Flux Compactifications and Branes in 6D Supergravity

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Motivation

What can we learn from 6D supergravity?

- toy model for string compactifications:
  chiral fermions, anomaly cancellation, flux compactifications, susy breaking, moduli stabilization...
- extend study of 5D brane worlds:
  higher codimension, susy
- tantalizing numerology for hierarchy problems:
  $M_\Lambda \sim M_W^2/M_{Pl} \sim 1/r$
- intermediate step in the compactification from 10D to 4D:
  orbifold GUTs

Overview

- Hierarchies and Supersymmetric Large Extra Dimensions
- Flux compactifications and moduli stabilization
- Brane worlds in 6D
- Conclusions

The Cosmological Constant

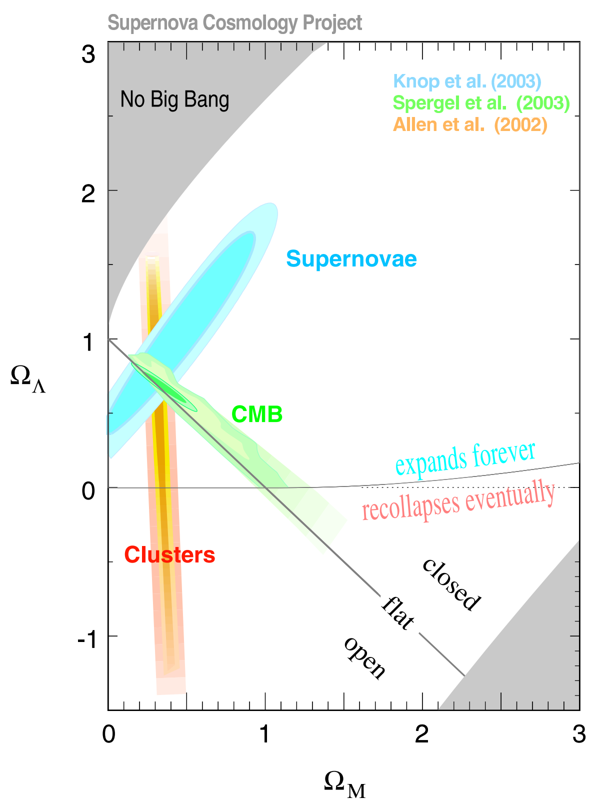
$$\underbrace{R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R - \Lambda g_{\mu\nu}}_{} = -8\pi G\, T_{\mu\nu}$$

Most general local, coordinate invariant, divergenceless, symmmetric rank-2 tensor we can construct from metric and its 1st and 2nd derivatives

- Λ is a parameter of the theory which must be fixed by observation
- The vacuum energy density $T^{vac}_{\mu\nu} = \rho\, g_{\mu\nu}$ is indistinguishable from Λ

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -8\pi G\, T_{\mu\nu} - 8\pi G \rho\, g_{\mu\nu}$$

Dark Energy



Cosmological Observations:

- Supernovae $\Rightarrow$ accelerated expansion
- CMB $\Rightarrow$ universe is flat

imply **Dark Energy** with equation of state:

$$p < -\rho/3$$

So far all data is consistent with $p = -\rho$

$\Rightarrow$ **Cosmological Constant** with $\rho \sim (10^{-3}\text{eV})^4$

Vacuum Energy

Embed cosmology into fundamental theory of short-distance physics

- Field of mass m contributes to vacuum energy:

$$\rho_{vac} = \int_0^M \frac{4\pi k^2 dk}{(2\pi)^3} \frac{1}{2} \sqrt{k^2 + m^2} \sim \frac{M^4}{16\pi^2}$$

So if we believe GR up to $M \sim 1\,\text{TeV} \Rightarrow$

$$\rho_{vac} \sim 1\,\text{TeV}^4$$

Some 50 orders of magnitude too big!

The 6D Brane World

Vacuum energy on brane worlds can curve the extra dimensions rather than their instrinsic 4D spacetime.

