Higgs as a pseudo Goldstone boson

Workshop Seminar WS 12/13

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1 Introduction

The idea of a composite Higgs dates back to the eighties and a good way to look at it is as an interpolation between the Standard Model Higgs and Technicolor theories. In this class of theories the Higgs arises as a pseudo Goldstone boson from a broken global symmetry in a strongly interacting sector. The Higgs is naturally lighter than the strong scale of the theory because of its Goldstone nature. The main advantages with respect to the Standard Model are the fact that it solves the hierarchy problem and it provides a dynamical prescription for EWSB. Compared to Technicolor the separation of scales between the electroweak and strong sector allows one to pass EWPTs and other collider constraints more easily. These points together with some phenomenological implications will be addressed in this lecture. These notes are mainly based on the TASI lecture notes [1] and recent lectures given at Trieste [3] and CERN [2] by Contino. The interested reader can find more details there.

2 EWSB and Unitarity

In order to discuss the Higgs and possible extensions we need to understand its role in the Standard Model and we need to address the problems which arise without the Higgs. For this reason let the Higgs be absent and consider the massless Standard Model Lagrangian

$$\mathcal{L}_0 = -\frac{1}{4} W^a_{\mu\nu} W^{a\,\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} - \frac{1}{4} G^a_{\mu\nu} G^{a\mu\nu} + \sum_{j=1}^{3} \bar{\Psi}^{(j)} i \not{D} \Psi^{(j)}. \quad (1)$$

Of course experiments also dictate masses for the fermions and the electroweak gauge bosons. We introduce them in a slightly alternative way including the Goldstone bosons that correspond to the longitudinal polarizations of the massive gauge bosons

$$\mathcal{L}_{mass} = \frac{v^2}{4} \mathrm{Tr}\left[(D_\mu \Sigma)^\dagger (D^\mu \Sigma) \right] - \frac{v}{\sqrt{2}} \sum_{i,j} \begin{pmatrix} \bar{u}_L^{(i)} \bar{d}_L^{(i)} \end{pmatrix} \Sigma \begin{pmatrix} \lambda^u_{ij}\, u_R^{(j)} \\ \lambda^d_{ij}\, d_R^{(j)} \end{pmatrix} + h.c. \quad (2)$$

The chiral field Σ contains the Goldstone bosons in a non-linear way as

$$\Sigma(x) = \exp\left[i\sigma^a \chi^a(x)/v \right], \qquad D_\mu \Sigma = \partial_\mu \Sigma - ig \frac{\sigma^a}{2} W^a_\mu \Sigma + ig' \Sigma \frac{\sigma^3}{2} B_\mu\,. \quad (3)$$

In this chiral form the electroweak symmetry $SU(2)_L \times U(1)_Y$ is manifest since Σ transforms as

$$\Sigma(x) \to U_L(x)\, \Sigma(x)\, U_Y^\dagger(x). \quad (4)$$

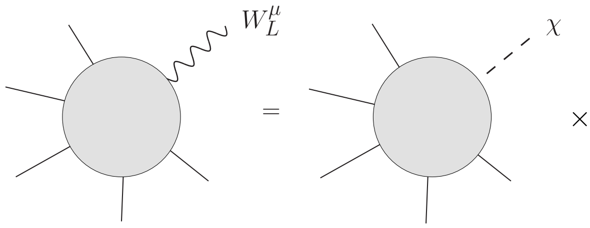
We see that the symmetry is nonlinearly realized on the Goldstone bosons χ^a, indicating that the symmetry is hidden or spontaneously broken by the mass terms. In the non-trivial vacuum, that is $\langle \Sigma \rangle = 1$, the familiar masses for the gauge bosons and fermions are reproduced. A nice feature we can also immediately see is the invariance under global $SU(2)_L \times SU(2)_R$ transformations

$$\Sigma(x) \to U_L\, \Sigma(x)\, U_R^\dagger, \quad (5)$$

for vanishing g' and $\lambda^u_{ij} = \lambda^d_{ij}$. This symmetry is broken to the diagonal by $\langle \Sigma \rangle = 1$ resulting in the custodial symmetry $SU(2)_c$ giving a ρ parameter equal to one at tree level.

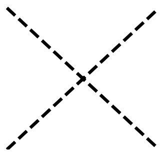
2.1 Unitarity

This theory has a problem with perturbative unitarity, since it predicts amplitudes that grow with energy. Then at some (high) energy perturbation theory breaks down and the theory is not valid anymore. These amplitudes which grow with energy occur in the scattering of the longitudinal modes of the massive gauge bosons. Let us sketch the argument using the equivalence theorem which states that the amplitude for a longitudinal gauge boson is equal to the amplitude for its respective Goldstone boson at sufficiently high energies:



$$\times \left(1+O\left(\frac{m_W^2}{E^2}\right)\right).$$

Then at leading order in E/m_W we find for the scattering of two longitudinal W's



