Four-point functions of R-currents in AdS/CFT-correspondence

Anna-Maria Mischler

II. Institut für Theoretische Physik
Universität Hamburg

in collaboration with J. Bartels and M. Salvadore

DESY Zeuthen
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Outline

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AdS/CFT-correspondence

- AdS/CFT-correspondence is a relation between a conformal field theory in d dimensions and string theory in d+1-dimensional Anti de Sitter space
- the case of interest is d=4:
  then the CFT is $\mathcal{N}=4$ SYM theory with gauge group $SU(N)$ and string theory is Type IIB superstring theory in an $AdS_5 \times S_5$ background
- in the low energy limit of string theory supergravity is a good approximation
- $\mathcal{N}=4$ SYM lives on the 4-dim boundary of AdS_5

Maldacena, 1997; Witten, 1998

Duality

- the coupling constant of SYM is g_{YM} and of string theory g_s
- they are related by $g_{YM}^2 = g_s$
- for the supergravity limit one needs $g_s \to 0$, $N \to \infty$ and $\lambda = g_s N$ fixed and large
- in a $SU(N)$ gauge theory the limit corresponds to the 't Hooft limit, $N \to \infty$, where $\lambda = g_{eff} = g_{YM}^2 N$ is fixed
- hence the AdS/CFT-correspondence is a duality because it relates the strong coupling regime of CFT to the weakly coupled string theory and vice versa

Map

- an operator $\mathcal{O}$ with conformal dimension Δ in $\mathcal{N} = 4$ SYM, the theory at the boundary, corresponds to a field ϕ of mass m in supergravity theory in $AdS_5 \times S_5$
- the value of ϕ at the boundary is ϕ_0
- meaning in the gauge theory: field ϕ_0 is a source for the operator $\mathcal{O}$, the field couples to $\mathcal{O}$ via $\int_{S^d} \phi_0 \mathcal{O}$

Partition function $Z_{\mathcal{O}}[\phi_0]$

$$Z_{\mathcal{O}}[\phi_0] = \int \mathcal{D}[\text{SYM fields}] \exp(-S_{\mathcal{N}=4\,SYM} + \int d^4x \mathcal{O}(x)\phi_0(x))$$

$$\Rightarrow \langle \mathcal{O}(x_1) \dots \mathcal{O}(x_n) \rangle = \frac{\delta^n}{\delta\phi_0(x_1) \dots \delta\phi_0(x_n)} Z_{\mathcal{O}}[\phi_0]|_{\phi_0 = 0}$$

Witten prescription

- $Z_{\mathcal{O}}[\phi_0]_{\mathrm{CFT}}$

$$= \int \mathcal{D}[\text{SYM fields}] \exp\left(-S_{\mathcal{N}=4\,SYM} + \int d^4x\, \mathcal{O}(x)\phi_0(x)\right)$$

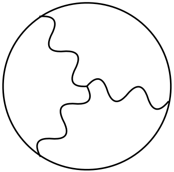
$$= Z_{\mathrm{class}}[\phi_0]_{\mathrm{AdS}} = \exp(-S_{\mathrm{sugra}}[\phi[\phi_0]])$$

n-point functions in AdS/CFT

$$\langle \mathcal{O}(x_1)\ldots\mathcal{O}(x_n)\rangle = \frac{\delta^n}{\delta\phi_0(x_1)\ldots\delta\phi_0(x_n)} \exp(-S_{\mathrm{sugra}}[\phi[\phi_0]])|_{\phi_0=0}$$

Witten diagrams

- computing n-point functions means evaluating Witten diagrams
- three-point function as one example:



- the boundary of AdS_5 is represented by a circle, wavy lines are gauge boson propagators
- two kind of propagators: bulk-to-boundary propagators, between an interior point and a boundary point, and bulk-to-bulk propagators, between two interior points

Outline

$\mathcal{N} = 4$ SYM

- non-Abelian gauge theory in four dimensions with $\mathcal{N} = 4$ supersymmetry
- contains a vector multiplet in the adjoint representation of the gauge group $SU(N_C)$
- theory offers a $SU_R(4)$ global symmetry called R-symmetry $\Rightarrow$ R-currents
- the fields are
  - 1 vector field A_μ, scalar of $SU_R(4)$
  - 4 chiral spinors λ_I in the fundamental representation of $SU_R(4)$
  - 6 real scalars X_M in the vector represenation of $SU_R(4)$

