Theoretical aspects of flavour mixing and unstable particles

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Motivation

- mixing and instability are fundamental properties of elementary particles that, in general, concur in nature
- important to construct a comprehensive renormalization scheme that simultaneously takes into account both properties in a physically consistent and mathematically rigorous way
- of relevance for future high-precision tests of electroweak physics at the LHC and the ILC

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Short history

- the renormalizability of the SM without quark-flavour mixing was proven in 1972 and was decorated by a Nobel price in 1999
- the elements of mixing matrices appear as basic parameters in the bare Lagrangian $\Rightarrow$ subject to renormalization, too
- first realized for the Cabibbo angle in the SM with two fermion generations [W.J. Marciano and A. Sirlin, 1975]
- the extension to the CKM matrix of the three-generation SM was attacked fifteen years later [A. Denner and T. Sack, 1990]

Status of the research

- a consistent on-shell renormalization condition for the CKM matrix, which complies with the criteria of UV finiteness, gauge independence, unitarity and flavour democracy was proposed just recently [K.P.O. Diener and B.A. Kniehl, 2001]
- the concept was then generalized to extensions of the SM, in which the lepton sector contains Majorana neutrinos [K.P.O. Diener and B.A. Kniehl, 2001]
- more recently, an explicit on-shell framework to renormalize the CKM matrix at the one-loop level was proposed [B.A. Kniehl, A. Sirlin, 2006]

CKM renormalization: main idea

- a simple generalisation of Feynman's approach in QED
- based on the separation of external-leg mixing corrections into gauge-independent self-mass (*sm*) and gauge dependent wave-function renormalization (*wfr*) contributions
- the renormalization of the two contributions is carried out in two equivalent frameworks:
  - (i) the renormalized quark matrices are diagonal and the nondiagonal mass counterterm matrices are employed to cancel all the divergent *sm* contributions, and also their finite parts up to Hermiticity constraints
  - (ii) the diagonalization of the complete mass matrices derived in the first approach

Mass counterterms generation

- the bare mass term: $-\overline{\psi}'_R m'_0 \psi'_L + \mathrm{H.c.}$
- write $m'_0 = m' - \delta m'$
- take a biunitary transformation of $\overline{\psi}', \psi'$ that diagonalizes m'
- in the new framework the mass term reads

$$-\overline{\psi}(m - \delta m^{(-)} a_- - \delta m^{(+)} a_+)\psi = -\overline{\psi}_R(m - \delta m^{(-)})\psi_L - \overline{\psi}_L(m - \delta m^{(+)})\psi_R$$

- m is real, diagonal and positive
- $\delta m^{(\pm)}$ are arbitrary nondiagonal matrices subject to the Hermiticity condition

$$\delta m^{(+)} = \delta m^{(-)\dagger}$$

- adjust $\delta m^{(\pm)}$ to cancel, as much as possible, the *sm* contributions to the self-energy corrections to the external legs

Mass counterterms generation: W-decay

- the *sm* corrections are fully cancelled in the amplitude associated with V_{ud}, the most accurately measured CKM parameter
- the residual finite contributions to other channels are very small
- the proof of gauge independence and finiteness of the remaining one-loop corrections reduces to that in the unmixed, single-generation case

Alternative framework

- diagonalization of the complete mass matrix obtained before

$$m - \delta m^{(-)} a_- - \delta m^{(+)} a_+$$

- by means of a biunitary transformation

$$\psi_L = U_L \hat{\psi}_L,\ \psi_R = U_R \hat{\psi}_R$$

- at one-loop level it is sufficient to approximate

$$U_L = 1 + i h_L,\ U_R = 1 + i h_R$$

- the Hermitian matrices h_L, h_R are chosen such that after applying the biunitary transformation the complete mass matrix is diagonal

CKM counterterm: W-decay

- $Wq_i\bar{q}_j$ interaction

$$\mathcal{L}_{Wq_i\bar{q}_j} = -\frac{g_0}{\sqrt{2}}\overline{\psi}_L^U V\gamma^\lambda\psi_L^D W_\lambda + H.c.$$

