Axions

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Werkstattseminar

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Plan:

1) Theta-term in QCD

2) Strong CP violation

   • $d_n(\bar{\theta})$

3) Solutions of strong CP puzzle

4) Axions from UV completions

5) Axion Cold Dark Matter

1) Theta-term in QCD

Most general gauge invariant Lagrangian of QCD up to dim 4 operators:

$$\mathcal{L} = -\frac{1}{4} G^a_{\mu\nu} G^{a,\mu\nu} + \bar{q}(i\cancel{D} - \mathcal{M}_q) q + \underbrace{\theta \frac{\alpha_s}{8\pi} G^a_{\mu\nu} \tilde{G}^{a,\mu\nu}}_{\text{Theta-term}}$$

$(\tilde{G}_{\mu\nu} = \frac{1}{2} \varepsilon_{\mu\nu\lambda\rho} G^{\lambda\rho})$

Theta-term

- total derivative:

$$G\tilde{G} = \partial_\mu K^\mu$$

Chern-Simons current

→ does not contribute in perturbation theory

- K^μ not invariant under "large", topol. non-trivial gauge transformations

→ Θ angular par., $-\pi \le \theta \le \pi$

→ plays role non-perturbatively

- Θ belongs to the fundamental parameters of QCD, on similar footing as α_s and the quark masses m_u, m_d, ..., which have to be determined experimentally.

- In fact, the actual physical parameter is

$$\bar{\Theta} = \Theta + \arg\det M_q$$

Real quark mass | Phase from Yukawa coupling | Angle variable | CP-odd quantity $\sim \mathbf{E}\cdot\mathbf{B}$

$$\mathcal{L}_{QCD} = \sum_q \bar{\psi}_q (\mathrm{i}D - m_q e^{\mathrm{i}\theta_q})\psi_q - \tfrac{1}{4} G_{\mu\nu a} G_a^{\mu\nu} - \Theta \frac{\alpha_s}{8\pi} G_{\mu\nu a} \tilde{G}_a^{\mu\nu}$$

Remove phase of mass term by chiral transformation of quark fields

$$\psi_q \to e^{-\mathrm{i}\gamma_5 \theta_q/2}\psi_q$$

$$\mathcal{L}_{QCD} = \sum_q \bar{\psi}_q (\mathrm{i}D - m_q)\psi_q - \tfrac{1}{4} GG - \underbrace{(\Theta - \arg\det M_q)}_{-\pi \le \bar{\Theta} \le +\pi} \frac{\alpha_s}{8\pi} G\tilde{G}$$

- $\bar{\Theta}$ can be traded between quark phases and $G\tilde{G}$ term
- No physical impact if at least one $m_q = 0$

$\bar{\Theta}$ unphysical if at least one $m_q = 0$

- The chiral anomaly

$$\partial_\mu \left(\bar{q} \gamma_\mu \gamma_5 q \right)$$

$$= n_f \frac{\alpha_s}{8\pi} G\tilde{G}$$

$$+ 2i \left[\bar{q}_R m_q q_L - h.c. \right]$$

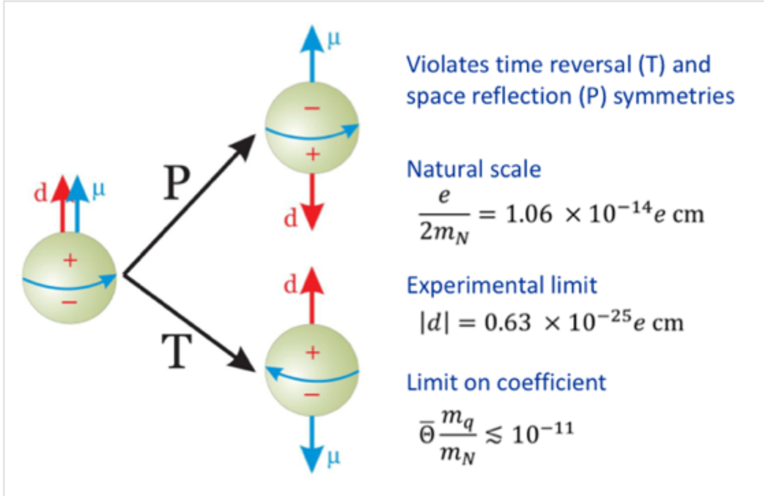
allows to shuffle contributions between $G\tilde{G}$ $\leftrightarrow$ imaginary quark masses

