MODULI STABILISATION

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▶ OUTLINE

$\boxed{1}$ WHY BOTHER?

Theories with extended spacetimes $(D>4)$ provide a framework to go beyond GR + SM

↳ these include (but are not restricted to) the low energy limits of the various string theories

they generally include several planck coupled scalar d.o.f., characterizing the size and shape of the extra-dimensions, the string coupling, background gauge d.o.f. ...

↳ **MODULI FIELDS**

4 reasons to study moduli stabilisation:

① Need to explain why we seem to live in 4D

② Planck coupled scalar d.o.f. are cosmologically dangerous: cosmological moduli problem

③ 5th force constraints

④ In string models the low energy parameters like masses and

couplings depend on the compactification geometry (on the moduli)
Until we can stabilise the moduli very little knowledge can be extracted from these models

▸ THE COSMOLOGICAL MODULI PROBLEM

there are 2 ways in which a massive planck coupled field can mess up cosmological evolution:

1. If ϕ is stable it will store too much energy and will overclose the universe (Polonyi Problem)
   ↳ upper bound on m_ϕ: $\boxed{m_\phi \lesssim 10^{-30}\,eV}$

2. If ϕ decays into visible sector it will have to do so before BBN or otherwise it will alter the predictions for the abundance of elements
   ↳ $\boxed{m_\phi > 10^{7}\ MeV}$

▸ 5TH FORCE CONSTRAINTS

moduli as planck coupled scalars would mediate unobserved 5th forces between visible sector particles.
Experimental constraints from solar system to cosmological scales forbid moduli to have masses in the range

$$[10^{-17}, 10^{-2}]\,eV$$

∴ Consistency of the higher dimensional theories with the 4D world ⊕ cosmological evolution of planck coupled scalar fields requires a mechanism to give masses to moduli fields.

↳ MODULI STABILISATION

2 A SIMPLE TOY MODEL

↳ FREUND-RUBIN COMPACTIFICATION AND THE ROLE OF FLUXES

Consider a 6 dimensional Einstein-Maxwell theory with action

$$\boxed{S = \int d^6x \sqrt{|G|} \left(M_{(6)}^4 R_{(6)} - M_{(6)}^2 |F_2|^2 \right),}$$

where $M_{(6)}$ is the 6D Planck mass, $R_{(6)}$ the 6D scalar curvature and F_2 the field strength tensor.

Let the metric G be given as a direct product of a 4D non-compact space and a 2D compact manifold with total volume $R^2 l_6^2$:

$$G_{MN} = \left(\begin{array}{c|c} g_{\mu\nu} & 0 \\ \hline 0 & R^2 h_{mn} \end{array} \right) \qquad \text{with} \begin{cases} \mu, \nu = 0,1,2,3 \\ m, n = 4,5 \end{cases}$$

Note that the 6D planck mass (and its inverse, the planck length $l_6 = 1/M_6$) is the only dimensionful parameter in the theory. In particular the volume of the compact space is

$$V = \int d^2x \sqrt{g_2} = R^2 \int d^2y \sqrt{h} \underset{\uparrow}{=} R^2 l_6^2$$

metric h has unit volume by construction

where $R = R(x)$ is a dimensionless quantity. This will show up in the 4D compactified theory as a scalar field

↳ MODULUS FIELD

The point of this exercise is to show that by turning on F_2 one can generate a potential for $R(x)$ and under certain circumstances even stabilise it.

From the block diagonal form of the metric it follows that

$$\sqrt{|G|} = \sqrt{|g|} \, R^2 \sqrt{|h|}$$

and also that

$$R_{(6)} = G^{MN} R_{MN} = G^{MN} R^{P}_{MPN} = G^{\mu\nu} R^{P}_{\mu P\nu} + G^{mn} R^{P}_{mPn}$$
$$= g^{\mu\nu}\left(R^{\alpha}_{\mu\alpha\nu} + R^{p}_{\mu p\nu}\right) + R^{-2} h^{mn}\left(R^{\alpha}_{m\alpha n} + R^{p}_{mpn}\right)$$

then $$R_{(6)} \supset R_{(4)}(g) + R^{-2} R_{(2)}(h) + \dots$$

the action can then be written as:

$$S = \int d^4x \sqrt{|g|} \int R^2 \sqrt{h}\, d^2y\, M_{(6)}^4 \left(R_{(4)}(g) + R^{-2} R_{(2)}(h) + \dots\right)$$

$$= \int d^4x \sqrt{|g|}\, R^2 M_{(6)}^2 R_{(4)}(g) + M_{(6)}^4 \chi(M_2) + \dots$$

where we have once more used the fact that the metric h has unit volume and defined the integrated curvature of M_2 as

$$\int \sqrt{h}\, R_{(2)}(h)\, d^2y \equiv \chi(M_2)$$

take R_0 to be the background value of R (we will want to identify it with the minimum later):

$$S = \underbrace{M_{(6)}^2 R_0^2}\int d^4x \sqrt{|g|} \left(\frac{R}{R_0}\right)^2 R_{(4)}(g) + \frac{M_{(6)}^2}{R_0^2} \chi(M_2) + \dots$$

from here we can read off the typical relation between the 4D and 6D Planck masses:

$$\boxed{M_{(4)}^2 = M_{(6)}^2 R_0^2}$$

‼ Aside: LARGE extra dimensions (ADD) as a "solution" to the hierarchy problem:

