

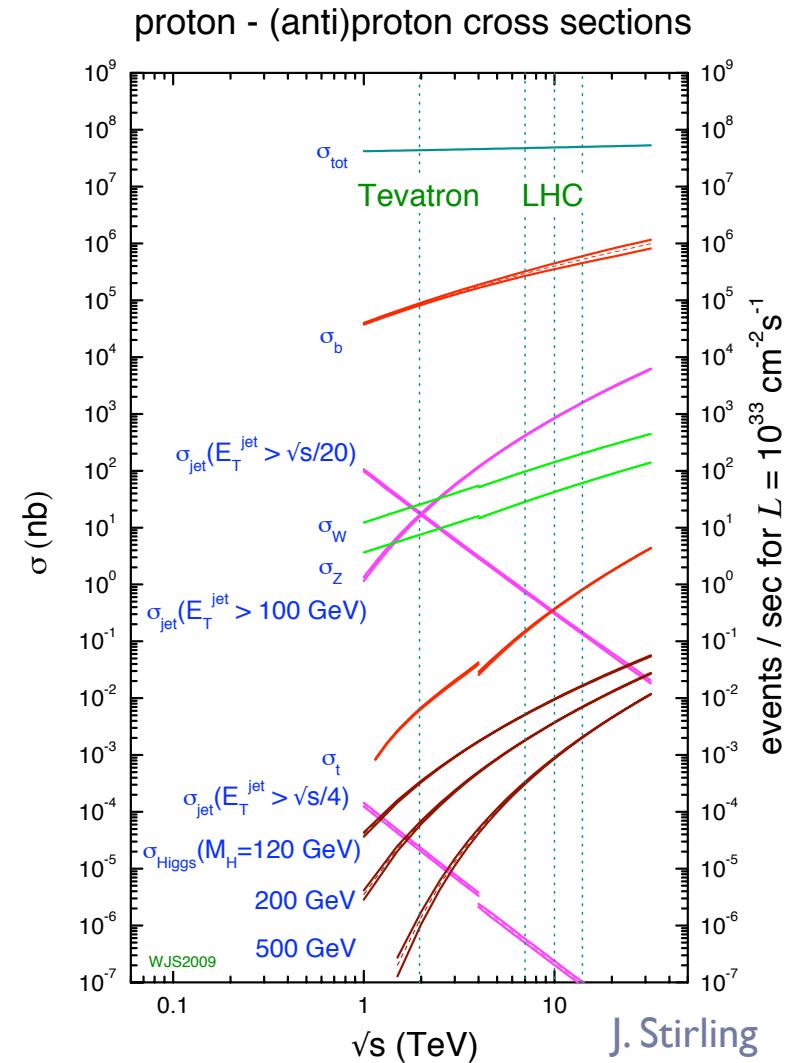
# Hard QCD Predictions for the LHC

Thomas Gehrmann

QCD@LHC 2013, DESY Hamburg, September 2013

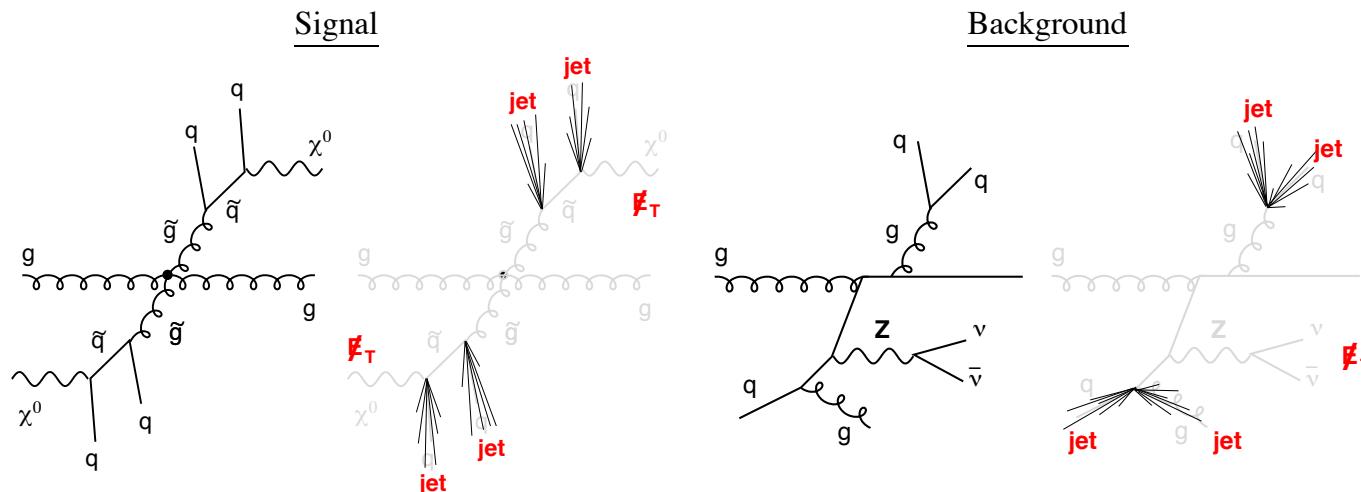
# Benchmark processes at LHC

- ▶ Large production rates for Standard Model processes
  - ▶ jets
  - ▶ top quark pairs
  - ▶ vector bosons
- ▶ Allow precision measurements
  - ▶ masses
  - ▶ couplings
  - ▶ parton distributions
- ▶ Require precise theory



# Multi-particle production at LHC

- ▶ LHC brings new frontiers in energy and luminosity
- ▶ Production of short-lived heavy states (Higgs, top, SUSY...)
  - ▶ detected through their decay products
  - ▶ yield multi-particle final states involving jets, leptons,  $\gamma$ ,  $E_T$
- ▶ Search for new effects in multi-particle final states
- ▶ Need precise predictions for hard scattering processes



Example: SUSY  
signature  $4j + E_T$



# Hard QCD at LHC

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- ▶ NLO: methods, results, directions
- ▶ Parton showers, resummation, matching
- ▶ NNLO: precision QCD
- ▶ Precision frontier: aims and ideas

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# ► NLO: methods, results, directions

# NLO multi-particle production

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## ▶ Why NLO?

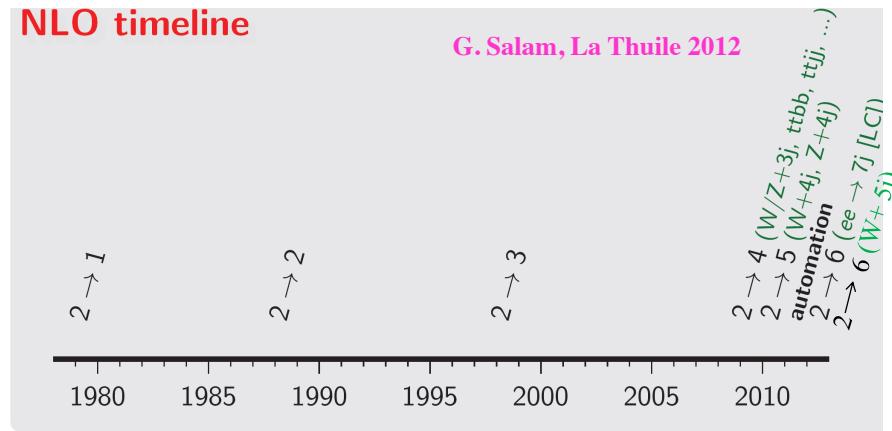
- ▶ reduce scale uncertainty of LO theory prediction
- ▶ reliable normalization and shape
- ▶ accounts for effects of extra radiation
- ▶ jet algorithm dependence

## ▶ Typical observations

- ▶ sizable NLO corrections
- ▶ corrections not constant, but kinematics-dependent
- ▶ remaining uncertainty at NLO typically 10-20%

# NLO multi-parton production

- ▶ Enormous progress in getting NLO predictions for  $2 \rightarrow (4, 5, 6!)$  processes over the last years