2) Determination of $\bar{\theta}$

- Theta-term ($G\tilde{G} \sim E \cdot B$) violates P an CP

→ leads to CP violation in flavour conserving interactions in contrast to CKM phase which leads to CP violation in flavour changing interactions

- Electric dipole moment (EDM) of neutron: very sensitive probe of CP violation in flavour-conserving interactions



- neutral non-rel. particle placed in E and B field described by

$$H = -\underset{\substack{\uparrow \\ \text{MDM}}}{\mu}\,\frac{\vec{B}\cdot\vec{S}}{S} - \underset{\substack{\uparrow \\ \text{EDM}}}{d}\,\frac{\vec{E}\cdot\vec{S}}{S}$$

- Nonzero $d_n \rightarrow$ both $\cancel{P}$ and $\cancel{T}$

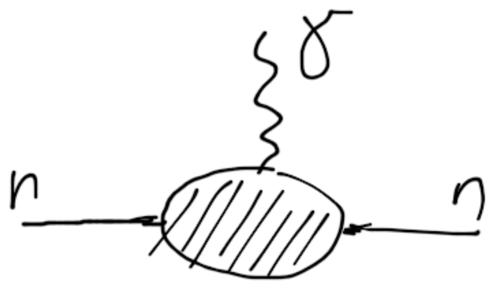
- Conclusion about $\cancel{CP}$ relies on validity of CPT

- Operator definition:

$$H_{T,P\text{-odd}} = -d_n \vec{E} \cdot \frac{\vec{S}_n}{S_n}$$

$\Longrightarrow$

$$\mathcal{L} = -d_n \frac{i}{2} \bar{\psi}_n \sigma^{\mu\nu} \gamma_5 \psi_n F_{\mu\nu}$$



- Calculations of $d_n(\bar{\theta})$:

• educated guess:

$$d_n(\bar{\theta}) \sim e\bar{\Theta} \frac{m_*}{m_n^2}$$

$$\sim \bar{\Theta}\,(6\times 10^{-17})\,e\,\mathrm{cm}$$

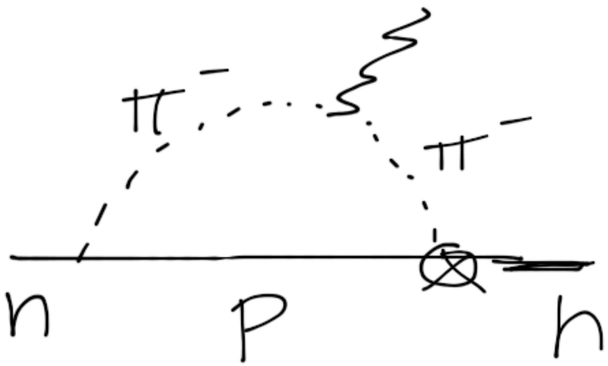
with reduced quark mass

$$m_* = \frac{m_u m_d}{m_u + m_d}$$

- Chiral estimate :

[Crewther, di Vecchia, Veneziano, Witten '79]

$$d_n(\bar\theta) = \frac{e\bar\theta\, m_*}{f_\pi^2} \times \left(\frac{0.9}{4\pi^2}\ln\left(\frac{\Lambda}{m_\pi}\right) + C\right)$$



- Sum rules

$$d_n(\bar\theta) = 1.2 \pm 0.5 \times 10^{-16}\ \bar\theta\ e\,cm$$

[Pospelov, Ritz 00]

update :