Gravity is as strong as the other forces ($M_{(6)} \sim M_{EW}$) but it looks weaker because it is "diluted" due to the existence of LARGE extra-dimensions ($\Leftrightarrow$ large R)

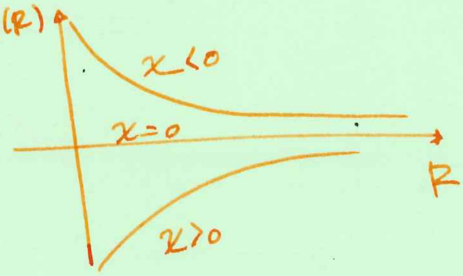
$$R_0 \sim \frac{M_{(4)}}{M_{(6)}} \sim 10^{16}$$

Note that unless we can provide a mechanism that naturally stabilises R around these values, LED is just a reparametrization of the hierarchy problem.

↳ Moduli stabilisation.

A further conformal transformation is required to cast the action into the 4D Einstein frame: $g_{\mu\nu} = \left(R/R_0\right)^{-2} f_{\mu\nu}$

$$S = \int d^4x \sqrt{|f|}\left[M_{(4)}^2 R_{(4)}(f) + \underbrace{\frac{M_{(4)}^4}{R^4}\chi(M_2)}_{\text{curvature contribution to the Radion's potential}} + \text{Kinetic terms} + \text{Flux terms}\right]$$



So by a suitable choice of the compactification manifold we can generate a potential for the radial modulus but <u>further ingredients</u> are necessary to stabilise it (i.e. generate a potential with a minimum)

$\longrightarrow$ <u>turn on FLUXES</u>

$$F_{MN} = \begin{cases} F_{\mu\nu} & \times \\ F_{\mu m} & \times \\ F_{mn} & \checkmark \end{cases} \quad \text{by requiring 4D Poincaré invariance}$$

then $-M_{(6)}^2 \int d^6x \sqrt{G}\,|F_2|^2 = -M_{(6)}^2 \int d^4x \sqrt{g}\, R^2 \int dy \sqrt{h}\,|F_2|^2 =$

$$= -\int d^4x \sqrt{|g|}\, R^2 \int d^2y \sqrt{h}\, M_{(6)}^2 F_{..}F_{..}h^{..}h^{..}R^{-4}$$

E-frame

$$= -\int d^4x \sqrt{|f|}\, \frac{M_{(6)}^2}{R_0^{-4}} R^{2-4-4} \underbrace{\int d^2y \sqrt{h}\, F_{..}F_{..}h^{..}h^{..}}_{\sim N^2 \times M_{(6)}^2}$$

$$= -\int d^4x \sqrt{|f|}\, \frac{N^2 M_{(4)}^4}{R^6}$$

$\rightsquigarrow$ Flux contribution to the modulus potential

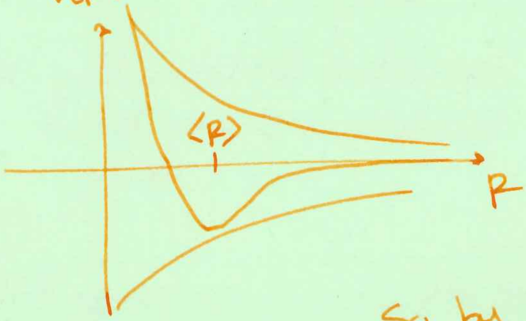
The full 4D action is then:

$$S = \int d^4x \sqrt{|f|}\left[R_{(4)}(f) - 8(\partial R)^2 \underbrace{+\frac{\chi(M)}{R^4} - \frac{N^2}{R^6}}_{-V(R),\ \text{modulus potential}}\right]$$

$$\boxed{V(R) = \frac{N^2}{R^6} - \frac{\chi(M)}{R^4}}$$ (in units of the 4D Planck Mass)

for $\chi(M) \leq 0$ no minimum $\rightarrow$ must include more structure (D-branes, O-planes, ...)

for $\chi(M) > 0$



$$\boxed{\langle R \rangle = \sqrt{\frac{3}{2\chi(M)}}\, N}$$

So by playing fluxes against integrated curvature of the compact space we manage to stabilise the radial modulus.

3 Type IIB Flux Compactifications

We have seen in the F.R. example how by playing curvature against fluxes one can stabilise the size of the compact space and end up with a theory that at low energies looks 4D.

Let us now turn to the more involved case of type IIB SUGRA (with local sources). This theory can be viewed as the low energy limit of the IIB string. We will see that the same principles of stabilisation by fluxes will also work here.