- after applying the biunitary transformation

$$\mathcal{L}_{Wq_i\bar{q}_j} = -\frac{g_0}{\sqrt{2}}\overline{\hat{\psi}}_L^U (V - \delta V)\gamma^\lambda\hat{\psi}_L^D W_\lambda + H.c.$$

with $\delta V = i(h_L^U V - V h_L^D)$

- $V - \delta V$ satisfies the unitarity condition
- V is finite and unitary $\Rightarrow$ identified with the renormalized CKM matrix
- δV identified with the CKM counterterm

Important to emphasize:

- both the renormalized CKM matrix V and its bare counterpart $V_0 = V - \delta V$ are explicitly gauge independent and satisfy the unitarity constraints

$$V^\dagger V = 1 \text{ and } V_0^\dagger V_0 = 1$$

  through the order of calculation

Effects on the partial hadronic decay widths of W boson

Partial hadronic decay widths (in GeV) of the W boson

Partial width	Born	One loop		
		[1]	[2]	$\overline{\mathrm{MS}}$-scheme†
$\Gamma(W \to ud)$	0.64898424	0.67583988	0.67583980	0.67583622
$\Gamma(W \to us) \times 10$	0.33949480	0.35353374	0.35353286	0.35354191
$\Gamma(W \to ub) \times 10^4$	0.10869466	0.13570269	0.12622422	0.11479695
$\Gamma(W \to cd) \times 10$	0.33931882	0.35348251	0.35346800	0.35346414
$\Gamma(W \to cs)$	0.64731343	0.67434688	0.67431437	0.67431636
$\Gamma(W \to cb) \times 10^2$	0.10863766	0.11758566	0.11883431	0.11708201
$\Gamma(W \to \mathrm{hadrons})$	1.36527628	1.42207781	1.42205523	1.42203548

[1] P. Gambino, P.A. Grassi and F. Madricardo, 1999

[2] B.A. Kniehl and A. Sirlin, 2006

† the 't Hooft mass scale was chosen to be $\mu = m_W$

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Short history

- during the past decade, essential progress in the gauge-independent treatment of unstable particles was made
- the concept of pole mass, based on the complex-valued position of the propagator's pole, was introduced in the standard electroweak theory
- this had important implications for the phenomenology of Z and W bosons and for the top quarks
- the gauge independence of the pole mass of unstable particles in the SM was formally proven to all orders [P. Gambino and P.A. Grassi, 2000]
- the usefulness of the pole definition of wave-function renormalization for the order-by-order removal of UV divergences in the renormalized self-energy was emphasized [M.L. Nekrasov, 2002]
- however, the possibility of particle mixing was not taken into account

Status of the research

- the differences between the pole and on-shell definitions of mass and total decay width of the SM Higgs boson was quantitatively investigated
- the on-shell renormalization scheme leads to considerable gauge dependences for large values of the Higgs boson mass [B.A. Kniehl and A. Sirlin, 1998]
- a pole definition of wave-function renormalization for unstable particles, valid to all orders of perturbation theory [B.A. Kniehl and A. Sirlin, 2002]
- it leads to exact expressions for renormalized self-energies and propagators, which are thus manifestly devoid of UV singularities at all orders
- the power-like infrared singularities that are generated in the on-shell scheme by the emission of massless gauge bosons from unstable particles are eliminated

Status of the research

- physical and unphysical threshold singularities which can arise from the wave-function renormalization in the on-shell scheme disappear
- a definition of decay branching fraction and partial decay width of unstable particles in gauge theories was also proposed [P.A. Grassi, B.A. Kniehl and A. Sirlin, 2002]
- it satisfies the basic principles of additivity and gauge independence
- very recently, the concepts of pole mass, width and propagators were extended to unstable fermions [B.A. Kniehl, A. Sirlin, 2008]
- these concepts are of grate significance given the fact that, with the exception of the electron, the lightest neutrino, and the proton (or, at the elementary level, the u quark), all known fundamental fermions are unstable particles

On-shell formulation

- the mass and widths of unstable scalar bosons are conventionally defined in an on-shell formulation as

$$M_{\mathrm{os}}^2 = M_0^2 + \mathrm{Re}A(M_{\mathrm{os}}^2)$$

$$M_{\mathrm{os}}\Gamma_{\mathrm{os}} = -\frac{\mathrm{Im}A(M_{\mathrm{os}}^2)}{1 - \mathrm{Re}A'(M_{\mathrm{os}}^2)}$$

- the two equations become gauge-dependent at NNLO
- Γ_{os} leads to serious unphysical singularities if $A(s)$ is not analytic in the neighbourhood of M_{os}^2