Rubakov & Shaposhnikov '83
...
Arkani-Hamed, Dimopoulos, Kaloper & Sundrum '05
Kachru, Schulz & Silverstein '05

A 3-brane in 6D induces a conical defect in the transverse dimensions:

Sundrum '98
Chen, Luty & Ponton '00

$$\delta = \frac{T_3}{M_6^4}$$

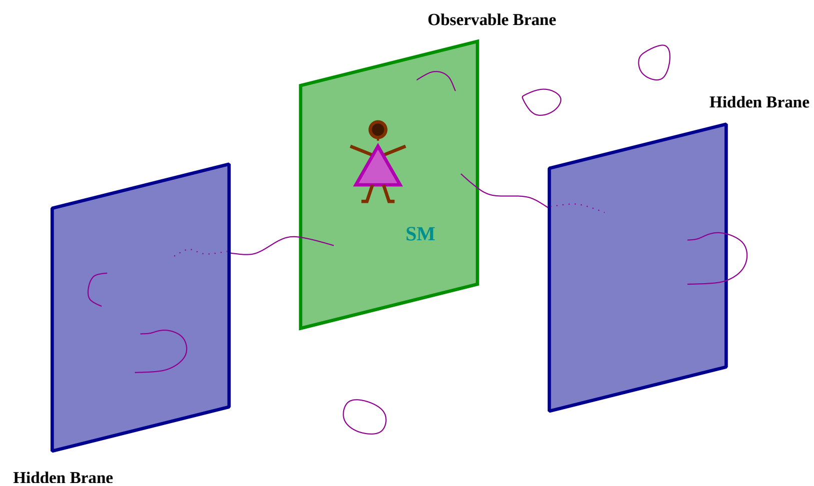
Tantalizing Numerology

Observed scale of Dark Energy:

$$\Lambda \sim \left(\frac{M_W^2}{M_{Pl}}\right)^4 \sim \frac{1}{r^4}$$

Sundrum '98

Chen, Luty & Ponton '00

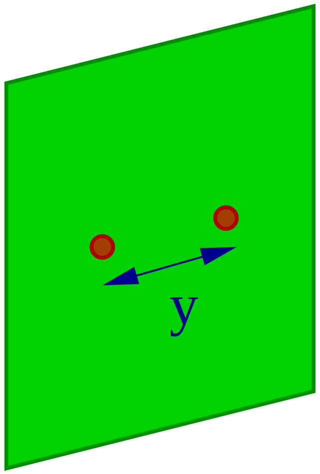


SM is 4D down to $\sim 10^{-19} m$ but gravity has been tested only down to submillimeters...

Arkani-Hamed, Dimopoulos & Dvali '98

Antoniadis, Arkani-Hamed, Dimopoulos & Dvali '98

(S)LED and the Hierarchy Problems



Gravity in unwarped brane world with n extra dimensions of size r

$$V(y) \sim \frac{m_1 m_2}{M_s^{n+2}} \frac{1}{y^{n+1}} \qquad y << r$$

$$V(y) \sim \frac{m_1 m_2}{M_s^{n+2} r^n} \frac{1}{y} \qquad y >> r$$

$$\Rightarrow M_{Pl}^2 \sim M_s^{n+2} r^n$$

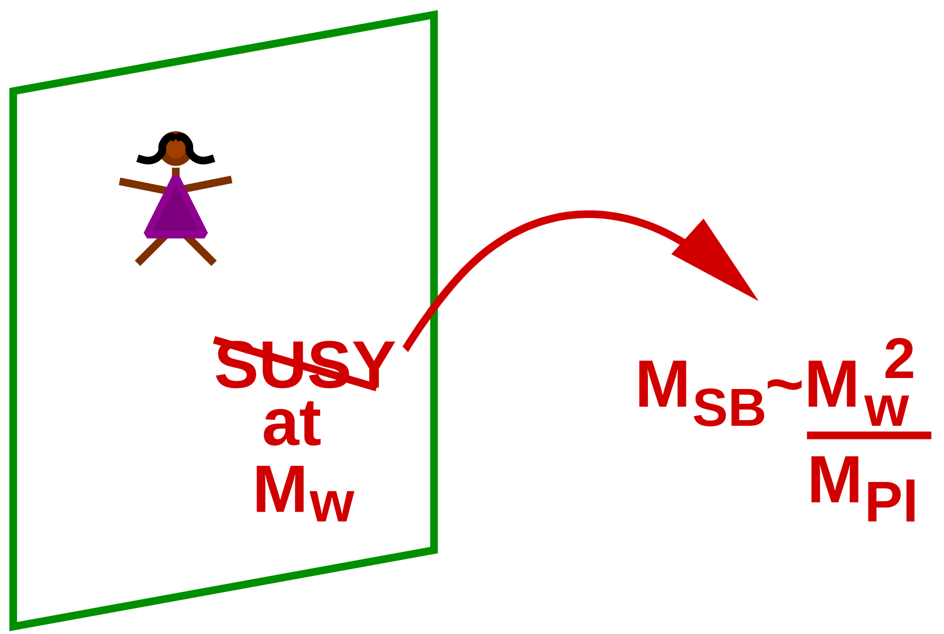
For $M_s \sim M_W$: $\Rightarrow r \sim 10^{32/n-16} mm$