Recall that the closed string sector of the IIB string includes:

- <u>NS-NS sector</u>: ϕ, g_{MN}, B_2
- <u>R-R sector</u>: l, C_2, C_4

At low energies these combine to give the following effective action:

$$\boxed{S_{IIB} = \frac{2\pi}{l_s^8}\int d^{10}x\sqrt{|g_{10}|}\left\{\underbrace{R}_{\text{gravity}} - \underbrace{\frac{\partial_M\tau\,\partial^M\bar{\tau}}{2[\mathrm{Im}(\tau)]^2}}_{\substack{\text{axio-dilaton's kinetic term}\\ \tau = l + ie^{-\phi}}} - \underbrace{\frac{G_3\cdot\bar{G}_3}{12\,\mathrm{Im}\tau} - \frac{\tilde{F}_5^{\,2}}{4.5!}}_{\substack{\text{flux terms}\\ G_3 = F_3 - \tau H_3}}\right\} + \underbrace{S_{cs}}_{\text{chern-simons action}} + \underbrace{S_{loc}}_{\text{local sources (D branes...)}}}$$

Like in the Freund-Rubin example we want the higher dimensional theory to look effectively 4D at low energies

↳ choose spacetime to be: $\mathcal{M}_{10} = \mathcal{M}_4 \times \mathcal{M}_6$

$\mathcal{M}_4$ ↓ 4D non-compact space

$\mathcal{M}_6$ ↳ 6D compact space!

<u>What do we want from the 4D-theory?</u>

- We would like the 4D theory to be <u>supersymmetric</u>. This forces

<u>$\mathcal{M}_6 \equiv$ Calabi-Yau Manifold</u>

- We would also like to stabilise the compact space by turning on fluxes like in the F.R case

We want to find if these two goals can be achieved simultaneously, that is if we can turn on fluxes to stabilise the geometry and still have a compact space that is CY.

↳ this is not trivial since fluxes gravitate (i.e. they show up in Einstein's Equations) it is not clear that if we start with a flux-less CY compactification, that we will preserve this geometry even after turning on flux.

Giddings, Kachru and Polchinski have shown in [0105097] that we can consistently turn on G_3-flux in the compact space and stabilise (some of the) moduli. The resulting geometry is no longer a CY, but it is close. The backreaction of fluxes generates warping, so the metric takes the form

$$ds_{10}^2 = \underbrace{e^{2A(y)}}_{\text{WARP FACTOR}} \eta_{\mu\nu} dx^\mu dx^\nu + \underbrace{e^{-2A(y)}}_{\text{WARP FACTOR}} \underbrace{\tilde{g}_{mn}}_{\text{CY metric}} dy^m dy^n$$

So we can still use the mathematics of CY-spaces to study the problem.

Consistency of these warped compactifications requires:

- the warp factor to be related to the 4-form potential

$$\boxed{e^{4A} = \alpha^2}$$

note that $\tilde{F}_5 = (1+*)\left[d\alpha \wedge dx^0 \wedge dx^1 \wedge dx^2 \wedge dx^3\right]$

- G_3 is imaginary self-dual:

$$\boxed{i G_3 = *_6 G_3}$$

- compactification must have negative tension objects:

$$\boxed{\frac{1}{4}\left(T^{m}{}_{m} - T^{\mu}{}_{\mu}\right)^{loc} = T_3 \rho_3^{loc}}$$

For a long time the existence of such objects was unknown and this lead to a well known no-go theorem [Maldacena-Nunez] stating that only solutions with no fluxes and constant warp factor were allowed in (pure) IIB SUGRA.

↳ Examples of such objects are D3-branes and O3-planes

Note:

From last week we have learned that compactification of IIB theory on a CY manifolds yields $\mathcal{N}=2$ SUSY.

From a pheno point of view this is too much, we would rather have just $\mathcal{N}=1$.

↳ Solution: Introduce orientifold planes which effectively project out the extra degrees of freedom, leaving the field content of a $\mathcal{N}=1$ theory.

For details on this splitting see Louis and Grimm {0412277, 0403067}

What are the consequences of the ISD condition?

In general we would have

$$G_3 = G_{(3,0)} + G_{(2,1)} + G_{(1,2)} + G_{(0,3)}$$

but the (3,0) and (1,2) components are not consistent with ISD. So we are left with

$$\boxed{G_3 = G_{(2,1)} + G_{(0,3)}}$$

Even though this may seem a condition on the fluxes, it is a condition on the moduli instead. Recall that the axio dilaton enters the definition of G_3 and that $*_6$ involves the metric and hence the geometric moduli.

↳ ISD ⇒ {complex structure, Axio dilaton} are stabilised

The Kähler moduli remain unfixed at this level.

One must note that this is not the whole story. In the spirit of perturbation theory there will be many more terms that can be added to the 10D action that are consistent with the symmetries we want to have (diff. invariance, T-duality in the NS-sector...).

These terms will be suppressed by further powers of l_s and g_s and are expected to be subleading but can play an important role in the phenomenology (particularly in the Kähler moduli sector).

