de Sitter vacua in large volume scenarios

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(Soon in the arXiv)

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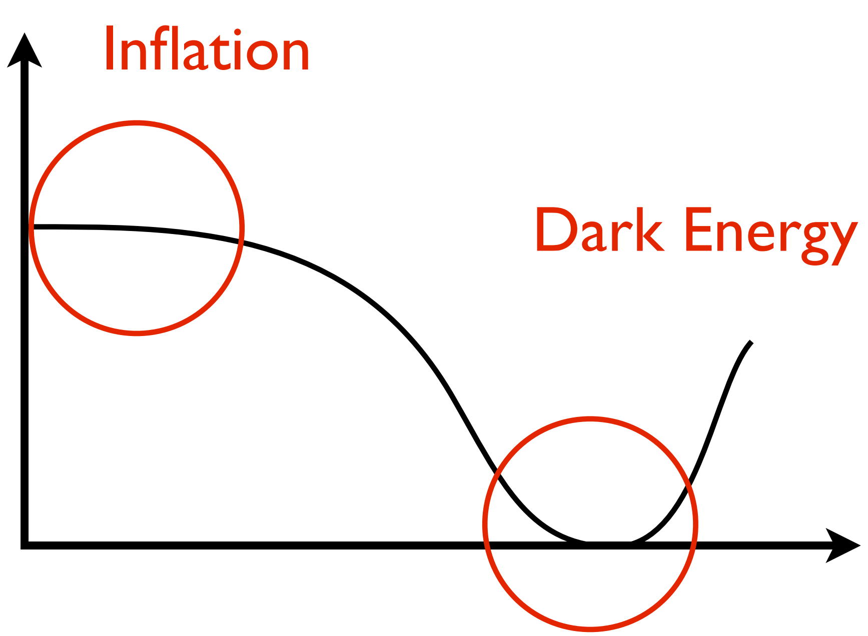
Outline

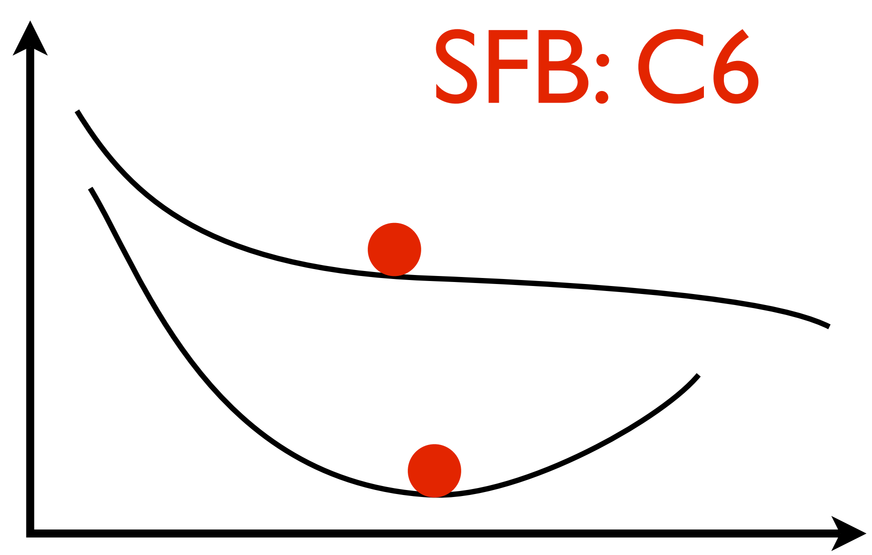
- Motivation
- Large volume scenarios
- Stability of vacua in supergravity
- de Sitter vacua in large volume scenarios
- Cosmological Implications
- Conclussions

