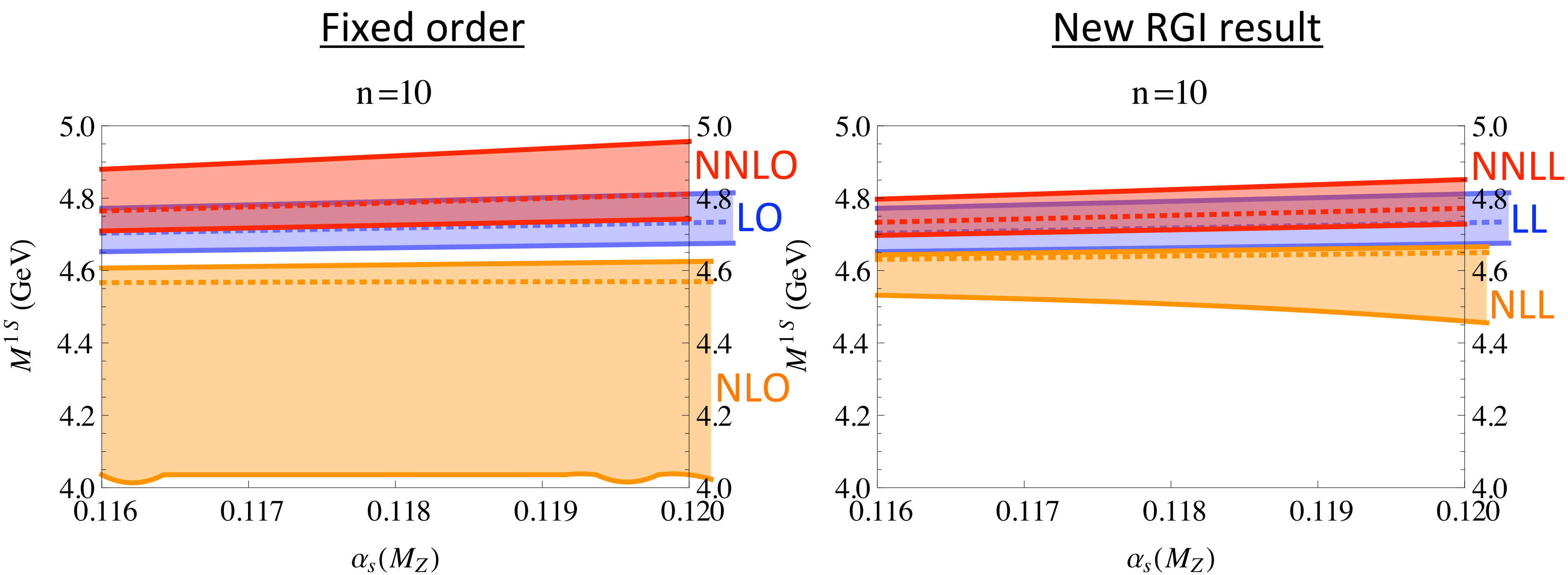
$$h = 1 \qquad \nu_* = 1/\sqrt{n} + 0.2$$



scale variation: $0.75 \leq h \leq 1/0.75$ $\quad m\nu_*^2/2 \leq \mu_{\text{usoft}} \leq 2m\nu_*^2$

Results

Comparison with (previous) fixed order results: [Hoang, Ruiz-Femenia, MS, '12]