Example of higher order terms:

$$\frac{2\pi}{l_s^8}\int d^{10}x\sqrt{|g_{10}|}\, l_s^6\left[R^4 + R^3\left(G_3G_3 + \bar{G}_3\bar{G}_3 + F_5^2 + \partial\tau\partial\bar{\tau} + \nabla^2\tau\right) + R^2\left(G^4 + \cdots\right) + G_3^8 + \cdots\right] \quad \text{at order } \alpha'^3 (\Leftrightarrow l_s^6)$$

We will come back to these higher order corrections to the action in part $\boxed{4}$ when we look at mechanisms for stabilisation of the Kähler moduli.

For the moment let us look at the same compactifications in the language of 4D $\mathcal{N}=1$ SUSY.

THE 4D PERSPECTIVE

Keeping in mind the lessons from the FR example of the previous section we note that

$$S_{10D} \sim \int_{M_4\times CY} d^{10}x\sqrt{|g_{10}|}\left[\underbrace{R_{10}}_{\substack{R_4 + \text{geometric}\\ \text{moduli kinetic}\\ \text{terms}}} + \underbrace{|G_3|^2}_{\substack{\text{flux generated}\\ \text{potential}}} + \underbrace{\frac{\partial\tau\partial\bar{\tau}}{\mathrm{Im}(\tau)^2}}_{\substack{\text{Axio-dilaton}\\ \text{kinetic term}}}\right]$$

The reduction of the 10D to 4D is a rather lengthy process so we skip to the final result

$$S_4 = \int d^4x\left[R - K_{A\bar{B}}\,\partial\Phi_A\cdot\partial\bar{\Phi}_B + V\right] \quad \text{where}$$

$\Phi_A = \{ T_A, U_A, S \}$ $\qquad$ $T_A = \tau_A + i\, b_A$

T_A: kähler; U_A: complex structure; S: dilaton

$K_{A\bar B}$ is the Kähler metric (the metric in moduli space) and is given by the 2nd derivative of the kähler potential K: $\boxed{K_{A\bar B} = \frac{\partial^2 K}{\partial \Phi_A \partial \bar\Phi_B}}$,

$K_{A\bar B}$ is found by reduction of the 10D Ricci scalar assuming a spacetime of the form $M_4 \times CY(3)$.

recall that the c.s. and kähler moduli are the coefficients of the metric perturbation in CY(3):

$$\begin{cases} \delta g_{a\bar b} = t^m\, \omega^m_{a\bar b} \\ \delta g_{ab} \sim z^k \end{cases}$$

t^m: kähler; z^k: complex structure

the scalar potential V for the moduli originates from

SCALAR POTENTIAL

$$\int d^{10}x \sqrt{|g_{10}|}\, |G_3|^2 = \int d^4x\, d^6y \sqrt{|g_4|}\sqrt{|g_6|}\, |G_3|^2 = \int d^4x \sqrt{|g_4|} \boxed{\int d^6y \sqrt{|g_6|}\, |G_3|^2}$$

turn on G_3 on CY only
$(G_{\mu\nu\rho} = G_{\mu m n} = G_{\mu\nu n} = 0)$

$V = \int d^6y \sqrt{|g_6|}\, |G_3|^2 \longrightarrow$ using the fact that $G_3 = G_{(2,1)} + G_{(0,3)}$ and the geometry of the CY it can be shown that V takes the form

$$\boxed{V_F = e^K \left(K^{A\bar B} D_A W \overline{D_B W} - 3|W|^2 \right)} \quad \text{with} \quad \boxed{W = -\int_M G_3 \wedge \Omega}$$

Flux superpotential

i.e. the standard $\mathcal{N}=1$ SUGRA F-term potential, where

$$D_A W = \partial_A W + W \partial_A K \quad \text{and} \quad \boxed{K = -2\log \mathcal{V} - \log(-i(\tau - \bar\tau)) - \log\left(-i \int_M \Omega \wedge \bar\Omega\right)}$$

kähler potential

the theory is by construction supersymmetric, with susy being possibly broken by the vacuum/flux choice. The dilaton and C.S. are stabilised supersymmetrically.

since G_3 is ISD $G_3 = G_{(2,1)} + G_{(0,3)}$ then if the $(0,3)$ component is $\neq 0$ the kähler moduli will break SUSY.

to see this note that the F-term in the T-sector is:

$$D_T W = \partial_T W + W \partial_T K$$

since $W = \int_{M} G_3 \wedge \Omega$ is independent of T we find

$$D_T W = W \partial_T K \quad \text{so } D_T W = 0 \Leftrightarrow W = 0 \rightarrow \text{Condition for } \underline{\underline{\text{unbroken SUSY}}}$$

$$-W = \underbrace{\int_M G_{(2,1)} \wedge \Omega_{(3,0)}}_{\substack{=0 \\ \text{by the properties} \\ \text{of the } \wedge}} + \underbrace{\int_M G_{(0,3)} \wedge \Omega_{(3,0)}}_{\neq 0 \text{ in general}}$$

So we see that for SUSY to be preserved $G_{(0,3)}$ has to vanish. otherwise it will be <u>broken</u> by the <u>Kähler moduli</u>.