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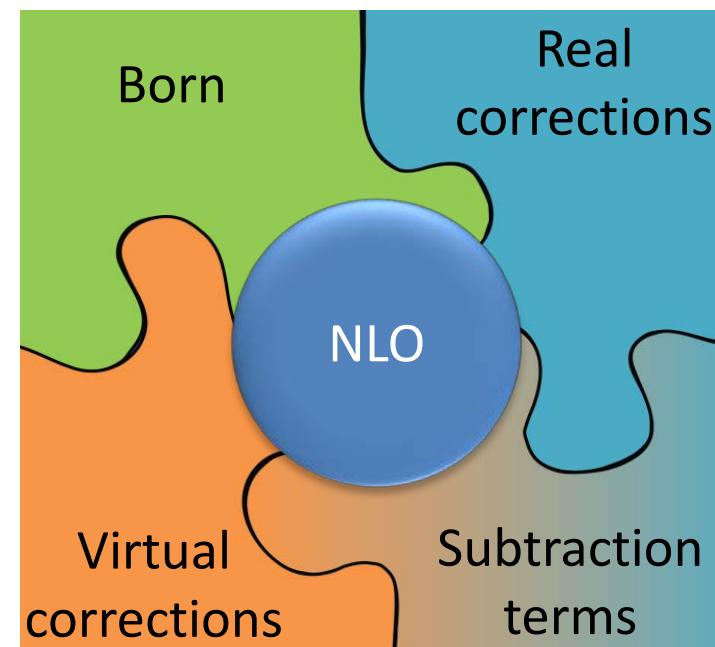
| Process ( $V \in \{Z, W, \gamma\}$ )          | Comments   |
|---|--|
| Calculations completed since Les Houches 2005 |  |
| 1. $pp \rightarrow VV$ jet                    | WW jet completed by Dittmaier/Kallweit/Uwer [27, 28].<br>Campbell/Ellis/Zanderighi [29].<br>$Z\gamma$ jet completed by Binot/Gleisberg/Karg/Kauer/Sanguineti [30].   |
| 2. $pp \rightarrow Higgs+2$ jets              | NLO QCD to the $gg$ channel completed by Campbell/Ellis/Zanderighi [31];<br>NLO QCD+EW to the VBF channel completed by Ciccolini/Demler/Dittmaier [32, 33].  |
| 3. $pp \rightarrow VVV$                       | Interference QCD-EW in VBF channel [34, 35].<br>$ZZZ$ completed by Lazopoulos/Melnikov/Petrucciani [36] and $WWZ$ by Hankele/Zeppenfeld [37], see also Binot/Ossola/Papadopoulos/Pittau [38].<br>VBFNLO [39, 40] meanwhile also contains $WWW, ZZW, WW\gamma, ZZ\gamma, WZ\gamma, W\gamma\gamma, Z\gamma\gamma, \gamma\gamma\gamma$ , $WZj, W\gamma j, \gamma\gamma j, \gamma\gamma\gamma j$ . |
| 4. $pp \rightarrow tt\bar{b}\bar{b}$          | relevant for $t\bar{t}H$ , computed by Brederenberg/Dameri/Dittmaier/Pozzorini [41, 42] and Bevilacqua/Catani/Papadopoulos/Pittau/Worek [43].  |
| 5. $pp \rightarrow V+3$ jets                  | $V+3$ jets calculated by the Blackhat/Sherpa [44] and Frixione [45] collaborations.<br>$Z+3$ jets by Blackhat/Sherpa [46].   |
| Calculations remaining from Les Houches 2005  |  |
| 6. $pp \rightarrow t\bar{t}+2$ jets           | relevant for $t\bar{t}H$ , computed by Bevilacqua/Catani/Papadopoulos/Worek [47, 48].  |
| 7. $pp \rightarrow VV\bar{b}\bar{b}$ ,        | Pozzorini et al. [25].   |
| 8. $pp \rightarrow VV+2$ jets                 | Bevilacqua et al. [23]. $W^+W^-+2$ jets [49]. $W^+W^-+2$ jets [50].<br>VBF contributions calculated by (Bozzi)Jager/Oleari/Zeppenfeld [51, 52, 53].  |
| NLO calculations added to list in 2007        |  |
| 9. $pp \rightarrow bbbb$                      | Binot et al. [54, 55].   |
| NLO calculations added to list in 2009        |  |
| 10. $pp \rightarrow V+4$ jets                 | top pair production, various new physics signatures.<br>Blackhat/Sherpa: $W+4$ jets [22], $Z+4$ jets [20].<br>see also HEJ [56] for $W+n$ jets.  |
| 11. $pp \rightarrow Whhj$                     | top new physics signatures, Reina/Schutzmeier [11].  |
| 12. $pp \rightarrow t\bar{t}t\bar{t}$         | various new physics signatures.<br>Blackhat/Sherpa [19].   |
| also: $pp \rightarrow 4$ jets                 |  |

- ▶ Made possible by
  - ▶ Improved techniques for loop amplitudes
  - ▶ Crucial: a high level of automation

K. Melnikov, MITP, 2013

# NLO automation

- ▶ Well-defined interfaces (Binoth Les Houches accord)
  - ▶ combine different ingredients from different codes
- ▶ One-loop amplitudes
  - ▶ BlackHat (Z. Bern, L. Dixon, F. Febres Cordero, S. Höche, H. Ita, D. Kosower, D. Maitre, K. Ozeren)
  - ▶ GoSam (G. Cullen, N. Greiner, G. Heinrich, G. Luisoni, P. Mastrolia, G. Ossola, T. Reiter, F. Tramontano)
  - ▶ OpenLoops (F. Cascioli, P. Maierhöfer, S. Pozzorini)
  - ▶ NJet (S. Badger, B. Biedermann, P. Uwer, V. Yundin)
  - ▶ MadLoop/aMC@NLO (R. Frederix et al.)
  - ▶ CutTools (G. Ossola, C. Papadopoulos, R. Pittau)
- ▶ Real radiation, subtraction terms and phase space (infrastructure)
  - ▶ Sherpa (F. Kraus et al.)
  - ▶ Madgraph/MadEvent (F. Maltoni et al.)
  - ▶ HelacNLO (G. Bevilacqua, C. Papadopoulos et al.)
  - ▶ MCFM (J. Campbell, K. Ellis, C. Williams)



# Automation in NLO computations

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- ▶ **Impressive list of results:**
  - ▶ multiple jets (up to 4) (Blackhat + Sherpa; Njet)
  - ▶ gauge boson and up to 5 jets (Blackhat + Sherpa)
  - ▶ two gauge bosons with up to 2 jets (T. Melia et al.; VBFNLO: F. Campanario, M. Kerner, L.D. Ninh, D. Zeppenfeld; GoSam + MadEvent)
  - ▶ Three gauge bosons (VBFNLO: G. Bozzi, F. Campanario, C. Englert, M. Rauch, D. Zeppenfeld)
  - ▶ Top quarks with jets (up to 2) (A. Denner, S. Dittmaier, S. Kallweit, S. Pozzorini; G. Bevilacqua, M. Czakon, C. Papadopoulos, M. Worek)
  - ▶ Top quarks with a gauge boson (A. Lazopoulos, K. Melnikov, F. Petriello; K. Melnikov, M. Schulze, A. Scharf; HelacNLO: A. Kardos, Z. Trocsanyi, C. Papadopoulos; MCFM: J. Campbell, K. Ellis)
  - ▶ Higgs with a top quark pair and one jet (GoSam + Sherpa + MadEvent: H. van Deurzen, G. Luisoni, P. Mastrolia, E. Mirabella, G. Ossola, T. Peraro)
  - ▶ Higgs and up to 3 jets (GoSam + Sherpa + MadEvent: G. Cullen, H. van Deurzen, N. Greiner, G. Luisoni, P. Mastrolia, E. Mirabella, G. Ossola, T. Peraro, F. Tramontano)
- ▶ **Address rich phenomenology with few examples**

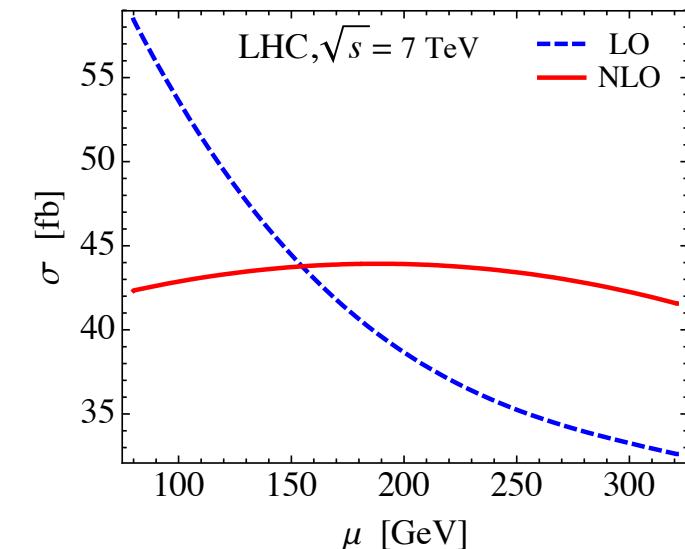
# $W^+W^- + 2$ jet production at NLO

- ▶ Background to BSM searches and for  $H \rightarrow WW$  decay
- ▶ Interplay with electroweak vector boson fusion process  
(VBFNLO: D. Zeppenfeld et al.)
- ▶ Two NLO QCD calculations completed recently  
(T. Melia, K. Melnikov, R. Rontsch, G. Zanderighi; N. Greiner, G. Heinrich, P. Mastrolia, G. Ossola, T. Reiter, F. Tramontano)

- ▶ Including  $W$ -boson decays:

$$pp \rightarrow W^+ (\rightarrow \nu_e e^+) W^- (\rightarrow \mu^- \bar{\nu}_\mu) jj$$

- ▶ Scale variation : Use  $\mu = \mu_F = \mu_R$ ,

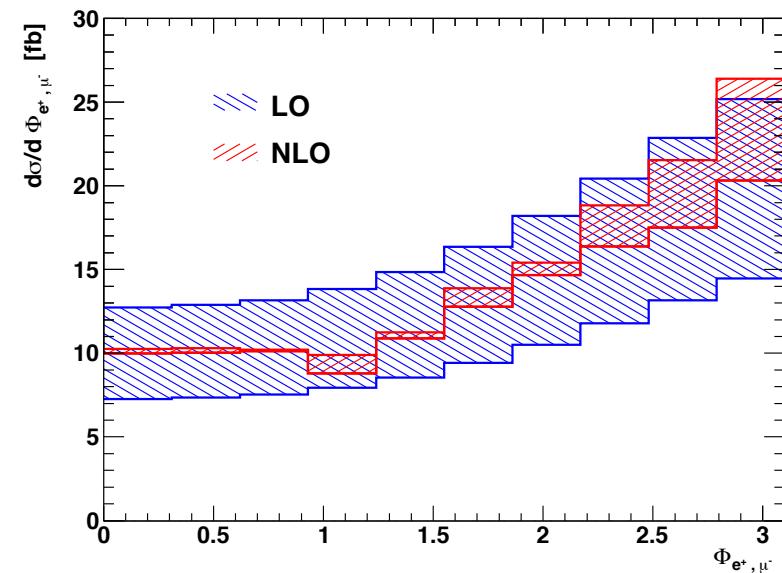


- ▶ Observe: NLO corrections stabilize scale dependence

# $W^+W^- + 2$ jet production at NLO

- ▶ Distribution in the lepton opening angle  $\Phi_{e^+, \mu^-}$
- ▶ Vary  $\mu = \mu_F = \mu_R$  in  $M_W < \mu < 4 M_W$