- Gravity is weak due to dilution in large extra dimensions
- If r is tied to Λ then there must be two submillimeter extra dimensions

Supersymmetric Large Extra Dimensions

Aghababaie, Burgess, S.L.P & Quevedo '02
Burgess '04

- Change gravity at the scale of Λ
- Separation of SUSY breaking scales



SLED and the Cosmological Constant

Explain not only why the cosmological constant is zero, but why it is $(10^{-120} M_{Pl})^4$!

- Supersymmetry is badly broken on the brane:

$$T_3 \sim M_W^4$$

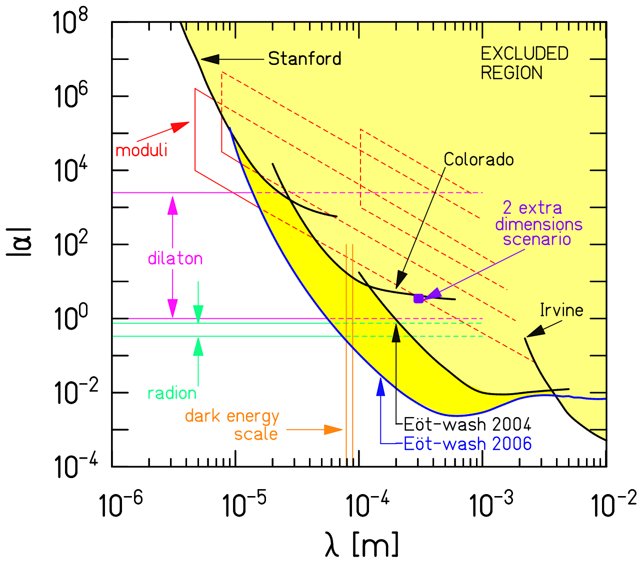
This localized vacuum energy is absorbed by the bulk curvature.

- Bulk susy-breaking is gravitationally suppressed:

$$M_{SB} = \frac{M_W^2}{M_{Pl}}$$

If bulk contribution to vacuum energy is $\mathcal{O}(M_{SB}^4)$ then we are there...

Current bounds on Extra Dimensions



Deviations from Newton's Law:

$$V(r) = -G\frac{m_1 m_2}{r}\left(1 + \alpha e^{-r/\lambda}\right)$$

Adelberger, Heckel & Nelson '03

- Current bounds from table-top experiments:

$$L < 44\mu m, \; M_6 > 3.2\text{TeV}$$

Kapner et al '06

- Stronger bounds come from supernovae:

$$L \lesssim 10\mu m, \; M_6 > 8.9\,\text{TeV}$$

for unwarped extra dimensions

Hannestad & Raffelt '01

6D $\mathcal{N} = 2$ Supergravity

6D chiral gauged supergravity in the bulk:

$$\begin{aligned} S_{bulk} &= \int d^6X\sqrt{-G}\left[\frac{1}{4}R - \frac{1}{4}\partial_M\sigma\,\partial^M\sigma - \frac{1}{4}G_{\alpha\beta}(\Phi)D_M\Phi^\alpha D^M\Phi^\beta\right.\\ &\quad - \frac{e^{-2\sigma}}{12}H_{MNP}\,H^{MNP} - \frac{e^{-\sigma}}{4}F^I_{MN}\,F^{I\,MN} - 2\,g_1^2\,v(\Phi)\,e^\sigma\\ &\quad \left. + \quad \text{fermions}\right] \end{aligned}$$