Motivation

A satisfactory realisation of Dark Energy and Inflation in String Theory requires a better understanding of the properties of de Sitter vacua

$$V > 0$$

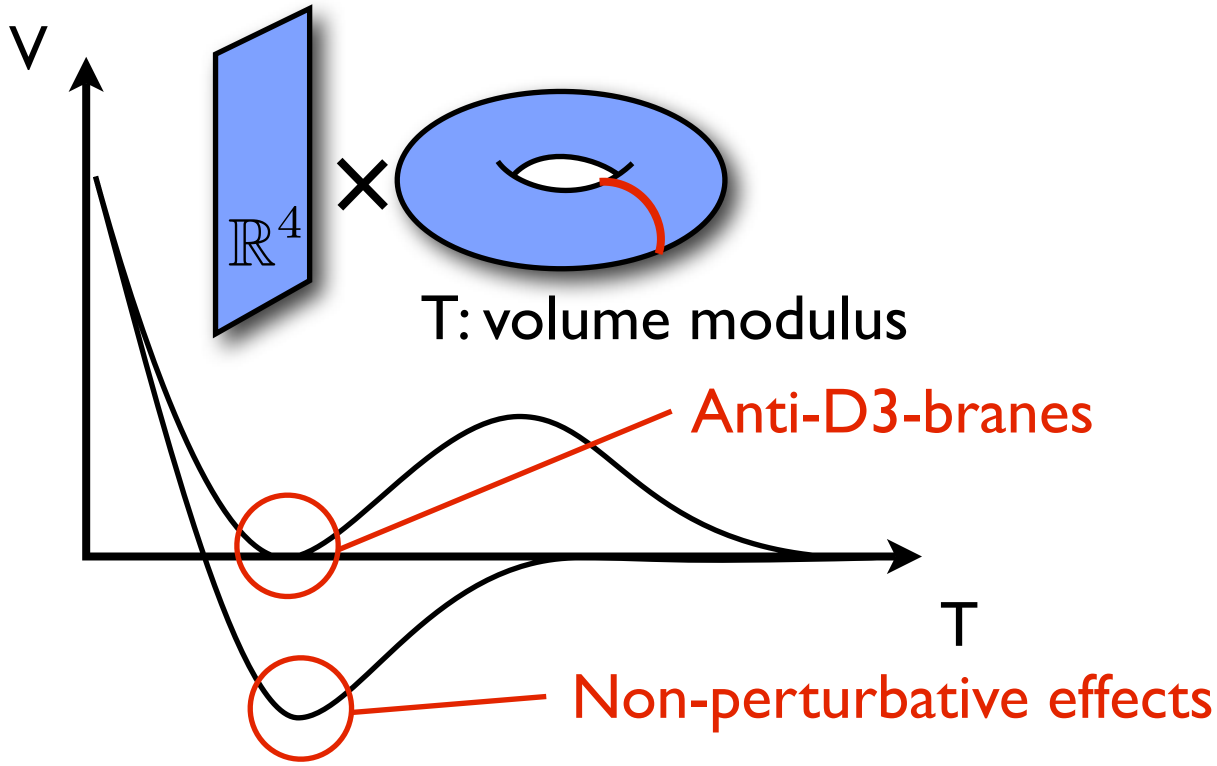




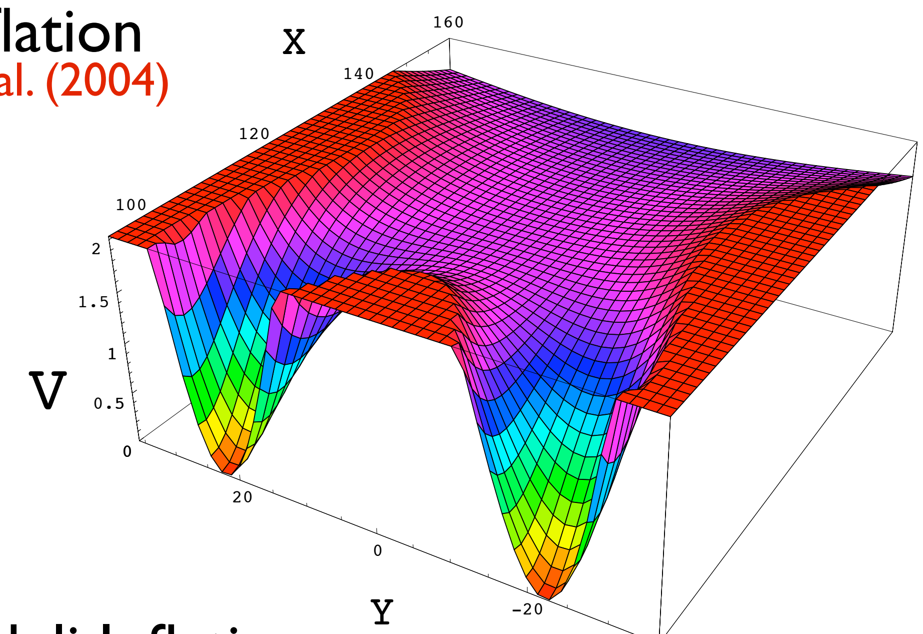
What are the properties of allowed de Sitter vacua?