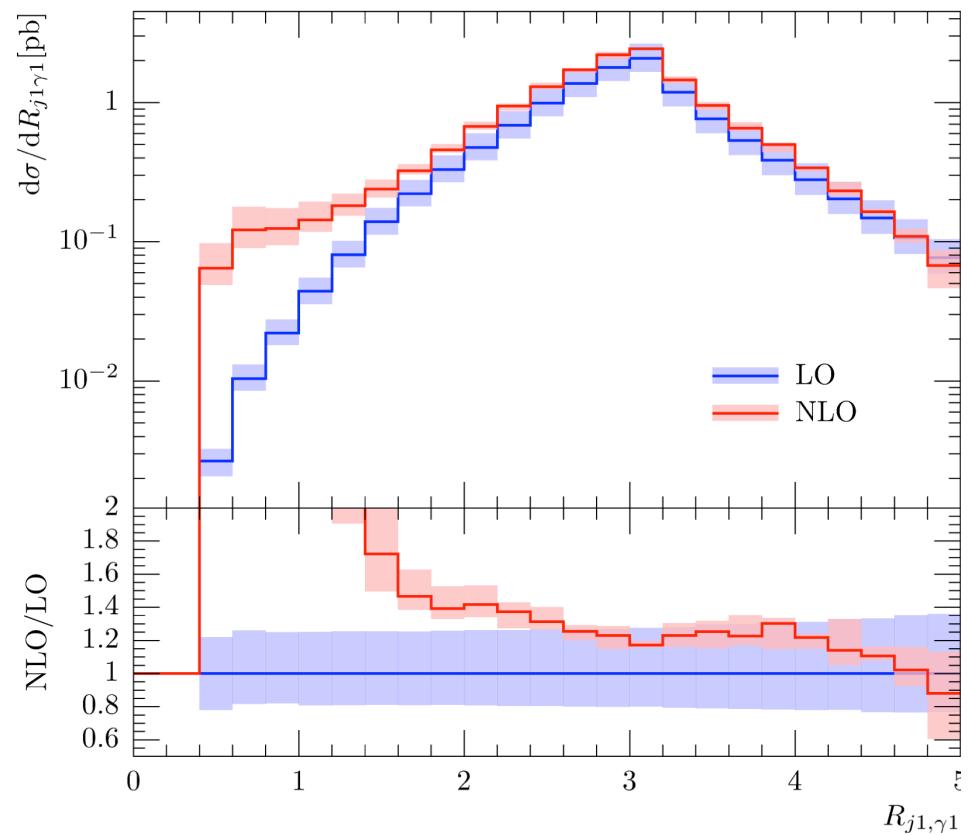
- ▶ NLO predictions within LO uncertainty band
- ▶ Relevant for designing cuts for the determination of  $HWW$  coupling
  - ▶ QCD process: peaked at  $\pi$
  - ▶ Higgs signal: peaked at 0



# $\gamma\gamma + 2$ jet production at NLO

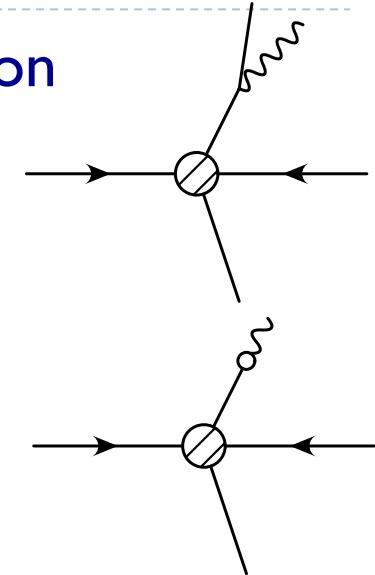
- ▶ Diphoton-plus-two-jet important Higgs background
- ▶ Currently determined from sideband data
- ▶ Insufficient for multiple differential measurements
- ▶ NLO: GoSam+MadEvent  
(G. Heinrich, N. Greiner, TG)
- ▶ Photon isolation:  
dynamical cone (S. Frixione)

$$E_{\text{had,max}}(r_\gamma) = \epsilon p_T^\gamma \left( \frac{1 - \cos r_\gamma}{1 - \cos R} \right)^n$$



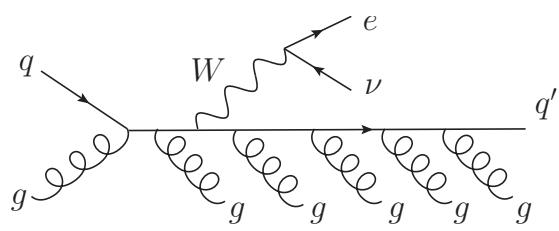
# Photon production mechanisms

- ▶ Direct process: photon produced in hard interaction
  - ▶ perturbatively calculable
  - ▶ collinear quark-photon contributions present
- ▶ Fragmentation of parton into photon:
  - ▶ described by a non-perturbative parton-to-photon fragmentation function
  - ▶ absorbs collinear singularities from direct process
  - ▶ requires non-perturbative input
- ▶ Fixed cone isolation (used in experiment)
  - ▶ both processes contribute
  - ▶ fragmentation contributions reduced but not eliminated
- ▶ Smooth cone isolation (preferred by theorists)
  - ▶ no collinear nor fragmentation contributions
- ▶ Ongoing discussion (Les Houches 2013)

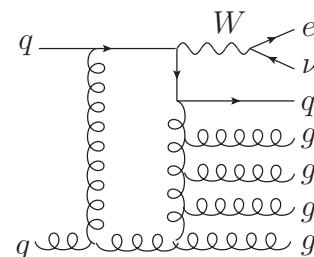


# W+5 jets at NLO

- ▶ First  $2 \rightarrow 6$  NLO calculation at a hadron collider
- ▶ Using Blackhat + Sherpa
  - ▶ Blackhat: virtual one-loop corrections using on-shell methods
  - ▶ Sherpa: real emission, subtraction, phase space integration



Example diagram for real emission  
( $2 \rightarrow 8$ ) at tree level



Example diagram for virtual emission  
( $2 \rightarrow 7$ ) at one-loop (octagon)

- ▶ Computation at the actual frontier of NLO complexity
  - ▶ Considered impossible until few years ago

# W+5 jets at NLO

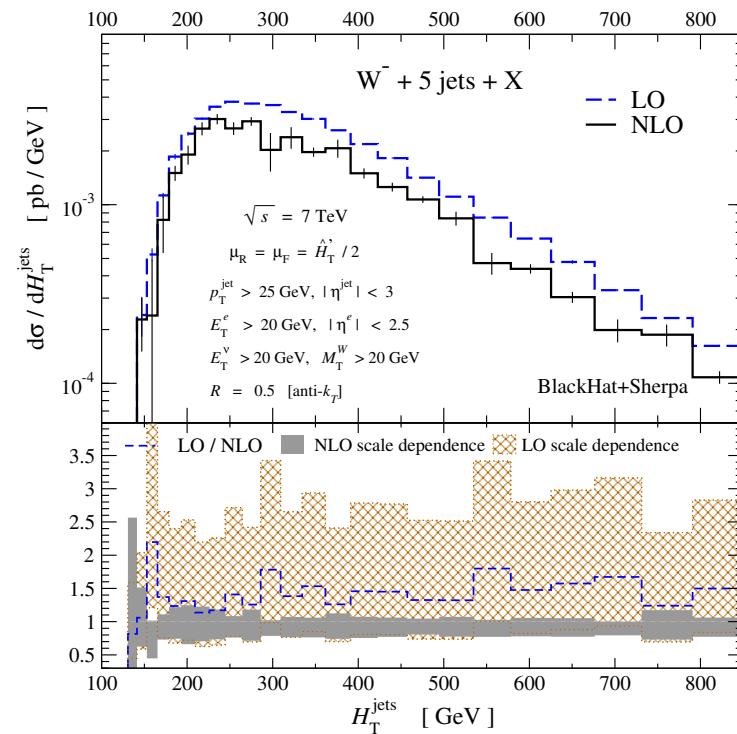
- ▶ Distribution in  $H_T^{\text{jets}}$  (sum of jet transverse energies)

- ▶ Dynamical scale choice

$$\mu_R = \mu_F = \hat{H}'_T / 2$$

$$\hat{H}'_T \equiv \sum_m p_T^m + E_T^W$$

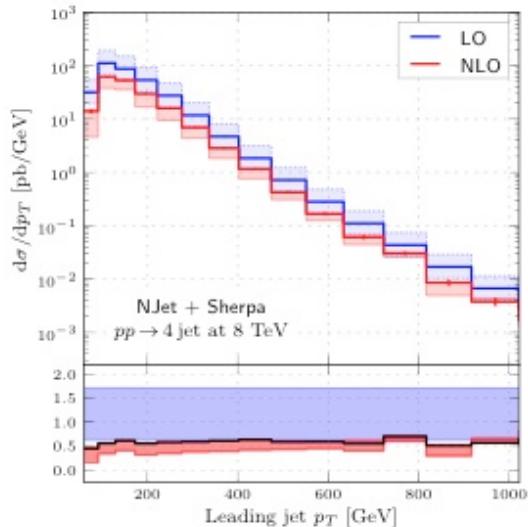
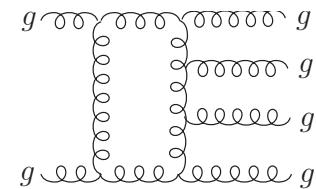
- ▶ scale variation  $\mu/2 \dots 2\mu$
- ▶ Observe:
  - ▶ Scale dependence reduced at NLO
  - ▶ ratio NLO/LO constant over full kinematical range
- ▶ NLO helps to motivate the scale choice



# pp $\rightarrow$ 4 jets at NLO

- ▶ Two calculations using on-shell methods for loop amplitudes

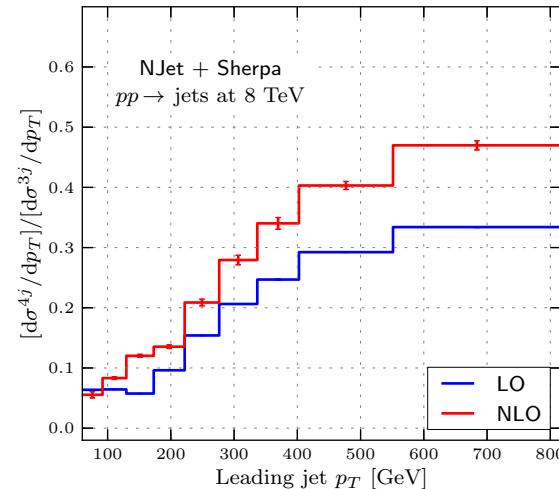
- ▶ Blackhat+Sherpa (Z. Bern, L. Dixon, F. Febres Cordero, S. Höche, H. Ita, D. Kosower, D. Maitre, K. Ozeren)
- ▶ NJET+Sherpa (S. Badger, B. Biedermann, P. Uwer, V. Yundin)



