Multi-parton dynamics working group – QCD@LHC2013

Intrinsic transverse momenta at high energies

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ABSTRACT

Intrinsic transverse momentum at high energies

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Transverse Momentum Dependent (TMD) distribution functions also take into account the intrinsic transverse momentum (p_T) of the partons. The p_T-integrated analogues can be linked directly to quark and gluon matrix elements using Operator Product Expansion in QCD, involving operators of definite twist. TMDs also involve operators of higher twist, which are not suppressed by powers of the hard scale, however. In this talk I will address the relevance of both quark and gluon TMDs and address theoretical issues related to gauge links that ensure color gauge invariance, universality of the functions and TMD-factorization.

[some of the recent work is in collaboration with Maarten Buffing and Asmita Mukherjee]

Content

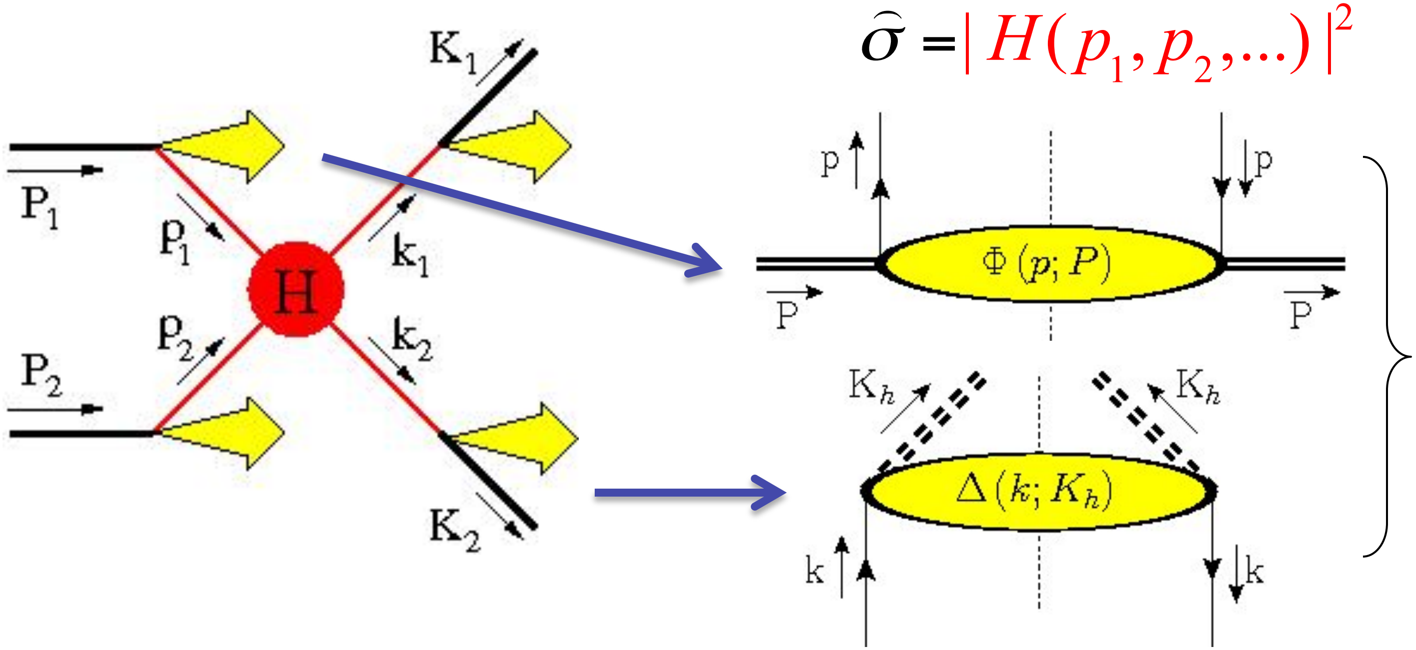
- PDFs and PFFs as matrix elements
- $q(x)$, $G(x)$, $\Delta q(x)$, $\Delta g(x)$, $\delta q(x)$, …………., $q(x,p_T)$, ……… ???
- Use theoretical framework: QCD
  - Extension of OPE resummed into PDFs to TMDs (definite rank)
  - Distribution and fragmentation functions (time reversal)
- The reward
  - Novel hadronic info on spin and orbital structure
  - Possible use of proton as tool (playing with partons)

PDFs and PFFs

Basic use of PDFs and PFFs (also for TMDs) is in a factorized description of high energy scattering processes

$$\hat{\sigma} = |H(p_1, p_2, ...)|^2$$

calculable



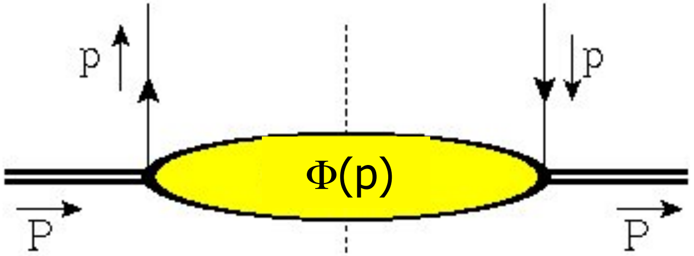
defined (!)
&
portable

$$\sigma(P_1, P_2, ...) = \iiint ...dp_1...\Phi_a(p_1, P_1; \mu) \otimes \Phi_b(p_2, P_2; \mu)$$

$$\otimes\, \hat{\sigma}_{ab,c...}(p_1, p_2, ...; \mu) \otimes \Delta_c(k_1, K_1; \mu)....$$

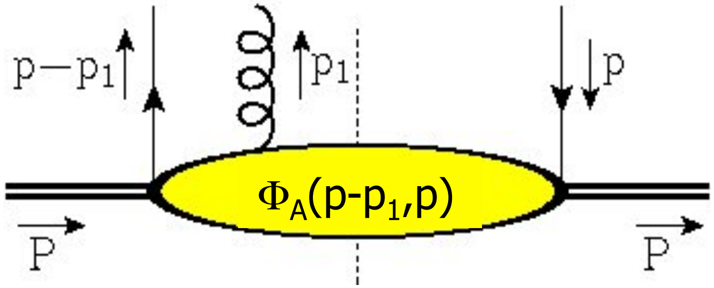
Soft part: hadron correlators

- At high energies interference terms suppressed and the soft parts combine into forward matrix elements of parton fields describing distribution (and fragmentation) parts



$$\Phi_{ij}(p;P) = \Phi_{ij}(p\,|\,p) = \int \frac{d^4\xi}{(2\pi)^4}\, e^{i\,p.\xi} \left\langle P \left| \bar{\psi}_j(0)\, \psi_i(\xi) \right| P \right\rangle$$

- Also needed are multi-parton correlators (time-ordering?)



$$\Phi^{\alpha}_{A;ij}(p-p_1,p_1\,|\,p) = \int \frac{d^4\xi\, d^4\eta}{(2\pi)^8}\, e^{i(p-p_1).\xi + ip_1.\eta} \left\langle P \left| \bar{\psi}_j(0)\, A^{\alpha}(\eta)\, \psi_i(\xi) \right| P \right\rangle$$