Example: KKLT
Kallosh et.al. (2003)

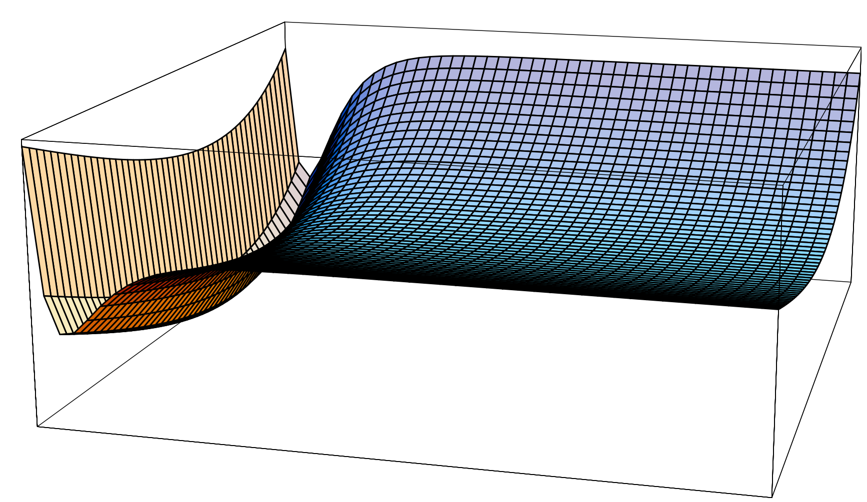
⟷ **Inflationary models (via closed strings)**



Racetrack Inflation
Blanco-Pillado et.al. (2004)



Kahler Moduli Inflation
Conlon & Quevedo (2005)

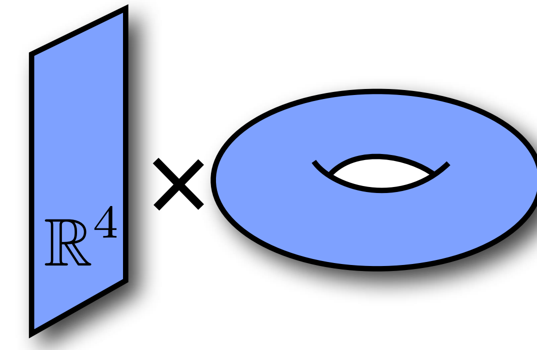


Problem:

SUSY explicitly broken by anti-D3 branes!

(Little control on the theory)

What if we do not include anti-D3-branes?