[Hisano et al. 12]

$$d_n(\bar\theta) = 4.2 \times 10^{-17}\bar\theta\ e\,cm$$

- **CP puzzle:**

expectation

$$d_n(\bar{\theta}) \sim 10^{-16}\,\bar{\theta}\ \mathrm{ecm}$$

experimental limit:

$$|d_n| < 2.9\times 10^{-26}\ \mathrm{ecm}$$

i.e.:

$$\boxed{|\bar{\theta}| \lesssim 10^{-10}}$$

3) Solutions of strong CP puzzle

- $m_u = 0$

  inconsistent with quark mass ratios inferred from hadron phenomenology and lattice

- Engineering $\bar{\theta} \simeq 0$:

  assume that at high scales P and CP exact, spontaneously broken at $\Lambda_{P(CP)}$

$$\bar{\theta}_{E \gg \Lambda_{P(CP)}} \equiv 0$$

Model engineering problem is then to ensure that corrections below $\Lambda_{P(CP)}$ to $\bar{\theta}$,

$$\bar{\theta}_{E < \Lambda_{P(CP)}} \sim \arg\det(M_u M_d)$$

are small while still allowing for $O(1)$ CKM phase,

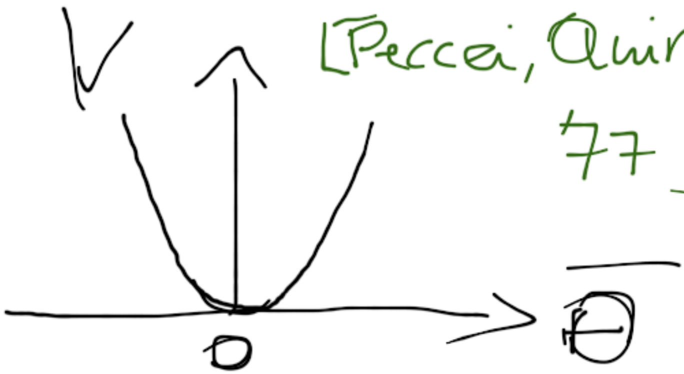
$$\theta_{CKM} \sim \arg\det\left[M_u M_u^\dagger, M_d M_d^\dagger\right]$$

Models of this kind:

R. N. Mohapatra and G. Senjanovic, "Natural Suppression Of Strong P And T Non-invariance," Phys. Lett. **B79**, 283 (1978).
A. Nelson, "Naturally Weak CP Violation," Phys. Lett. **136B** (1984) 387.
S. M. Barr, "Solving The Strong CP Problem Without The Peccei-Quinn Symmetry," Phys. Rev. Lett. **53** (1984) 329.
G. Hiller and M. Schmalz, "Solving The Strong CP Problem With Supersymmetry," hep-ph/0105254.
K. S. Babu, B. Dutta and R. N. Mohapatra, "Solving the strong CP and the SUSY phase problems with parity symmetry," Phys. Rev. D **65**, 016005 (2002).

- <u>Dynamical relaxation</u>

- If $\bar{\theta}$ were a field $\bar{\theta}(x)$ rather than a parameter, then QCD dynamics would lead to $\langle \bar{\theta} \rangle = 0$.



[Peccei, Quinn 77]

- Effective potential of $\bar{\theta}(x)$ can be obtained from chiral Lagrangian

- Eliminate $\bar{\theta} G\tilde{G}$ in favor of phase of e.g. up quark mass, $m_u \to m_u e^{i\bar{\theta}}$
- The low energy dynamics of this theory is described by the chiral Lagrangian:

$$\mathcal{L} = \frac{f_\pi^2}{4} \operatorname{tr}\left[\partial_\mu U \partial^\mu U^{-1}\right] + \frac{f_a^2}{2} \partial_\mu \bar{\theta} \partial^\mu \bar{\theta} + \frac{f_\pi^2}{2} \mu \operatorname{tr}\left[M U + \bar{M} U^{-1}\right]$$