- Multi-parton correlators Φ_D, Φ_F, etc. with $D^{\alpha}(\eta)$, $F^{n\alpha}(\eta)$, …

Give a meaning to (integration) variables

- In high-energy processes other momenta are available providing a hard scale P.P′ ~ s = Q^2 >> M^2 (light-like vector P.n = 1, e.g. n = P′/P.P′)
- Expand integration variables

$$p = \underset{\underset{\sim Q}{\nearrow}}{x P^\mu} + \underset{\underset{\sim M}{\uparrow}}{p_T^\mu} + \underset{\underset{\sim M^2/Q}{\nwarrow}}{\sigma n^\mu} \qquad x = p^+ = p.n \sim 1$$

$$\sigma = p.P - xM^2 \sim M^2$$

- Use $\hat{\sigma} \sim \delta(x - x_B) + \alpha_s \ldots$ to identify x (and get corrections)
- Additional (soft - hard) scale accessible through non-collinearities, e.g. in DIS γ^*+p is not aligned with produced hadron or momenta inside a jet identifies transverse scale, linked to convolution of p_T's involved.
- You can't measure them all (integrate over 'virtuality' σ), etc.

$$\Phi(p) = \Phi(x, p_T, \sigma) \;\Rightarrow\; \Phi(x, p_T) \;\Rightarrow\; \Phi(x) \;\Rightarrow\; \Phi$$

(Un)integrated correlators

$$\Phi(x,p_T,\sigma) = \int \frac{d^4\xi}{(2\pi)^4}\, e^{i\,p.\xi} \left\langle P\middle|\overline{\psi}(0)\,\psi(\xi)\middle|P\right\rangle$$

- unintegrated

$$\Phi(x,p_T;n) = \int \frac{d(\xi.P)d^2\xi_T}{(2\pi)^3}\, e^{i\,p.\xi} \left\langle P\middle|\overline{\psi}(0)\,\psi(\xi)\middle|P\right\rangle_{\xi.n=0}$$

- TMD (light-front)

- σ = p⁻ integration renders time-ordering automatic, allowing factorization of forward anti-parton–target scattering amplitude
- Involves operators of twists starting at a lowest value (which is usually called the 'twist' of a TMD)

$$\Phi(x) = \int \frac{d(\xi.P)}{(2\pi)}\, e^{i\,p.\xi} \left\langle P\middle|\overline{\psi}(0)\,\psi(\xi)\middle|P\right\rangle_{\xi.n=\xi_T=0} \quad or \quad \xi^2=0$$

- collinear (light-cone)

- Involves operators of a definite twist. Evolution via splitting functions (moments are anomalous dimensions)

$$\Phi = \left\langle P\middle|\overline{\psi}(0)\,\psi(\xi)\middle|P\right\rangle_{\xi=0}$$

- local

- Local operators with calculable anomalous dimension

New information in TMD's: $f(x,p_T)$

- Quarks in polarized nucleon: $S = S_L\left(\frac{P}{M} + Mn\right) + S_T \qquad S_L^2 + S_T^2 = -1$

$$\Phi^q(x,p_T) \;\propto\; x f_1^q(x,p_T^2)\,\not{P} \;+\; S_L x g_{1L}^q(x,p_T^2)\,\not{P}\gamma_5$$
$$+x h_{1T}^q(x,p_T^2)\not{S}_T\not{P}\gamma_5 \;+\; \ldots$$

unpolarized quarks, q(x)

T-polarized quarks in T-polarized N (δq)

chiral quarks in L-polarized N (Δq)

compare

$$u(p,s)\bar{u}(p,s) = \tfrac{1}{2}(\not{p} + m)(1 + \gamma_5\not{s})$$

- … but also

$$\Phi^q(x,p_T) \;\propto\; \ldots + \frac{(p_T \cdot S_T)}{M} x g_{1T}^q(x,p_T^2)\not{P}\gamma_5 \;+\; \ldots$$

spin ←→ spin

chiral quarks in T-polarized N

New information in TMD's: $f(x,p_T)$

- … and T-odd functions

$$\Phi^q(x,p_T) \;\propto\; \ldots + i h_1^{\perp q}(x,p_T^2)\frac{\not p_T}{M}\not P + i\frac{(p_T \times S_T)}{M} x f_{1T}^{\perp q}(x,p_T^2)\not P + \ldots$$

T-polarized quarks in unpolarized N (Boer-Mulders)

unpolarized quarks in T-polarized N (Sivers)

compare

$$u(p,s)\bar{u}(p,s) = \tfrac{1}{2}(\not p + m)(1+\gamma_5 \not s)$$

spin ←→ orbit

- Note that there are also parts that lack simple partonic interpretation

$$\Phi(x,p_T) \;\propto\; \ldots + M\,x e^q(x,p_T^2) + \ldots$$

Higher-twist

parton mass? But these are suppressed and linked to quark-gluon correlators via EQM

Color gauge invariance

- Gauge invariance in a non-local situation requires a gauge link U(0,ξ)

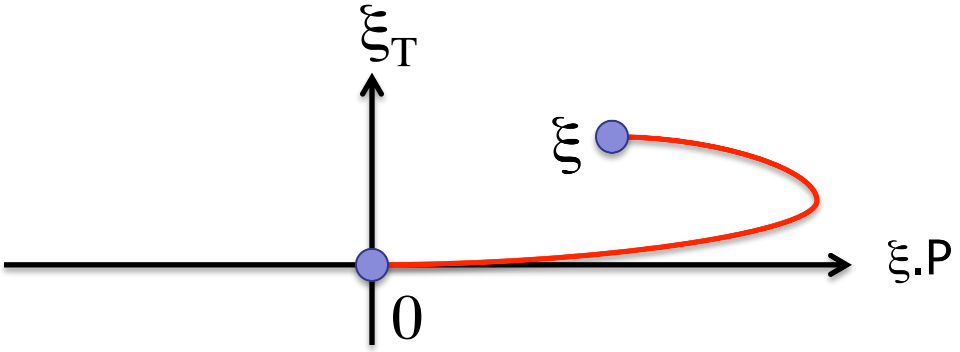
$$\bar{\psi}(0)\psi(\xi) = \sum_n \frac{1}{n!}\xi^{\mu_1}\ldots\xi^{\mu_N}\bar{\psi}(0)\,\partial_{\mu_1}\ldots\partial_{\mu_N}\psi(0)$$

$$U(0,\xi) = \mathcal{P}\ \exp\left(-ig\int_0^\xi ds^\mu A_\mu\right)$$

$$\bar{\psi}(0)U(0,\xi)\psi(\xi) = \sum_n \frac{1}{n!}\xi^{\mu_1}\ldots\xi^{\mu_N}\bar{\psi}(0)\,D_{\mu_1}\ldots D_{\mu_N}\psi(0)$$

- Introduces path dependence for Φ(x,p_T)

$$\Phi^{[U]}(x,p_T) \ \Rightarrow\ \Phi(x)$$



Which gauge links?