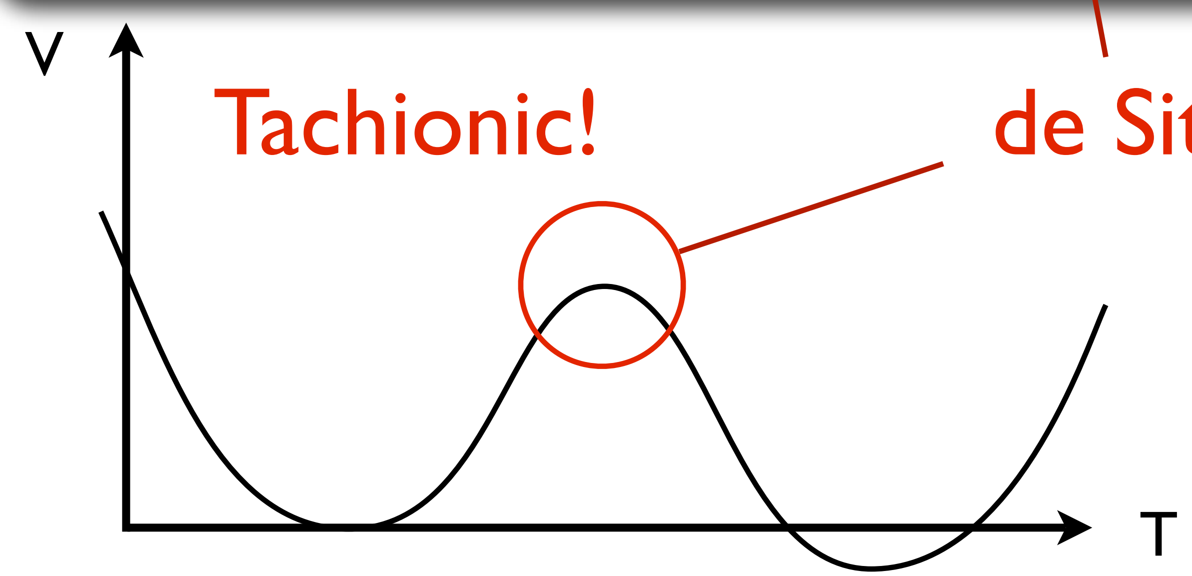
SUGRA: with a single volume modulus

$$V(T) = e^K \left(K^{T\bar{T}} |W_T - K_T W|^2 - 3|W|^2 \right)$$

Kahler potential ⟶ $K = -3\ln(T + \bar{T})$ Superpotential

Vacua with $V_T = 0$ & $V \geq 0$ characterised by $V_{T\bar{T}} \leq 0$

Brustein & Alwis (2004)



The problem is in the Kahler!

$$K = -3\ln(T + \bar{T})$$

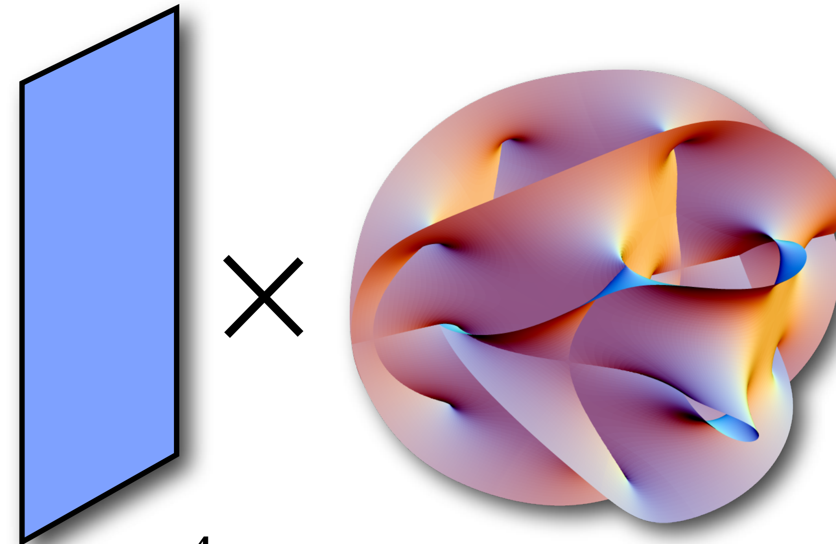
The big challenge: To find a de Sitter vacuum with stabilised moduli and spontaneous SUSY breaking (F-term uplifting)

Questions to be addressed in the rest of this talk:

- What is the generalisation of $V_{T\bar{T}} \leq 0$
- What are the necessary cond's for F-term uplifting?
- Are there sufficient cond's for F-term uplifting?

In this talk I do not address contributions from charged sectors (no D-terms)

Large volume scenarios



We want to consider compactifications on Calabi-Yau three-folds.

Recall P. Smyth's talk

$\mathbb{R}^4 \times \mathrm{CY}$

$\mathcal{N} = 1$ SUGRA effective description

$$S = \int \left[\frac{1}{2} R - K_{m\bar{n}} \partial \Phi^m \partial \bar{\Phi}^{\bar{n}} - V(\Phi, \bar{\Phi}) \right]$$

$$V_F(\Phi, \bar{\Phi}) = e^G (\bar{G}_{\bar{m}} K^{\bar{m}n} G_n - 3)$$

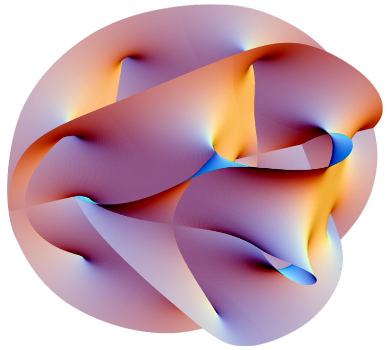
Kahler potential

Superpotential

$$K_{m\bar{n}} = \partial_m \partial_{\bar{n}} G \qquad \& \qquad G = K + \ln W + \ln \bar{W}$$