The scalar potential factorises to:

$$V_F/_{e^K} = \underbrace{K^{S\bar{S}} D_S W \overline{D_S W} + K^{U\bar{U}} D_U W \overline{D_U W}}_{\substack{=0 \text{ since } D_S W = D_U W = 0 \\ \text{the fluxes stabilise dilaton and} \\ \text{c.s. at a SUSY point.}}} + K^{T\bar{T}} D_T W \overline{D_T W} - 3|W|^2$$

$$= K^{T\bar{T}} D_T W D_{\bar{T}} \bar{W} - 3|W|^2 = |W|^2 \left(K^{T\bar{T}} \partial_T K \partial_{\bar{T}} K - 3\right)$$

potential for the Kähler moduli

However $K^{T\bar{T}} \partial_T K \partial_{\bar{T}} K = 3$ so $\boxed{V_{\text{Kähler}} \equiv 0}$

NO-SCALE STRUCTURE

<u>Kähler moduli are exactly flat directions at this level.</u>

Higher order corrections (like the α'^3 discussed before) will play an important role in the stabilisation of the size of the compact space.

4 Towards (almost) Realistic Phenomenology

So far we have seen that by turning on fluxes in the compact space one is able to stabilise some of the moduli. In particular the dilaton and the complex structure are fixed but the Kähler moduli are not.

↳ Intuitively, we have stabilised the string coupling and the shape of the compact space but crucially not its size.

At this point we should remind ourselves that we are dealing with an effective theory and that there are further effects that have not been taken into account.

↳ there are 2 expansion parameters in this theory

- the string length l_s ($\Leftrightarrow$'y α' ; $l_s = 2\pi\alpha'^{1/2}$)
- the string coupling g_s

In particular the Kähler potential and the superpotential will necessarily include higher order terms:

$$\begin{cases} K = K_{tree} + K_{pert} + K_{non\text{-}pert} \\ W = W_{tree} + W_{np} \end{cases}$$

$\longrightarrow$ protected by non-renormalisation theorem $\Rightarrow$ no perturbative correction

▶ Perturbative Corrections to K

It is usually expected that $K_p \gg K_{np}$. Noting that ⊛ we can write

$$K = K_{tree} + K_{\alpha'} + K_{g_s}$$

↳ "string loop corrections"

↳ α' corrections

We will ~~[illegible]~~ not discuss string loop corrections here. For further reading see the works of Cicoli, Conlon, Quevedo { 0708.1873, 0805.1029

The dominant α' correction enters at order $(\alpha')^3$ and has the form

$$\sim \int d^{10}x \sqrt{|g|}e^{-2\phi} \alpha'^3 R^4 \quad \text{in the 10D-string frame.}$$

Why are there no α' and $(\alpha')^2$ corrections?

Hard to see from an EFT point of view but

$\boxed{1}$ no α', α'^2 terms consistent with SUSY action.
↳ Bergshoeff / Roo, 1989

$\boxed{2}$ these terms (α', α'^2) just do not show up in worldsheet computations
↳ Ellis / Jetzer / Mizrachi, 1987

BBHL taught us in 0204254 that the $\alpha'^3 R^4$ term will give rise to a correction to the Kahler potential of the Kahler moduli of the form

$$K = -2\log\left(\mathcal{V} + \underbrace{\frac{\xi}{2g_s^{3/2}}}\right)$$

proportional to the Euler number of the compact space $\chi(M) = 2(h_{1,1} - h_{2,1})$

This term breaks the no-scale structure, but by itself does ~~not~~ stabilise the Kahler moduli:

$$\delta V_{\alpha'} \sim \frac{3\xi |W|^2}{4 g_s^{3/2} \mathcal{V}^3} \rightarrow \underline{\underline{\text{runaway potential}}}$$

▶ Non-perturbative corrections to W

W_{tree} depends on dilaton and C.S. but not on Kahler

the dependence on T-moduli can be introduced through non-perturbative effects like

- Euclidean D3 brane
- Gaugino condensation

$$S_{ED3} = \frac{1}{(2\pi)^3 \alpha'^2} \int_{\Sigma_i} -\sqrt{g} + i C_4 \Longrightarrow \text{enters path integral as } e^{-S_{ED3}}.$$

noting that $\int_{\Sigma_i} \sqrt{g} = \mathrm{vol}(\Sigma_i) \sim T_i$ where T_i is the kähler modulus parametrizing the size of Σ_i cycle. thus generates a term

$$\boxed{e^{-T_i} \sim W_{ED3}}$$

Note that for the ED3 to contribute to W rather than to K, it must have exactly 2 ~~b~~ fermionic zero modes to give rise to a $\int d^4 u \, d^2\theta \, (\dots)$ term

$$S_{D7} \supset \int_{M_4 \times \Sigma_i} \sqrt{g}\, e^{-\phi} F_{\mu\nu} F^{\mu\nu}$$ → the gauge coupling of the theory is

$$\frac{1}{g^2_{YM}} = \frac{\mathrm{Re}(T_i)}{2\pi}$$

with the right choice of fluxes, A stack of N_c branes gives rise to a pure $\mathcal{N}=1$ $SU(N_c)$ YM theory.