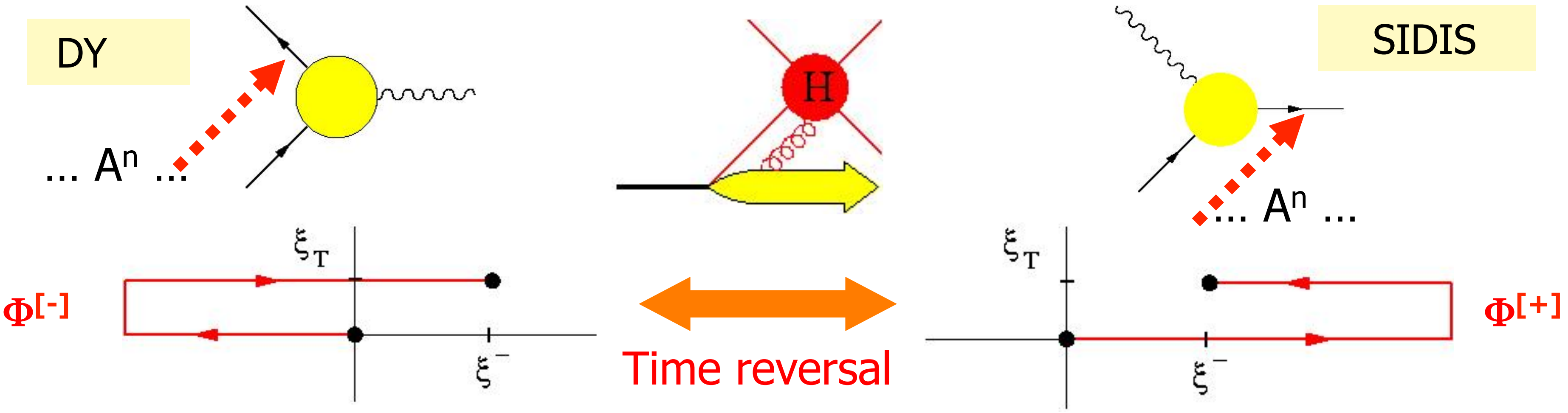
$$\Phi_{ij}^{q[C]}(x, p_T; n) = \int \frac{d(\xi.P)\,d^2\xi_T}{(2\pi)^3}\, e^{i\,p.\xi} \left\langle P \middle| \overline{\psi}_j(0)\, U_{[0,\xi]}^{[C]}\, \psi_i(\xi) \middle| P \right\rangle_{\xi.n=0}$$

TMD

$$\Phi_{ij}^{q}(x; n) = \int \frac{d(\xi.P)}{(2\pi)}\, e^{i\,p.\xi} \left\langle P \middle| \overline{\psi}_j(0)\, U_{[0,\xi]}^{[n]}\, \psi_i(\xi) \middle| P \right\rangle_{\xi.n=\xi_T=0}$$

collinear

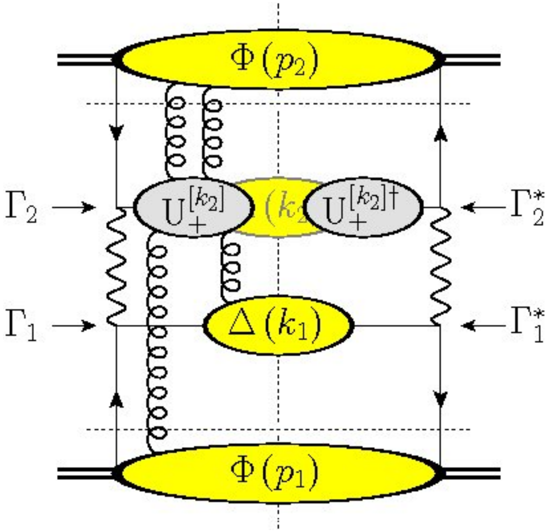
- Gauge links for TMD correlators process-dependent with simplest cases

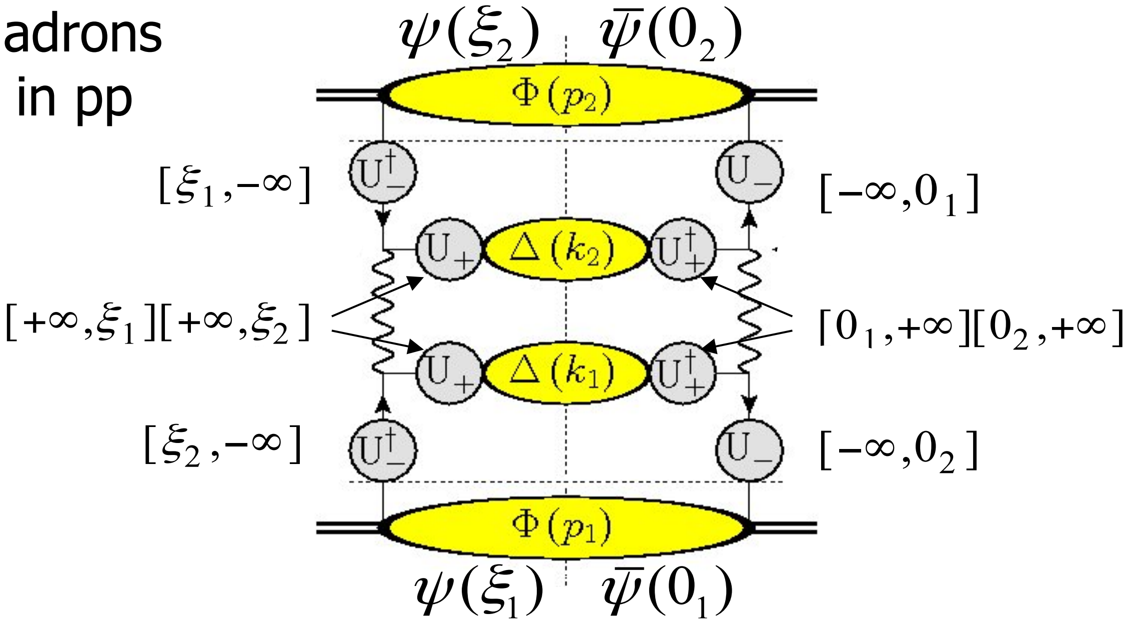


AV Belitsky, X Ji and F Yuan, NP B 656 (2003) 165
D Boer, PJM and F Pijlman, NP B 667 (2003) 201

Which gauge links?

- With more (initial state) hadrons color gets entangled, e.g. in pp





- Outgoing color contributes future pointing gauge link to $\Phi(p_2)$ and future pointing part of a loop in the gauge link for $\Phi(p_1)$

T.C. Rogers, PJM, PR D81 (2010) 094006

- Can be color-detangled if only p_T of one correlator is relevant (using polarization, …) but must include Wilson loops in final U

MGA Buffing, PJM, JHEP 07 (2011) 065

- May require multi-hadron contributions (T.C. Rogers, 2013)

Which gauge links?