Nishino & Sezgin '84

Points to note:

- Classical scaling symmetry: eoms are invariant under $G_{MN} \to \lambda G_{MN}$ and $e^\sigma \to e^\sigma/\lambda$
- Chiral fermions $\Rightarrow$ in general anomolous: restricts the matter content of the theory *e.g.* $E_7 \times E_6 \times U(1)_R$

Anomaly Cancellation

For special gauge groups and hyperino reps ($n_H = n_V + 244$) the anomalies cancel via a Green-Schwarz mechanism:

Gauge Group	Hyperino Rep
$E_7 \times E_6 \times U(1)_R$	$(\mathbf{912}, \mathbf{1})_0$
$E_7 \times G_2 \times U(1)_R$	$(\mathbf{56}, \mathbf{14})_0$
$F_4 \times Sp(9) \times U(1)_R$	$(\mathbf{52}, \mathbf{18})_0$
$E_6 \times Sp(1)_R$	$325\,(\mathbf{1}, \mathbf{1})$
$SU(2) \times U(1)_R$, $SU(2) \times U(1)_{R+}$, $U(1) \times Sp(1)_{R+}$, $SU(2) \times Sp(1)_{R+}$, $Sp(1)_{R+}$, $SU(3) \times U(1)_R$	*many anomaly cancelling reps*
many models with drone U(1)'s e.g: $E_7 \times U(1)^{22} \times U(1)_R$	$2\,(\mathbf{133})_0 + 2\,(\mathbf{56})_0$

Randjbar-Daemi, Salam, Sezgin & Stradthdee '85
Avramis, Kehagias & Randjbar-Daemi '05
Avramis & Kehagias '05
Suzuki & Tachikawa '05

Salam-Sezgin Model

Salam & Sezgin '84

Sphere-monopole background

$$ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu + r^2 \left(d\theta^2 + \sin^2\theta d\phi^2\right)$$

$$\sigma = const \qquad A_\phi = \frac{c}{2}\left(\cos\theta \mp 1\right) Q$$

- Dirac quantization $\Rightarrow$ $c\, g\, e^I = N^I$, with N^I integers
- Equations of motion $\Rightarrow$
  - $c = \pm \frac{1}{g_1}$
  - $r^2 e^\sigma = 1/g_1^2$
  - modulus $r^2 e^{-\sigma}$ (guaranteed by classical scaling symmetry)
- SUSY transformations $\Rightarrow$ $\mathcal{N} = 2 \to \mathcal{N} = 1$ susy breaking

Salam-Sezgin Low Energy Dynamics

Aghababaie, Burgess, S.L.P & Quevedo '02

Dimensionally reducing the 6D Lagrangian:

$$\mathcal{L}_B \sim \sqrt{-G}\left[\frac{1}{2}R - \frac{e^{-\sigma}}{4}\, F^I_{MN}\, F^{I\,MN} - 2\, g_1^2\, e^{\sigma}\right]$$

on the sphere-monopole leads to **4D effective potential**:

$$V = 2g_1^2\,\frac{e^{\sigma}}{r^2}\left(1 - \frac{1}{g_1^2 r^2 e^{\sigma}}\right)^2$$

- dilaton stabilized $r^2 e^{\sigma} = 1/g_1^2$
- flat four dimensions for all values of volume modulus $r^2 e^{-\sigma}$
- volume can be stabilized by considering perturbative and non-perturbative corrections to the classical theory.

