Dilaton and moduli in string theory

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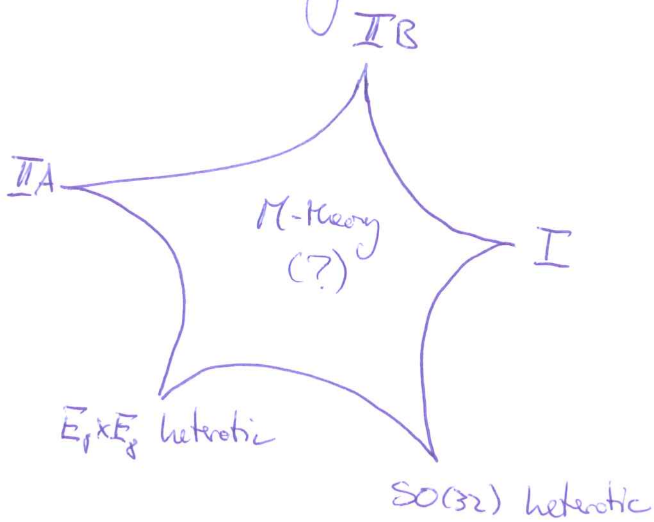
(I) Dilaton in 10d supergravity

- Motivation of 10d SUGRA: dimensional reduction of (unique) 11d SUGRA (c.f. Appendix 2), or as low-energy limit of certain string theories:



Field content

	IIA	IIB	Het.
bos.	G_{MN}, B_2, ϕ	G_{MN}, B_2, ϕ	G_{MN}, B_2, ϕ
	C_1, C_3	C_0, C_2, C_4	A_1^a (SO(32) or $E_8 \times E_8$)
ferm.	$\psi_\mu^{1,2}, \lambda^{1,2}$	$\psi_\mu^{1,2}, \lambda^{1,2}$	$\psi_\mu, \lambda, \chi^a$
	$\Downarrow$	$\Downarrow$	$\Downarrow$
	$N=2$ susy, non-chiral (1,1)	$N=2$ susy, chiral (0,2)	$N=1$ susy

$\rightsquigarrow$ Each one of these string theories / supergravity theories has a dilaton = scalar degree of freedom.

Bosonic action(s)

- $$S^{bos}_{IIA} = S_1 \cancel{\ast} - \frac{1}{4k_{10}^2}\left[\int d^{10}x \sqrt{-G}\left(|F_2|^2 + |\tilde{F}_4|^2\right) + \int B_2 \wedge F_4 \wedge F_4\right]$$

with $$S_1 = \frac{1}{2k_{10}^2}\int d^{10}x \sqrt{-G}\, e^{-2\phi}\left(R + 4\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}|H_3|^2\right)$$

$$F_p = dC_{p-1} \; ; \; H_3 = dB_2 \; ; \; \tilde{F}_4 = dC_3 - C_1 \wedge F_3$$

$$\int |F_p|^2 = \int F_p \wedge \ast F_p$$

- $$S_{IIB} = S_1 - \frac{1}{4k_{10}^2}\left[\int d^{10}x \sqrt{-G}\left(|F_1|^2 + |\tilde{F}_3|^2 + \frac{1}{2}|\tilde{F}_5|^2\right) + \int C_4 \wedge H_3 \wedge F_3\right]$$

with $$\tilde{F}_3 = F_3 - C_0 \wedge H_3$$

$$\tilde{F}_5 = F_5 - \frac{1}{2}C_2 \wedge H_3 + \frac{1}{2}B_2 \wedge F_3$$

$$F_5 = \ast F_5$$

- $$S_{het} = S_1 - \frac{1}{2g_{10}^2}\int d^{10}x \sqrt{-G}\, e^{-2\phi}\, \mathrm{Tr}\left(|F_2|^2\right)$$

with $F_2 = dA_1$.

$\rightsquigarrow$ Dilaton vev determines gravitational coupling in type II theories and gravitational + gauge coupling in heterotic theories.

$\longrightarrow$ Otherwise only kinetic term!

- Note: For connection to (pseudo) Goldstone boson of broken scale invariance, see Appendix 1.

II Moduli in Calabi-Yau ($\mathbb{C}Y$) compactifications

- To compactify six dimensions, consider spacetime to be a product manifold:

$$M_{10} = M_4 \times X_6$$

↳ internal space, determines 4d eff. theory.