↳ undergoes gaugino condensation at a scale Λ_{strong} (determined by looking ~~at~~ for the energy scale at which the gauge coupling blows up)

thus than gives rise to $\boxed{W_{D7} = \Lambda^3_{strong} = A\, e^{2\pi T_i / N_c}}$

Both ED3 and gaugino cond give rise to $\boxed{W_{np} = \sum_i A_i\, e^{-a_i T_i}}$

↓

Introduces kähler moduli dependence in the superpotential.

With these corrections to the Effective Action we are in a position to go beyond No-scale and to stabilise the Kähler moduli. Let us start by looking at the

▶ KKLT scenario

let $K = -2 \log \mathcal{V}$; $W = W_0 + W_{np}$

And assume the simplest geometry possible, with a single kähler modulus parametrizing the volume of the compact space: $\mathcal{V} = \left(\frac{T + \bar{T}}{2}\right)^{3/2}$

let T support one non-perturbative effect: $W_{np} = A e^{-aT}$.

recall that W_0 stabilises S and U supersymmetrically : $\begin{cases} D_U W_0 = 0 \\ D_S W_0 = 0 \end{cases}$

we look for a solution for the Kahler sector that also ~~breaks~~ preserves SUSY : $D_T W\Big|_{T_0} = 0 \Leftrightarrow \partial_T W_{np} + (W_0 + W_{np})\, \partial_T K = 0$

$$\Leftrightarrow -a W_{np} + (W_0 + W_{np}) \frac{(-2)}{\mathcal{V}} \frac{3}{2} \left(\frac{T+\bar{T}}{2}\right)^{1/2} \frac{1}{2} = 0$$

$$\Leftrightarrow -a W_{np} - \frac{3}{2}(W_0 + W_{np}) \frac{1}{\left(\frac{T+\bar{T}}{2}\right)} = 0 \Leftrightarrow \boxed{W_0 = -W_{np}\left(+\frac{2}{3} a \left(\frac{T+\bar{T}}{2}\right) + 1\right)}$$

or

$$W_0 = -A e^{-aT_0}\left[\frac{2}{3} a \left(\frac{T_0 + \bar{T}_0}{2}\right) + 1\right] \quad (*)$$

→ If we tune the fluxes (W_0) such that this holds, we have a SUSY AdS min.

The AdS nature of the vacuum can be seen by plugging W_0 into the F-term potential

$$V_F = e^K \Big[\underbrace{(K^{-1})^{A\bar{B}} D_A W \, D_{\bar{B}} \bar{W}}_{=0 \text{ due to requirement of SUSY}} - 3|W|^2 \Big]$$

$$= -3|W|^2 e^K = -\frac{a^2 A^2 e^{-2a_0 T_0}}{6 T_0} < 0 \rightarrow \underline{\underline{\text{AdS}}}$$

An important observation is required here. The Action that we started with is valid in the limit ~~when~~ of large volumes:, ~~that is, in the limit of~~ $T \gg 1$. For $T \sim \mathcal{O}(1)$ the higher order corrections to S will be as important as (and possibly more important than) the terms we are considering here.

↳ must make sure $T \gg 1$ which by (*) implies that the flux superpotential W_0 must be <u>rather small.</u>

<u>Example</u> from KKLT's paper: $W_0 \sim 10^{-4}$, $A = 1$, $a = 0.01 \Rightarrow T_0 \sim 100$

$$\Updownarrow$$

$$\boxed{\mathcal{V} \sim 10^3\, l_s^6}$$

At this stage the vacuum is still <u>SUSY AdS</u> → not the ~~SUSY~~ dS that we'd like to find.

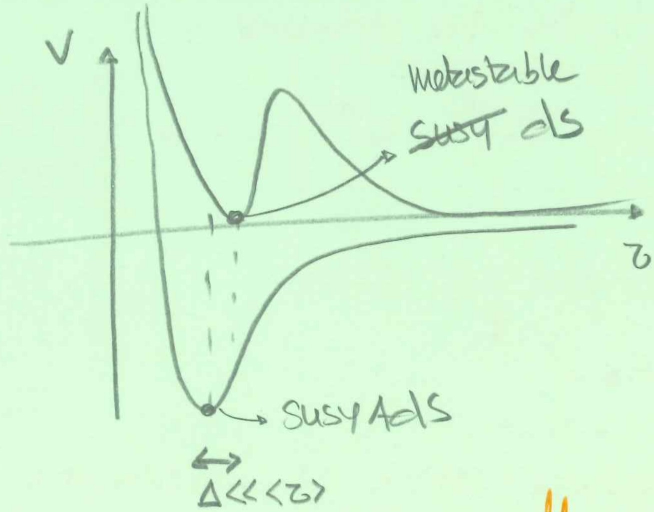
↳ solution: Add $\overline{D3}$ brane to simultaneously uplift the AdS to dS and break SUSY (hard)

the addition of $\overline{D3}$ translates into an extra term in the scalar potential:

$$\delta V \sim \frac{D}{\mathrm{Re}(T)^3} \sim 1/\mathcal{V}^2$$

the full potential is then:

$$\boxed{V = \frac{aAe^{-a\tau}}{2\tau^2}\left(\frac{1}{3}\tau a A e^{-a\tau} + W_0 + Ae^{-a\tau}\right) + \frac{D}{\tau^3}}$$