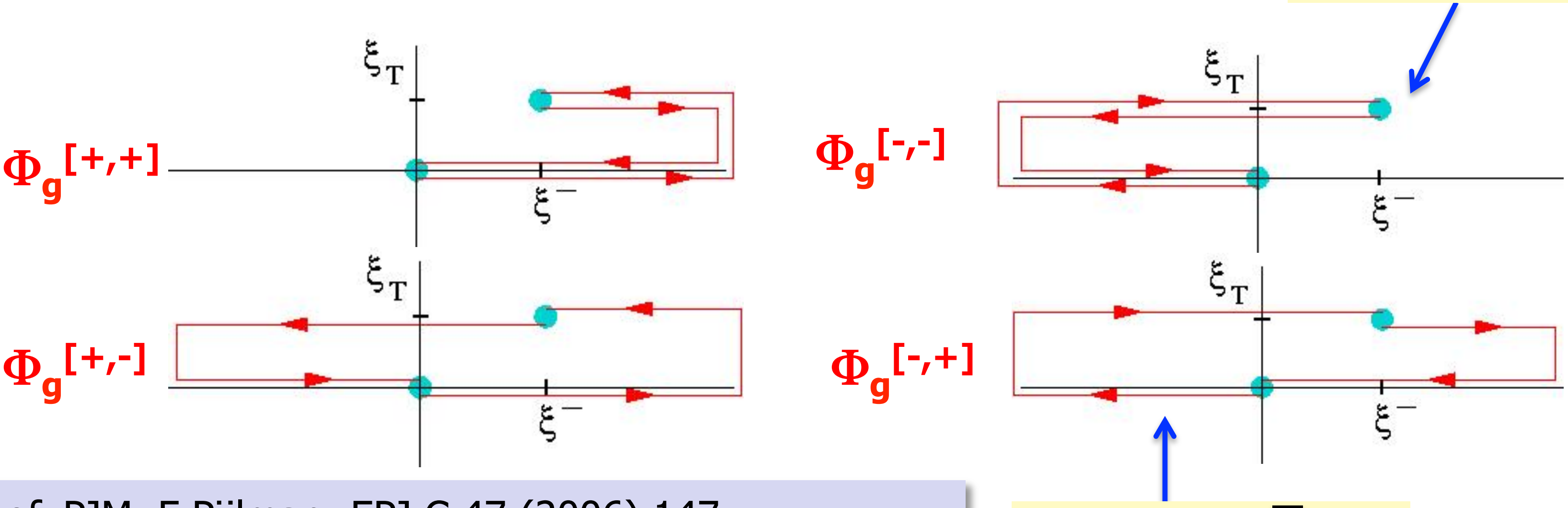
$$\Phi_g^{\alpha\beta[C,C']}(x,p_T;n) = \int \frac{d(\xi.P)\,d^2\xi_T}{(2\pi)^3}\, e^{i\,p.\xi} \left\langle P\middle| U^{[C]}_{[\xi,0]} F^{n\alpha}(0)\, U^{[C']}_{[0,\xi]} F^{n\beta}(\xi) \middle|P\right\rangle_{\xi.n=0}$$

- The TMD gluon correlators contain two links, which can have different paths. Note that standard field displacement involves C = C’

$$F^{\alpha\beta}(\xi) \to U^{[C]}_{[\eta,\xi]} F^{\alpha\beta}(\xi)\, U^{[C]}_{[\xi,\eta]}$$

- Basic (simplest) gauge links for gluon TMD correlators:

gg → H



in gg → $Q\bar{Q}$

C Bomhof, PJM, F Pijlman; EPJ C 47 (2006) 147
F Dominguez, B-W Xiao, F Yuan, PRL 106 (2011) 022301

Summarizing: color gauge invariant correlators

- So it looks that at best we have well-defined matrix elements for TMDs but including multiple possiblities for gauge links and each process or even each diagram its own gauge link (depending on flow of color)
- Leading quark TMDs:

$$\Phi^{[U]}(x,p_T;n) = \Big\{ f_1^{[U]}(x,p_T^2) - f_{1T}^{\perp[U]}(x,p_T^2)\,\frac{\epsilon_T^{p_T S_T}}{M} + g_{1s}^{[U]}(x,p_T)\gamma_5 + h_{1T}^{[U]}(x,p_T^2)\,\gamma_5\,\not{S}_T + h_{1s}^{\perp[U]}(x,p_T)\,\frac{\gamma_5\,\not{p}_T}{M} + i h_1^{\perp[U]}(x,p_T^2)\,\frac{\not{p}_T}{M}\Big\}\frac{\not{P}}{2},$$

- Leading gluon TMDs:

$$2x\,\Gamma^{\mu\nu[U]}(x{,}p_T) = -g_T^{\mu\nu}\, f_1^{g[U]}(x{,}p_T^2) + g_T^{\mu\nu}\frac{\epsilon_T^{p_T S_T}}{M}\, f_{1T}^{\perp g[U]}(x{,}p_T^2) + i\epsilon_T^{\mu\nu}\, g_{1s}^{g[U]}(x{,}p_T) + \left(\frac{p_T^\mu p_T^\nu}{M^2} - g_T^{\mu\nu}\frac{p_T^2}{2M^2}\right) h_1^{\perp g[U]}(x{,}p_T^2) - \frac{\epsilon_T^{p_T\{\mu}\, p_T^{\nu\}}}{2M^2}\, h_{1s}^{\perp g[U]}(x{,}p_T) - \frac{\epsilon_T^{p_T\{\mu}\, S_T^{\nu\}} + \epsilon_T^{S_T\{\mu}\, p_T^{\nu\}}}{4M}\, h_{1T}^{g[U]}(x{,}p_T^2).$$

Operator structure in collinear case (reminder)

- Collinear functions and x-moments

$$\Phi^q(x) = \int \frac{d(\xi.P)}{(2\pi)} e^{i\,p.\xi} \left\langle P \middle| \bar{\psi}(0) U_{[0,\xi]}^{[n]} \psi(\xi) \middle| P \right\rangle_{\xi.n=\xi_T=0}$$

$$x^{N-1}\,\Phi^q(x) = \int \frac{d(\xi.P)}{(2\pi)} e^{i\,p.\xi} \left\langle P \middle| \bar{\psi}(0) (\partial^n)^{N-1} U_{[0,\xi]}^{[n]} \psi(\xi) \middle| P \right\rangle_{\xi.n=\xi_T=0}$$

x = p.n

$$= \int \frac{d(\xi.P)}{(2\pi)} e^{i\,p.\xi} \left\langle P \middle| \bar{\psi}(0) U_{[0,\xi]}^{[n]} (D^n)^{N-1} \psi(\xi) \middle| P \right\rangle_{\xi.n=\xi_T=0}$$

- Moments correspond to local matrix elements of operators that all have the same twist since dim(D^n) = 0

$$\Phi^{(N)} = \left\langle P \middle| \bar{\psi}(0) (D^n)^{N-1} \psi(0) \middle| P \right\rangle$$

- Moments are particularly useful because their anomalous dimensions can be rigorously calculated and these can be Mellin transformed into the splitting functions that govern the QCD evolution.