$$\mathcal{A}(\chi^+\chi^- \to \chi^+\chi^-) = \frac{1}{v^2}(s+t)\,. \tag{6}$$

The growth with energy originates from the derivative interaction among four Goldstone bosons. The unitarity bound prescribes that the elastic amplitudes a_l of each l-th partial wave must satisfy $\mathrm{Im}(a_l) = |a_l|^2 + |a_l^{in}|^2$. For elastic scattering a_l is constraint to lie on the unitary circle $\mathrm{Re}^2(a_l) + (\mathrm{Im}(a_l) - 1/2)^2 = 1/4$. Then for tree level scattering the amplitude is real and an imaginary part only arises for the 1-loop level. Perturbativity breaks down when both parts are of the same order, which gives us breakdown when $\mathrm{Re}(a_l) > \pi/2$ or $1/2$. Projecting on partial wave amplitudes, using the Legendre polynomials ($P_0(x) = 1$, $P_1(x) = x$, $P_2(x) = 3x^2/2 - 1/2$, etc.), we find

$$a_l = \frac{1}{32\pi}\int_{-1}^{+1} d\cos\theta\ \mathcal{A}(s,\theta)P_l(\cos\theta). \tag{7}$$

The s-wave amplitude then reads

$$a_0(W_L^+W_L^- \to W_L^+W_L^-) \simeq \frac{1}{32\pi}\,\frac{s}{v^2}\,, \tag{8}$$

which leads to the bound $\Lambda \simeq \sqrt{s} \leq 4\pi v$.

Ultimately the loss of perturbative unitarity can be traced back to the non-renormalizability of the chiral Lagrangian in equation (2). This again is related to the fact that the chiral Lagrangian is an effective field theory which breaks down at some energy scale. For chiral theories in general we have that the break down scale is $\Lambda = 4\pi v$, as we saw in the lecture on sigma models. Now we have two possibilities: either cure the problem by introducing new degrees of freedom which restore perturbative unitarity or let the theory become strongly coupled at some higher energy. Both scenarios indicate the emergence and therefore need of new physics and we conclude that there has to be some symmetry breaking dynamics as an UV-completion of the chiral Lagrangian.

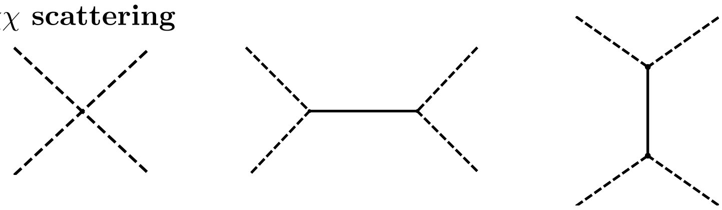
2.2 Higgs Model

Now we use some knowledge from the sixties and the seventies and add a scalar h which is a singlet under $SU(2)_L \times SU(2)_R$ with arbitrary couplings to gauge bosons and fermions. So instead of the chiral mass Lagrangian we now have [4] (to quadratic order in the scalar h)

$$\begin{aligned} \mathcal{L}_H =& \frac{1}{2}\left(\partial_\mu h\right)^2 + V(h) + \frac{v^2}{4}\mathrm{Tr}\left[\left(D_\mu\Sigma\right)^\dagger\left(D_\mu\Sigma\right)\right]\left(1 + 2a\,\frac{h}{v} + b\,\frac{h^2}{v^2} + \ldots\right) \\ & - \frac{v}{\sqrt{2}}\sum_{i,j}\left(\bar{u}_L^{(i)}\bar{d}_L^{(i)}\right)\Sigma\left(1 + c\,\frac{h}{v} + \cdots\right)\begin{pmatrix}\lambda_{ij}^u\, u_R^{(j)} \\ \lambda_{ij}^d\, d_R^{(j)}\end{pmatrix} + h.c. \end{aligned} \tag{9}$$

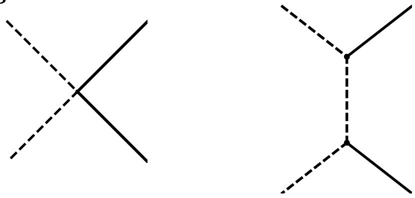
where a, b and c are arbitrary couplings and $V(h)$ is the potential for the scalar field. Now let's look at the possible amplitudes which could be growing with energy and look at the effect of the added scalar field h.