The Brane

In the presence of 3-brane source:

$$S_{brane} = -T_3 \int d^4x \sqrt{-g}$$

with induced metric $g_{\mu\nu} = G_{MN}(Y(x))\partial_\mu Y^M(x)\partial_\nu Y^N(x)$

Einstein's equation $\Rightarrow$

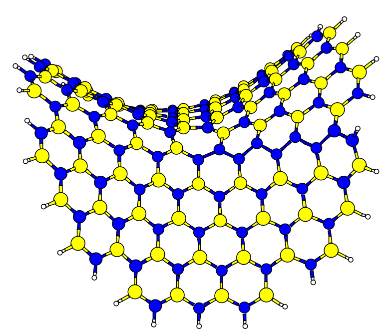
$$R_2 = R_2^{smth} + 2\sum_i T_3^i \frac{\delta^{(2)}(y - y_i)}{\sqrt{g_2}}$$

Negative Tension Branes

Negative deficit angles are also possible:

Like nanocones with negative disclination angles in condensed matter physics:

Azevedo, Mazzoni, Chacham & Nunes '04



Explicit Brane Solutions

Gibbons, Guven & Pope '03

Aghababaie, Burgess, Cline, Firouzjahi, S.L.P, Quevedo, Tasinato & Zavala '03

General solution with:

1. Maximal symmetry in 4D
2. Axial symmetry in 2D
3. At most conical defects

$$ds^2 = e^{A(\rho)}\,\eta_{\mu\nu}\,dx^\mu\,dx^\nu + d\rho^2 + e^{B(\rho)}\,d\phi^2$$

$$\mathcal{A} = \mathcal{A}_\phi(\rho)\,Q\,d\phi\,, \qquad \sigma = \sigma(\rho)$$

$$H_{MNP} = 0\,, \qquad \Phi^\alpha = 0$$

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$$\mathcal{A} = \mathcal{A}_\phi(\rho)\,Q\,d\phi\,, \qquad \sigma = \sigma(\rho)$$

$$H_{MNP} = 0\,, \qquad \Phi^\alpha = 0$$

Solution has conical defects at $\rho = 0$ and $\rho = \bar{\rho}$ with deficit angles:

$$\delta,\ \bar{\delta}$$

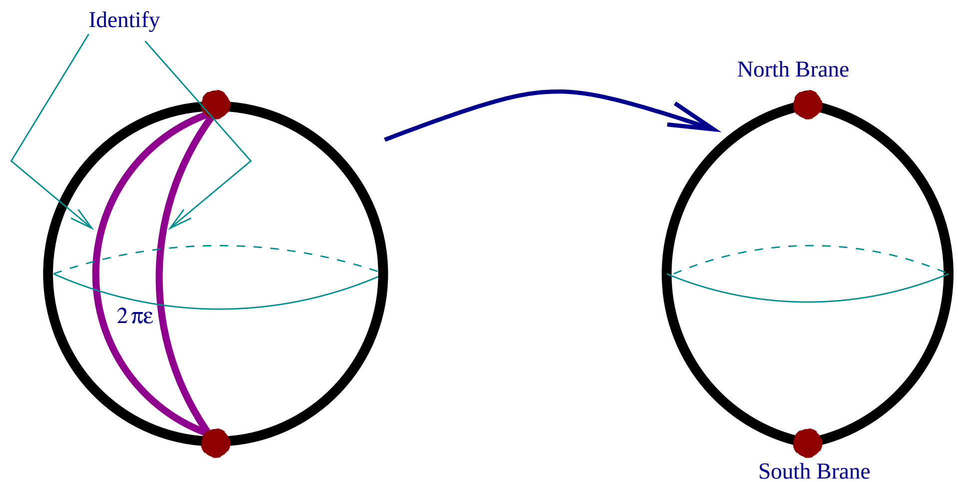
which are integration constants $< 2\pi$ related by Dirac quantization:

$$e^I \left(1 - \frac{\delta}{2\pi}\right)^{1/2} \left(1 - \frac{\bar{\delta}}{2\pi}\right)^{1/2} \frac{g}{g_1} = N^I$$

Sphere Limit

As $\bar{\delta} \to \delta$ the warp factor goes to one $\Rightarrow$ **unwarped rugby ball**.

Carroll & Guica '02
Navarro '02
Aghababaie, Burgess, S.L.P & Quevedo '02



As both $\delta, \bar{\delta} \to 0 \Rightarrow$ **classic sphere-monopole compactification**.

Randjbar-Daemi, Salam & Strathdee '83
Salam & Sezgin '84

4D Effective Theory

Analysis of all linearized perturbations is almost complete.