Dynamical scale:

$$\mu_R = \mu_F = \mu = \hat{H}_T/2$$

$$\hat{H}_T = \sum_{i=1}^{N_{\text{parton}}} p_{T,i}^{\text{parton}}.$$

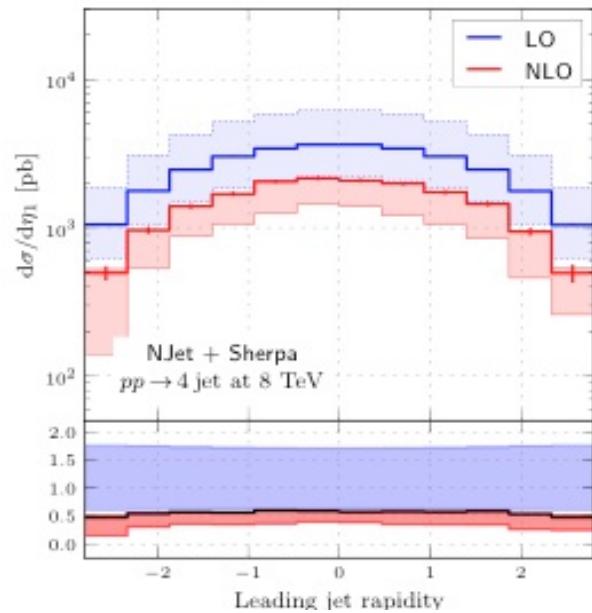


- ▶ NLO prediction with central scale  $\hat{H}_T/2$

- ▶ 4-to-3 jet ratio increases at NLO

# pp → 4 jets at NLO

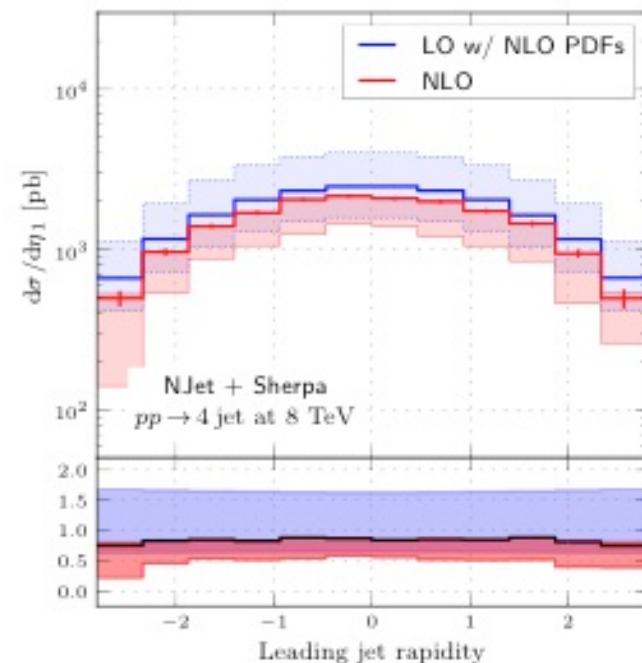
- ▶ To disentangle NLO effects from parton distributions and genuine NLO corrections from hard scattering process
  - ▶ Use NLO partons for both NLO and LO predictions



Dynamical scale

$$\mu_R = \mu_F = \hat{H}'_T / 2$$

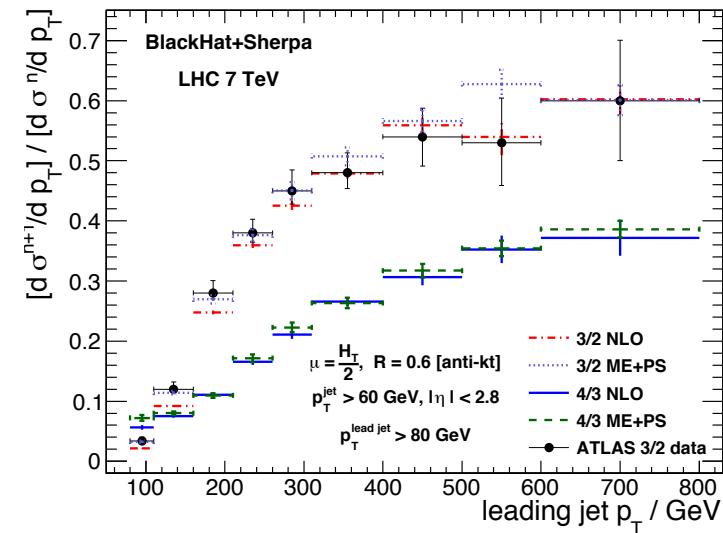
$$\hat{H}_T = \sum_{i=1}^{N_{\text{parton}}} p_{T,i}^{\text{parton}}.$$



- ▶ LO with NLO partons closer to full NLO than pure LO

# Jet ratios at NLO

- ▶ Systematic uncertainties (th. and exp.) cancel in ratios
  - ▶ Predictions more reliable
  - ▶ Can be used in data-driven background estimation
- ▶ Jet ratio as function of leading jet  $p_T$ 
  - ▶ NLO and parton shower both agree with data for large  $p_T$
  - ▶ Parton shower (multiple emission) better at low  $p_T$ 
    - Large uncertainty on parton shower not shown



Observe: 3/2 ratio below the data at small  $p_T$

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► Parton showers, resummation, matching

# Fixed order versus parton shower

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- ▶ Fixed order calculations
  - ▶ Expansion in powers of the coupling constant
  - ▶ Correctly describes hard radiation pattern
  - ▶ Final states are described by single hard particles
  - ▶ NLO: up to two particles in a jet, NNLO: up to three..
  - ▶ Soft radiation poorly described
- ▶ Parton shower (keynote of Jeppe Andersen)
  - ▶ Exponentiates multiple soft radiation (leading logarithms)
  - ▶ Describes multi-particle dynamics and jet substructure
  - ▶ Allows generation of full events (interface to hadronization)
  - ▶ Basis of multi-purpose generators (SHERPA, HERWIG, PYTHIA)
  - ▶ Fails to account for hard emissions
- ▶ Ideally: combine virtues of both approaches

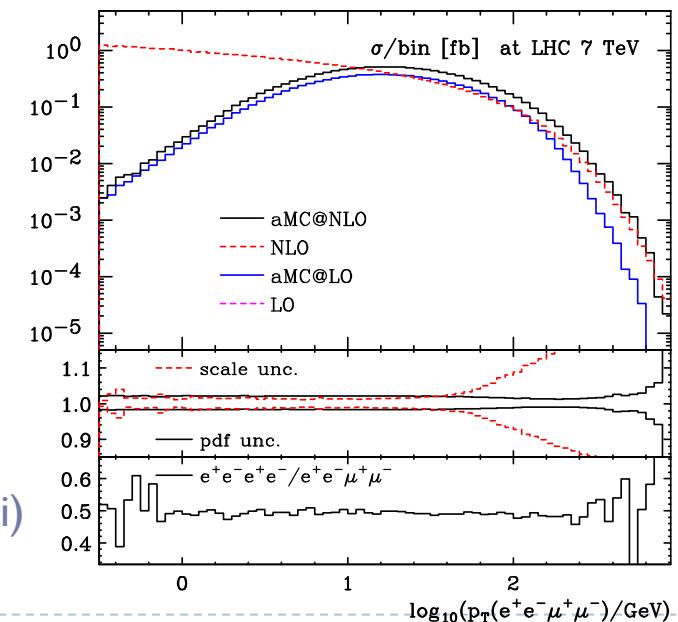
# Merging of fixed order and parton shower

## ► Merging multiplicities

- Combine fixed-order matrix elements at different multiplicity with vetoed shower
- Leading order prescriptions: CKKW (S. Catani, F. Krauss, R. Kuhn, B. Webber) and MLM (M. Mangano)
- Has become standard for parton shower simulations

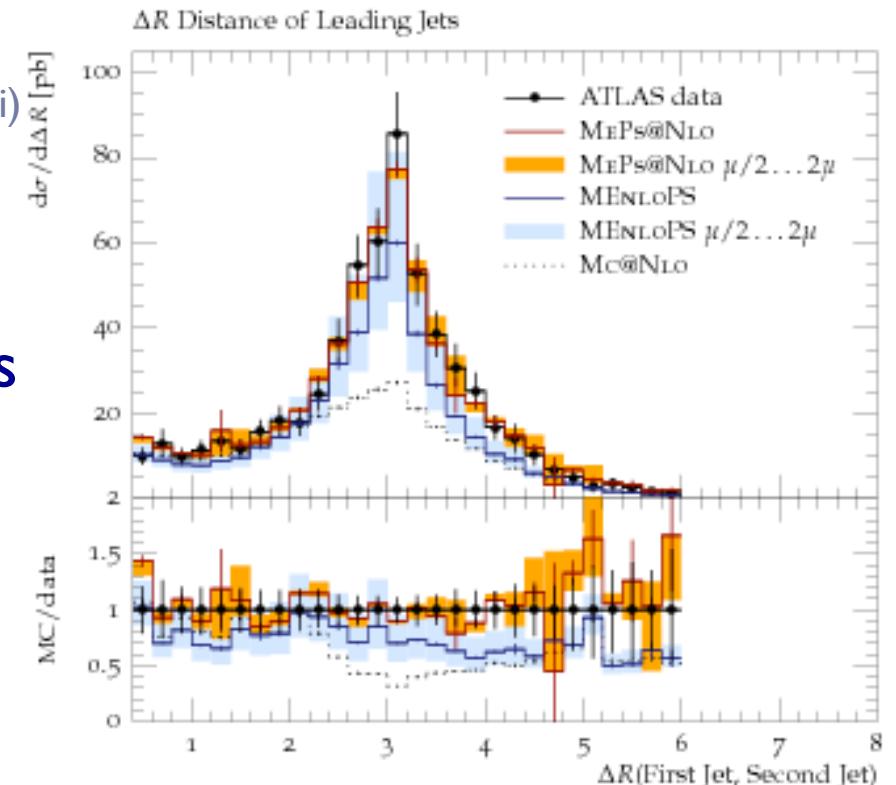
## ► Merging NLO with parton shower

- Combine fixed-multiplicity NLO calculation with parton shower
- Accomplished for many processes (MC@NLO: S. Frixione, B. Webber; POWHEG: P. Nason, C. Oleari et al.)
- Automation: aMC@NLO (R. Frederix, S. Frixione, V. Hirschi, F. Maltoni, R. Pittau, P. Torrielli)