$\chi\chi \to \chi\chi$ **scattering**



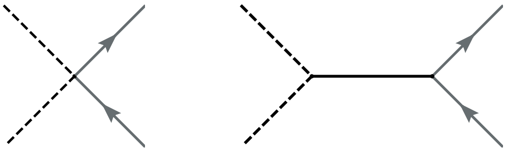
$$\begin{aligned} \mathcal{A}(\chi^+\chi^- \to \chi^+\chi^-) &= \frac{1}{v^2}\left[s - a^2\frac{s^2}{s - m_h^2} + (s \leftrightarrow t)\right] \\ &= \frac{s+t}{v^2}\left(1 - a^2\right) + O\left(\frac{m_h^2}{E^2}\right). \end{aligned}$$

$\chi\chi \to hh$ **scattering**



$$\mathcal{A}(\chi^+\chi^- \to hh) = \frac{s}{v^2}\left(b - a^2\right) + O\left(\frac{m_h^2}{E^2}\right).$$

$\chi\chi \to \psi\bar{\psi}$ **scattering**



$$\mathcal{A}(\chi^+\chi^- \to \psi\bar{\psi}) = \frac{m_\psi\sqrt{s}}{v^2}\,(1-ac) + O\left(\frac{m_h^2}{E^2}\right)\,.$$

From the $\chi\chi \to \chi\chi$ scattering result we see that now the theory is perturbative until a higher scale

$$\Lambda_a = \frac{4\pi v}{\sqrt{1-a^2}}, \tag{10}$$

and similarly for the other amplitudes. The different amplitudes are fully unitarized for $a^2 = 1$, $b = a^2$ and $ac = 1$, implying that $a = b = c = 1$ unitarizes the whole Lagrangian. It is exactly this parameter space point (assuming vanishing higher order terms) which coincides with the Standard Model Higgs and for this point the Lagrangian (9) can be rewritten in terms of the familiar Higgs doublet

$$H(x) = \frac{1}{\sqrt{2}}\, e^{i\sigma^a\chi^a(x)/v}\begin{pmatrix} 0 \\ v + h(x)\end{pmatrix}. \tag{11}$$

Notice that the renormalizability of the Standard Model Lagrangian provides us with a unitary description for all energies. In general, however, theories can predict different values for a, b and c as for example we will see for the composite Higgs. In the end experiments, hopefully the LHC, will determine the couplings of the Higgs and thereby its nature and that of EWSB.

2.3 Technicolor

Although the Higgs model might be the most straightforward method to solve the unitarity problem, there is already another example in nature which does the same, namely QCD. At low energies QCD breaks a global $SU(2)_L \times SU(2)_R$ chiral symmetry to the vectorial $SU(2)_V$ via the known condensates, like the pions. This has already been discussed in the first lecture in this workshop series, but we will indicate the main features to get an idea how it works, after all the composite Higgs shares some similarities with QCD/Technicolor. Technicolor basically is an upscaled version of QCD with a possibly different gauge group, which at least also has the same global chiral symmetry. Hence let us have a look at QCD in terms of unitarity and EWSB. The chiral Lagrangian for the pions is given by

$$\mathcal{L}_\pi = \frac{f_\pi^2}{4}\,\mathrm{Tr}\Big[(\partial_\mu\Sigma)^\dagger\,(\partial^\mu\Sigma)\Big]\,, \qquad \Sigma(x) = \exp(i\sigma^a\pi^a(x)/f_\pi)\,, \tag{12}$$

where $f_\pi = 92$ MeV is the pion decay constant. This Lagrangian however also suffers from the same unitarity problems as before, however we know that there is no light scalar Higgs-like resonance unitarizing the theory. Instead the tower of resonances in QCD, which is exchanged in pion-pion scattering at high energies enforces unitarity.

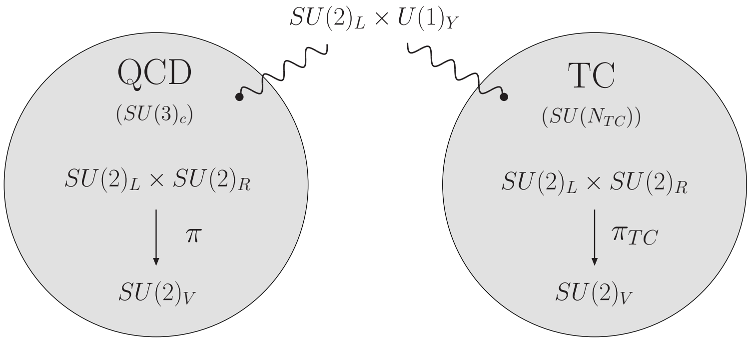


Figure 1: Cartoon of a new Technicolor sector and QCD with part of their global symmetries gauged by the weak interactions.

Now let us turn on the weak interactions and look at the effects. We have a global symmetry breaking of $SU(2)_L \times SU(2)_R \times U(1)_B \to SU(2)_V \times U(1)_B$ of which only the $SU(2)_L \times U(1)_Y$ part is gauged. In this way an explicit breaking of the global symmetry is introduced and the QCD vacuum breaks the electroweak invariance and the pions are eaten to give mass to the W and the Z. To see this explicitly we gauge the chiral Lagrangian

$$\mathcal{L}_\pi = \frac{f_\pi^2}{4} \mathrm{Tr}\left[(D_\mu \Sigma)^\dagger (D^\mu \Sigma) \right]. \tag{13}$$

Then expanding around the vacuum $\langle \Sigma \rangle = 1$ we find for the gauge boson masses

$$\mathcal{L}_{\mathrm{mass}} = \frac{g^2 f_\pi^2}{4} W_\mu^+ W^{\mu -} + \frac{g^2 + g'^2}{8} f_\pi^2 Z_\mu Z^\mu. \tag{14}$$

Which gives too low masses for the gauge bosons, however one could imagine another upscaled version of QCD with $F_\pi \simeq v$. This is Technicolor, where one has an $SU(N_{TC})$ gauge group with a global $SU(2)_L \times SU(2)_R$ invariance broken down to $SU(2)_V$ at low energies due to confinement, see figure 1.