- In order to have $N=1$ supersymmetry in four dimensions, X_6 must be a CY manifold: Kähler manifold (= complex Hermitean manifold with closed Kähler form) with vanishing first Chern class $c_1 = \frac{1}{2\pi} \mathrm{tr} R$.

↳ Equivalent definitions:

1. X_6 admits a Ricci-flat metric.
2. The metric on X_6 has holonomy group $SU(3)$.
   ↳ Since under $SO(6)_{\text{Lorentz}} \to SU(3)$ the spinor decomposes as $4 \to 3 + 1$ ↳ single covariantly constant spinor $\Longrightarrow N=1$ SUSY in 4d!
3. X_6 has a globally defined and nowhere vanishing holomorphic three-form (denoted Ω_3)

[4. X_6 has a trivial canonical bundle]

- Remember definition of Dolbeault cohomology:

$$H^{p,q}(M) = \frac{\text{closed } (p,q)\text{-forms on } M}{\text{exact } (p,q)\text{-forms on } M}.$$

and $\dim H^{p,q} = h^{p,q}$ Hodge numbers

Moduli as metric deformations

- Remember we have a metric g which satisfies $\mathrm{Ric}(g) = 0$.
  ↪ Can we deform g such that $\mathrm{Ric}(g+\delta g) = 0$?
- Yes, two kinds of possible deformations:

$$\delta g = \underbrace{\delta g_{ij}\, dz^i dz^j}_{②} + \underbrace{\delta g_{i\bar{j}}\, dz^i dz^{\bar{j}}}_{①} + \text{c.c.}$$

① For Ricci-flatness, $\delta g_{i\bar{j}}\, dz^i dz^{\bar{j}}$ must be harmonic
→ element of $H^{1,1}(X_6)$

Remember Kähler form $J = i g_{i\bar{j}}\, dz^i dz^{\bar{j}}$, $g_{i\bar{j}} = \partial_i \partial_{\bar{j}} K(z,\bar{z})$

⇒ "Kähler moduli"

② $g' = g + \delta g_{ij}$ is no longer Hermitean
↪ put back to Hermitean form via non-holomorphic coordinate transformations.
⟹ g' is hermitean with respect to a different complex structure on X_6
→ "Complex structure moduli"

In addition, construct a $(2,1)$-form $\Omega_{ijk}\, g^{k\bar{k}}\, \delta_{\bar{k}\bar{e}}\, dz^i dz^j dz^{\bar{e}}$
$\in H^{2,1}(X_6)$

- Count CY moduli via Hodge numbers $h^{2,1}, h^{1,1}$. For any CY manifold:

$$\begin{array}{ccccccc}
 & & & h^{3,3} & & & \\
 & & h^{3,2} & & h^{2,3} & & \\
 & h^{3,1} & & h^{2,2} & & h^{1,3} & \\
h^{3,0} & & h^{2,1} & & h^{1,2} & & h^{0,3} \\
 & h^{2,0} & & h^{1,1} & & h^{0,2} & \\
 & & h^{1,0} & & h^{0,1} & & \\
 & & & h^{0,0} & & &
\end{array}
=
\begin{array}{ccccccc}
 & & & 1 & & & \\
 & & 0 & & 0 & & \\
 & 0 & & h^{1,1} & & 0 & \\
1 & & h^{2,1} & & h^{2,1} & & 1 \\
 & 0 & & h^{1,1} & & 0 & \\
 & & 0 & & 0 & & \\
 & & & 1 & & &
\end{array}$$

III Moduli and dilaton in 4d effective action

↪ c.f. Francesco's talk.

- Spectrum of 4d $N=1$ Supergravity:

  gravity multiplet : $g_{\mu\nu}$, ψ_μ
  vector multiplets : V_μ, λ
  chiral multiplets : φ, χ

↪ effective (bosonic) Lagrangian:

$$\mathcal{L}^{4d}_{bos} = -\frac{1}{2}R - g_{i\bar{j}} D_\mu \varphi^i D^\mu \bar{\varphi}^{\bar{j}} + \frac{1}{4} \operatorname{Re} f_{ab} F^a_{\mu\nu} F^{\mu\nu, b}$$

$$+ \frac{1}{4} \operatorname{Im} f_{ab} F^a \tilde{F}^b - V,$$

where
- $g_{i\bar{j}} = \partial_i \partial_{\bar{j}} K(\varphi, \bar{\varphi})$ Kähler metric
- f_{ab} gauge kinetic function
- $V = e^K \left(g^{i\bar{j}} D_i W D_{\bar{j}} \bar{W} - 3|W|^2 \right) + \frac{1}{2} (\operatorname{Re} f)^{-1}_{ab} D^a D^b$

  with $D_i W = \partial_i W + (\partial_i K) W$.