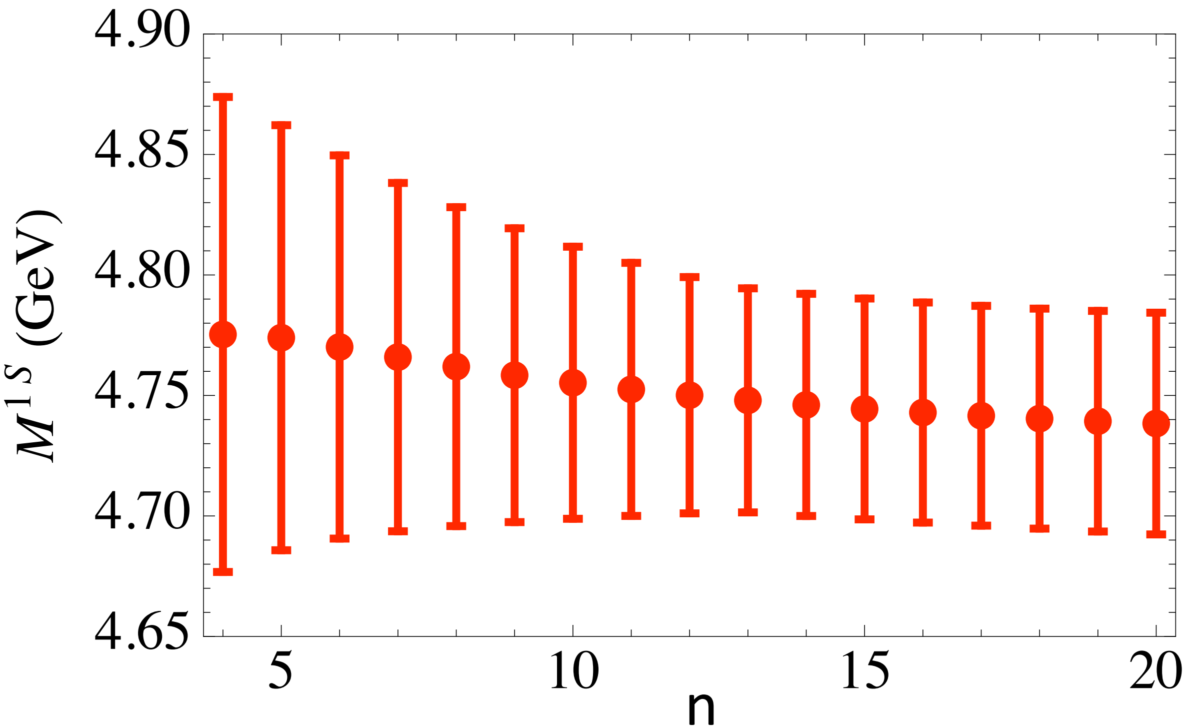
RGI result (n=10):
$\alpha_s = 0.1183 \pm 0.0010$

$$M_b^{1S} = 4.755 \pm 0.057_{\text{pert}} \pm 0.009_{\alpha_s} \pm 0.003_{\text{exp}}\ \text{GeV}$$

$$\overline{\text{MS}}: \quad \bar{m}_b(\bar{m}_b) = 4.235 \pm 0.055_{\text{pert}} \pm 0.003_{\text{exp}}\ \text{GeV}$$

Results

Comparison of n-th moment fits: [Hoang, Ruiz-Femenia, MS, '12]



RGI result (n=10): $M_b^{1S} = 4.755 \pm 0.057_{\rm pert} \pm 0.009_{\alpha_s} \pm 0.003_{\rm exp}$ GeV

$\alpha_s = 0.1183 \pm 0.0010$

Finite charm mass correction $\approx -(20-30)$ MeV (fixed order) [Hoang, Manohar, '00] [Hoang, '00]

Summary

Precise prediction of heavy quark threshold production in vNRQCD:

✓ $\sigma_{\text{tot}} \sim \text{Im}\Big[c_1(\nu)^2 \cdot G(0,0,E,\nu) \Big] + \ldots$

✓ $G(0,0,E,\nu)$ known to NNLL, New: $c_1(\nu)$ to NNLL

✓ RG improvement important!

Applications:

- $t\,\bar{t}$ threshold production @ ILC → $m_t, y_t, \alpha_s, \Gamma_t$

  $\Delta\sigma_{\text{tot}}/\sigma_{\text{tot}} \sim 2-3\,\%$ detailed error analysis → W.I.P

- Bottom mass from Υ sum rules: $M_b^{1S} = 4.755 \pm 0.058\,\text{GeV}$
- $e^+e^- \to t\bar{t}H$, $e^+e^- \to \tilde{t}\bar{\tilde{t}}$, ...

Backup

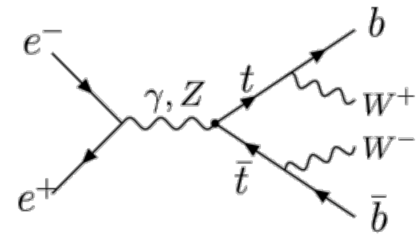
$$e^+e^- \longrightarrow t\,\bar{t}$$

Top-antitop threshold: EW effects

Backup

- Power counting: $\boxed{\Gamma_t/m_t \sim \alpha_{EW} \sim \alpha_s^2 \sim v^2 \ll 1}$