with $U = \exp\left[2i \frac{\tau \pi}{f_\pi}\right] \in SU(2)_F$

$$\mu = \frac{m_\pi^2}{m_u + m_d}$$

$$M = \begin{pmatrix} m_u e^{i\bar{\theta}} & 0 \\ 0 & m_d \end{pmatrix}$$

$V(\bar{\theta}) =$

$$\min_{\substack{U \\ \text{for fixed} \\ \bar{\theta}}} \left[\frac{f_\pi^2 \mu}{2} \operatorname{tr}\left[M U + \bar{M} U^{-1}\right]\right]$$

$$= \frac{f_\pi^2 m_\pi^2}{2} \frac{m_u m_d}{(m_u + m_d)^2} \bar{\theta}^2 + \mathcal{O}(\bar{\theta}^4)$$

$\Rightarrow$ Minimum at $\bar{\theta} = 0$

$\Rightarrow$

- Dynamical $\bar{\theta}(x) \equiv \dfrac{\bar{a}(x)}{f_a}$

wipes out strong problem:

$$\langle \bar{\theta} \rangle = \frac{\langle \bar{a} \rangle}{f_a} = 0$$

- How to realize a dynamical $\bar{\theta}$ parameter?

Add to SM a boson with satisfying shift symmetry

$$a(x) \longrightarrow a(x) + \text{const.}$$

which is only violated by axionic coupling to gluons,

$$\mathcal{L} = \frac{1}{2}\partial_\mu a \partial^\mu a + \frac{a}{f_a}\frac{\alpha_s}{8\pi} G\tilde{G}$$

Then the constant $\bar{\theta}$ can be eliminated by

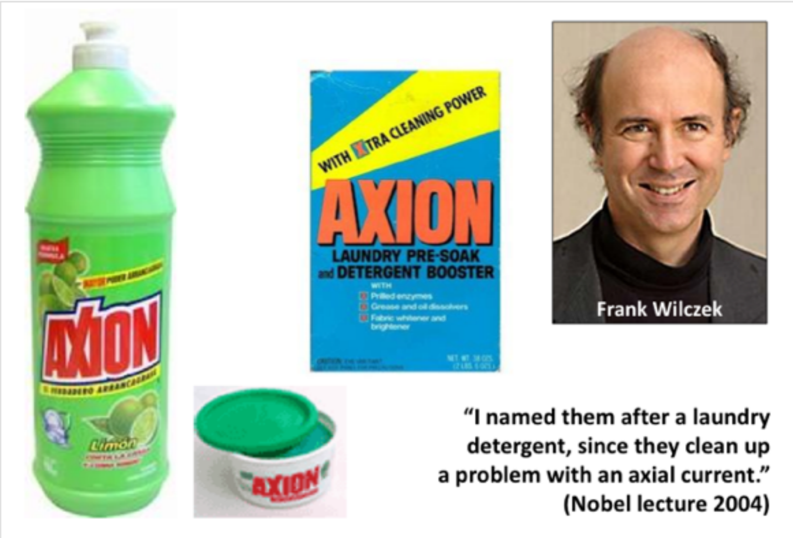
$$\bar{\theta} + \frac{a(x)}{f_a} \equiv \frac{\bar{a}(x)}{f_a} \equiv \bar{\theta}_{eff}(x)$$

and the QCD dynamics leads to $\langle \bar{a} \rangle = 0$, as demonstrated before.

$a(x)$ ... axion field

name of detergent in US

[Wilczek 78]



"I named them after a laundry detergent, since they clean up a problem with an axial current."
(Nobel lecture 2004)

- Mass of elementary particle excitation around $\langle \bar{a} \rangle = 0$, the axion, can be read off the quadratic term in V.

$$m_a = \frac{m_\pi f_\pi}{f_a} \frac{\sqrt{m_u m_d}}{(m_u + m_d)}$$

[Weinberg 78]