Moduli dependence: Heterotic case

- Spectrum: $h^{1,1}$ Kähler moduli $T_A = v_A + i\alpha_A$

  ↪ from $\delta g_{i\bar{j}}$ ↪ from $\delta B_{i\bar{j}}$

  $h^{2,1}$ complex structure moduli

  $z_a \triangleq (\delta g_{ij})_a$ + 1 chiral superfield S.

a) $f_{ab} = \delta_{ab} \left(e^{-2\phi} + i \frac{a}{8\pi^2} \right) \equiv \delta_{ab} \frac{S}{8\pi^2}$

$\Rightarrow \operatorname{Re} \langle S \rangle \sim e^{-2\langle \phi \rangle}$ determines gauge coupling.

May receive one-loop order corrections + non-perturbative instanton corrections.

b) Tree-level superpotential has moduli as flat directions in vacuum state. → Yukawa couplings may depend on moduli! May receive non-pert. corrections.
→ Francesco.

c) Kähler potential may depend on all kinds of moduli:

i) Kähler: $K_1 = -\ln V = -\ln \int_{X_6} J \wedge J \wedge J = -3 \ln (T + \bar{T})$
→ for single K. modulus describing size of the compactification.

ii) CX. S.: $K_2 = -\ln \left(-i \int_{X_6} \Omega(z) \wedge \bar{\Omega}(\bar{z}) \right)$

iii) dilaton: $K_3 = \ln (S + S^+)$

→ $K_{mod} = K_1 + K_2 + K_3$.

Moduli dependence: Type IIB case

- Compactify on Calabi-Yau orientifold to obtain $N=1$ in 4d:

Take CY manifold X_6 and mod out orientifold projection

$$\mathcal{O} = (-1)^{F_L} \Omega_p \sigma^*$$

with: F_L: no. of left-moving space-time fermions

- Ω_p: World-sheet parity ($\phi, g_{\mu\nu}, C_2$ even; B_2, C_0, C_4 odd)
- σ^*: pull-back of internal space symmetry σ, holomorphic, $\sigma^2 = 1$

⇝ acts on forms as follows:

$\sigma^* \phi = \phi$; $\sigma^* C_0 = C_0$; $\sigma^* g_{\mu\nu} = g_{\mu\nu}$; $\sigma^* C_2 = -C_2$;
$\sigma^* B_2 = -B_2$; $\sigma^* C_4 = C_4$; $\sigma^* \Omega = -\Omega$
→ presence of O3/O7 planes

- Cohomology groups split into "even" and "odd" eigenspaces of σ^*:

$$H^{(p,q)} = H_+^{(p,q)} \oplus H_-^{(p,q)}$$

- Spectrum:

1 chiral multiplet $\quad \tau = C_0 + i e^{-\phi}$

$h_-^{(2,1)}$ chiral multiplets $\quad z^k \quad$ (cx.s. moduli)

$h_-^{(1,1)}$ chiral multiplets $\quad (b^a, c^a)\,;\ G^a = c^a - \tau b^a$

from B_2 $\quad$ from C_2

$h_+^{(1,1)}$ chiral multiplets $\quad (v^\alpha, \rho_\alpha)\,;\ T_\alpha = \frac{3i}{2}\rho_\alpha + \frac{3}{4} f(v_\alpha)$

from $\delta g_{i\bar{j}}$ $\quad$ from C_4

\+ gravity & vector multiplets

$\rightarrow$ gauge kinetic function depends on z^k, not on τ or ϕ.

Kähler potential:

$$K_{cs} = -\ln\left(-i \int_{*} \Omega(z) \wedge \bar{\Omega}(\bar{z})\right) \quad \text{unchanged}$$

$$K_K = -\ln(-i(\tau - \bar{\tau})) - 2\ln\left(\mathcal{V}(\tau, T, G)\right)$$

not explicitly solvable for v^α.