- Physical final state: $e^+e^- \to W^+W^-b\bar{b}$

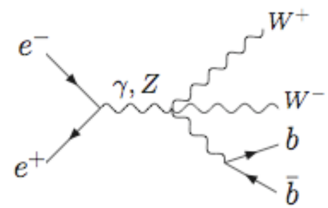


- Apply loose invariant mass cuts on reconstructed tops/antitops:

$$p_{t,\bar{t}}^2 = (m_t \pm \Delta M_t)^2 = m_t^2 + \Lambda^2 \qquad \boxed{m_t\Gamma_t \ll \Lambda^2 \lesssim m_t^2}$$

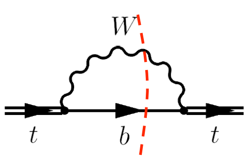
→ no effect on resonant contributions!

→ non-resonant background suppressed:



LO: $E = \sqrt{s} - 2m_t \to E + i\Gamma_t$ (replacement rule)

unstable top propagator: $\dfrac{i}{E/2 + p^0 - \mathbf{p}^2/(2m) + i\Gamma_t/2}$

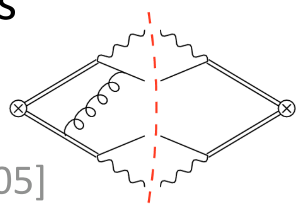


[Fadin, Khoze, '87]

Backup

Beyond LO:

- QED: "Coulomb photon" → trivial extension of QCD corrections



- Gluon exchange with final state → negligible at NLO and NNLO
  [Fadin, Khoze, Martin '94] [Hoang, Reisser '05]
  [Melnikov, Yakovlev '94] [Beneke, Jantzen, Ruiz-Femenia '10]

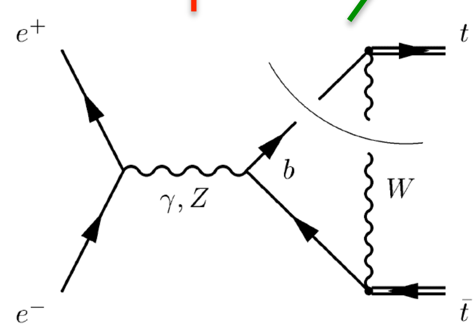
- Corrections to current matching:

$$c_1(1) = c_{1,\mathrm{LL}}^{\mathrm{born}} + c_{1,\mathrm{NLL}}^{\mathrm{QCD}} + c_{1,\mathrm{NNLL}}^{\mathrm{QCD}} + i\, c_{1,\mathrm{NNLL}}^{\mathrm{bW,abs}} + c_{1,\mathrm{NNLL}}^{\mathrm{EW}} + \ldots$$

[Kuhn, Guth '92]

complex matching

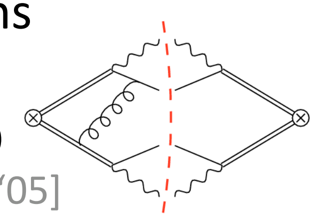
[Hoang, Reisser '06]



Backup

Beyond LO:

- QED: "Coulomb photon" $\longrightarrow$ trivial extension of QCD corrections



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- Corrections to current matching:

$$c_1(1) = c_{1,\mathrm{LL}}^{\mathrm{born}} + c_{1,\mathrm{NLL}}^{\mathrm{QCD}} + c_{1,\mathrm{NNLL}}^{\mathrm{QCD}} + i\, c_{1,\mathrm{NNLL}}^{\mathrm{bW,abs}} + c_{1,\mathrm{NNLL}}^{\mathrm{EW}} + \ldots$$

$$\Rightarrow \quad \sigma_{\mathrm{tot}} \sim \mathrm{Im}\left[c_1(\nu)^2\; G(0,0,E+i\Gamma_t,\nu)\right] \sim \frac{\alpha_s \Gamma_t}{\epsilon} + \text{finite}$$