SUSY broken if and only if $D_i W \equiv W_i + K_i W = W G_i \neq 0$

Kahler Potential:
(Type IIB)



volume — dilaton — complex structure

$$K = -2\ln \mathcal{V} - \ln[S + \bar{S}] + K_{\mathrm{c.s.}}$$

The volume $\mathcal{V}$ is a function of $\tau^i = \rho^i + i\chi^i$

$$\mathcal{V} = \frac{1}{6}\mathcal{K}^{ijk} t_i t_j t_k \quad \text{with} \quad \rho^i = \partial_{t_i}\mathcal{V} = \frac{1}{2}\mathcal{K}^{ijk} t_j t_k$$

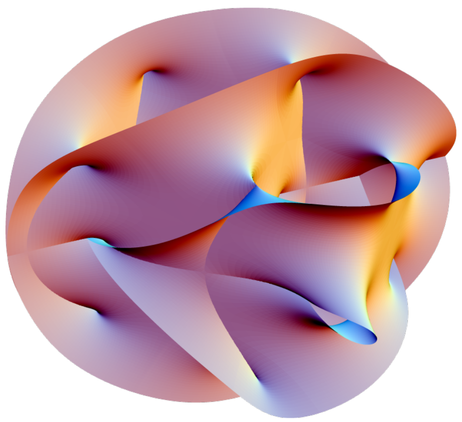
Intersection numbers $\mathcal{K}^{ijk}$: Constants parameterising the CY

The no-scale property is satisfied $K^i K_i = 3$

See for example: Grimm & Louis (2004)

Complex structure $K_{\mathrm{c.s.}} = -\ln i \int_{\mathrm{CY}} \Omega \wedge \bar{\Omega}$

The Superpotential (Type IIB)



- Non-perturbative effects
- Fluxes: geometric & non geom.

$$W = W_{\rm n.p.} + W_{\rm flux}$$

$$W_{\rm n.p.} \propto e^{-\sum_i a_i \tau^i}$$

Witten (1996); Tripathy & Trivedi (2003)

$$W_{\rm flux} = \frac{i}{6} m_0 \mathcal{K}^{ijk}\tau^i\tau^j\tau^k + \frac{1}{2} p_i \mathcal{K}^{ijk}\tau^j\tau^k - i q_i \tau^i + e_0$$
$$+ \frac{1}{6} n_0 S \mathcal{K}^{ijk}\tau^i\tau^j\tau^k + \frac{i}{2} S m_i \mathcal{K}^{ijk}\tau^j\tau^k - S e_i \tau^i + i S h_0$$

Shelton et.al. & Grana et.al. & Benmachiche et.al. (2006)

Integers given by fluxes

Brief comment on moduli stabilisation

$$V = e^G (K^{ij} G_i \bar{G}_{\bar{j}} - 3) + e^G K^{\alpha\beta}_{\text{c.s.}} G_\alpha \bar{G}_{\bar{\beta}} + e^G (S + \bar{S})^2 |G_S|^2$$

Volume moduli — Complex Structure — Dilaton

Provided that $W_i = 0$ then:
Dilaton and c.s. can be stabilised at $G_\alpha = 0$ & $G_S = 0$

Giddings, Kashru & Polshinski (2001)

Fluxes allow to construct Minkowski vacua with all moduli stabilised. These vacua are supersymmetric

Micu, Palti, & Tasinato (2007)

What about de Sitter vacua?