Pole formulation

- way to solve the problems with the on-shell formulation: use the complex-valued position of the propagator's pole

$$\bar{s} = M_0^2 + A(\bar{s})$$

- writting $\bar{s} = M^2 - iM\Gamma \Rightarrow$ the pole formulation

$$M^2 = M_0^2 + \mathrm{Re}A(\bar{s})$$

$$M\Gamma = -\mathrm{Im}A(\bar{s})$$

- equivalently one may write

$$M\Gamma = -Z\mathrm{Im}A(M^2),\ Z = \frac{1}{1 + \mathrm{Im}\left(A(\bar{s}) - A(M^2)\right)/(M\Gamma)}$$

- consequence: gauge independent formulation and the threshold singularities are solved

Renormalized propagator of unstable fermions

- using the decomposition: $iS^{(r)}(p) = i\left[S_+^{(r)}(p)a_+ + S_-^{(r)}(p)a_-\right]$
- the renormalized propagator is:

$$S_\pm^{(r)}(p) = \frac{\not p\left[1 - B_\pm(p^2)\right] + \sqrt{Z_\mp/Z_\pm}\,M_0\left[1 + A_\mp(p^2)\right]}{|Z_\pm|\,D(p^2)}$$

- and the renormalization constants:

$$Z_\pm = \frac{1 + R_\pm}{2F\left[1 - B_\pm(M^2)\right]},\quad R_\pm = \frac{1 + A_\pm(M^2)}{1 + A_\mp(M^2)},\quad F = 1 - M^2\frac{f'(M^2)}{f(M^2)}$$

- where

$$\Sigma(p) = \Sigma_+(p)a_+ + \Sigma_-(p)a_-,\ \Sigma_\pm = \not p B_\pm(p^2) + M_0 A_\pm(p^2)$$

$$D(p^2) = \left[1 - B_+(p^2)\right]\left[1 - B_-(p^2)\right]\left[p^2 - M_0^2 f(p^2)\right]$$

$$f(p^2) = \frac{\left[1 + A_+(p^2)\right]\left[1 + A_-(p^2)\right]}{\left[1 - B_+(p^2)\right]\left[1 - B_-(p^2)\right]}$$

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Summary

- review of the status of the research in both directions: flavour mixing renormalization and unstable particles
- an explicit on-shell renormalization framework for the CKM matrix, based on the separation of the external-leg mixing corrections into gauge-independent *sm* and gauge-dependent *wfr* contributions
- first results on its effects on the partial hadronic decay widths of the W boson
- pole mass and width of unstable scalar bosons and the comparison with the on-shell formulation
- closed form for the renormalized propagator of unstable fermions with no mixing

Outlook

- central objective
  - develop a general renormalization scheme that reconciles mixing and instability of elementary particles in a consistent and rigorous way
  - explore its phenomenological implications for precision tests of the SM, the MSSM and alternative theories of new physics
- step-by-step
  - the top-quark decay $t \to bl^+\nu_l$ in the SM will serve as a simple starting point for the study of the interplay between flavour mixing and instability
  - incorporation of Majorana neutrinos and bosons
  - generalisation to all orders: a general pole scheme of mixing renormalization for unstable fermions and bosons valid to all orders

BACKUP SLIDES

Flavour mixing renormalization

QED

- self-energy contribution to an outgoing fermion

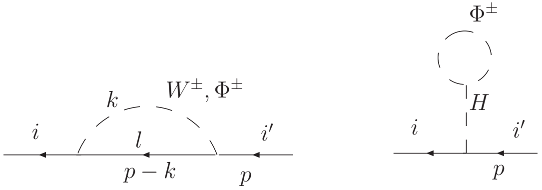
$$\Delta\mathcal{M}^{\mathrm{leg}} = \bar{u}(p)\Sigma(\not{p})\frac{1}{\not{p}-m}$$

with $\Sigma(\not{p}) = \underbrace{A + B}_{\text{divergent constants}}(\not{p}-m) + \underbrace{\Sigma_{\mathrm{fin}}(\not{p})}_{\text{finite part proportional to } (\not{p}-m)^2 \text{ in the vicinity of } \not{p}=m}$, the self-energy