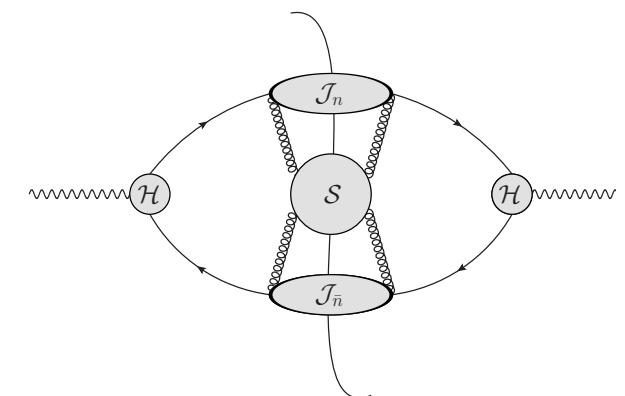
# Merging of fixed order and parton shower

- ▶ Combining NLO computations for different multiplicities and interfacing with parton showers (proof-of-principle)
  - ▶ SHERPA (S. Höche, F. Krauss, M. Schönherr, F. Siegert)
  - ▶ MINLO  
(K. Hamilton, P. Nason, C. Oleari, G. Zanderighi)
  - ▶ UNLOPS (L. Lönnblad, S. Prestel)
  - ▶ FxFx (S. Frixione, R. Frederix)
- ▶ Yields combined event samples
- ▶ Improves especially jet-jet correlations
- ▶ Work in progress



# Resummation

- ▶ Parton shower: leading logarithmic accuracy (LL)
- ▶ Resummation of higher-order logarithms
  - ▶ NLL: largely automated (CAESAR:A. Banfi, G. Salam, G. Zanderighi)
  - ▶ NNLL and beyond: process-by-process calculations
- ▶ Methods
  - ▶ Laplace-space resummation  
(CSS: J. Collins, D. Soper, G. Sterman)
  - ▶ Soft-collinear effective theory (SCET: C. Bauer, S. Fleming, D. Pirjol, I. Rothstein, I. Stewart; M. Beneke, A. Chapovsky, M. Diehl, T. Feldmann)
  - ▶ Systematic extension beyond NLL



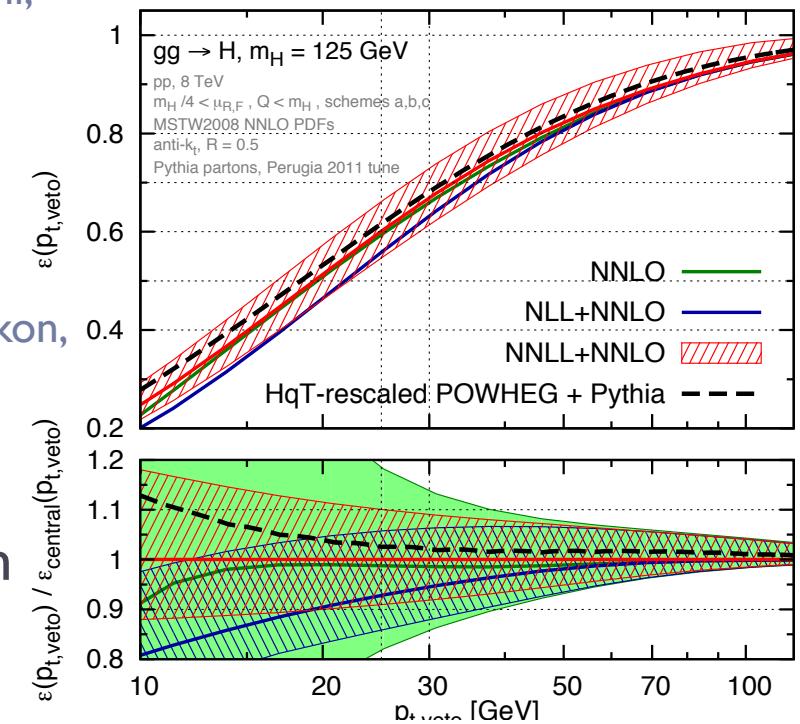
# Resummation

## ▶ Recent NNLL results

- ▶ Higgs boson  $p_T$  distribution (D. de Florian, G. Ferrera, M. Grazzini, D. Tommasini; V. Ahrens, T. Becher, M. Neubert, L.L. Yang)
- ▶ Jet veto cross sections (A. Banfi, P.F. Monni, G. Salam, G. Zanderighi; T. Becher, M. Neubert)
- ▶ Jet- $p_T$  distributions in Higgs events (I. Stewart, F. Tackmann, J. Walsh, S. Zuberi)
- ▶ Top quark pair production (M. Beneke, P. Falgari, S. Klein, C. Schwinn, M. Cacciari, M. Czakon, M. Mangano, A. Mitov, P. Nason, V. Ahrens et al.)

## ▶ Impact of NNLL

- ▶ extended range of theory prediction
- ▶ reduction of scale uncertainty



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► NNLO: towards precision QCD

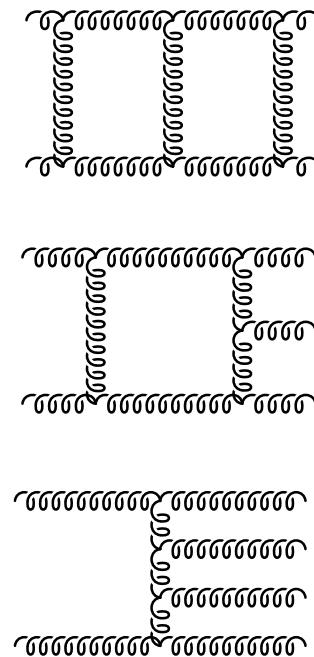
# NNLO observables at hadron colliders

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- ▶ NNLO predictions
  - ▶ expected to have a per-cent level accuracy
  - ▶ yielding first reliable estimate of theoretical uncertainty
- ▶ For processes measured to few per cent accuracy
  - ▶ jet production
  - ▶ vector boson (+jet) production
  - ▶ top quark pair production
- ▶ For processes with potentially large perturbative corrections
  - ▶ New channels and/or phase space regions open up
    - ▶ Higgs or vector boson production

# NNLO calculations

- ▶ Require three principal ingredients (here:  $\text{pp} \rightarrow 2\text{j}$ )
  - ▶ two-loop matrix elements
    - ▶ explicit infrared poles from loop integral
      - known for all massless  $2 \rightarrow 2$  processes
    - ▶ one-loop matrix elements
      - ▶ explicit infrared poles from loop integral
      - ▶ and implicit poles from single real emission
        - usually known from NLO calculations
    - ▶ tree-level matrix elements
      - ▶ implicit poles from double real emission
        - known from LO calculations
  - ▶ Infrared poles cancel in the sum
  - ▶ Challenge: combine contributions into parton-level generator
    - ▶ Need a method to extract implicit infrared poles



# Real radiation at NNLO: methods

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## ► Sector decomposition

(T. Binoth, G. Heinrich; C. Anastasiou, K. Melnikov, F. Petriello)

### ► $p\bar{p} \rightarrow H, p\bar{p} \rightarrow V$ , including decays

(C. Anastasiou, K. Melnikov, F. Petriello; S. Bühler, F. Herzog, A. Lazopoulos, R. Müller)

## ► Sector-improved subtraction schemes

(M. Czakon; R. Boughezal, K. Melnikov, F. Petriello)

### ► $p\bar{p} \rightarrow t\bar{t}$ (M. Czakon, P. Fiedler, A. Mitov)

### ► $p\bar{p} \rightarrow H+j$ (R. Boughezal, F. Caola, K. Melnikov, F. Petriello, M. Schulze)

## ► $q_T$ -subtraction (S. Catani, M. Grazzini)

### ► $p\bar{p} \rightarrow H, p\bar{p} \rightarrow V, p\bar{p} \rightarrow \gamma\gamma, p\bar{p} \rightarrow VH$

(S. Catani, L. Cieri, D. de Florian, G. Ferrera, M. Grazzini, F. Tramontano)

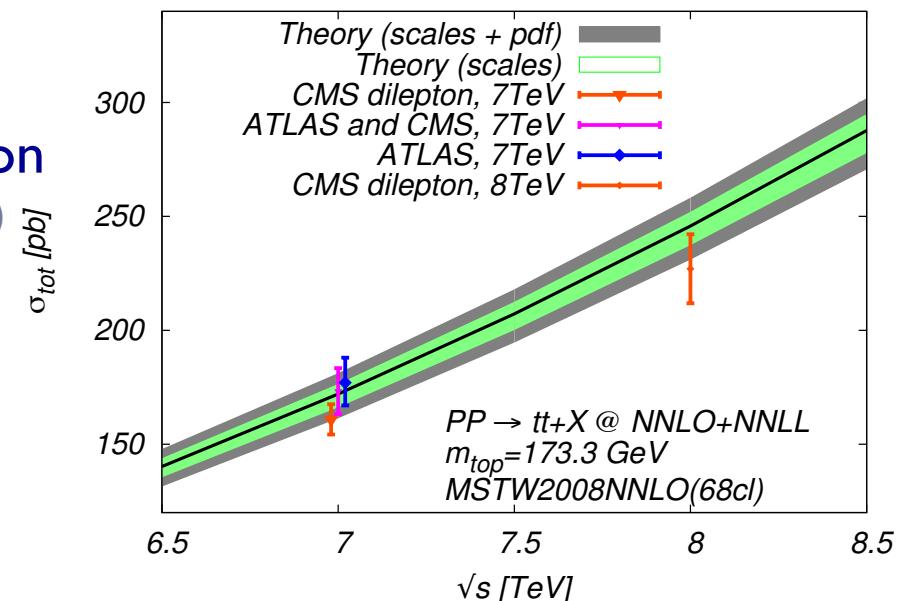
## ► Antenna subtraction (A. Gehrmann-De Ridder, E.W.N. Glover, TG)