So we have seen two ways to resolve the unitarity problem and generate a viable mechanism for EWSB, one weakly coupled and one strongly coupled. However, both of them are not very satisfying. The Higgs model has a hierarchy problem and it is generally believed that a more symmetric theory like SUSY is there to address this. Although Technicolor has no Higgs and therefore no hierarchy problem, it is roughly excluded by experimental searches. First of all a light bosonic resonance is found, moreover Technicolor predicts too high contributions to FCNC's and the S parameter.

3 Composite Higgs

An interesting interpolation between the Higgs model and Technicolor is the composite Higgs paradigm, where the Higgs is a bound state from a strongly interacting sector [5]. In particular the Higgs will emerge as a pseudo Goldstone boson of an enlarged global symmetry of this strong sector, this will assure that it is naturally lighter than the other resonances of the strong sector. First we will discuss the general principles which are necessary for a successful construction of the Higgs as a pseudo Goldstone boson and then we will present the minimal custodially invariant example.

The composite Higgs paradigm is based on two requirements

- The Higgs is a composite pseudo Goldstone boson of some global symmetry breaking $\mathcal{G} \to \mathcal{H}_1$ at a scale f in a strongly coupled theory.
- The electroweak gauging of $\mathcal{G}$ does not trigger a Higgs mechanism at tree level, instead at loop level a Higgs potential is generated leading to EWSB.

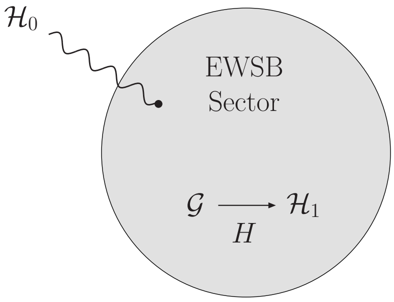


Figure 2: Cartoon of a strongly interacting EWSB sector with global symmetry $\mathcal{G}$ broken down to $\mathcal{H}_1$ at low energy. The subgroup $\mathcal{H}_0 \subset \mathcal{G}$ is gauged by external vector bosons.

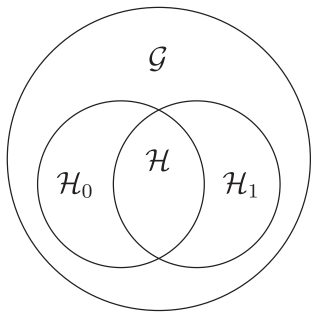


Figure 3: The pattern of symmetry breaking.

A look at the symmetry structure reveals the general features:

- $\mathcal{G} \to \mathcal{H}_1$ global symmetry breaking
- $\mathcal{H}_0 \subset \mathcal{G}$ gauged subgroup
- $\mathcal{H} = \mathcal{H}_1 \cap \mathcal{H}_0$ unbroken gauge group
- GBs: $n = \dim(\mathcal{G}) - \dim(\mathcal{H}_1)$
- Eaten GBs: $n_0 = \dim(\mathcal{H}_0) - \dim(\mathcal{H})$
- $n - n_0$ are pseudo Goldstone bosons

To realize the Higgs as a pseudo Goldstone boson two more conditions need to be realized:

- The SM group $\mathcal{G}_{\text{SM}}$ must be embedded in the unbroken group $\mathcal{H}_1$.
- $\mathcal{G}/\mathcal{H}_1$ contains at least one doublet of $SU(2)_L$.

For simplicity the external gauge group can be identified with the Standard Model one, that is $\mathcal{H}_0 = \mathcal{G}_{\text{SM}} = SU(2)_L \times U(1)_Y$. This implies that at tree level $\mathcal{G}_{\text{SM}} \subset \mathcal{H}_1$. However, at one loop level an EWSB potential is generated which implies that $\mathcal{G}_{\text{SM}} \to U(1)_{\text{em}}$ which we can understand as a misalignment of the true vacuum from the gauged subgroup.