- A, referred to as *sm*, has a pole at $\not{p}=m$ and is gauge independent $\Rightarrow$ cancelled by the mass counterterm
- B, referred to as *wfr*, is regular at $\not{p}=m$ but gauge dependent $\Rightarrow$ combined with the proper vertex diagrams leading to a gauge-independent result

CKM matrix

- unlike the QED case, one encounters not only diagonal terms but also off-diagonal external-leg contributions generated by the Feynman diagrams below



$\Longrightarrow$ self-energy corrections to an external-leg are of the form

$$\Delta\mathcal{M}_{ii'}^{\text{leg}} = \bar{u}_i(p)\Sigma_{ii'}(\not{p})\frac{1}{\not{p} - m_{i'}}$$

- all other contributions (Z^0, Φ^0, γ, H) as well as additional tadpole diagrams lead to diagonal expressions of the usual kind

Renormalization framework

- evaluate the contributions in R_ξ gauge, treating the i and i' quarks on an equal footing, e.g.,

$$\begin{aligned} 2\not{p}a_- &= \not{p}a_- + a_+\not{p} \\ &= (\not{p} - m_i)a_- + a_+(\not{p} - m_{i'}) + m_i a_- + m_{i'} a_+ \end{aligned}$$

- the contributions to $\Sigma_{ii'}(\not{p})$ can be classified in four classes:
  (i) terms with a left factor $(\not{p} - m_i)$
  (ii) terms with a right factor $(\not{p} - m_{i'})$
  (iii) terms with a left factor $(\not{p} - m_i)$ and a right factor $(\not{p} - m_{i'})$
  (iv) constant terms not involving $\not{p}$

Terms of class (iii)

- generic form: $(\not{p} - m_i)\ \Gamma_{ii'}(p^2)\ (\not{p} - m_{i'})$
- when inserted into the self-energy corrections to an external leg,

$$\Delta\mathcal{M}_{ii'}^{\mathrm{leg}} = \bar{u}_i(p)\Sigma_{ii'}(\not{p})\frac{1}{\not{p} - m_{i'}} = \bar{u}_i(p)(\not{p} - m_i)\ \Gamma_{ii'}(p^2)\ (\not{p} - m_{i'})\frac{1}{\not{p} - m_{i'}}$$

$$\Delta\mathcal{M}_{ii'}^{\mathrm{leg}} = \underbrace{\bar{u}_i(p)(\not{p} - m_i)}_{= 0 \text{ due to Dirac's equation}}\ \Gamma_{ii'}(p^2)$$

- these kind of terms vanish

Terms of classes (i) and (ii)

- generic form: $(\not{p} - m_i)\, \Gamma_{ii'}(p^2)$ and $\Gamma_{ii'}(p^2)\, (\not{p} - m_{i'})$ respectively
- contain gauge-dependent parts
- when inserted into the self-energy corrections to an external leg,

$$\Delta\mathcal{M}^{\mathrm{leg}}_{ii'} = \begin{cases} \bar{u}_i(p)\Gamma_{ii'}(p^2)\delta_{ii'} \\ \\ \bar{u}_i(p)\Gamma_{ii'}(p^2) \end{cases}$$

- nonsingular as $\not{p} \to m_{i'}$
- suitable to be combined with the proper vertex diagrams
- identified with *wfr* contributions

Terms of class (iv)

- generic form: $\Gamma_{ii'}(p^2)$
- are gauge-independent
- when inserted into the self-energy corrections to an external leg,

$$\Delta\mathcal{M}^{\mathrm{leg}}_{ii'} = \bar{u}_i(p)\Sigma_{ii'}(\not p)\frac{1}{\not p - m_{i'}} = \bar{u}_i(p)\Gamma_{ii'}(p^2)\frac{1}{\not p - m_{i'}}$$

- retain the virtual-quark propagator and are singular in this limit
- identified with *sm* contributions