IV References

- Books by Polchinski + Green, Schwarz, Witten
- B. Greene: string theory on CY manifolds, arXiv 9702135
- P. Candelas, X. C. de la Ossa: Moduli space of CY manifolds, Nucl. Phys. B355 (1991)
- S. Giddings, S. Kachru, J. Polchinski: Hierarchies from fluxes in string compactifications, Phys. Rev. D66 (2002)
- T.W. Grimm, J. Louis: The effective action of N=1 CY orientifolds, Nucl. Phys. B699 (2004)

A.1 Dilaton and scale invariance

- Consider general relativity in D dimensions:

$$S = -\frac{1}{2k^2}\int d^Dx \sqrt{g}\, g^{MN} R_{MN}$$

- Classically, ~~t~~ there is a $\overset{\text{Weyl}}{\text{scaling}}$ $g_{MN} \xrightarrow{(*)} t^{-2} g_{MN}$ which leaves R_{MN} invariant and transforms S as

$$S \longrightarrow t^{-(D-2)} S.$$

$\longrightarrow$ normalization of S is invariant in the classical theory.

- In quantum theory, (*) is not a symmetry. E.g. the path integral

$$Z = \int e^{iS/\hbar}$$ is not invariant.

- The discussed supergravity actions have the same classical scale invariance (when fermions are included, $\psi \to t^{1/2}\psi$ and $e_M^a \to e_M^a \cdot t^{-1}$ for the vielbein), once the dilaton transforms as

$$\phi \longrightarrow t^2 \phi.$$

- Note that while $\langle\phi\rangle$ is arbitrary, we must have $\langle\phi\rangle \neq 0$.

$\rightarrow$ 10d SUGRA scale invariance is spontaneously broken.

$\Rightarrow$ In classical theory, massless dilaton ϕ = Goldstone boson of spont. broken scale invariance.

- In quantum theory $\rightarrow$ pseudo-Goldstone boson.

$\longrightarrow$ mass?

$\hookrightarrow$ $m_\phi = 0$ as long as SUSY is unbroken, as well as $\underline{V(\phi)=0}$.

A.2 10d SUGRA from 11d SUGRA

- 11d SUGRA is unique and has maximal spacetime supersymmetry
- The field content is very simple:

$$\underset{\text{metric}}{G_{MN}}, \quad \underset{\text{3-form}}{A_3}, \quad \underset{\text{gravitino}}{\Psi_M}$$

↳ bosonic action:

$$S_{11}^{bos} = \frac{1}{2k_{11}^2}\int d^{11}x\sqrt{-G}\left(R - \frac{1}{2}|F_4|^2\right) - \frac{1}{12k_{11}^2}\int A_3 \wedge F_4 \wedge F_4$$

- Perform dimensional reduction via compactification of 11th dimension on a circle of radius r:

$$G_{MN} \longrightarrow G_{\mu\nu}, \; A_\mu, \; \underbrace{\sigma}_{\text{scalar}}, \qquad \text{i.e.}$$

$$\begin{aligned} ds^2 &= G^{11}_{MN}\, dx^M dx^N \\ &= G^{10}_{\mu\nu}\, dx^\mu dx^\nu + \exp(2\sigma)\left(dx^{10} + A_\nu dx^\nu\right)^2 \end{aligned}$$

$$A_{MNP} \longrightarrow A_{\mu\nu\rho}, \; A_{\mu\nu 10} \equiv A_{\mu\nu}$$

$$\Rightarrow S_{11} \longrightarrow S_{10} = S_1 + S_2 + S_3$$

with
$$S_1 = \frac{1}{2k_{10}^2}\int d^{10}x\sqrt{-G}\left(e^{\sigma}R - \frac{1}{2}e^{3\sigma}|F_2|^2\right)$$

$$S_2 = -\frac{1}{4k_{10}^2}\int d^{10}x\sqrt{-G}\left(e^{-\sigma}|F_3|^2 + e^{\sigma}|\tilde{F}_4|^2\right)$$

$$S_3 = -\frac{1}{4k_{10}^2}\int A_2 \wedge F_4 \wedge F_4$$

- Here, $\kappa_{10}^2 = \dfrac{\kappa_{11}^2}{2\pi r}$. Performing the redefinitions

$$G_{\mu\nu} \longrightarrow e^{-\sigma} G_{\mu\nu}\,, \quad \sigma \longrightarrow \frac{2\phi}{3}$$

this yields the action of $\underline{\mathrm{II}}$A supergravity!