S.L.P, Randjbar-Daemi & Salvio '06
Tolley, Burgess, de Rham & Hoover '06
S.L.P, Randjbar-Daemi & Salvio '07
S.L.P, Randjbar-Daemi & Salvio *in progress*

Fluctuations that are scalars wrt to 4D observer:

- Bulk:
  $$\{\delta G^{\mu}_{\mu}, \delta G_{\rho\rho}, \delta G_{\phi\phi}, \delta G_{\rho\phi}, \delta\sigma, \delta B_{\mu\nu}, \delta B_{\rho\phi}, \delta\mathcal{A}_{\rho}Q, \delta\mathcal{A}_{\phi}Q, \delta\mathcal{A}^{I}_{\rho}T^{I}, \delta\mathcal{A}^{I}_{\phi}T^{I}, \delta\Phi\}$$
- Brane:
  $$\{\delta Y^{\rho}, \delta Y^{\phi}\}$$

Give rise to Kaluza-Klein tower of 4D effective scalar fields.

Example of linearized analysis

Scalar fluctuations of gauge fields orthogonal to monopole bkgd:

$$\mathcal{A}^I = \langle \mathcal{A}^I \rangle + V^I$$

have bilinear action (in lightcone gauge):

$$S_2(V,V) = -\frac{1}{2}\int d^6X\sqrt{-G}\,Tr\left[-\partial_\mu V_i \partial_\mu V^i + D_i V_j D^i V^j + R_{ij} V^i V^j \right. \\ \left. +2g\,F_{ij} V^i \times V^j\right]$$

where the covariant derivative is defined by

$$D_\phi V_j = \nabla_\phi V_j - ig\,e\,\mathcal{A}_\phi\,V_j\,.$$

Make Kaluza-Klein expansion:

$$V_j(X) = \sum_{n=0}^{\infty}\sum_{m=-\infty}^{\infty} V_{j\,mn}(x) f_n(\rho) e^{im\phi}$$

Effective Schroedinger Problem

Problem reduces to two decoupled effective Schroedinger equations:

$$\left(-\partial_\xi^2 + U_\pm(\xi)\right) V_\pm(x,\xi) = M^2 V_\pm(x,\xi)$$

with boundary conditions:

$$\int d^4x d\xi \partial_\xi \left[\delta V_\pm^\dagger \left(\partial_\xi - f(\xi)\right) V_\pm\right] = 0$$

and normalizability conditions:

$$-\frac{1}{2}\int d^4x d\xi \left[\partial_\mu V_\pm^\dagger \partial^\mu V_\pm\right] < \infty$$

where

$$V_\pm(t,\rho) = \frac{1}{\sqrt{2}}\left(e^{B/4} V_{\rho m}(t,\rho) \pm i e^{-B/4} V_{\phi m}(t,\rho)\right)$$

Explicit solutions can be found in terms of hypergeometric functions $F(a,b,c,\xi)$

S.L.P, Randjbar-Daemi & Salvio '07

Mass Spectrum

S.L.P, Randjbar-Daemi & Salvio '07

- For $m \leq -1/\omega$ and $m \leq N + 1/\bar{\omega}$

$$M^2 = \frac{4}{r_0^2}\left\{n(n+1) - \left(n+\frac{1}{2}\right)[m\omega + (m-N)\bar{\omega}] + m(m-N)\omega\bar{\omega}\right\}$$

- For $-1/\omega < m \leq N + 1/\bar{\omega}$

$$M^2 = \frac{4}{r_0^2}\left\{\left(n+\frac{3}{2}\right)^2 - \frac{1}{4} + \left(n+\frac{3}{2}\right)[m\omega - (m-N)\bar{\omega}]\right\}$$

- For $N + 1/\bar{\omega} < m \leq -1/\omega$

$$M^2 = \frac{4}{r_0^2}\left\{n(n-1) - \left(n-\frac{1}{2}\right)[m\omega - (m-N)\bar{\omega}]\right\}$$

- For $m > -1/\omega$ and $m > N + 1/\bar{\omega}$

$$M^2 = \frac{4}{r_0^2}\left\{n(n+1) + \left(n+\frac{1}{2}\right)[m\omega + (m-N)\bar{\omega}] + m(m-N)\omega\bar{\omega}\right\}$$

for $n = 0, 1, 2, \ldots$, where $\omega = (1 - \frac{\delta}{2\pi})^{-1}$ and $\bar{\omega} = (1 - \frac{\bar{\delta}}{2\pi})^{-1}$.