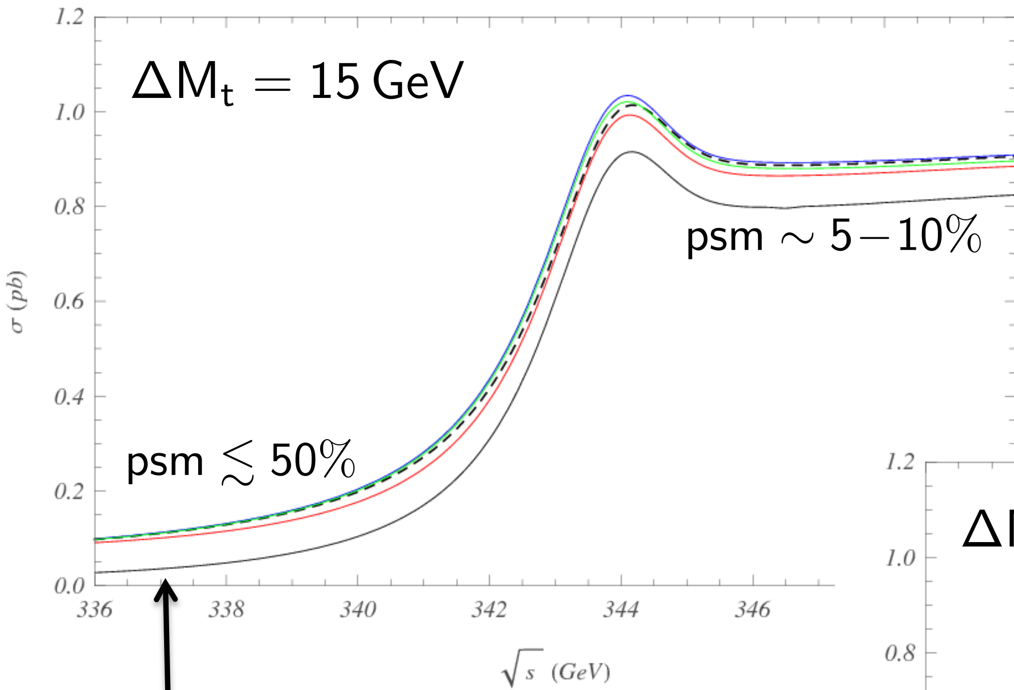
phase space divergence $\downarrow$ $i\,C(\nu)\,\cdot$

sums phase space logs in $C(\nu)$ at NLL ✔

[Hoang, Reisser, Ruiz-Femenia '10]

- "Phase space matching" for $C(\nu)$ to allow for Λ cuts: NLO, NNLO, N^{3}LO ✔

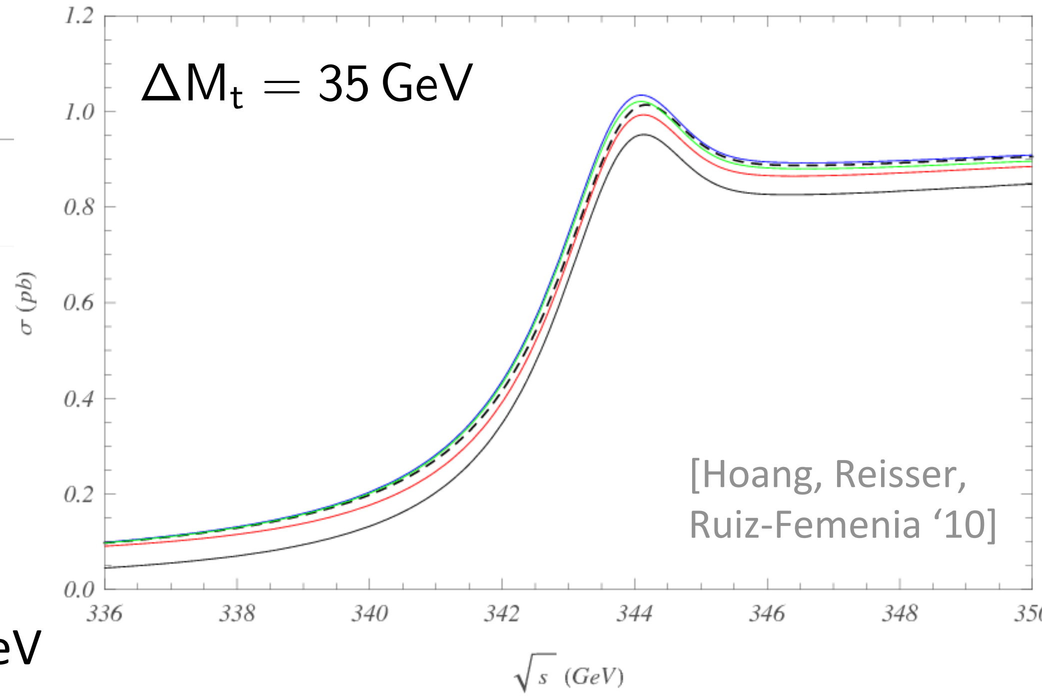
Backup



dashed line: NNLL pure QCD prediction (add step by step)