Operator structure in TMD case

- For TMD functions one can consider transverse moments

$$\Phi(x,p_T;n) = \int \frac{d(\xi.P)d^2\xi_T}{(2\pi)^3} e^{i\,p.\xi} \left\langle P \middle| \overline{\psi}(0)\, U^{[\pm]}_{[0,\xi]}\, \psi(\xi) \middle| P \right\rangle_{\xi.n=0}$$

$$p_T^{\alpha}\Phi^{[\pm]}(x,p_T;n) = \int \frac{d(\xi.P)d^2\xi_T}{(2\pi)^3} e^{i\,p.\xi} \left\langle P \middle| \overline{\psi}(0) U_{[0,\pm\infty]} D_T^{\alpha} U_{[\pm\infty,\xi]} \psi(\xi) \middle| P \right\rangle_{\xi.n=0}$$

$$p_T^{\alpha_1} p_T^{\alpha_2}\Phi^{[\pm]}(x,p_T;n) = \int \frac{d(\xi.P)d^2\xi_T}{(2\pi)^3} e^{i\,p.\xi} \left\langle P \middle| \overline{\psi}(0) U_{[0,\pm\infty]} D_T^{\alpha_1} D_T^{\alpha_2} U_{[\pm\infty,\xi]} \psi(\xi) \middle| P \right\rangle_{\xi.n=0}$$

- Upon integration, these transverse moments involve collinear twist-3 (and higher) multi-parton correlators

MGA Buffing, A Mukherjee, PJM, PRD 86 (2012) 074030 , Arxiv: 1207.3221 [hep-ph]

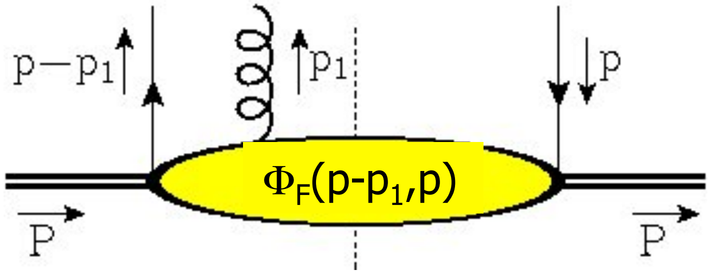
Operator structure in TMD case

- For first transverse moment one needs quark-gluon correlators

$$\Phi_D^\alpha(x-x_1,x_1\,|\,x) = \int \frac{d\xi.P\,d\eta.P}{(2\pi)^2}\,e^{i(p-p_1).\xi+ip_1.\eta}\left\langle P\middle|\overline{\psi}(0)\,D_T^\alpha(\eta)\,\psi(\xi)\middle|P\right\rangle_{\xi.n=\xi_T=0}$$

$$\Phi_F^\alpha(x-x_1,x_1\,|\,x) = \int \frac{d\xi.P\,d\eta.P}{(2\pi)^2}\,e^{i(p-p_1).\xi+ip_1.\eta}\left\langle P\middle|\overline{\psi}(0)\,F^{n\alpha}(\eta)\,\psi(\xi)\middle|P\right\rangle_{\xi.n=\xi_T=0}$$

- In principle multi-parton, but we need



$$\Phi_D^\alpha(x) = \int dx_1\;\Phi_D^\alpha(x-x_1,x_1\,|\,x)$$

$$\Phi_A^\alpha(x) = PV\int dx_1\,\frac{1}{x_1}\,\Phi_F^{n\alpha}(x-x_1,x_1\,|\,x)$$

$$\boxed{\tilde{\Phi}_\partial^\alpha(x) = \Phi_D^\alpha(x) - \Phi_A^\alpha(x)}$$

T-even (gauge-invariant derivative)

$$\boxed{\Phi_G^\alpha(x) = \pi\,\Phi_F^{n\alpha}(x,0\,|\,x)}$$

T-odd (soft-gluon or gluonic pole)

Efremov, Teryaev; Qiu, Sterman; Brodsky, Hwang, Schmidt; Boer, Teryaev, M; Bomhof, Pijlman, M

Operator structure in TMD case

- Transverse moments can be expressed in these particular collinear multi-parton twist-3 correlators (which are not suppressed!)

$$\Phi_\partial^{\alpha[U]}(x) = \int d^2p_T\; p_T^\alpha \Phi^{[U]}(x,p_T;n) = \underbrace{\tilde{\Phi}_\partial^\alpha(x)}_{\text{T-even}} + C_G^{[U]}\underbrace{\Phi_G^\alpha(x)}_{\text{T-odd}}$$

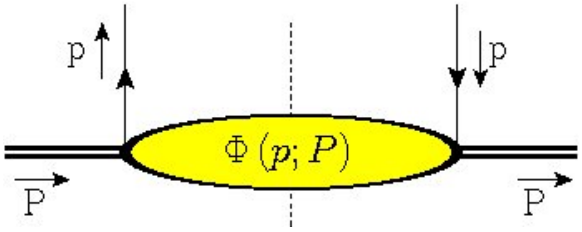
$$\Phi_{\partial\partial}^{\alpha\beta[U]}(x) = \overbrace{\tilde{\Phi}_{\partial\partial}^{\alpha\beta}(x)}^{\text{T-even}} + C_{GG,c}^{[U]}\,\overbrace{\Phi_{GG,c}^{\alpha\beta}(x)}^{\text{T-even}} + C_G^{[U]}\overbrace{\left(\tilde{\Phi}_{\partial G}^{\alpha\beta}(x) + \tilde{\Phi}_{G\partial}^{\alpha\beta}(x)\right)}^{\text{T-odd}}$$

$\mathrm{Tr}_c(GG\,\psi\overline{\psi})$ $\quad$ $\mathrm{Tr}_c(GG)\,\mathrm{Tr}_c(\psi\overline{\psi})$

- $C_G^{[U]}$ calculable gluonic pole factors

U	$U^{[\pm]}$	$U^{[+]}\,U^{[\square]}$	$\frac{1}{N_c}\,\mathrm{Tr}_c(U^{[\square]})\,U^{[+]}$
$\Phi^{[U]}$	$\Phi^{[\pm]}$	$\Phi^{[+\square]}$	$\Phi^{[(\square)+]}$
$C_G^{[U]}$	± 1	3	1
$C_{GG,1}^{[U]}$	1	9	1
$C_{GG,2}^{[U]}$	0	0	4

Distributions versus fragmentation



- Operators:

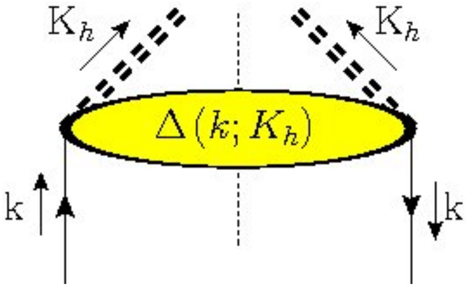
$$\Phi^{[U]}(p\,|\,p) \sim \left\langle P\,|\,\bar{\psi}(0)U_{[0,\xi]}\psi(\xi)\,|\,P\right\rangle$$

$$\Phi_{\partial}^{\alpha[U]}(x) = \tilde{\Phi}_{\partial}^{\alpha}(x) + C_G^{[U]}\,\Phi_G^{\alpha}(x)$$

T-even

T-odd (gluonic pole)

$$\boxed{\Phi_G^{\alpha}(x) = \pi\Phi_F^{n\alpha}(x,0\,|\,x) \neq 0}$$



- Operators:

out state

$$\Delta(k\,|\,k) \sim \sum_X \left\langle 0\,|\,\psi(\xi)\,|\,K_h X\right\rangle\left\langle K_h X\,|\,\bar{\psi}(0)\,|\,0\right\rangle$$

$$\boxed{\Delta_G^{\alpha}(x) = \pi\Delta_F^{n\alpha}(\tfrac{1}{Z},0\,|\,\tfrac{1}{Z}) = 0}$$

$$\Delta_{\partial}^{\alpha[U]}(x) = \tilde{\Delta}_{\partial}^{\alpha}(x)$$

T-even operator combination,
but still T-odd functions!