Vacua stability in SUGRA'S

$$V = e^G(\bar{G}_{\bar{\imath}} K^{\bar{\imath}j} G_j - 3)$$

Locally stable vacua: $V_m = G_m V + e^G(G_m + \bar{G}^i \nabla_m G_i) = 0$

Mass matrix $M = \begin{pmatrix} V_{m\bar{n}} & V_{mn} \\ V_{\bar{m}\bar{n}} & V_{\bar{m}n} \end{pmatrix}$ has to be positive definite

$$\begin{aligned} V_{m\bar{n}} \quad = \quad & K_{m\bar{n}} V - G_m \bar{G}_{\bar{n}} V + e^G K_{m\bar{n}} \\ & + e^G (\bar{\nabla}_{\bar{n}} \bar{G}_{\bar{\jmath}}) K^{\bar{\jmath} i} (\nabla_m G_i) - e^G \bar{G}^i G^{\bar{\jmath}} R_{\bar{n} i \bar{\jmath} m} \end{aligned}$$

$$R_{\bar{n} i \bar{\jmath} m} = \bar{\nabla}_{\bar{n}} \bar{\nabla}_{\bar{\jmath}} K_{im} \qquad \nabla_i G_j = \partial_i G_j - \Gamma^k_{ij} G_k$$

Positive definite mass matrix implies $u^{\bar{m}} V_{\bar{m}n} \bar{u}^n > 0$

In particular, if $u_i = e^{-G/2} G_i$ then $\lambda \equiv e^{-G} \bar{G}^m V_{m\bar{n}} G^{\bar{n}} > 0$

It turns out that λ depends only on G_i and the Riemman tensor describing the Kahler geometry

$$\lambda = 2\bar{G}^m G_m - \bar{G}^m G^{\bar{n}} \bar{G}^i G^{\bar{j}} R_{\bar{n} i \bar{j} m}$$

- There is a state in the spectrum with $m^2 \propto \lambda \begin{matrix} > 0 \\ < 0\,? \end{matrix}$

Thus Minkowski vacua with broken SUSY require the following necessary condition:

$$\bar{G}^i G_i = 3 \qquad \& \qquad \bar{G}^m G^{\bar{n}} \bar{G}^i G^{\bar{j}} R_{\bar{n} i \bar{j} m} < 6$$

Gomez-Reino & Scrucca (2004)

We can extend the previous analysis for de Sitter vacua

$$\lambda = -e^{-G}\frac{2}{3}V\bar{G}^{m}G_{m} + \sigma$$

Sign independent of G_i direction

Sign independent of G_i magnitud

$$\sigma = \frac{2}{3}(\bar{G}^{m}G_{m})^2 - \bar{G}^{m}G^{\bar{n}}\bar{G}^{i}G^{\bar{j}}R_{\bar{n}i\bar{j}m} \qquad V > 0$$

Thus, the problem reduces to studying the sign of σ

$\mathcal{K}^{111} \neq 0$

$\mathcal{K}^{112} \neq 0$

$\mathcal{K}^{123} \neq 0$

Example: Toroidal compactifications

$$K = -\sum_i n_i \ln(\tau^i + \bar{\tau}^i) \qquad \sum_i n_i - 3 = 0$$

$$\frac{3}{2}\sigma = (\sum_i x_i)^2 - 3\sum_i \frac{x_i^2}{n_i} \leq 0 \qquad x_i = \frac{n_i|G_i|^2}{(\tau^i + \bar{\tau}^i)^2} = \alpha n_i$$

What about non-trivial compactifications with cycles?

The complete spectrum of the theory $(\mathcal{N} = 1)$

Assume N (complex) moduli:

Moduli problem

- 1 modulus with mass² dictated by λ
- N-1 moduli with masses² dictated by $\nabla_i G_j$
- N moduli with masses² dictated by $\nabla_i \nabla_j G_k$

But recall that in type IIB string theory:

$$W_{\rm flux} = \frac{i}{6} m_0 \mathcal{K}^{ijk} \tau^i \tau^j \tau^k + \frac{1}{2} p_i \mathcal{K}^{ijk} \tau^j \tau^k - i q_i \tau^i + e_0$$
$$+ \frac{1}{6} n_0 S \mathcal{K}^{ijk} \tau^i \tau^j \tau^k + \frac{i}{2} S m_i \mathcal{K}^{ijk} \tau^j \tau^k - S e_i \tau^i + i S h_0$$

Thus 2N-1 moduli can be stabilised by wisely choosing W !!!

de Sitter vacua in large vol. scenarios (Type IIB)

Let us come back to string compactifications $K = -2 \ln \mathcal{V}$

$$\mathcal{V} = \frac{1}{6}\mathcal{K}^{ijk} t_i t_j t_k \quad \text{with} \quad \rho^i = \partial_{t_i} \mathcal{V} = \frac{1}{2}\mathcal{K}^{ijk} t_j t_k$$