The alternative way to view it, is via a two step breaking $\mathcal{G} \xrightarrow{f} \mathcal{H}_1 \xrightarrow{v} \mathcal{H}_2$, where only the $U(1)_{\text{em}}$ part of $\mathcal{H}_2$ is gauged. As a short summary the following features are to be stressed:

- The Higgs boson is a Goldstone boson from the global symmetry breaking $\mathcal{G}/\mathcal{H}_1$ in a strong sector.
- The gauging of the global symmetry $\mathcal{G}_{\text{SM}} \subset \mathcal{G}$ introduces an explicit breaking of $\mathcal{G}$ and makes the Higgs a pseudo Goldstone boson.
- Then loops of SM fermions and gauge bosons generate a Higgs potential which gives the Higgs a small mass and can break electroweak symmetry.
- The EWSB breaking scale v is dynamically generated and can be much smaller than the strong breaking scale f.
- Strong resonance masses around $m_\rho \sim g_\rho f$ and Higgs mass around $m_h \sim g_{\text{SM}} v$, where we generally have $g_{\text{SM}} \lesssim 1 \lesssim g_\rho \lesssim 4\pi$.
- The ratio $\xi = (v/f)^2$ acts as a suppression scale for precision observables: in the limit of $\xi \to 0$ we obtain the Standard Model, whereas in the limit of $\xi \to 1$ we obtain a Technicolor like theory with a light Higgs.

3.1 Minimal Custodial Model: $SO(5)/SO(4)$

Now we want to construct the minimal model which can accommodate EWSB, the Higgs doublet and does respect the custodial symmetry. Hence we need a coset which at least contains four real Goldstone bosons: the simplest solution would then be $SU(3)/SU(2) \times U(1)$ giving the required Goldstone bosons. However the constraint of custodial symmetry will not be satisfied here, custodial symmetry is related to an approximate $SU(2)_L \times SU(2)_R$ which is broken to the diagonal by EWSB. So we need at least an unbroken custodial symmetry and the minimal choice then is $SO(4) \simeq SU(2)_L \times SU(2)_R$. We then find the minimal symmetry breaking $SO(5)/SO(4) = S^4$, where the coset is a sphere in five dimensions containing the dim $SO(5)-$ dim $SO(4) = 10 - 6 = 4$ Goldstones.

For a construction with a realistic embedding of the hypercharge we need an extra $U(1)_X$ symmetry [6]. We now have $SO(5) \times U(1)_X / SO(4) \times U(1)_X$, the factor $U(1)$ does not play any role in the symmetry breaking. Then we gauge the $SU(2)_L \times U(1)_Y$ part of the unbroken $SU(2)_L \times SU(2)_R \times U(1)_X$ which gives a hypercharge $Y = T_{3R} + X$.

Now let us analyze the Goldstone bosons and their parametrization both for the strong sector breaking and for EWSB. This detailed form contains the information about the symmetry breaking and will give us information about the couplings of the Goldstones once used in effective Lagrangians. The Goldstone bosons living on the $SO(5)/SO(4)$ coset can be parameterized in the usual CCWZ formalism as

$$\Phi(x) = \exp\left(i\sqrt{2}\pi^{\hat{a}}(x)T^{\hat{a}}/f\right)\begin{pmatrix}0\\0\\0\\0\\1\end{pmatrix}, \tag{15}$$

where $T^{\hat{a}}_{ij} = -\frac{i}{\sqrt{2}}\left(\delta^{\hat{a}}_i\delta^5_j - \delta^{\hat{a}}_j\delta^5_i\right)$ are the broken $SO(5)/SO(4)$ generators. The explicit expression for $\Phi(x)$ is then readily calculated and by defining $\pi = \sqrt{(\pi^{\hat{a}})^2}$ and $\hat{\pi}^{\hat{a}} = \pi^{\hat{a}}/\pi$ we find

$$\Phi(x) = \begin{pmatrix}\sin(\pi/f)\begin{pmatrix}\hat{\pi}^1\\\hat{\pi}^2\\\hat{\pi}^3\\\hat{\pi}^4\end{pmatrix}\\ \cos(\pi/f)\end{pmatrix}. \tag{16}$$

Then if the potential generated by Standard Model loop contributions triggers EWSB symmetry breaking we can parametrize the $SO(4)/SO(3)$ Goldstone bosons (those which are eaten by the W and the Z) in the usual way $\Sigma(x) = \exp\left(i\sigma^i\chi^i(x)/v\right)$. If we then expand around the vacuum, which is given by $\langle\pi\rangle = \theta \cdot f$ and thereby replace $\pi(x) \to \theta \cdot f + h(x)$, we find

$$\Phi(x) = \begin{pmatrix}\sin(\theta + h(x)/f)\,\Sigma(x)\begin{pmatrix}0\\0\\0\\1\end{pmatrix}\\ \cos(\theta + h(x)/f)\end{pmatrix}. \tag{17}$$

This is how the fields are embedded in the two step symmetry breaking $SO(5) \xrightarrow{f} SO(4) \xrightarrow{v} SO(3)$, this form can then be used to derive the couplings of the Goldstone bosons.