Collins, Metz; Meissner, Metz; Gamberg, M, Mukherjee, PR D 83 (2011) 071503

Classifying Quark TMDs

- Collecting right moments gives expansion into full TMDs of definite rank

$$
\begin{aligned}
\Phi^{[U]}(x,p_T) = \Phi(x,p_T^2) + p_{Ti}\tilde{\Phi}^{i}_{\partial}(x,p_T^2) + p_{Tij}\tilde{\Phi}^{ij}_{\partial\partial}(x,p_T^2) + \ldots \\
+ \sum_c C^{[U]}_{G,c}\left[p_{Ti}\Phi^{i}_{G,c}(x,p_T^2) + C^{[U]}_{G,c}\,p_{Tij}\tilde{\Phi}^{ij}_{\{\partial G\}}(x,p_T^2) + \ldots\right] \\
+ \sum_c C^{[U]}_{GG,c}\left[p_{Tij}\Phi^{ij}_{GG,c}(x,p_T^2) + \ldots\right]
\end{aligned}
$$

Classifying Quark TMDs

factor	**TMD RANK**			
	0	1	2	3
1	$\Phi(x,p_T^2)$	$\tilde{\Phi}_{\partial}(x,p_T^2)$	$\tilde{\Phi}_{\partial\partial}(x,p_T^2)$	$\tilde{\Phi}_{\partial\partial\partial}(x,p_T^2)$
$C_{G,c}^{[U]}$		$\Phi_{G,c}(x,p_T^2)$	$\tilde{\Phi}_{\{G\partial\},c}(x,p_T^2)$	$\tilde{\Phi}_{\{G\partial\partial\},c}(x,p_T^2)$
$C_{GG,c}^{[U]}$			$\Phi_{GG,c}(x,p_T^2)$	$\tilde{\Phi}_{\{GG\partial\},c}(x,p_T^2)$
$C_{GGG,c}^{[U]}$				$\Phi_{GGG,c}(x,p_T^2)$

MGA Buffing, A Mukherjee, PJM, PRD2012 , Arxiv: 1207.3221 [hep-ph]

Classifying Quark TMDs

factor	**QUARK TMD RANK UNPOLARIZED HADRON**			
	0	1	2	3
1	f_1			
$C_G^{[U]}$		$h_1^\perp$		
$C_{GG,c}^{[U]}$				

- Only a finite number needed: rank up to 2(S$_{\text{hadron}}$+s$_{\text{parton}}$)
- Example: quarks in an unpolarized target needs only 2 functions

$$\Phi(x,p_T^2) = \left(f_1(x,p_T^2)\right)\frac{\not{P}}{2}$$

T-even

$$\Phi_G^\alpha(x,p_T^2) = \left(i h_1^\perp(x,p_T^2)\frac{\gamma_T^\alpha}{M}\right)\frac{\not{P}}{2}$$

T-odd

[B-M function]

Classifying Quark TMDs

factor	**QUARK TMD PDFs RANK SPIN ½ HADRON**			
	0	1	2	3
1	f_1, g_1, h_1	$g_{1T}, h_{1L}^{\perp}$	$h_{1T}^{\perp(A)}$	
$C_G^{[U]}$		$h_1^{\perp}, f_{1T}^{\perp}$		
$C_{GG,c}^{[U]}$			$h_{1T}^{\perp(B1)}, h_{1T}^{\perp(B2)}$	

Three pretzelocities:

$$A:\ \bar{\psi}\partial\partial\psi = Tr_c\left[\partial\partial\psi\bar{\psi}\right]$$

$$B1:\ Tr_c\left[GG\psi\bar{\psi}\right]$$

$$B2:\ Tr_c\left[GG\right]Tr_c\left[\psi\bar{\psi}\right]$$

Classifying Quark TMDs

factor	**QUARK TMD PDFs RANK SPIN ½ HADRON**			
	0	1	2	3
1	$f_1,\ g_1,\ h_1$	$g_{1T},\ h_{1L}^{\perp}$	$h_{1T}^{\perp(A)}$	
$C_G^{[U]}$		$h_1^{\perp},\ f_{1T}^{\perp}$		
$C_{GG,c}^{[U]}$			$h_{1T}^{\perp(B1)},\ h_{1T}^{\perp(B2)}$	

factor	**QUARK TMD PFFs RANK SPIN ½ HADRON**			
	0	1	2	3
1	$D_1,\ G_1,\ H_1$	$D_{1T}^{\perp}, G_{1T}, H_1^{\perp}, H_{1L}^{\perp}$	$H_{1T}^{\perp}$	

Classifying Quark TMDs

factor	QUARK TMD RANK VECTOR POLARIZED SPIN ½ HADRON			
	0	1	2	3
1	g_1, h_1	$g_{1T}, h_{1L}^{\perp}$	$h_{1T}^{\perp(A)}$	
$C_{G,c}^{[U]}$		$f_{1T}^{\perp}$		
$C_{GG,c}^{[U]}$			$h_{1T}^{\perp(B1)}, h_{1T}^{\perp(B2)}$	
$C_{GGG,c}^{[U]}$				

factor	QUARK TMD RANK TENSOR POLARIZED SPIN 1 HADRON			
	0	1	2	3
1	$f_{1LL}, \cancel{h_{1LT}}$	f_{1LT}	$f_{1TT}^{(A)}$	
$C_{G}^{[U]}$		$h_{1LL}^{\perp}, g_{1LT}, h_{1TT}$	$h_{1LT}^{\perp}, g_{1TT}$	$h_{1TT}^{\perp(A)}$
$C_{GG,c}^{[U]}$			$f_{1TT}^{(Bc)}$	
$C_{GGG,c}^{[U]}$				$h_{1TT}^{\perp(Bc)}$

Classifying Gluon TMDs

factor	**GLUON TMD RANK UNPOLARIZED HADRON**			
	0	1	2	3
1	f_1		$h_1^{\perp(A)}$	
$C_{GG,c}^{[U]}$			$h_1^{\perp(Bc)}$	

factor	**GLUON TMD RANK SPIN ½ HADRON**			
	0	1	2	3
1	$f_1,\, g_1$	g_{1T}	$h_1^{\perp(A)}$	
$C_{G,c}^{[U]}$		$f_{1T}^{\perp(Ac)},\, h_{1T}^{(Ac)}$	$h_{1L}^{\perp(A,c)}$	$h_{1T}^{\perp(Ac)}$
$C_{GG,c}^{[U]}$			$h_1^{\perp(Bc)}$	
$C_{GGG,c}^{[U]}$				$h_{1T}^{\perp(Bc)}$