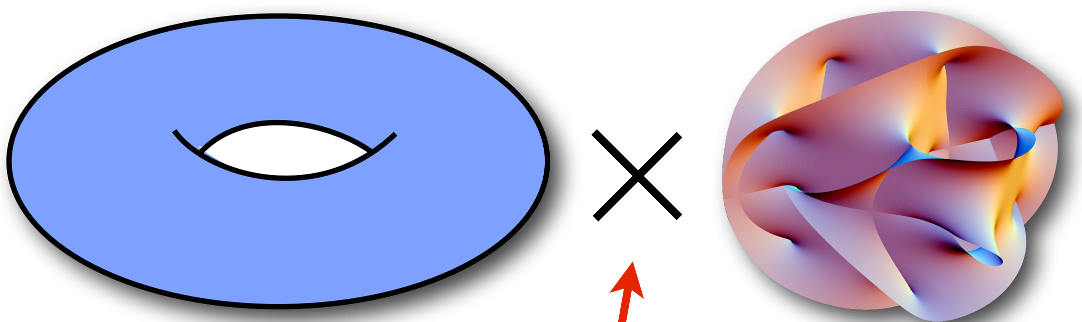
The Kahler metric is found as $K^{ij} = 8 e^{-K} \mathcal{K}^{ijk} K_k + K^i K^j$

And σ can be computed

$$\begin{aligned}\sigma \quad = \quad & -128 e^{-2K} G_i \bar{G}_{\bar{j}} G_n \bar{G}_{\bar{m}} \mathcal{K}^{ijp} K_{pq} \mathcal{K}^{mnq} + \frac{2}{3}(\bar{G}^m G_m)^2 \\ & +16 e^{-K} (\mathcal{K}^{ijk} G_i \bar{G}_{\bar{j}} G_k K^n \bar{G}_{\bar{n}} + \mathcal{K}^{ijk} G_i \bar{G}_{\bar{j}} \bar{G}_{\bar{k}} K^n G_n) \\ & +|K^{ij} G_i G_j - (K^i G_i)^2|^2 + 2|K^i G_i|^4 - 4|K^i G_i|^2 \bar{G}^m G_m\end{aligned}$$

We can obtain a no-go theorem for de Sitter vacua on several non-trivial backgrounds!

Example: Consider CY's of the form



$T^2 : \tau^1$ $\qquad$ $M_4 : \tau^a$

$K_{1a} = 0$

$$\mathrm{CY} = T^2 \times M_4 \quad (M_4 = T^4,\ K3)$$

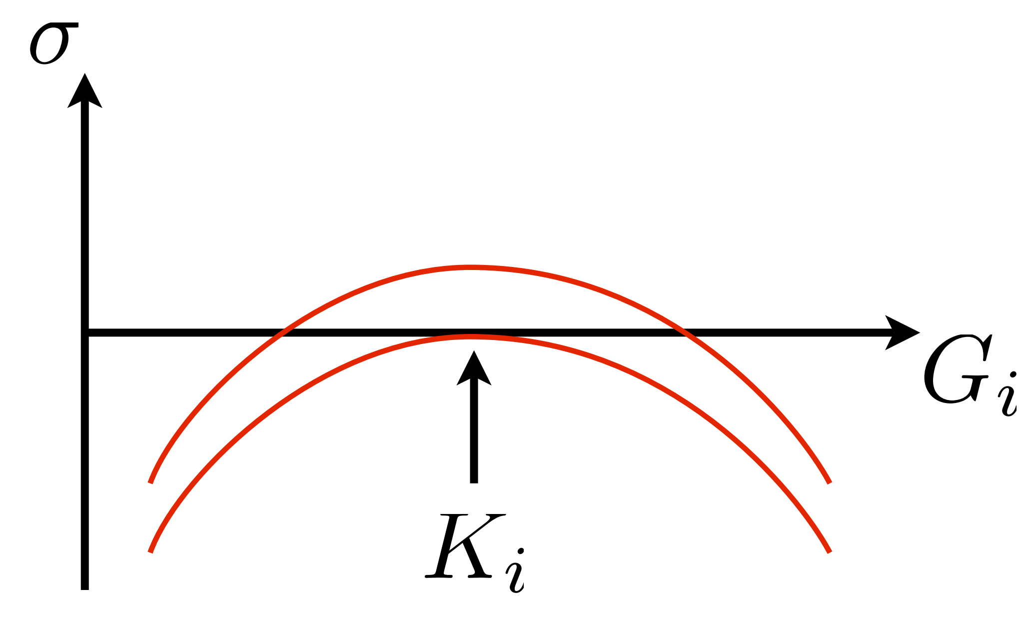
$$\sigma \leq -\frac{1}{3}(K^{ab}\bar{G}_{\bar{a}}G_b - 2|K^1 G_1|^2)^2$$