+ NNLL QED effects
+ NNLL EW current matching (real)
+ NNLL EW current matching (absorbtive)
+ NLL+NNLL+N^{3}LL phase space matching contributions (psm)

large psm correction due to unphysical phase space in pure QCD prediction

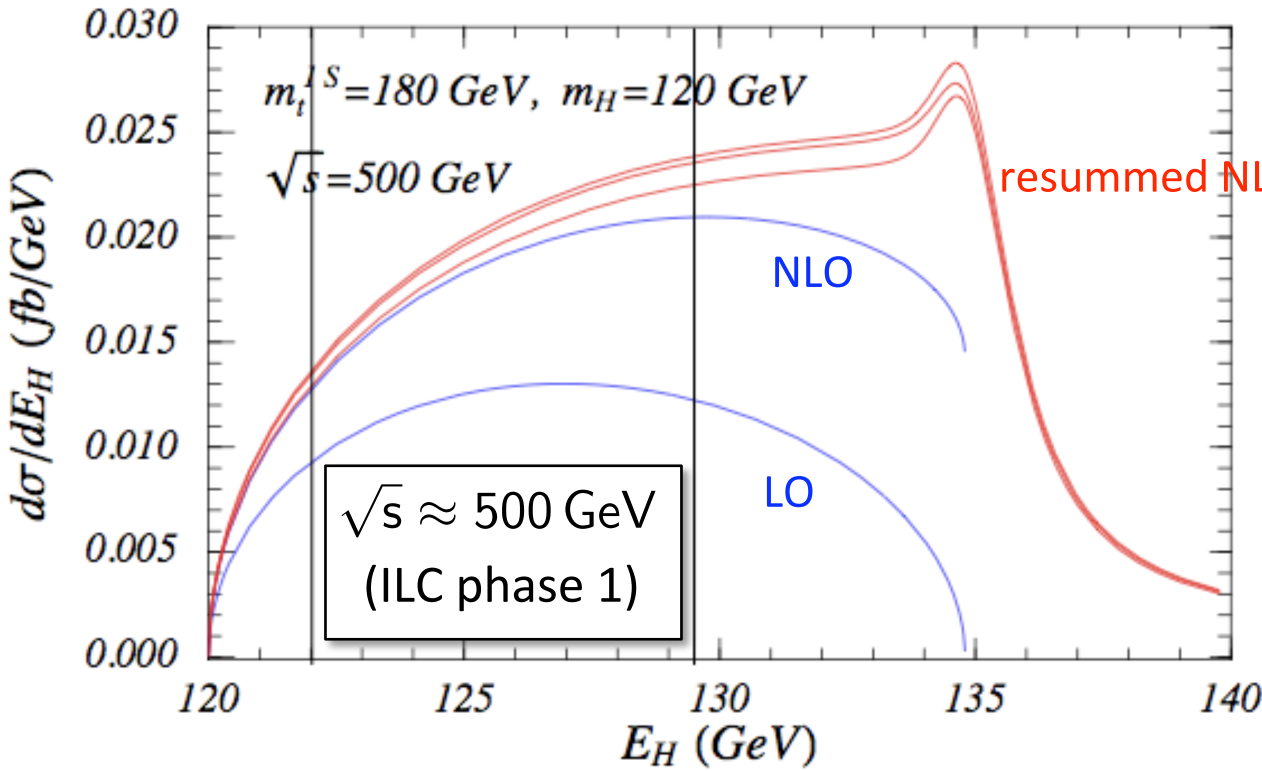


→ Shift in peak position: 30-50 MeV

$$\boxed{e^+e^- \longrightarrow t\,\bar{t}\,H}$$

Associated Higgs production

Backup



[Farrell, Hoang; 2006]

- For light Higgs ($m_H \approx 120$ GeV): full $t\,\bar{t}$ phase space nonrelativistic!

  → must sum $(\alpha_s/v)^n, (\alpha_s \ln v)^n$ terms → recycle $t\,\bar{t}$ results (vNRQCD)

  → factor 2 enhancement over tree level (+ factor 2 from polarized beams)

- realistic studies: $(\delta y_t/y_t)^{ILC}_{500GeV} \sim 30\% \;\rightarrow\; 10 - 15\%$? [Juste, '02,'06]