4 Higgs Couplings

A very interesting feature with regard to the LHC is the strength of the couplings between the Higgs and the other Standard Model particles. Which are tested and will be tested with more accuracy in the future. This will be a distinguishing feature in the future for models and therefore we would like to know how the Higgs couples. For the minimal model $SO(5)/SO(4)$ we will use an effective description for the the couplings at low energy with respect to the strong sector. The Lagrangian and therefore the couplings are then fully determined by symmetry arguments. And if we use the specific form for the Goldstone bosons from the previous section we will be able to give the coupling strength as a function of only one parameter which is a measure for the separation of the to symmetry breaking scales f and v.

4.1 Couplings to Gauge Bosons

The $SO(5)$ invariant effective Lagrangian describing the couplings between gauge bosons and scalars is equal to the chiral Lagrangian with covariant derivatives as given by the CCWZ prescription [7]

$$\mathcal{L} = \frac{f^2}{2} \left(D_\mu \Phi\right)^T \left(D^\mu \Phi\right). \tag{18}$$

We can plug in the previous result (17) and the result reads

$$\begin{aligned}
\mathcal{L} =& \frac{f^2}{2} \left[D_\mu \sin\left(\theta + h(x)/f\right) \Sigma\right]^T \left[D^\mu \sin\left(\theta + h(x)/f\right) \Sigma\right] \\
&+ \frac{f^2}{2} \partial_\mu \cos\left(\theta + h(x)/f\right) \partial^\mu \cos\left(\theta + h(x)/f\right) \\
\supset& \frac{f^2}{2} \left[\left(D_\mu \Sigma\right)^T \left(D^\mu \Sigma\right)\right] \sin^2\left(\theta + h(x)/f\right) \\
&+ \frac{f^2}{2} \partial_\mu \sin\left(\theta + h(x)/f\right) \partial^\mu \sin\left(\theta + h(x)/f\right) + \{\sin \to \cos\} \\
\supset& \frac{1}{2} \partial_\mu h \partial^\mu h + \frac{f^2}{4} \mathrm{Tr}\left[\left(D_\mu \Sigma\right)^\dagger \left(D^\mu \Sigma\right)\right] \sin^2\left(\theta + h(x)/f\right)
\end{aligned} \tag{19}$$

In order to have successful EWSB we need to reproduce the electroweak gauge boson masses, this will give us a relation between the misalignment θ and the parameters v and f. For this purpose only look at the vacuum term where where $\langle \Sigma \rangle = 1$ and retain only the θ terms, for the W mass we find

$$m_W^2 = \frac{g^2 f^2}{4} \sin^2\theta \qquad \Longrightarrow \qquad \sin^2\theta = \frac{v^2}{f^2} = \xi. \tag{20}$$

Now that we have related the misalignment parameter θ to the electroweak vev we can expand (19) around the vacuum to find the couplings for the Higgs to gauge bosons

$$\begin{aligned}\mathcal{L} \supset\ & \frac{f^2}{4}\mathrm{Tr}\left[(D_\mu\Sigma)^\dagger (D^\mu\Sigma)\right]\left[\sin^2\theta + \frac{h(x)}{f}\sin 2\theta + \frac{h(x)^2}{f^2}\cos 2\theta + \cdots\right] \\ =\ & \frac{v^2}{4}\mathrm{Tr}\left[(D_\mu\Sigma)^\dagger (D^\mu\Sigma)\right]\left[1 + 2\frac{h(x)}{v}\sqrt{1-\xi} + \frac{h(x)^2}{v^2}(1-2\xi) + \cdots\right].\end{aligned} \tag{21}$$

From which we can extract the coefficients

$$a = \sqrt{1-\xi}, \quad b = 1 - 2\xi. \tag{22}$$

4.2 Couplings to Fermions

For the derivation of the Higgs to fermion couplings a few assumptions are necessary, naively we have three options to couple the fermions to the composite sector (to which the Higgs belongs).

- **Total compositeness:** The Standard Model fermions are totally composite and couple directly to the strong sector: this has been ruled out by LEP.
- **Bilinear coupling:** The Standard Model fermions are elementary and couple directly to the strong sector: this has problems with flavor observables.
- **Partial compositeness:** The composite and Standard Model fermions mix, providing couplings for the Standard Model fermions to the strong sector [8].