KKLT manages to generate a phenomenologically viable minimum, however:

→ requires small W_0 for control over the SUGRA approximation. While $W_0 \ll 1$ is certainly possible, it is known that $W_0 \sim \mathcal{O}(1-10)$ are much more likely.

→ when more than 1 Kähler modulus is present in the theory KKLT requires one n.p. effect per cycle (one e^{-T_i} for each T_i). This is a very non-trivial requirement on the geometry.

→ the breaking of ~~susy~~ susy by $\overline{D3}$ is explicit. Is it valid to use supersymmetric framework?

→ large values of N_c (or ⇔'ly small values of a_i) required may be inconsistent with the limit on the entropy of dS space in quantum gravity

LARGE Volume Scenario

In the KKLT treatment we have ignored the α'^3 correction to K. It can be shown to be subleading ~~[illegible]~~ near the minimum

$$V_{KKLT} \sim -3\, m_{3/2}^2 M_p^2 \sim \frac{M_p^4}{\mathcal{V}^2} \gg \frac{M_p^4}{\mathcal{V}^3} \sim V_{\alpha'^3}$$

But α'^3-term is there, so can it give us some new vacua other than KKLT, and if so what are their properties?

Consider the α'^3-corrected action:

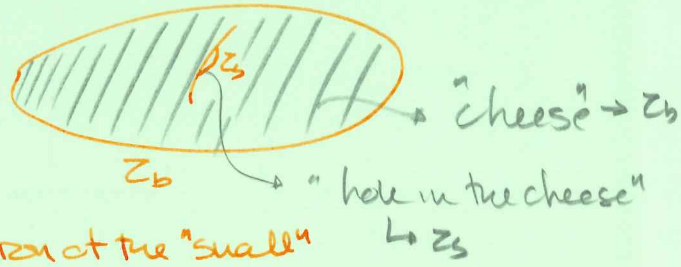
$$K = -2\log(\mathcal{V} + \hat{\xi}) \quad ; \quad W = W_0 + \sum_i A_i e^{-a_i T_i}$$

take a geometry with the volume

$$\boxed{\mathcal{V} = \tau_b^{3/2} - \tau_s^{3/2}} \longrightarrow \text{Swiss cheese manifold}$$

and let τ_s support the n-p effect

$$W_{np} = A_s e^{-aT_s}$$



Computing the potential (after minimisation of the "small" axion)

$$\boxed{V = C_1 \frac{\sqrt{\tau_s}\, e^{-2a\tau_s}}{\mathcal{V}} \underset{\substack{\uparrow \\ \text{from the small} \\ \text{axion minimisation}}}{-} C_2 \frac{a\tau_s\, e^{-a\tau_s}}{\mathcal{V}^2} + \underbrace{C_3 \frac{1}{\mathcal{V}^3}}_{(\alpha')^3 \text{ term}}}$$

For positive C_3 the minimum lies at
↳ (manifolds with $h_{21} > h_{11}$)

$$\boxed{\tau_b^{3/2} \sim \mathcal{V} \sim e^{1/g_s}} \quad ; \quad \boxed{\tau_s \sim 1/g_s}$$

~~[illegible]~~ recalling that g_s must be $\ll 1$ for the perturbative expansion to make sense we see that $\tau_s \gg 1$ and that

$$\boxed{\text{the volume is exponentially LARGE}}$$

Note that this is achieved without any tunning of the flux superpotential.

Example: $\begin{cases} W_0 = 10 \\ A = 1 \\ a = 2\pi \\ g_s = 0.027 \\ \xi = 1.31 \\ \lambda = \frac{1}{9\sqrt{2}} \end{cases} \Longrightarrow \boxed{\begin{matrix} \tau_s \sim 5 \\ \mathcal{V} \sim 2\times 10^{15} \end{matrix}}$

good for TeV scale susy since (naively)

$$m_{3/2} \sim \frac{M_p}{\mathcal{V}} \sim \frac{2\times10^{18}\,\text{GeV}}{2\times10^{15}} = \underline{1\,\text{TeV}}$$

the LVS minimum is AdS and non-supersymmetric → the Kähler moduli get non-vanishing F-terms

uplifting is still required → proceed as in KKLT by adding $\overline{D3}$ (or including dilaton-dependent non perturbative terms).

not as problematic as in KKLT as the LVS minimum is already non-susy.