Stability Conditions

- For stability we require $M^2 > 0$

  Exact and complete mass spectrum $\Rightarrow$

  - Abelian monopole embeddings are stable
  - Monopole embeddings in non-Abelian sectors are generically unstable
  - **Negative tension branes ameliorate the instability**

- Where does the instability lead us?

Burgess, S.L.P & Zavala *in progress*

Mass Gaps

S.L.P, Randjbar-Daemi & Salvio '06

Conical defects give rise to non-conventional behaviour for mass gaps

Volume of internal manifold:

$$V_2 = 4\pi \frac{1}{\bar{\omega}} \left(\frac{r_0}{2}\right)^2$$

Mass gap:

$$M_{GAP}^2 = \frac{1}{r_0^2} \left(a + b\omega + c\bar{\omega}\right)$$

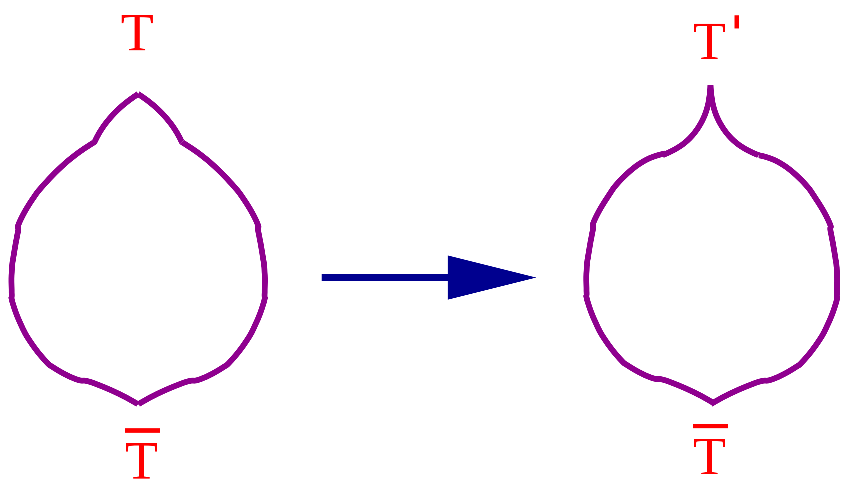
Take volume $\rightarrow \infty$ by taking $\bar{\omega} \rightarrow 0$ and mass gap remains finite!

SM fields, in addition to gravity, could "feel" the extent of LED

Open Questions

Classical dynamics:

- topological constraint $\Rightarrow$ relation between brane tensions: *are there flat solutions for arbitrary brane tensions?*



Quantum dynamics:

- Are the choices required for 4D flat cosmologies stable against renormalization?
- Do bulk contributions to the vacuum energy cancel down to ${M_{SB}}^4$?
- Why are the extra dimensions so large?

Cosmology at Colliders!

Burgess '04
Burgess, Matias & Quevedo '04

Dynamical Dark Energy *and*:

- Deviations from inverse square law for gravity at $r/2\pi \sim 1\,\mu m$
- A particular scalar-tensor theory of gravity at large distances
- Potential atrophysical signals (and bounds) due to energy loss into extra dimensions by stars and supernovae.
- Distinctive missing energy signals at the LHC due to emission of particles into the extra dimensions

Predictions at the bounds of experiments in gravity, cosmology, astrophysics and accelerators!

Conclusions

- 6D supergravity provides a laboratory in which to explore compactifications and codimension two branes
- Supersymmetric Large Extra Dimensions may provide a technically natural solution to the cosmological constant problem:
  - change gravity at the scale of Λ
  - SUSY helps in non-conventional way
- Explicit calculations reveal surprising dynamics for 6D brane worlds:
  - negative tension branes can relax stability constraints
  - conical defects allow a large mass gap for large volume compactifications
- Several open questions...
- Many and diverse predictions within reach of upcoming experiments