MGA Buffing, A Mukherjee, PJM, Arxiv: 1306.6513 [hep-ph]

Bessel transforms

SKIP

- Terms in p_T expansion of TMDs involve

$$\frac{p_{T i_1\ldots i_m}}{M^m}\,\widetilde{\Phi}^{i_1\ldots i_m}_{\ldots}(x,p_T^2) \qquad \text{or} \qquad \widetilde{\Phi}^{(m/2)}_{\ldots}(x,p_T^2)\,e^{\pm i m\varphi_p}$$

- Use azimuthal integration to get actual p_T^2-dependent TMD PDFs

$$\tilde{\Phi}_{\partial}^{\alpha(1)}(x,p_T^2) = \int \frac{d\varphi}{2\pi}\, p_T^{\alpha}(\varphi)\left[\Phi^{[+]}(x,p_T)+\Phi^{[-]}(x,p_T)\right]$$

$$\Phi_{G}^{\alpha(1)}(x,p_T^2) = \int \frac{d\varphi}{2\pi}\, p_T^{\alpha}(\varphi)\left[\Phi^{[+]}(x,p_T)-\Phi^{[-]}(x,p_T)\right]$$

- Relevant for lattice calculations and experimental analysis
- In general this produces (m/2) moments of the functions

$$\tilde{\Phi}_{\partial}^{\alpha(m/2)}(x,p_T^2) \equiv \left(\frac{-p_T^2}{2M}\right)^{m/2} \tilde{\Phi}_{\partial}^{\alpha}(x,p_T^2)$$

Boer, Gamberg, Musch, Prokudin, JHEP 1110 (2011), 021; Arxiv: 1107.5294 [hep-ph]

Bessel transforms

SKIP

- The universal TMDs of definite rank are natural objects that can be studied in impact parameter space

$$\frac{p_{T i_1\ldots i_m}}{M^m}\,\widetilde{\Phi}^{i_1\ldots i_m}_{\ldots}(x,p_T^2) \qquad \text{or} \qquad \widetilde{\Phi}^{(m/2)}_{\ldots}(x,p_T^2)\,e^{\pm i m\varphi_p}$$

- Azimuthal averaging gives for rank m (m/2)-moments, that can be naturally studied using Bessel transforms

$$\widetilde{f}^{(m/2)}_{\ldots}(x,|p_T|) = \int_0^\infty db\,\sqrt{|p_T|b}\,J_m(|p_T|b)\,f^{(m/2)}_{\ldots}(x,b)$$

- b_T-space useful for phenomenology (Boglione) as well as evolution (Collins, Rogers)

Boer, Gamberg, Musch, Prokudin, JHEP 1110 (2011), 021; Arxiv: 1107.5294 [hep-ph]

Summary for Sivers hunters

SKIP

- Which TMD?

$$f_{1T}^{\perp}$$

- Transverse polarized target, leading in 1/Q, need azimuthal dependence
- T-odd, single spin asymmetry!
- Rank 1

$$\Phi_G^{\alpha(1)}(x,p_T^2) = \int \frac{d\varphi}{2\pi}\, p_T^{\alpha}(\varphi)\frac{1}{2}\left[\Phi^{[+]}(x,p_T) - \Phi^{[-]}(x,p_T)\right]$$

- Different gauge links: different processes (DY, SIDIS)

Summary for Pretzelocity hunte

SKIP

- Which TMD? $h_{1T}^{\perp ....}$
- Transverse polarized target, leading in 1/Q, need azimuthal dependence
- T-even, double spin asymmetry!
- Rank 2!

$$\Phi^{\alpha\beta(2)}_{GG,1}(x,p_T^2) = \int \frac{d\varphi}{2\pi}\, p_T^{\alpha\beta}(\varphi)\frac{1}{8}\left[\Phi^{[+Box]}(x,p_T) - \Phi^{[+]}(x,p_T)\right]$$

$$\Phi^{\alpha\beta(2)}_{\partial\partial}(x,p_T^2) = \int \frac{d\varphi}{2\pi}\, p_T^{\alpha\beta}(\varphi)\ \Phi^{[\pm]}(x,p_T)\ -\ \Phi^{\alpha\beta(2)}_{GG,1}(x,p_T^2)$$

$$\Phi^{\alpha\beta(2)}_{GG,2}(x,p_T^2) = \int \frac{d\varphi}{2\pi}\, p_T^{\alpha\beta}(\varphi)\ \Phi^{[+(Box)]}(x,p_T)\ -\ \Phi^{\alpha\beta(2)}_{\partial\partial}(x,p_T^2) - \Phi^{\alpha\beta(2)}_{GG,1}(x,p_T^2)$$

- Various gauge links: different processes (DY, SIDIS, multi-final states), …

Where do we stand with TMDs (schematic)

- Collinear high-energy processes

$$\boxed{\sigma(x_1,x_2,z)=\Phi^i(x_1)\Phi^j(x_2)f_C\hat{\sigma}^C_{ij\to k\ldots}(x_1,x_2,z)\Delta^k(z)}$$

collinear PDF for parton i

color factor like $1/N_c$

Partonic x-section

collinear PFF for parton k

Where do we stand with TMDs (schematic)

- Collinear high-energy processes

$$\boxed{\sigma(x_1,x_2,z) = \Phi^i(x_1)\Phi^j(x_2)\,f_C\,\hat{\sigma}^{C}_{ij\to k\ldots}(x_1,x_2,z)\Delta^k(z)}$$

- Azimuthal dependences (involving a single hadron):

$$\boxed{\sigma(x_1,x_2,z,q_T) = \Phi^{i[U(C)]}(x_1,q_T)\Phi^j(x_2)\,f_C\,\hat{\sigma}^{[C]}_{ij\to k\ldots}(x_1,x_2,z)\Delta^k(z)}$$

$$\underbrace{\Phi^{i[U]}}_{\text{gauge-link dependent TMD PDF for parton i}}(x,p_T) = \Phi^i(x,p_T^2) + p_T^\alpha \underbrace{\tilde{\Phi}_\partial^{i\alpha}}_{\text{rank-1 TMDs (T-even)}}(x,p_T^2) + \underbrace{C_G^{[U]}}_{\text{gluonic pole factor}}\, p_T^\alpha \underbrace{\Phi_G^{i\alpha}}_{\text{rank-1 ETQS TMDs (T-odd)}}(x,p_T^2) + C_{GG}^{[U]}\, p_T^{\alpha\beta} \underbrace{\Phi_{GG}^{i\alpha\beta}}_{\text{rank-2 ETQS TMDs (T-even)}}(x,p_T^2) + \ldots$$

Where do we stand with TMDs (schematic)

- Collinear high-energy processes:

$$\sigma(x_1,x_2,z) = \Phi^i(x_1)\Phi^j(x_2)f_C\hat{\sigma}^C_{ij\to k\ldots}(x_1,x_2,z)\Delta^k(z)$$

- Azimuthal dependences (involving several hadrons):

$$\sigma(x_1,x_2,z,q_T) = f_C^{[U_1U_2]}\,\Phi^{i[U_1(C)]}(x_2,p_{2T})\otimes\Phi^{j[U_2(C)]}(x_1,p_{1T})\hat{\sigma}^{[C]}_{ij\to k\ldots}(x_1,x_2,z)\Delta^k(z,k_T)$$

gauge-link process-dependent color factors

Where do we stand with TMDs (schematic)