### ► $e^+e^- \rightarrow 3j$ (A. Gehrmann-De Ridder, E.W.N. Glover, G. Heinrich, TG; S. Weinzierl)

### ► $p\bar{p} \rightarrow 2j$ (A. Gehrmann-De Ridder, E.W.N. Glover, J. Pires, TG)

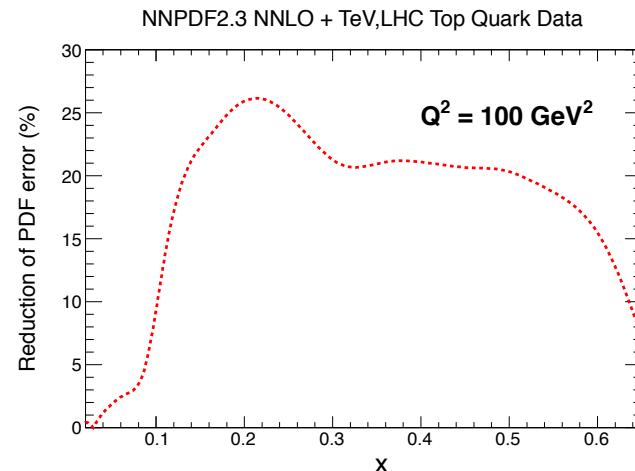
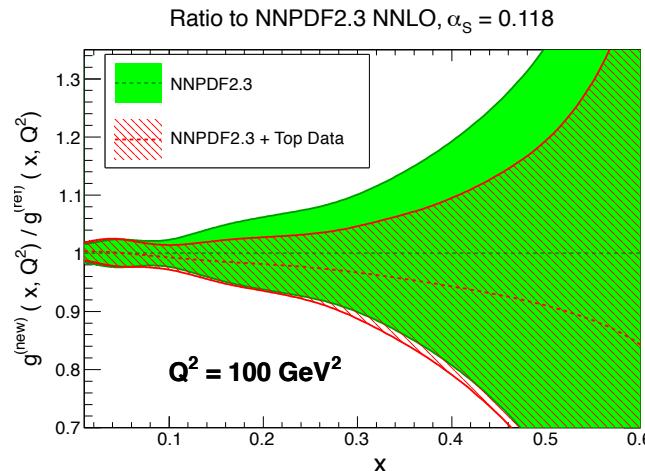
# Top quark pair production at LHC

- ▶ Large production cross section at the LHC ( $\sim 250\text{pb}$  at  $8\text{TeV}$ )
  - ▶ Expected experimental error of  $\sim 5\%$  for  $\sigma_{t\bar{t}}$
  - ▶ NLO+NLL predictions yield an uncertainty of  $\sim 10\%$
- ▶ NNLO accuracy of theory needed
- ▶ Calculation for the total cross section completed (M. Czakon, P. Fiedler, A. Mitov)
  - ▶ From a purely numerical code
  - ▶ based on sector-improved subtraction
    - ▶ numerical cancellation of infrared poles
- ▶ Observe: theoretical and experimental uncertainties comparable (% level)
- ▶ Differential distributions in progress



# Top quark pair production at NNLO

- ▶ Impact on the determination of parton distributions  
(keynote of Mandy Cooper-Sarkar)
  - ▶ Top production at LHC mainly from  $qg$  and  $gg$  processes
  - ▶ Total cross section sensitive on gluon distribution
  - ▶ Inclusion into NNLO global parton distribution fit  
(M. Czakon, M. Mangano, A. Mitov, J. Rojo)



- ▶ Observe: reduced uncertainty on gluon at large  $x$

# Higgs+jet production at the LHC

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- ▶ Essential to establish the properties of the newly discovered Higgs boson
- ▶ Experiments select events according to number of jets
  - ▶ Different backgrounds for different jet multiplicities
  - ▶  $H+0\text{jet}$  and inclusive  $H$  production known at NNLO  
(C.Anastasiou, K. Melnikov, F. Petriello; S.Catani,M. Grazini)
  - ▶  $H+1\text{jet}$  and  $H+2\text{jet}$  known at NLO
  - ▶  $H+0\text{jet}$  and  $H+1\text{jet}$  samples of comparable sizes
- ▶ NNLO for  $H+1\text{jet}$  needed
  - ▶ gluons-only total cross section completed  
(R. Boughezal, F. Caola, K. Melnikov, F. Petriello, M. Schulze)
  - ▶ Full calculation and differential distributions in progress

# Higgs+jet production at NNLO

- ▶ First results for H+jet total cross section (gluons only)

(R. Boughezal, F. Caola, K. Melnikov, F. Petriello, M. Schulze)

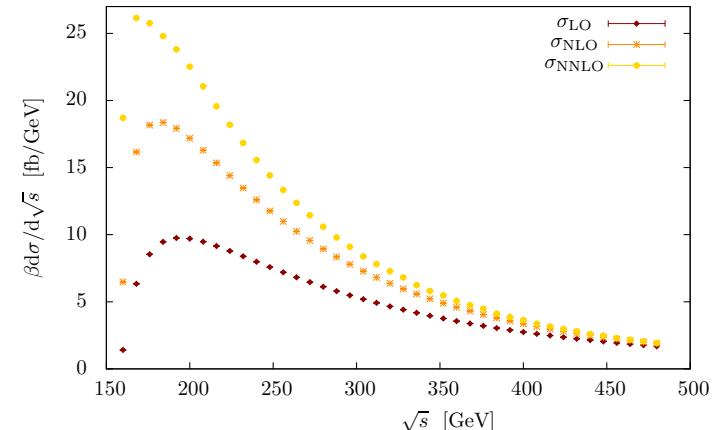
- ▶ using a purely numerical code
  - ▶ Based on sector-improved subtraction
    - numerical cancellation of infrared singularities
  - ▶ cross section multiplied by gluon luminosity

$$\beta \frac{d\sigma_{\text{had}}}{d\sqrt{s}} = \beta \frac{d\sigma(s, \alpha_s, \mu_R, \mu_F)}{d\sqrt{s}} \times \mathcal{L} \left( \frac{s}{s_{\text{had}}}, \mu_F \right),$$

▶ with  $\beta = \sqrt{1 - \frac{E_{th}^2}{s}}$ ,  $E_{th} \approx 158 \text{ GeV}$

- ▶ Observe large NNLO effects close to partonic threshold region

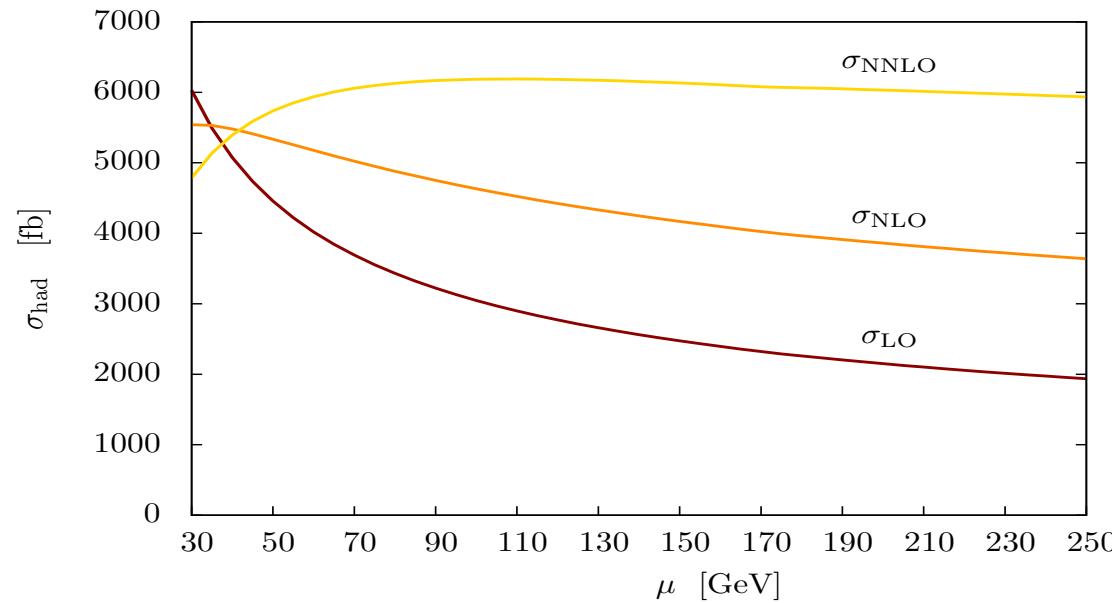
$P_{tj} > 30 \text{ GeV}$ ,  $k_T$ -alg.,  $R=0.5$