This last option is still experimentally viable, and we assume that every Standard Model fermion couples to composite fermionic operator with the same quantum numbers. These couplings are linear and mix the elementary and composite states for each quark generation q_L, u_R and d_R. Looking only at one elementary chiral field ψ_L and one composite heavy fermion χ we have the Lagrangian

$$\mathcal{L} = \bar{\psi}_L\, i\,\partial\!\!\!/\, \psi_L + \bar{\chi}\,(i\,\partial\!\!\!/ - m_*)\,\chi + \Delta_L \bar{\psi}_L \chi_R + h.c. \tag{23}$$

The left-handed component of the heavy fermion then mixes with the left-handed elementary field and the mass eigenstates are given by

$$\begin{pmatrix}\psi_L \\ \chi_L\end{pmatrix} \to \begin{pmatrix}\cos\varphi_L & -\sin\varphi_L \\ \sin\varphi_L & \cos\varphi_L\end{pmatrix}\begin{pmatrix}\psi_L \\ \chi_L\end{pmatrix}, \qquad \tan\varphi_L = \frac{\Delta_L}{m_*}. \tag{24}$$

And similar expressions can be derived for right-handed elementary particles with a mixing angle φ_R. The mixing results in a heavy mass eigenstate with mass $\sqrt{m_*^2 + \Delta_L^2}$ and light mass eigenstate with negligible mass to be identified with the SM fermion. However, the Standard Model fermion receives a mass through EWSB which now resides in the strong sector and is transmitted via the mixings. We find

$$y_{SM} = Y_* \sin\varphi_L \sin\varphi_R, \tag{25}$$

hence the fermion mass is proportional to the mixings, which indicates that light fermions are mainly elementary and heavy fermions mainly composite.

Now we turn back to the $SO(5)/SO(4)$ example, where we now have the freedom to specify how the composite operators transform under $SO(5)$. With the derivation of the couplings between the Higgs and the fermions in mind we choose here the spinorial representation, but note that a different representation will imply a different coupling. The mixing Lagrangian is now given by

$$\mathcal{L} = \lambda_q\, \bar{q}_L O_q + \lambda_u\, \bar{u}_R O_u + \lambda_d\, \bar{d}_R O_d + h.c. \tag{26}$$

The operators transform as spinors of $SO(5)$, and because of the linear coupling also the SM fermions do. A spinor of $SO(5)$ decomposes as a 4 of $SO(4)$ which is a $(2,1) + (1,2)$ of $SU(2)_R \times SU(2)_L$, so we can embed the SM fermions in the following way

$$\Psi_q = \begin{bmatrix} q_L \\ Q_L \end{bmatrix}, \qquad \Psi_u = \begin{bmatrix} q_R^u \\ \begin{pmatrix} u_R \\ d_R' \end{pmatrix} \end{bmatrix}, \qquad \Psi_d = \begin{bmatrix} q_R^d \\ \begin{pmatrix} u_R' \\ d_R \end{pmatrix} \end{bmatrix}, \tag{27}$$

Progressing along the same lines as the CCWZ formalism for the gauge Lagrangian, we write down an effective Lagrangian with the most general $SO(5)$ invariant couplings to fermions

$$\mathcal{L}_\Psi = \sum_{r=q,u,d} \bar{\Psi}_r\, i \not{\partial}\, \Psi_r + i\lambda f \sum_{r=u,d} \bar{\Psi}_q \Gamma^i \Phi^i \Psi_r. \tag{28}$$

In here Φ denotes the same Goldstone fields as in (17) and the Γ^i denote the spinorial representation of $SO(5)$

$$\Gamma^{\hat{a}} = \begin{bmatrix} 0 & \sigma^{\hat{a}} \\ \sigma^{\hat{a}\,\dagger} & 0 \end{bmatrix}, \qquad \Gamma^5 = \begin{bmatrix} \mathbf{1} & 0 \\ 0 & -\mathbf{1} \end{bmatrix}, \qquad \sigma^{\hat{a}} = \{\vec{\sigma}, -i\mathbf{1}\}, \tag{29}$$

and hence

$$\Gamma^i \Phi^i = \begin{pmatrix} \cos\left(\theta + h(x)/f\right) \mathbf{1} & -i \sin\left(\theta + h(x)/f\right) \Sigma(x) \\ i \sin\left(\theta + h(x)/f\right) \Sigma(x) & -\cos\left(\theta + h(x)/f\right) \mathbf{1} \end{pmatrix}. \tag{30}$$

Plugging these expressions into the Lagrangian and only keeping the q_L and the u_R we find

$$\mathcal{L}_\Psi \supset \bar{q}_L\, i\not{\partial}\, q_L + \bar{u}_R\, i\not{\partial}\, u_R + \lambda f \sin(\theta + h/f)\bar{q}_L \Sigma\, u_R. \tag{31}$$

Expanding the last term around the vacuum gives

$$\begin{aligned}\mathcal{L}_\Psi &\supset \lambda f \bar{q}_L \Sigma\, u_R \left(\sin\theta + \frac{h(x)}{f}\cos\theta + \cdots\right)\\ &= \lambda v \bar{q}_L \Sigma\, u_R \left(1 + \frac{h(x)}{v}\sqrt{1-\xi} + \cdots\right)\end{aligned} \tag{32}$$

Where λ must be identified with the Yukawa coupling and we omitted generation indices. The parameter for the Higgs to fermion coupling is easily identified as $c = \sqrt{1-\xi}$.