Axion Properties

Gluon coupling (generic)	$L_{aG} = \frac{\alpha_s}{8\pi f_a} G\tilde{G} a$	
Mass (generic)	$m_a = \frac{\sqrt{m_u m_d}}{m_u + m_d} \frac{m_\pi}{f_\pi f_a} \approx \frac{6\ \mu\text{eV}}{f_a/10^{12}\ \text{GeV}}$	
Photon coupling	$L_{a\gamma} = -\frac{g_{a\gamma}}{4} F\tilde{F} a = g_{a\gamma}\, \mathbf{E}\cdot\mathbf{B}\, a$ $g_{a\gamma} = \frac{\alpha}{2\pi f_a}\left(\frac{E}{N} - 1.92\right)$	
Pion coupling	$L_{a\pi} = \frac{C_{a\pi}}{f_\pi f_a}(\pi^0 \pi^+ \partial_\mu \pi^- + \cdots)\partial^\mu a$	
Nucleon coupling (axial vector)	$L_{aN} = \frac{C_N}{2 f_a} \overline{\Psi}_N \gamma^\mu \gamma_5 \Psi_N \partial_\mu a$	
Electron coupling (optional)	$L_{ae} = \frac{C_e}{2 f_a} \overline{\Psi}_e \gamma^\mu \gamma_5 \Psi_e \partial_\mu a$	

Constraints from astrophysics:

$$f_a \gtrsim 10^{8 \div 9}\ \text{GeV}$$

4) Axion from UV completions

- Axion from new Higgs fields:

Postulate new global U(1) symmetry, which is spontaneously broken at scale f_a through vev of a Higgs field

$$\phi = \frac{f_a + \rho(x)}{\sqrt{2}} e^{i\frac{a(x)}{f_a}}$$

Engineer that Nambu-Goldstone field $a(x)$ has anomalous coupling $\sim a G\tilde{G}$ arising from triangle graph $\rightarrow$ a is axion.

Simplest Invisible Axion: KSVZ Model

Ingredients: Scalar field Φ, breaks $U(1)_{PQ}$ spontaneously
Very heavy colored quark with coupling to Φ, provides $aG\tilde{G}$ term

$$\mathcal{L}_{KSVZ} = \left(\frac{i}{2}\overline{\Psi}\partial_\mu\gamma^\mu\Psi + \text{h.c.}\right) + \partial_\mu\Phi^\dagger\partial^\mu\Phi - V(|\Phi|) - h\left(\overline{\Psi}_L\Psi_R\Phi + \text{h.c.}\right)$$

Invariant under chiral phase transformations (Peccei Quinn symmetry)

$$\Phi \to e^{i\alpha}\Phi, \quad \Psi_L \to e^{i\alpha/2}\Psi_L, \quad \Psi_R \to e^{-i\alpha/2}\Psi_R$$

Mexican hat potential $V(|\Phi|)$, expand fields as

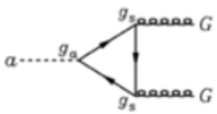
$$\Phi(x) = \frac{f_a + \rho(x)}{\sqrt{2}}\, e^{ia(x)/f_a}$$

Low-energy Lagrangian

$$\mathcal{L}_{KSVZ} = \left(\frac{i}{2}\overline{\Psi}\partial_\mu\gamma^\mu\Psi + \text{h.c.}\right) + \frac{1}{2}\left(\partial_\mu a\right)^2 - m\overline{\Psi}e^{\frac{i\gamma_5 a}{f_a}}\Psi, \quad \text{where} \quad m = hf_a/\sqrt{2}$$

Lowest-order interaction term induces $aG\tilde{G}$ term

$$\mathcal{L}_{aG} = -\frac{\alpha_s}{8\pi}\frac{a}{f_a}G\tilde{G}$$



Couples axion to QCD sector

Georg Raffelt, MPI Physics, Munich

ISAPP, Heidelberg, 15 July 2011

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Kim
Shifman, Vainshtein, Zakharov

– Axions from string theory:

- 4D low energy EFT predicts natural candidate for axion
- axions and ALPs arise as KK zero modes of anti-symmetric form fields bel. to massless spectrum of bosonic string
- axionic coupling to gauge fields $aG\tilde{G}$, $aF\tilde{F}$, ..., also predicted from dimensional reduction of higher dimensional action to four dimensions.