# Higgs+jet production at NNLO

- ▶ Scale dependence of the integrated total cross section

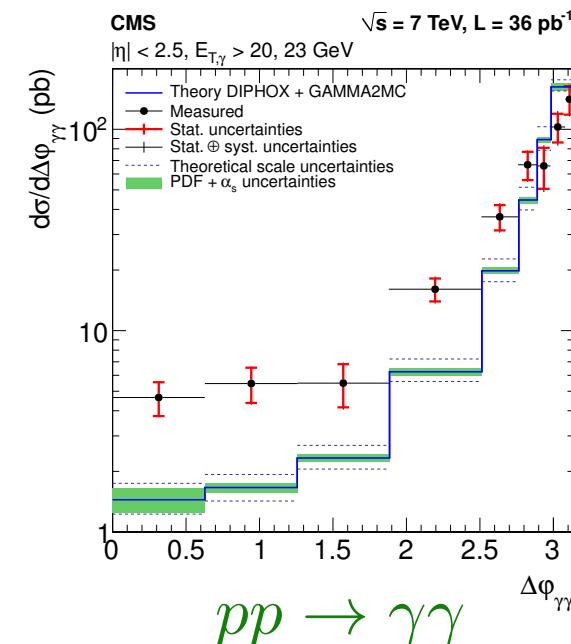
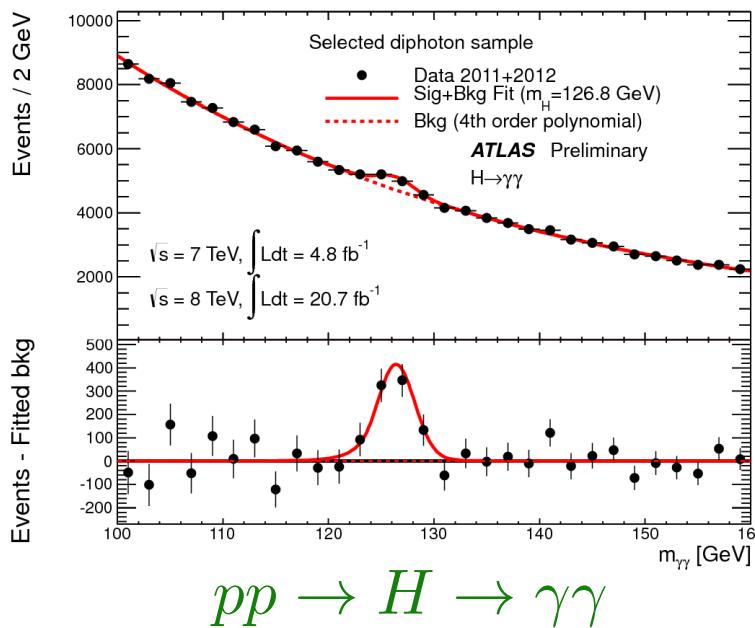
$$\mu = \mu_F = \mu_R$$



- ▶ Considerable stabilization at NNLO
- ▶ Corrections smallest for  $\mu = M_H/2$  as in inclusive case

# Di-photon production at the LHC

- ▶ Di-photon production: irreducible background for  $H \rightarrow \gamma\gamma$ 
  - ▶ at present determined from sideband data fits
- ▶ Discrepancy between NLO theory and data in some distributions



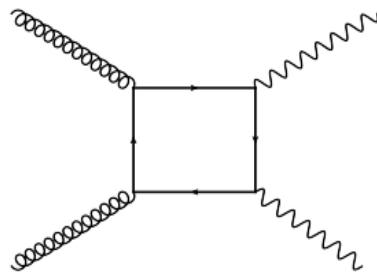
- ▶ Require precise theoretical predictions (NNLO)

# Di-photon production at the LHC

## ► New NNLO calculation: $2\gamma$ NNLO

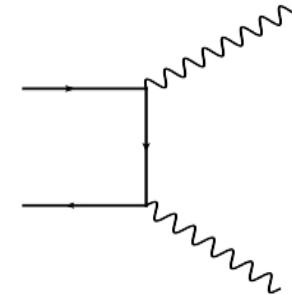
(S. Catani, L. Cieri, D. de Florian, G. Ferrera, M. Grazzini)

- parton-level event generator, based on  $\mathbf{q}_T$ -subtraction
  - Analytic cancellation of infrared poles
  - using a smooth isolation criterion to define photons
  - includes all  $\mathcal{O}(\alpha_s^2)$  corrections to direct photon production  $pp \rightarrow \gamma \gamma$
- First fully consistent inclusion of the Box contribution



$\mathcal{O}(\alpha_s^2)$ , gluon luminosity

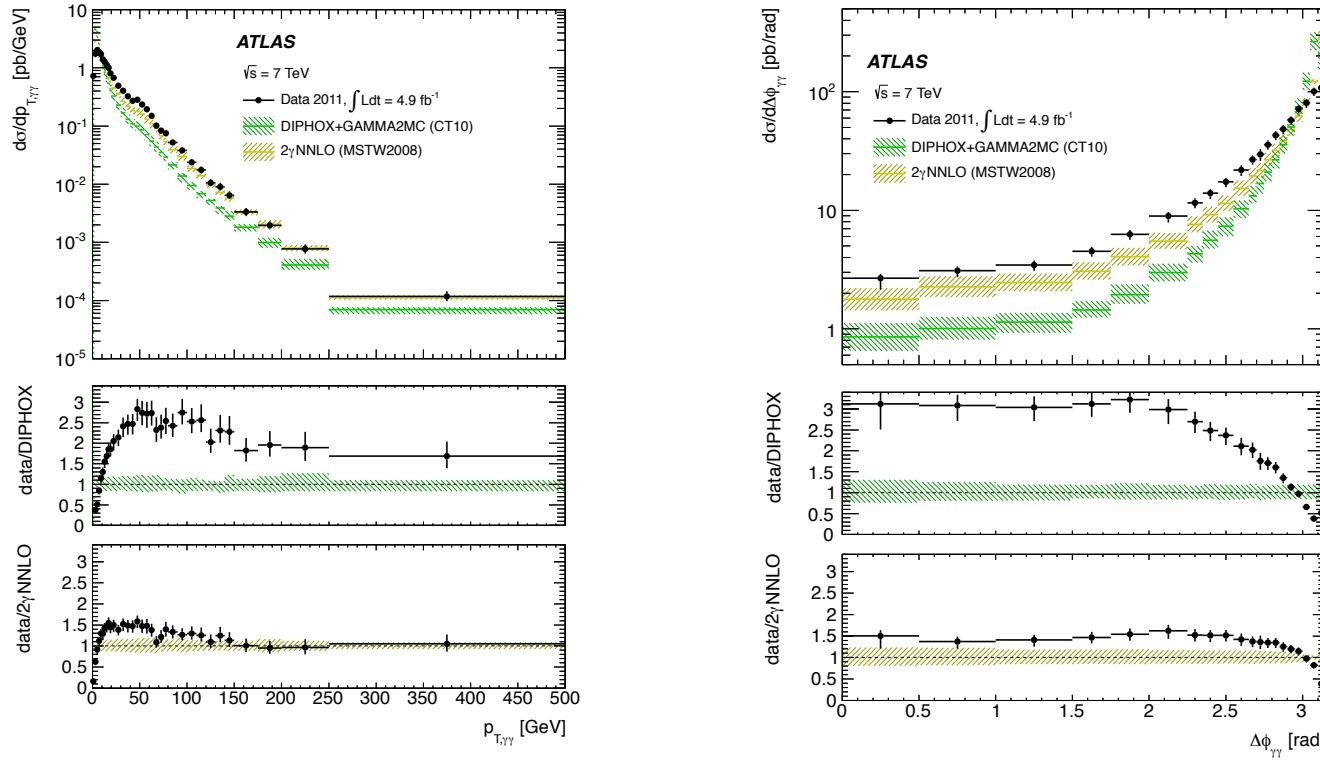
comparable size to



$\mathcal{O}(\alpha_s^0)$ , qq luminosity

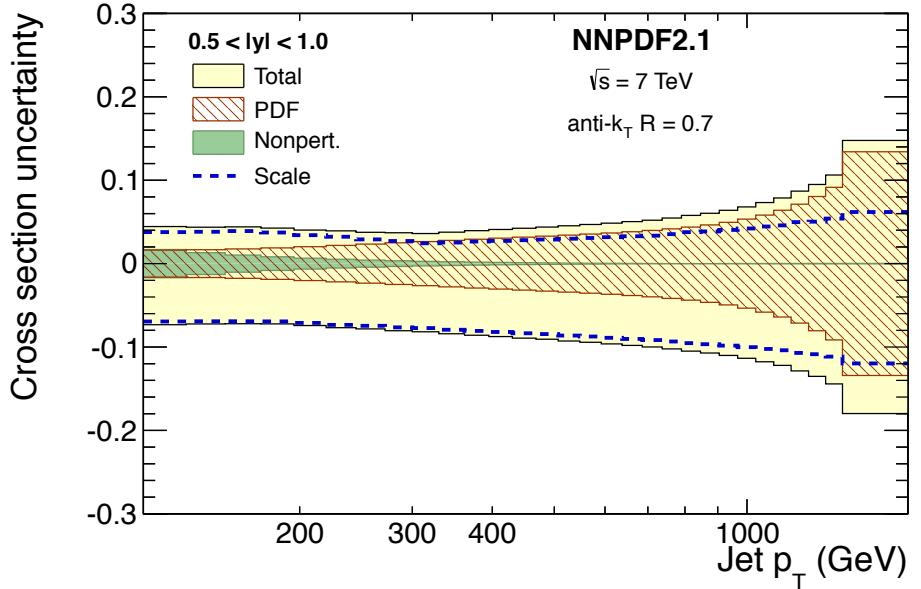
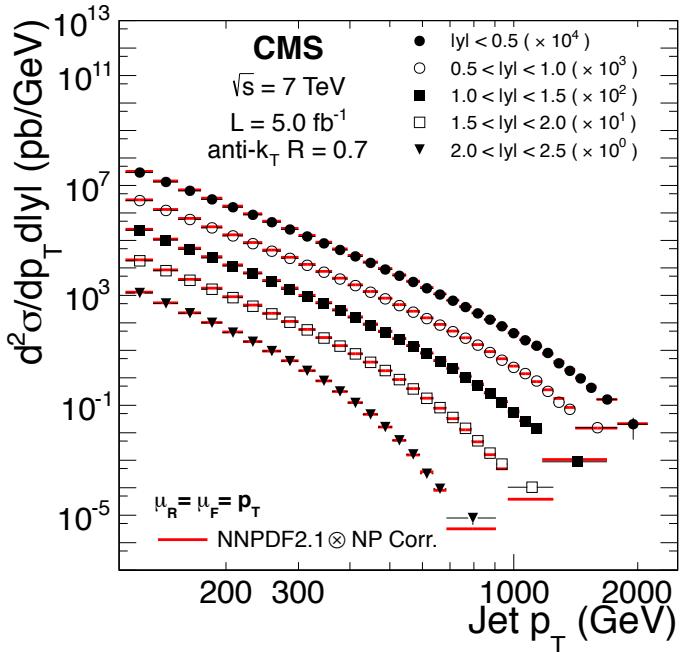
- Box also included in NLO-type codes (DIPHOX+gamma2MC, MCFM)  
(T. Binoth, J.P. Guillet, E. Pilon, M. Werlen; Z. Bern, L. Dixon, C. Schmidt; J. Campbell et al.)