4.3 Comparison with Experiment

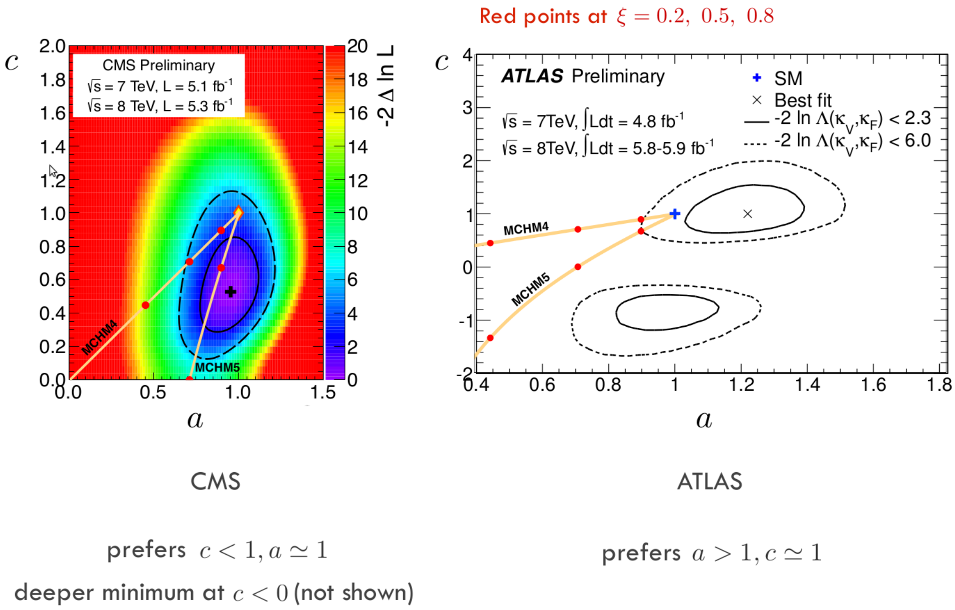


Figure 4: Experimental constraints on the a and c parameter from CMS and ATLAS, with the composite Higgs predictions.

From the previous sections we obtained for the Higgs couplings that

$$a = \sqrt{1-\xi}, \quad b = 1 - 2\xi, \quad c = \sqrt{1-\xi} \quad \text{where} \quad \xi = \frac{v^2}{f^2}. \tag{33}$$

These give clear predictions between the different Higgs couplings, which can be tested quite precisely in the future at the LHC. The best sensitivity is to the couplings a and c and both CMS and ATLAS have provided various likelihood plots for these parameters given in figure 4 taken from [2].

5 Dynamical Potential

One of the key features of composite models is the dynamical generation of the potential via loop diagrams and thereby a dynamical breaking of the electroweak symmetry. For this reason we need to make sure that the potential satisfies the following criteria:

- The minimum of the potential breaks both the global $SO(4)$ invariance as well as the electroweak gauge invariance.
- A light Higgs boson mass is required, therefore the induced breaking should be relatively small.
- From a phenomenological point of view the relation between the Higgs mass and fermionic partners is interesting.

Deriving the Higgs potential here is lengthy and beyond the scope of this lecture, but we can quote a recent result [9] here. There are two main contributions from Standard Model particles, those from gauge bosons and from the top quark. For the gauge boson contributions they find

$$V_{\text{gauge}}(h) = \alpha \sin^2 \frac{h}{f} + \beta \sin^4 \frac{h}{f} + \cdots , \tag{34}$$

where $\alpha > 0$ and β are complicated expressions. Since α is positive this potential can not induce EWSB, according to Witten's argument on vector-like gauge theories. However, also the top induces a potential, which is actually dominant

$$V_{\text{top}}(h) = \alpha \sin^2 \frac{h}{f} - \beta \sin^2 \frac{h}{f} \cos^2 \frac{h}{f} + \cdots . \tag{35}$$

Now we can have EWSB as long as $\alpha < \beta$ and $\beta \geq 0$, which can be realized. So we have found a way to dynamically generate an EW symmetry breaking potential for the Higgs arising as a pseudo Goldstone boson from a strongly coupled sector.

Higgs Mass Since the Higgs mass derives directly from the potential one can in principle derive it from symmetry principles, however this is quite involved. A NDA estimate is much more feasible here and for the potential one may find [4]

$$V(H) \sim \frac{m_\rho^4}{g_\rho^2} \times \frac{y_t g_\rho}{16\pi^2} \times \hat{V}(H/f), \tag{36}$$

which leads to a quartic coupling $\lambda \sim (g_\rho/4\pi)^3 4\pi y_t$. The resulting Higgs boson mass equals

$$m_h \sim \left(\frac{g_\rho}{4\pi}\right)^{3/2} \sqrt{4\pi v m_t} \sim 150 \text{ GeV}. \quad (37)$$

And we see that a relatively light Higgs is feasible within these theories. More involved calculations [9] give results depending on the mass of the heavy fermion partner, and a natural Higgs mass may be obtained if these partners are relatively light, that is $m_Q < 1$ TeV.

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