- Collinear high-energy processes:

$$\boxed{\sigma(x_1,x_2,z) = \Phi^i(x_1)\Phi^j(x_2)\,f_C\,\hat{\sigma}^C_{ij\to k\ldots}(x_1,x_2,z)\Delta^k(z)}$$

- Convoluted azimuthal dependences:

$$\boxed{\sigma(x_1,x_2,z,q_T) = f_C^{[U_1U_2]}\,\Phi^{i[U_1(C)]}(x_2,p_{2T})\otimes\Phi^{j[U_2(C)]}(x_1,p_{1T})\,\hat{\sigma}^{[C]}_{ij\to k\ldots}(x_1,x_2,z)\Delta^k(z,k_T)}$$

$$\Phi^{i[U]}(x,p_T) = \tilde{\Phi}^i(x,p_T^2) + p_T^\alpha\tilde{\Phi}_\partial^{i\alpha}(x,p_T^2) + C_G^{[U]}\,p_T^\alpha\tilde{\Phi}_G^{i\alpha}(x,p_T^2) + C_{GG}^{[U]}\,p_T^{\alpha\beta}\,\Phi_{GG}^{i\alpha\beta}(x,p_T^2) + \ldots$$

gauge-link dependent TMD PDF for parton i

rank-1 TMDs (T-even)

gluonic pole factor

rank-1 ETQS TMDs (T-odd)

rank-2 ETQS TMDs (T-even)

Where do we stand with TMDs (schematic)

- Collinear high-energy processes:

$$\boxed{\sigma(x_1,x_2,z) = \Phi^i(x_1)\Phi^j(x_2)\,f_C\,\hat{\sigma}^C_{ij\to k\ldots}(x_1,x_2,z)\Delta^k(z)}$$

- Convoluted azimuthal dependences:

$$\boxed{\sigma(x_1,x_2,z,q_T) = f_C^{[U_1U_2]}\,\Phi^{i[U_1(C)]}(x_2,p_{2T})\otimes\Phi^{j[U_2(C)]}(x_1,p_{1T})\,\hat{\sigma}^{[C]}_{ij\to k\ldots}(x_1,x_2,z)\Delta^k(z,k_T)}$$

$$\Phi^{i[U]}(x,p_T) = \tilde{\Phi}^i(x,p_T^2) + p_T^\alpha\tilde{\Phi}_\partial^{i\alpha}(x,p_T^2) + C_G^{[U]}p_T^\alpha\tilde{\Phi}_G^{i\alpha}(x,p_T^2) + C_{GG}^{[U]}p_T^{\alpha\beta}\Phi_{GG}^{i\alpha\beta}(x,p_T^2) + \ldots$$

$$\Delta^{k[U]}(x,k_T) = \tilde{\Delta}^k(z,k_T^2) + k_T^\alpha\tilde{\Delta}_\partial^{k\alpha}(x,k_T^2) + k_T^{\alpha\beta}\tilde{\Delta}_{\partial\partial}^{k\alpha\beta}(z,k_T^2) + \ldots$$

universal TMD
PFF for parton i

rank-1 TMDs
(T-even and odd)

rank-2 TMDs
(T-even and odd)

SKIP

Where do we stand with TMDs (sche

- Collinear high-energy processes:

$$\boxed{\sigma(x_1,x_2,z)=\Phi^i(x_1)\Phi^j(x_2)f_C\hat{\sigma}^C_{ij\to k\ldots}(x_1,x_2,z)\Delta^k(z)}$$

- Convoluted azimuthal dependences:

$$\boxed{\sigma(x_1,x_2,z,q_T)=f_C^{[U_1U_2]}\,\Phi^{i[U_1(C)]}(x_2,p_{2T})\otimes\Phi^{j[U_2(C)]}(x_1,p_{1T})\hat{\sigma}^{[C]}_{ij\to k\ldots}(x_1,x_2,z)\Delta^k(z,k_T)}$$

$$\Phi^{i[U]}(x,p_T)=\Phi^i(x,p_T^2)+p_T^\alpha\tilde{\Phi}_\partial^{i\alpha}(x,p_T^2)+C_G^{[U]}p_T^\alpha\Phi_G^{i\alpha}(x,p_T^2)+C_{GG}^{[U]}p_T^{\alpha\beta}\Phi_{GG}^{i\alpha\beta}(x,p_T^2)+\ldots$$

$$\Delta^{k[U]}(x,k_T)=\Delta^k(z,k_T^2)+k_T^\alpha\tilde{\Delta}_\partial^{k\alpha}(x,k_T^2)+k_T^{\alpha\beta}\tilde{\Delta}_{\partial\partial}^{k\alpha\beta}(z,k_T^2)+\ldots$$

- Deconvoluted azimuthal dependence

$$\boxed{\sigma(x_1,x_2,z,q_T)=f_C^{[U_1U_2]}\,\Phi^{i[U_1(C)]}(x_1,b_T)\otimes\Phi^{j[U_2(C)]}(x_2,b_T)\hat{\sigma}^{[C]}_{ij\to k\ldots}(x_1,x_2,z)\Delta^k(z,b_T)}$$

gauge-link process-dependent
color factors

Treatment of diffractive contributions?

$$\begin{aligned}\Phi^{[loop]}_{diff}(x,p_T;n) &= \int \frac{d(\xi.P)\,d^2\xi_T}{(2\pi)^3}\, e^{i\,p.\xi} \left\langle P \right| Tr\left[U^{[loop]}_{[0,\xi,0]} - 1\right] \left| P \right\rangle_{\xi.n=0} \\ &= \delta(x) \int \frac{d^2\xi_T}{(2\pi)^2}\, e^{i\,p_T.\xi_T} \left\langle P \right| Tr\left[U^{[loop]}_{[0,\xi,0]} - 1\right] \left| P \right\rangle_{\xi.n=0}\end{aligned}$$

- Simplest nonzero contribution is $\Phi_{GG}^{[loop]}(p_T)$, thus rank 2.
- Contributions are process-dependent!

See also: F Dominguez, B-W Xiao, F Yuan, PRL 106 (2011) 022301

Conclusions

- (Generalized) universality using definite rank functions: azimuthal dependence of transverse momentum multiplying functions $f(x,p_T^2)$.
- Rank 0 are the well-known collinear functions (three quark and two gluon spin distributions)
- Rank m is coupled to $\cos(m\phi)$ and $\sin(m\phi)$ azimuthal asymmetries. There are leading azimuthal asymmetries with m up to $2(S_{\text{hadron}}+s_{\text{parton}})$.
- Be careful: multiple universal distribution functions in azimuthal asymmetries (depending on color structure), e.g. three pretzelocities.
- In principle distinguishable in different experiments (differences depending on color flow in tree-level diagrams):

  gluon + gluon → colorless (distinguish CP+ from CP- Higgs)

  gluon-gluon → quark-antiquark pair.
- Novel information on hadron structure (comparison with lattice calc.)
- This is tree level: factorization studies are the next step