# ATLAS di-photon results



- ▶ Inclusion of NNLO corrections resolves discrepancy between NLO-type prediction and data
  - ▶ Despite the use of slightly different cone isolation criteria

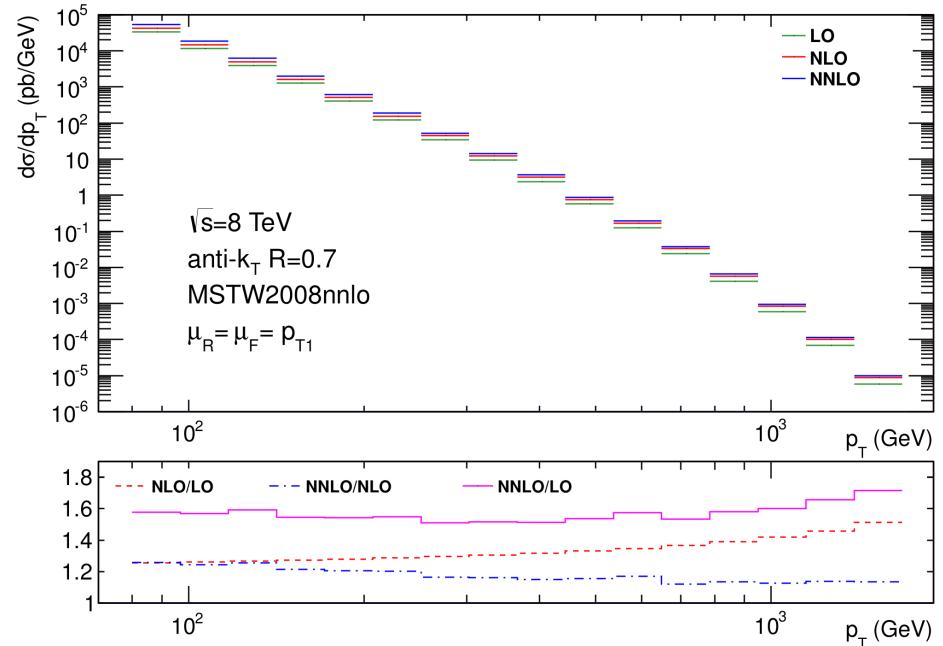
# Jet cross sections at LHC



- ▶ Jet data can be used to constrain parton distributions
- ▶ Scale and PDF uncertainties on NLO prediction of comparable size
- ▶ Need improved theory (NNLO)

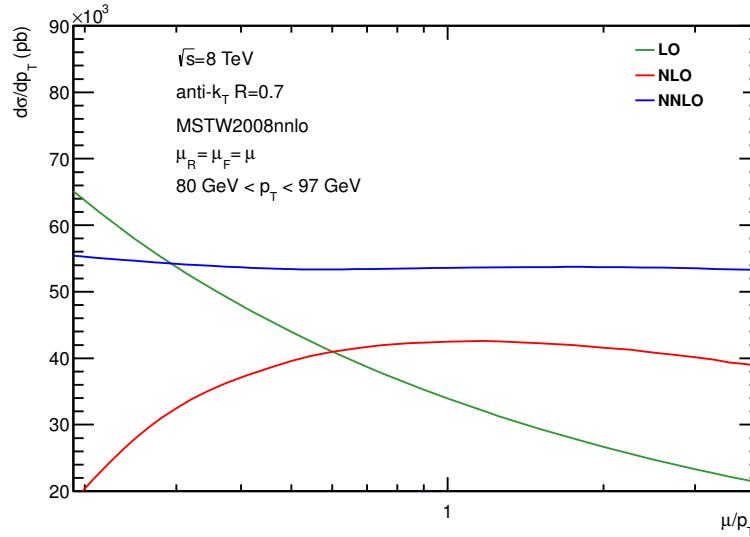
# $pp \rightarrow 2\text{jets}$ at NNLO

- ▶ First results at NNLO available
  - ▶  $gg \rightarrow gg$  subprocess at leading colour (LC)  
(A. Gehrmann-De Ridder, E.W.N. Glover, J. Pires, TG)
  - ▶ Developed a new parton-level event generator NNLOJET
  - ▶ using antenna subtraction
    - ▶ analytic cancellation of infrared poles
- ▶ Inclusive jet  $p_T$  distribution
  - ▶ NNLO/NLO differential K-factor flat over the whole  $p_T$  range



# $pp \rightarrow 2\text{jets}$ at NNLO

- ▶ Inclusive jet  $p_T$  distribution: scale dependence (gluons only, LC)  
(A. Gehrmann-De Ridder, E.W.N. Glover, J. Pires, TG)
  - ▶ Dynamical scale choice: leading jet  $p_T$
  - ▶ Same PDF for all fixed order predictions



- ▶ Stabilization at NNLO

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- ▶ Precision frontier: aims and ideas

# Towards NNLO automation

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- ▶ Methods for real radiation at NNLO becoming mature
  - ▶  $q_T$  subtraction
  - ▶ Sector-improved schemes
  - ▶ Antenna subtraction
- ▶ Issues
  - ▶ Automation of code generation
  - ▶ Numerical efficiency and stability

# Towards NNLO automation

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- ▶ Virtual two-loop amplitudes: analytically process-by-process
  - ▶ Current stockpile
    - ▶  $p\bar{p} \rightarrow 2j$  (C.Anastasiou, N. Glover, C. Oleari, M.Tejeda-Yeomans; Z. Bern, L. Dixon, A. De Freitas)
    - ▶  $p\bar{p} \rightarrow V+j$  (L. Garland, N. Glover, A. Koukoutsakis, E. Remiddi, TG)
    - ▶  $p\bar{p} \rightarrow V+\gamma$  (L.Tancredi, E.Weihs, TG)
    - ▶  $p\bar{p} \rightarrow H+j$  (N. Glover, M. Jaquier, A. Koukoutsakis, TG)
    - ▶  $p\bar{p} \rightarrow t\bar{t}$  (P.Bärnreuther, M. Czakon, P. Fiedler; R. Bonciani, A. Ferroglia, A. von Manteuffel, C. Studerus, TG)
  - ▶ In progress
    - ▶  $p\bar{p} \rightarrow VV$  (L.Tancredi, E.Weihs, TG)
- ▶ Research directions: towards different masses and  $2 \rightarrow 3$ 
  - ▶ Semi-numerical approaches (P. Bärnreuther, M. Czakon, P. Fiedler)
  - ▶ Classification of integral basis (H.Johansson, D. Kosower, K. Larsen)
  - ▶ Unitarity-based methods (P. Mastrolia, E. Mirabella, G. Ossola, T. Peraro)

# NNLO and beyond: techniques

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- ▶ **Seemingly simple task: check equality of two expressions**

- ▶ Becomes very tricky if complicated functions involved
    - ▶ e.g. Abel relation (1855)

$$\ln(1-x) \ln(1-y) = \text{Li}_2\left(\frac{x}{1-y}\right) + \text{Li}_2\left(\frac{y}{1-x}\right) - \text{Li}_2(x) - \text{Li}_2(y) - \text{Li}_2\left(\frac{xy}{(1-x)(1-y)}\right)$$

- ▶ **Systematic procedure for iterated rational integrals**

- ▶ Symbol and coproduct (A. Goncharov, M. Spradlin, A. Volovich, C. Vergu; C. Duhr)
  - ▶ Often allows huge simplifications (many pages → few lines)

- ▶ **starts to get used for loop integrals**

- ▶ simplification
  - ▶ analytical continuation
  - ▶ automated derivation of relations

# Beyond NNLO: observables

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- ▶ Hadronic R-ratio in  $e^+e^-$ 
  - ▶ Most precise QCD observable in  $Z$  and  $\tau$  decays
  - ▶ Known to  $O(\alpha_s^4)$  (P. Baikov, K. Chetyrkin, H. Kühn, J. Rittinger)
  - ▶ Produces most precise  $\alpha_s(M_Z) = 0.1198 \pm 0.0015$
- ▶ Gluon-fusion Higgs cross section at hadron colliders
  - ▶ Large NLO and NNLO corrections
  - ▶ Ultimate precision on Higgs couplings may require  $N^3LO$
  - ▶ Ingredients
    - ▶ Three-loop vertex functions (P. Baikov, K. Chetyrkin, A. Smirnov, V. Smirnov, M. Steinhauser; N. Glover, T. Huber, N. Ikizlerli, C. Studerus, TG)
    - ▶ Counterterms and lower-order expansions (C. Anastasiou, S. Bühler, C. Duhr, F. Herzog; M. Höschele, J. Hoff, A. Pak, M. Steinhauser, T. Ueda)
    - ▶ Triple real radiation (C. Anastasiou, C. Duhr, F. Dulat, B. Mistlberger)
    - ▶ Interplay of real and virtual corrections at  $N^3LO$
  - ▶ Major work in progress

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- ▶ Instead of a summary: Outlook

# Where do we stand?

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- ▶ Witnessed an NLO revolution
  - ▶ Previously unthinkable NLO multi-particle calculations now feasible due to technological breakthroughs
  - ▶ High-level of automation
  - ▶ Standardization of interfaces: combine different codes (providers)
  - ▶ Interface to experiment (codes, ntuples, histograms,...)?
- ▶ NLO and parton showers
  - ▶ Matching of individual processes (MC@NLO, POWHEG)
- ▶ Substantial progress on NNLO calculations
  - ▶ Several different methods available
  - ▶ Calculations on process-by-process basis
  - ▶ Codes typically require HPC infrastructure

# Future Directions

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- ▶ **NLO+PS as new standard for event generation**
  - ▶ Fully automated public codes
  - ▶ Consistent matching to parton shower
  - ▶ Matching of different multiplicities at NLO
  - ▶ Monte Carlo with NLO-accurate event samples
- ▶ **NNLO automation**
  - ▶ Uncover analytical structures to organize calculation of real and virtual corrections
  - ▶ Develop standard interfaces
  - ▶ Interface to experiment ?
- ▶ **Beyond NNLO**
  - ▶  $N^3LO$  precision for benchmark processes

- ▶ Apologises for all important work not covered here
- ▶ Be prepared for exciting times ahead with the LHC