We may go further and classify compactifications

$\sigma \leq 0$ Type IIB with orientifolds	$\sigma \geq 0$
$\mathcal{K}^{1ab} \neq 0$ $\quad$ $\mathcal{K}^{iii} \neq 0$ $\quad$ $\mathcal{K}^{123} \neq 0$ $\quad$ etc...	Heterotic compactifications $\quad$ $d_{iii} \neq 0$ $\quad$ etc...

Generic: There is a local maximum of σ at $G_i = K_i$

$$\sigma = -2 \left| \eta_i - \frac{1}{3}(K^m \eta_m) K_i \right|^2 + \mathcal{O}(\epsilon^3)$$



$$\eta_i = \epsilon_i + \bar{\epsilon} \qquad \epsilon_i = \frac{W_i}{W}$$

Question: How do we evade the no-go result?

- alpha'-corrections ⟶ $K = -2\ln(V + \xi)$
- mixing with dilaton ⟶ $K^i K_i = 4$
- adding matter sectors ⟶ $K = -2\ln\mathcal{V} + K_{\rm mat}(\phi, \bar{\phi}, \mathcal{V})$

Consequences for Cosmology PRELIMINARY!

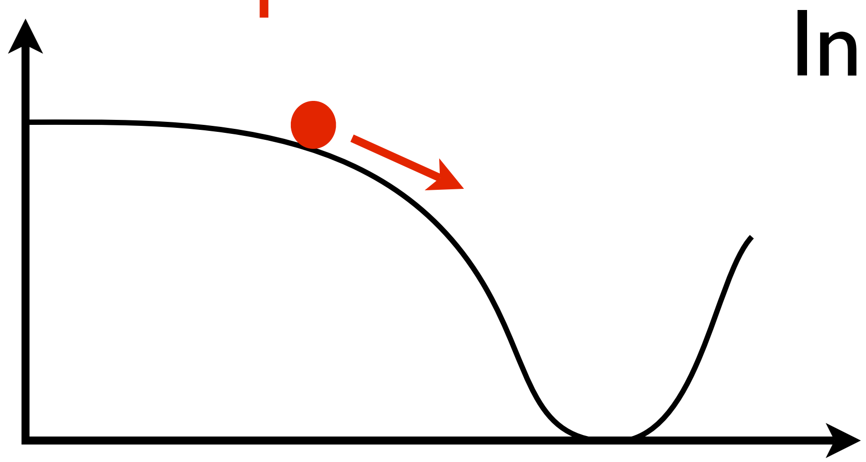
It is possible to extract a tight constraint on Inflation

$$\lambda = -e^{-G}\frac{2}{3}V\bar{G}^m G_m + \sigma + 2e^{-G}(\bar{G}^m V_m + G^{\bar{n}} V_{\bar{n}})$$

The η parameter can be computed $\eta = \dfrac{e^G \lambda}{V\bar{G}^m G_m}$

$$\eta \sim -\frac{2}{3} + \frac{2}{V\bar{G}^m G_m}(\bar{G}^m V_m + G^{\bar{n}} V_{\bar{n}}) \quad \& \quad \epsilon = \frac{V^m V_m}{V^2}$$

Slow roll parameters



Independent of the number of moduli!

Thus having η and ϵ both small irreconcilable!

Conclusions

- We have presented a stability analysis of vacua in string theory
- It was shown that compactifications of the form $\mathrm{CY} = T^2 \times M_4$ do not admit de Sitter vacua (many other examples don't admit de Sitter vacua)
- These results are avoided by breaking the no-scale property of the Kahler
- This analysis is relevant for Cosmology