Scattering of R-currents (1)

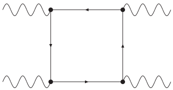
- interested in four-point correlation functions in $\mathcal{N}=4$ SYM

Momentum space four-point function

$$i(2\pi)^4\delta(p_A+p_B-p_{A'}-p_{B'})A_R(s,t) = \int\prod_i d^4x_i e^{-ip_A\cdot x_A-ip_B\cdot x_B+ip_{A'}\cdot x_{A'}+ip_{B'}\cdot x_{B'}}\times \langle J_R^{\mu A}(x_A)J_R^{\nu B}(x_B)J_R^{\mu' A'}(x_{A'})J_R^{\nu' B'}(x_{B'})\rangle$$

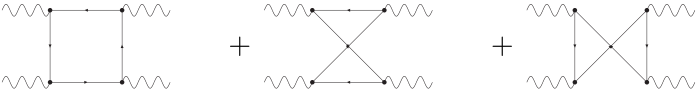
- corresponds to scattering of R-currents $J_R^{\mu A}(x)$
- R-current scattering is the analog of $\gamma^*\gamma^*$-scattering in QCD

$\gamma^*\gamma^*$-scattering in QCD

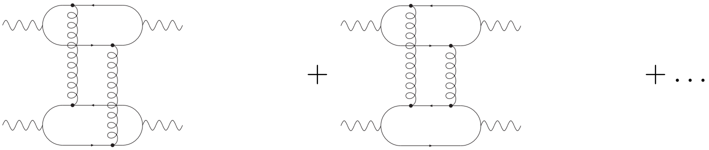
- elastic scattering of two virtual photons at high energies
- each photon splits into a quark-antiquark pair
- the decay of the photons is mediated by the electromagnetic current j^μ
- leads to the computation of the four-point function of j^μ
- one sample diagram is 
- the computation is performed in the Regge limit: $s \to \infty$ and $t < 0$, fixed
- in this limit: conjectured duality of the BFKL pomeron and the graviton, Kotikov, Lipatov et al., 2005
  integrability of BKP-states

Leading diagrams

- in the Regge limit one-loop diagrams are subleading $\sim \log^2 s$



- three-loop diagrams give the leading contribution to the amplitude $\sim \alpha_s^2 s$

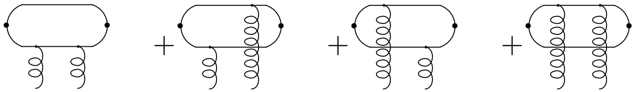


Impact factors

Amplitude for three-loop diagrams

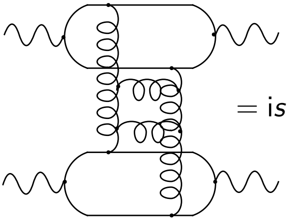
$$A(s,t) = is \int \frac{d^2\mathbf{k}}{(2\pi)^2} \frac{\Phi_A^{\lambda_A \lambda_{A'} aa'}(\mathbf{k}, \mathbf{q}-\mathbf{k}) \Phi_B^{\lambda_B \lambda_{B'} aa'}(\mathbf{k}, \mathbf{q}-\mathbf{k})}{\mathbf{k}^2 (\mathbf{q}-\mathbf{k})^2}$$

- the leading term of the amplitude is purely imaginary
- the amplitude $A(s,t)$ factorizes into two impact factors
- the impact factors are

$\Phi_{A,B}(\mathbf{k}_1, \mathbf{k}_2) =$ 

LLA resummation

- Leading-Logarithmic Approximation is summing up radiative corrections to $A(s,t)$

$$\mathrm{i}\,\mathrm{Im}\,A^{LLA}(s,t) = \sum$$  $$= \mathrm{i}s\,\Phi_A \otimes \mathcal{G}(s) \otimes \Phi_B$$

- $\mathcal{G}$(s) is the Green's function of the BFKL equation
- the four-point function satisfies Regge factorization in LLA
- the two exchanged gluons become reggeized
- the BFKL Pomeron is the bound state of the reggeized gluons