- for completeness, in the case of an outgoing up-type quark, the *sm* contributions read

$$
\begin{aligned}
\Delta\mathcal{M}^{\mathrm{sm}}_{ii'} = {} & \frac{\alpha}{8\pi s_W^2} V_{il} V^{\dagger}_{li'} \bar{u}_i(p) \left\{ m_i \left(1 + \frac{m_i^2}{2m_W^2}\Delta\right) \right. \\
& + \left[m_i a_- + m_{i'} a_+ + \frac{m_i m_{i'}}{2m_W^2} (m_i a_+ + m_{i'} a_-) \right] \left[I(m_i^2, m_l) - J(m_i^2, m_l) \right] \\
& \left. - \frac{m_l^2}{2m_W^2} (m_i a_- + m_{i'} a_+) \left[3\Delta + I(m_i^2, m_l) + J(m_i^2, m_l) \right] \right\} \frac{1}{\not{p} - m_{i'}},
\end{aligned}
$$

$$
\Delta = \frac{1}{n-4} + \frac{\gamma_E - \ln(4\pi)}{2} + \ln\left(\frac{m_W}{\mu}\right),
$$

$$
\{I(m_i^2, m_l); J(m_i^2, m_l)\} = \int_0^1 dx \{1; x\} \ln \frac{m_l^2 x + m_W^2(1-x) - p^2 x(1-x) - i\varepsilon}{m_W^2}
$$

- *sm* contribution:

$$\frac{\alpha}{8\pi s_W^2} V_{il} V_{li'}^\dagger \bar{u}_i(p) \left\{ m_i \left(1 + \frac{m_i^2}{2m_W^2} \Delta \right) \right.$$
$$+ \left[m_i a_- + m_{i'} a_+ + \frac{m_i m_{i'}}{2m_W^2} \left(m_i a_+ + m_{i'} a_- \right) \right] \left[I(m_i^2, m_l) - J(m_i^2, m_l) \right]$$
$$\left. - \frac{m_l^2}{2m_W^2} \left(m_i a_- + m_{i'} a_+ \right) \left[3\Delta + I(m_i^2, m_l) + J(m_i^2, m_l) \right] \right\} \frac{1}{\not{p} - m_{i'}},$$

- mass term:

$$-\bar{u}_i(p)(\delta m_{ii'}^{(-)} a_- + \delta m_{ii'}^{(+)} a_+) \frac{1}{\not{p} - m_{i'}}$$

- the mass counterterms are chosen to exactly cancel all the contributions in the $i' = i$, $(uc,\ ut,\ ct)$ channels
- there are residual *sm* contributions in the cu, tu, tc channels, but finite, gauge independent and numerically very small

Unstable particles

Unstable fermions

- of covariance grounds, the fermion self-energy is of the form

$$\Sigma(p) = \Sigma_+(p)a_+ + \Sigma_-(p)a_-,\ \Sigma_\pm(p) = \not{p}B_\pm(p^2) + M_0 A_\pm(p^2)$$

- the fermion propagator is: $iS(p) = \dfrac{i}{\not{p} - M_0 - \Sigma(p)}$
- evaluating the invers of the denominator one finds

$$S(p) = \frac{1}{D(p^2)}\left\{\left\{\not{p}\left[1 - B_+(p^2)\right] + M_0\left[1 + A_-(p^2)\right]\right\} a_+ \right.$$

$$\left. + \left\{\not{p}\left[1 - B_-(p^2)\right] + M_0\left[1 + A_+(p^2)\right]\right\} a_- \right\}$$

with

$$D(p^2) = \left[1 - B_+(p^2)\right]\left[1 - B_-(p^2)\right]\left[p^2 - M_0^2 f(p^2)\right]$$

$$f(p^2) = \frac{\left[1 + A_+(p^2)\right]\left[1 + A_-(p^2)\right]}{\left[1 - B_+(p^2)\right]\left[1 - B_-(p^2)\right]}$$

Unstable fermions

- introducing the definitions

$$\Sigma_{1,2}(p) = \frac{1}{2}\left[\Sigma_+(p) \pm \Sigma_-(p)\right]$$

$$A_{1,2}(p^2) = \frac{1}{2}\left[A_+(p^2) \pm A_-(p^2)\right]$$

- the propagator becomes

$$S(p) = \frac{1}{\not{p} - M_0 - \Sigma_{\text{eff}}(p)}\left[1 - \Sigma_P(p)\gamma_5\right]$$

where

$$\Sigma_{\text{eff}}(p) = \Sigma_1(p) + \frac{\Sigma_2(p)\left[\Sigma_2(p) - 2M_0 A_2(p^2)\right]}{C(p)}, \quad \Sigma_P(p) = \frac{\Sigma_2(p)}{C(p)}$$

$$C(p) = \not{p} - \Sigma_1(p) + M_0\left[1 + 2A_1(p^2)\right]$$