- Often an axiverse (# axions ~ # cycles): axion + many axion-like particles (ALPs):

$$\mathcal{L} \supset \frac{1}{2}\partial_\mu a_i \partial^\mu a_i - \frac{\alpha_s}{8\pi}\left(\bar{\theta} + C_{ig}\frac{a_i}{f_{a_i}}\right) G^b_{\mu\nu}\tilde{G}^{b,\mu\nu} - \frac{\alpha}{8\pi}C_{i\gamma}\frac{a_i}{f_{a_i}} F_{\mu\nu}\tilde{F}^{\mu\nu} + \sum_\Psi \left[\overline{\Psi}\gamma^\mu \frac{1}{2}(\tilde{X}^i_{\psi_R} + \tilde{X}^i_{\psi_L})\gamma_5\Psi + \overline{\Psi}\gamma^\mu\frac{1}{2}(\tilde{X}^i_{\psi_R} - \tilde{X}^i_{\psi_L})\Psi\right]\frac{\partial_\mu a_i}{f_{a_i}},$$

- $f_a \sim f_{a_i} \sim 10^{9 \div 16}$ GeV

Review:

AR, 1209.2299

5) Axion Cold Dark Matter ($f_a \gtrsim 10^9$ GeV)

- For $f_a > T_{RH}$, axion produced non-thermally via vacuum realignment mechanism: Preskill et al, Dine, Fischler, Abbott, Sikivie] 83

$$\ddot{a} + 3H(T)\dot{a} - m_a^2(T) = 0$$

- At early times, where $H(T) \gtrsim m_a(T)$, i.e. $T_{RH} > T \gtrsim 1$ GeV, axion field sits fixed at $a_i = \theta_i f_a$

- At late time, when $m_a(T) \gtrsim 3H(T)$, axion field starts quickly oscillating $\hat{=}$ coherent state of non-relativistic particles $\hat{=}$ Cold Dark Matter

Modern values for QCD parameters and temperature-dependent axion mass imply (Bae, Huh & Kim, arXiv:0806.0497)

$$\Omega_a h^2 = 0.195\,\Theta_i^2 \left(\frac{f_a}{10^{12}\mathrm{GeV}}\right)^{1.184} = 0.105\,\Theta_i^2 \left(\frac{10\,\mu\mathrm{eV}}{m_a}\right)^{1.184}$$

- $\Theta_i \sim 1$ implies $f_a \sim 10^{12}$ GeV and $m_a \sim 10$ μeV ("classic window")
- $f_a \sim 10^{16}$ GeV (GUT scale) or larger (string inspired) requires $\Theta_i \lesssim 0.003$ ("anthropic window")

- For $T_{RH} > f_a$:

axions produced nonthermally via:
- vacuum realignment
- string decay
- domain wall decay

$$\frac{\Omega_{a,DW+ST}}{\Omega_{\rm obs}} \begin{cases} \sim \left(\frac{40\mu\mathrm{eV}}{m_a}\right)^{1.184} & \text{Sikivie, Harari et at.} \\ \sim \left(\frac{400\mu\mathrm{eV}}{m_a}\right)^{1.184} & \text{Shellard, Davis et at. Kawasaki, Hiramatsu et at} \end{cases}$$

$$\frac{\Omega_{a,VR}}{\Omega_{\rm obs}} \sim \left(\frac{40\mu\mathrm{eV}}{m_a}\right)^{1.184}$$

Axion can be dominant part of CDM for

$10\,\mu eV \lesssim m_a \lesssim meV$

Literature:

- Review on $d_n(\bar\theta)$:

  Pospelov, Ritz, Ann. Phys. 318 (2005) 119 [hep-ph/0504321]

- Review on axion and ALPs :

  AR, 1210.5081