Balitsky, Fadin, Kuraev, Lipatov

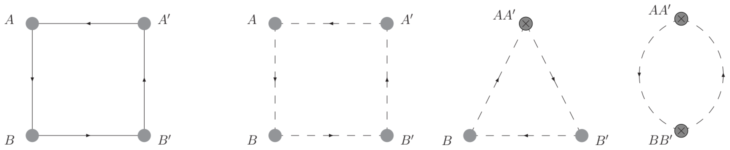
Scattering of R-currents (2)

- scattering of R-currents means evaluation of the amplitude

$$A_R(s,t) \sim \langle J_R^{\mu A}(x_A) J_R^{\nu B}(x_B) J_R^{\mu' A'}(x_{A'}) J_R^{\nu' B'}(x_{B'}) \rangle$$

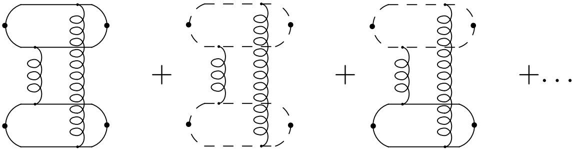
- fermions and scalars provide contributions to the amplitude in $\mathcal{N} = 4$ SYM
- QUESTIONS for R-current scattering:
  - Which diagrams are the leading ones?
  - Does the amplitude $A_R(s,t)$ factorize? If so:
  - What are the impact factors?

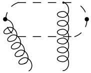
One-loop diagrams

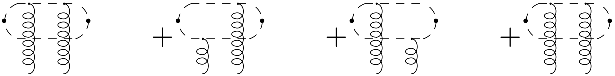


- contributions from fermions and scalars to the correlation function, altogether 3 fermionic and 12 scalar diagrams
- one-loop diagrams are again subleading at high energy $\sim \log^2 s$
- one-loop diagrams are UV finite because of supersymmetry, the divergencies of the fermionic and the scalar part cancel each other

Impact factors



- at high energy in $\mathcal{N} = 4$ SYM three-loop diagrams are dominating $\sim s$
- the amplitude factorizes, one gets separately fermion impact factors Φ_F and scalar impact factors Φ_S
- additional scalar diagrams like  are suppressed $\sim 1/s^2$
- the scalar impact factor is

$\Phi_S(\mathbf{k}_1, \mathbf{k}_2) =$ 

Results

- there are four impact factors $\Phi = \Phi_F + \Phi_S$ to compute: $\Phi^{LL'}$, $\Phi^{TT'}$, $\Phi^{LT'}$ and $\Phi^{TL'}$
- the results for $\Phi^{LL'}$ and $\Phi^{TT'}$ have got the same structure as the impact factors of $\gamma^*\gamma^*$-scattering
- a new feature is the vanishing of the helicity changing impact factors: $\Phi^{LT'} = \Phi^{TL'} = 0$
- the simplification is due to supersymmetry: the fermionic and scalar parts cancel each other

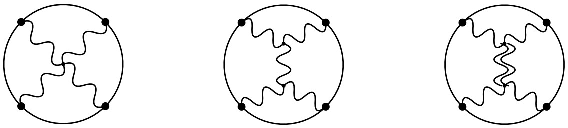
Outline

Partition function for J_R

- $SU(4)$ R-symmetry currents $J_R^{\mu A}$ of SYM are gauge invariant operators of type $\mathcal{O}$
- they correspond to $SU(4)$ gauge fields $A^{\mu A}$ on the supergravity side with the value $A_0^{\mu A}$ at the boundary
- one gets the partition function

$$
\begin{aligned}
Z_{J_R^A}[A_0] &= \int \mathcal{D}[\text{SYM fields}] \exp\left(-S_{\mathcal{N}=4\,\text{SYM}} + \int d^4x\, J_R^A A_0^A\right) \\
&= \exp\left(-S_{sugra}[A^A[A_0^A]]\right)
\end{aligned}
$$

Witten diagrams

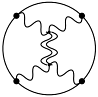


- Witten diagrams for the four-point function of R-currents
- there are only tree diagrams in the supergravity limit
- the currents are inserted at points of the boundary of AdS_5
- the interaction takes place in the interior
- one has boundary-to-bulk propagators for the gauge boson and bulk-to-bulk propagators for the gauge field and the graviton

Outline

Summary

- we computed the four-point function of R-currents in $\mathcal{N} = 4$ SYM in the Regge limit,
  computation of impact factors
- next steps:

  evaluation of corresponding Witten diagrams, 

  generalization to, e.g. six-point functions of R-currents (BKP Green's function, integrability)