Note that ~~other~~ the a posteriori uplifting that is required by those models is ~~a~~ poorly understood at best.

▶ **Kähler UpLiFTING** → note that $\delta V_{\overline{D3}} \sim 1/\mathcal{V}^3$ and that $\delta V_{\alpha'^3} \sim 1/\mathcal{V}^3$
can we use the α'^3 term to perform the uplifting?

take $K = -2\log(\mathcal{V} + \hat{\xi})$ and $W = W_0 + Ae^{-aT}$

$$= -2\log\left[\left(\frac{T+\bar{T}}{2}\right)^{3/2} + \hat{\xi}\right]$$

$$\hookrightarrow V(\tau) = \underbrace{\frac{e^{-2a\tau}(3aA + a^2A^2\tau)}{6\tau^2} + \frac{aAe^{-a\tau}W_0}{2\tau^2}}_{=V_{KKLT}\text{ (without uplifting)}} + \underbrace{\frac{3W_0^2\xi}{64\sqrt{2}\,\tau^{9/2}}}_{\sim \frac{C}{\mathcal{V}^3}}$$

keeping in mind that we are looking for vacua with $\tau \gg 1$ we neglect the first term and focus on the following potential for the Kähler moduli:

$$V(\tau) \sim \frac{a A e^{-a\tau} W_0}{2\tau^2} + \frac{3 W_0^2 \hat{\xi}}{64\sqrt{2}\,\tau^{9/2}}$$

↳ minimum lives at

$$\boxed{e^{-a\tau}\, a\tau^{7/2}\left(1+\frac{2}{a\tau}\right) = -\frac{27\, W_0\, \hat{\xi}}{64\sqrt{2}\, aA}}$$

and the vacuum energy is:

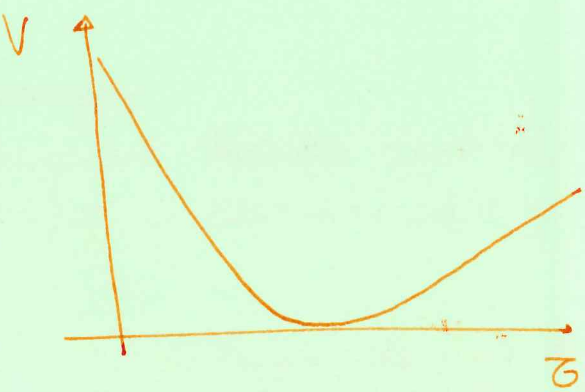
$$\langle V \rangle = \frac{3 W_0 \hat{\xi}}{64\sqrt{2}}\left(1 - \frac{9}{2a\tau\,(1+2/a\tau)}\right)$$

↳ so for a Minkowski minimum one must have $\boxed{\tau \sim \frac{9}{2a} = \frac{9 N_c}{4\pi}}$

that is for getting a controlled Minkowski vacuum one needs large N_c but there is no need to tune $W_0 \ll 1$ like in KKLT.

- Example:

$$\begin{cases} W_0 = -7.7 \\ a = 2\pi/100 \\ \hat{\xi} = 100 \\ A = 1 \end{cases} \Rightarrow \langle \tau \rangle = 39.78$$



↳ Kähler uplifting generates metastable dS vacua with broken SUSY ~~without~~ with only the background d.o.f., with no need to introduce anti D branes, and other ingredients.

Note that KKLT, LVS and Kähler uplifting should be seen as 3 distinct classes of vacua of the same underlying theory. They should all be realised in the string landscape, depending on the "local" hierarchies between W_{up}, W_0, δK_α

KKLT

LVS

Kähler up

schematically $$\boxed{V_F \sim \frac{W_{up}^2}{V} - \frac{W_{up} W_0}{V^2} + \frac{W_0^2}{V^3}}$$

KKLT: $W_{up} \sim W_0 \ll 1 \Rightarrow V_F \sim W_0^2 \left(\frac{1}{V} - \frac{1}{V^2} + \underbrace{\frac{1}{V^3}}\right)$

neglect as it is subleading

Kähler up: $W_{up} \ll W_0 \sim \mathcal{O}(1)$

$$\Rightarrow V_F \sim W_0 \left(-\frac{W_{up}}{V^2} + \frac{W_0}{V^3}\right)$$

LVS: $W_{up} \ll W_0$; $W_{up} \sim \mathcal{O}(1/V)$

$$\Rightarrow V_F \sim \underbrace{\frac{W_{up}^2}{V} - \frac{W_{up} W_0}{V^2} + \frac{W_0^2}{V^3}}$$

all 3 terms are of similar magnitude

▶ TAKE HOME MESSAGE:

Progress in moduli stabilisation over the last 10/15 years has finally helped to bridge the gap between higher dimensional constructions and 4D observations. We are now in a position in which we can start from a higher dimensional theory and see what it predicts in terms of 4D quantities: mass scales, SUSY breaking mechanism, couplings, cosmological history...